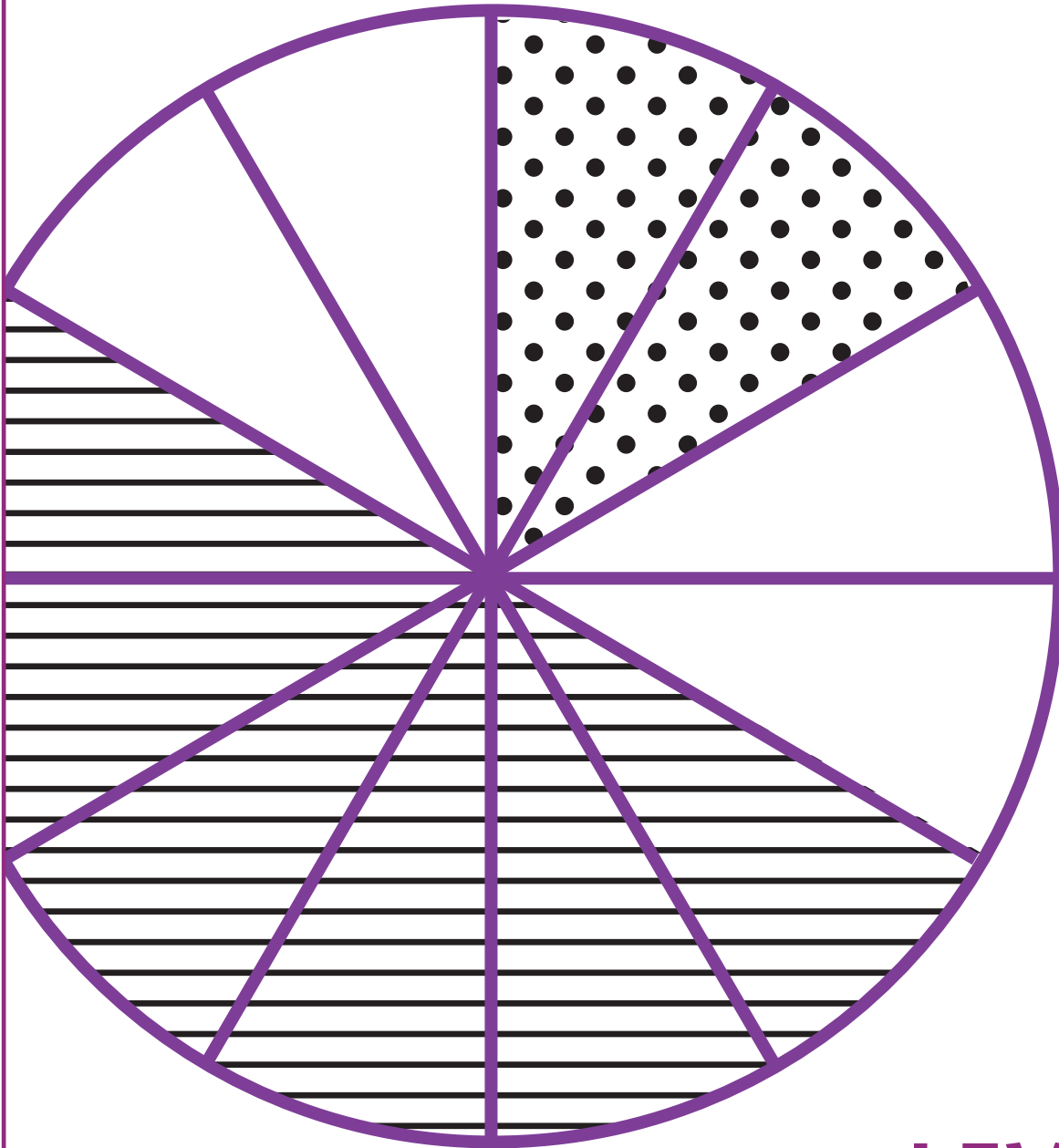


LITERACY FOUNDATIONS MATH



LEVEL 4
NUMBER SENSE

To the Student

This resource covers topics from the British Columbia Ministry of Education's Literacy Foundations Math Level 4. You may find this resource useful if you're a Literacy Foundations Math student, or a K–12 student in grades 7 – 9.

We have provided learning material, exercises, and answers for the exercises, which are located at the back of each set of related lessons. We hope you find it helpful.

Literacy Foundations Math Prescribed Learning Outcomes

The Literacy Foundations Math Prescribed Learning Outcomes (PLOs) are grouped into four areas: Number (A), Patterns and Relations (B), Shape and Space (C), and Statistics and Probability (D). For a complete list of the PLOs in Level 5, search for Literacy Foundations Math curriculum on the BC Ministry of Education's website.

PLOs Represented in This Resource

The PLOs represented in this Level 4 resource are as follows:

Number

A6, A7, A9, A11 – A18

Patterns and Relations

All topics, B1 – B3

Shape and Space

C1 – C5, C7

*C3 topics are represented with the exception of angle construction

Statistics and Probability

D2

PLOs Not Represented in This Resource

The PLOs for which no material is included in this resource are as follows:

Number

There is no material for A1 – A5, read and write numbers, place value, and patterns for multiplying by 10, etc.; A8, compare decimal numbers; nor A10, patterns for multiplying and dividing by $\frac{1}{10}$, etc.

Shape and Space

There is no material for C3, construct angles.

Statistics and Probability

There is no material for D1, graph data to solve problems.

Acknowledgements and Copyright

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Table of Contents

Lesson 1: Fractions	1
The Fraction "1"	6
Lowest Terms	12
Lesson 2: Adding and Subtracting Fractions	19
Lesson 3: Decimals	29
Lesson 4: Percent	75
Lesson 5: Ratios and Proportion	91
Answer Key	99

Lesson 1

Fractions

Learning Outcomes

By the end of this section you will be better able to:

- Describe a quantity using fractions
- Identify and calculate equivalent fractions
- Reduce fractions to lower terms

A fraction is a number that describes a piece of something. In this lesson, all of the fractions describe an amount less than one. $\frac{1}{4}$, $\frac{3}{8}$, and $\frac{13}{16}$ are examples of fractions. The number above the bar is called the numerator of the fraction; the number below the bar is called the denominator.

Consider the square shown in Figure 1 below. This square is divided into 4 equal parts; 1 of these 4 parts is shaded. The fraction $\frac{1}{4}$ tells us what portion of the square is shaded. Similarly, the fraction $\frac{3}{4}$ tells us what portion is not shaded.

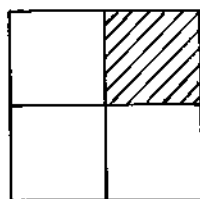


Figure 1

Now consider Figure 2. Here there are 12 small squares; 4 of them are shaded. We can say that $\frac{4}{12}$ of the rectangle is shaded. If, however, we consider the larger squares, we find that 1 of the larger squares is shaded, so we can also say that $\frac{1}{3}$ of the rectangle is shaded.

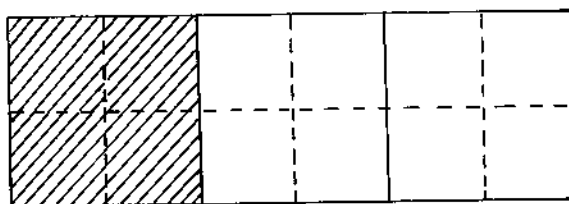




Figure 2

Self Test

1. Give three fractions that tell what part of the square in Figure 3 is marked .

2. Give two fractions that tell what part of the square is marked .

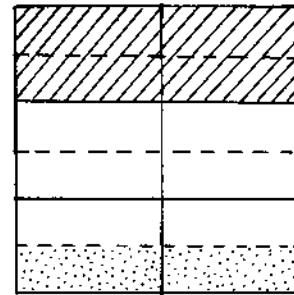


Figure 3

Answers

1. $\frac{1}{3}, \frac{2}{6}, \frac{4}{12}$
2. $\frac{2}{12}, \frac{1}{6}$

Now consider the set of circles and triangles shown in Figure 4.

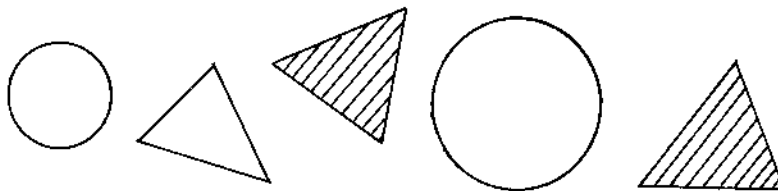


Figure 4

Of the 5 figures, 2 are circles; the fraction $\frac{2}{5}$ tells us what part of the set of figures is circles. Similarly, the fraction $\frac{3}{5}$ tells us what part of the set is triangles.

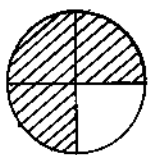
Self Test

1. In Figure 4, what fractional part of the set of figures is shaded?
2. In Figure 4, what fraction of the set of triangles is shaded?

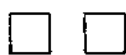
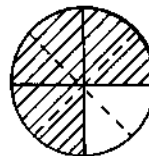
Answers

1. $\frac{2}{5}$
2. $\frac{2}{3}$

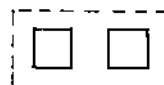
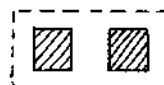
Equivalent Fractions



$\frac{3}{4}$ is equivalent to $\frac{6}{8}$



$\frac{2}{6}$ is equivalent to $\frac{1}{3}$

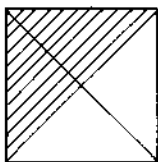


The fractions $\frac{3}{4}$ and $\frac{6}{8}$ represent equivalent portions of the circles; these fractions are called **equivalent fractions**. The fractions $\frac{2}{6}$ and $\frac{1}{3}$ are also equivalent; they represent equivalent parts of the sets of squares.

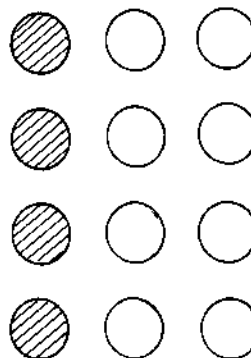
Self Test

Give the pair of equivalent fractions suggested in each case.

1.



2.



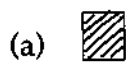
Answers

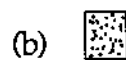
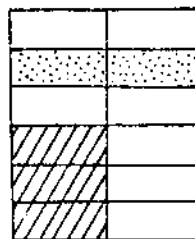
1. $\frac{2}{4}, \frac{1}{2}$

2. $\frac{4}{12}, \frac{1}{3}$ (or $\frac{8}{12}, \frac{2}{3}$)

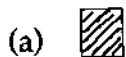
Exercises 1.1

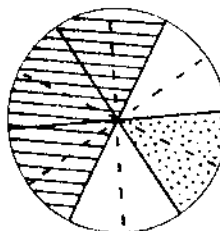
1. In each of (a) and (b) give two fractions that tell what part of the rectangle is marked as indicated.



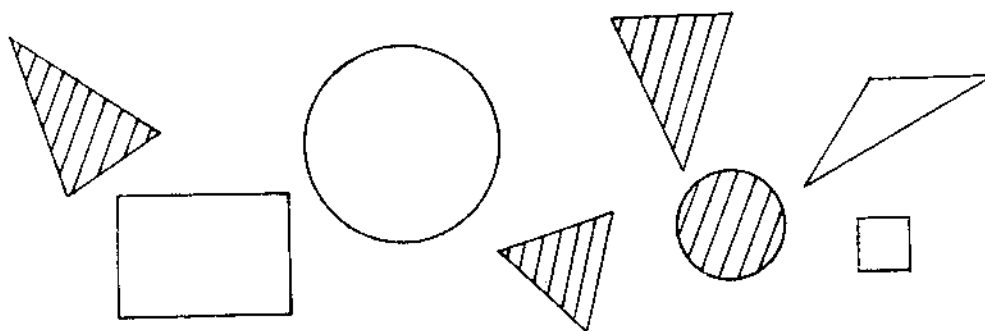


2. In each of (a) and (b), give two fractions that tell what part of the circle is marked as indicated.





3. This question refers to the set of figures shown below. (Express your answer in lowest terms.)



- (a) What fraction of the set of figures is triangles?

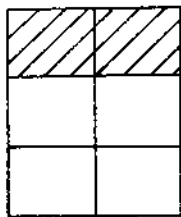
(b) What fraction of the set of figures is circles?

(c) What fraction of the triangles is shaded?

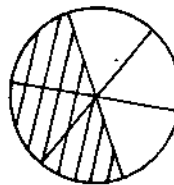
(d) What fraction of the circles is shaded?

4. Give two equivalent fractions suggested in each case.

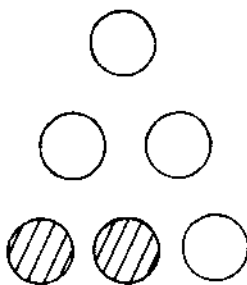
(a)



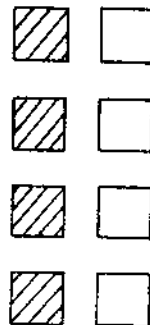
(b)



(c)



(d)





Turn to the Answer Key at the end of the module to check your work.

The Fraction “1”

We will now consider fractions in which the two numbers are the same, that is, in which the numerator is the same as the denominator. These fractions are rather special in that every one of them is equivalent to 1.

$\frac{2}{2}$ is equivalent to 1.

$\frac{3}{3}$ is equivalent to 1.

$\frac{4}{4}$ is equivalent to 1.

.....

.....

$\frac{29}{29}$ is equivalent to 1.

.....

.....

To see that this is, in fact, the case, consider the shaded portion of the rectangle in Figure 6(a).

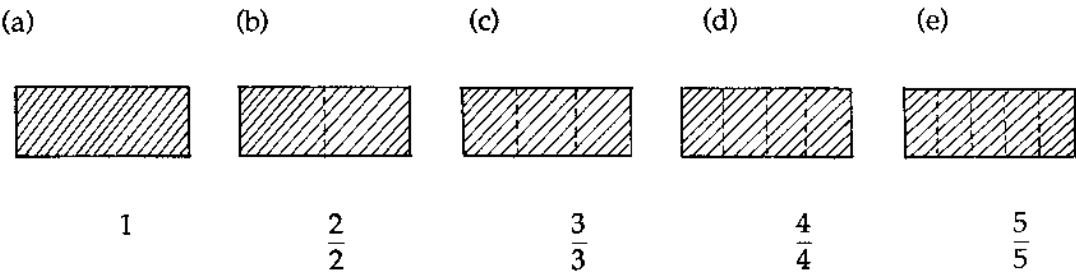


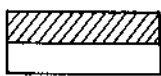
Figure 6

The entire rectangle is shaded, so we can therefore represent the shaded portion by the number 1. (There is 1 rectangle, and all of it is shaded.) Suppose now that we divide this same rectangle into 2 equal regions, Figure 6 (b). Now the fraction $\frac{2}{2}$ represents the shaded portion. (There are 2 regions and both are shaded.) Hence, we see that the fraction $\frac{2}{2}$ is equivalent to 1. If we divide the rectangle into 3 regions, we find that the fraction $\frac{3}{3}$ is also equivalent to 1.

Continuing in this way, we find that any fraction in which the numerator and denominator are the same is equal to 1.

More Equivalent Fractions

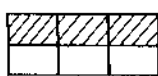
Consider these equivalent fractions.



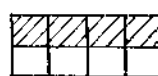
$$\frac{1}{2}$$



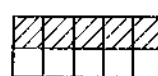
$$\frac{2}{4}$$



$$\frac{3}{6}$$



$$\frac{4}{8}$$



$$\frac{5}{10}$$

Each of the fractions $\frac{2}{4}$, $\frac{3}{6}$, $\frac{4}{8}$, $\frac{5}{10}$ and so on, are equivalent to the fraction $\frac{1}{2}$.

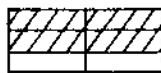
We can say *all of these fractions are equivalent* to $\frac{1}{2}$.

Self Test

What fraction are all the illustrations equivalent to?



$$\frac{2}{3}$$



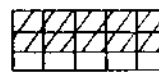
$$\frac{4}{6}$$



$$\frac{6}{9}$$



$$\frac{8}{12}$$



$$\frac{10}{15}$$

Answer

$$\frac{2}{3}$$

Notice that each of the numerators has the numerator, 2, of $\frac{2}{3}$ as a factor and that each of the denominators has the denominator, 3, of $\frac{2}{3}$ as a factor; that is, 2 will divide into each of the numerators, and 3 will divide into each of the denominators.

$$\frac{2}{3} \quad \frac{4}{6} \quad \frac{6}{9} \quad \frac{8}{12} \quad \frac{10}{15}$$

$$\frac{1 \times 2}{1 \times 3} \quad \frac{2 \times 2}{2 \times 3} \quad \frac{3 \times 2}{3 \times 3} \quad \frac{4 \times 2}{4 \times 3} \quad \frac{5 \times 2}{5 \times 3}$$

Note the pattern. The first fraction has 1 as factors in numerator and denominator, the second has 2 as factors, the third, 3 as factors, and so on. Do you see that the sixth fraction (although not shown) will have 6 as factors and will be

$$\frac{6 \times 2}{6 \times 3} = \frac{12}{18}$$

and that the seventh fraction will have 7 as factors and will be

$$\frac{7 \times 2}{7 \times 3} = \frac{14}{21}?$$

Now consider the fractions equivalent to $\frac{1}{2}$.

$$\frac{1}{2} \quad \frac{2}{4} \quad \frac{3}{6} \quad \frac{4}{8} \quad \frac{5}{10} \dots$$

$$\frac{1 \times 1}{1 \times 2} \quad \frac{2 \times 1}{2 \times 2} \quad \frac{3 \times 1}{3 \times 2} \quad \frac{4 \times 1}{4 \times 2} \quad \frac{5 \times 1}{5 \times 2} \dots$$

The sixth fraction will be $\frac{6 \times 1}{6 \times 2} = \frac{6}{12}$ the ninth will be $\frac{9 \times 1}{9 \times 2}$ and so on.

Self Test

1. Write the 12th fraction in the series of fractions equivalent to $\frac{1}{2}$.
2. Write the 8th fraction in the series of fractions equivalent to $\frac{1}{3}$.

Answers

1. $\frac{12 \times 1}{12 \times 2} = \frac{12}{24}$
2. $\frac{8 \times 1}{8 \times 3} = \frac{8}{24}$

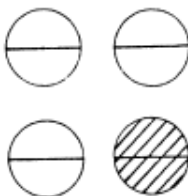
Exercises 1.2

1. Each figure suggests two equivalent fractions. Give these fractions.

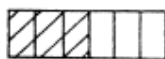
(a)



(b)



2. Give the three equivalent fractions that the figures suggests.



3. The fractions $\frac{3}{3}$, $\frac{17}{17}$, and $\frac{31}{31}$ are equivalent to _____.

4. Give the missing fractions, following the given pattern below.

$$\frac{1 \times 4}{1 \times 5}$$

↓

$$\frac{4}{5}$$

$$\frac{2 \times 4}{2 \times 5}$$

↓

$$\frac{8}{10}$$

a.

↓

$$\frac{12}{15}$$

$$\frac{4 \times 4}{4 \times 5}$$

↓

c. _____

b.

↓

$$\frac{20}{25}$$

$$\frac{6 \times 4}{6 \times 5}$$

↓

d.

5. Give the three indicated fractions for each group of equivalent fractions.

(a) $\frac{5}{6}$, $\frac{10}{12}$, $\frac{15}{18}$, _____, _____, _____, ...

(b) $\frac{3}{10}$, $\frac{6}{20}$, _____, $\frac{12}{40}$, _____, _____, ...

(c) $\frac{5}{8}$, $\frac{10}{16}$, $\frac{15}{24}$, _____, _____, $\frac{30}{48}$, _____, ...

(d) $\frac{1}{8}$, $\frac{2}{16}$, _____, _____, $\frac{5}{40}$, _____, ...



Turn to the Answer Key at the end of the module to check your work.

Lowest Terms

A fraction is in lowest terms if the numerator and the denominator have no common factor other than 1. For example, the fraction $\frac{8}{9}$ is in lowest terms since 1 is the only common factor of 8 and 9.

$$\frac{8}{9} \quad \begin{array}{l} \{1, 2, 4, 8\} \\ \{1, 3, 9\} \end{array} \quad \text{No common factor other than 1.}$$

The fraction $\frac{12}{15}$ is **not** in lowest terms since 1 and 3 are common factors of 12 and 15.

$$\frac{12}{15} \quad \begin{array}{l} \{1, 2, 3, 4, 6, 12\} \\ \{1, 3, 5, 15\} \end{array} \quad \text{Common factors 1 and 3.}$$

As further example, consider the fractions $\frac{16}{18}$ and $\frac{14}{15}$.

$$\frac{16}{18} \quad \begin{array}{l} \{1, 2, 4, 8, 16\} \\ \{1, 2, 3, 6, 9, 18\} \end{array} \quad \begin{array}{l} \text{Common factors 1 and 2.} \\ \text{This fraction is not in lowest terms.} \end{array}$$

$$\frac{14}{15} \quad \begin{array}{l} \{1, 2, 7, 14\} \\ \{1, 3, 5, 15\} \end{array} \quad \begin{array}{l} \text{No common factors other than 1.} \\ \text{This fraction is not in lowest terms.} \end{array}$$

Self Test

1. List the factors of 15.

2. List the factors of 20.

3. Is the fraction $\frac{15}{20}$ in lowest terms? Explain.

Answers

- 1, 3, 5, 15
- 1, 2, 4, 5, 10, 20
- No, because common factors of 15 and 20 are 1 and 5.

We can build a series of equivalent fractions from a given lowest-terms fraction. For example, given the fraction $\frac{1}{3}$, we can determine a series of six equivalent fractions.

$$\begin{array}{cccccc} \frac{1 \times 1}{1 \times 3}, & \frac{2 \times 1}{2 \times 3}, & \frac{3 \times 1}{3 \times 3}, & \frac{4 \times 1}{4 \times 3}, & \frac{5 \times 1}{5 \times 3}, & \frac{6 \times 1}{6 \times 3} \\ = & \frac{1}{3}, & \frac{2}{6}, & \frac{3}{9}, & \frac{4}{12}, & \frac{5}{15}, & \frac{6}{18} \end{array}$$

Note that we multiplied the numerator and the denominator of $\frac{1}{3}$ by 1, by 2, by 3, and so on. Similarly, given the fraction $\frac{1}{10}$, we can determine a series of four equivalent fractions.

$$\frac{1 \times 3}{1 \times 10}, \frac{2 \times 3}{2 \times 10}, \frac{3 \times 3}{3 \times 10}, \frac{4 \times 3}{4 \times 10} \longrightarrow \frac{3}{10}, \frac{6}{20}, \frac{9}{30}, \frac{12}{40}$$

If we are given a series of equivalent fractions, we can determine the equivalent lowest-terms fraction. For example, in the series

$\frac{2}{12}, \frac{3}{18}, \frac{4}{24}, \dots$ the lowest-term fraction is not given.

Given the series $\frac{2}{12}, \frac{3}{18}, \frac{4}{24}, \dots$
 $\rightarrow \frac{2 \times 1}{2 \times 6}, \frac{3 \times 1}{3 \times 6}, \frac{4 \times 1}{4 \times 6}, \dots$

and we see that the lowest-terms fraction of this is $\frac{1}{6}$.

Similarly, since $\frac{4}{18}, \frac{6}{27}, \frac{8}{36}, \dots$
 $\rightarrow \frac{2 \times 2}{2 \times 9}, \frac{3 \times 2}{3 \times 9}, \frac{4 \times 2}{4 \times 9}, \dots$

we see that the lowest-terms fraction of this is $\frac{2}{9}$.

Self Test

1. From the fraction $\frac{1}{4}$ build a series of five equivalent fractions.

2. Find the lowest-terms fraction of this series. $\frac{4}{10}, \frac{6}{15}, \frac{8}{20}$

Answers

$$1. \quad \frac{1 \times 1}{1 \times 4}, \frac{2 \times 1}{2 \times 4}, \frac{3 \times 1}{3 \times 4}, \frac{4 \times 1}{4 \times 4}, \frac{5 \times 1}{5 \times 4} \rightarrow \frac{1}{4}, \frac{2}{8}, \frac{3}{12}, \frac{4}{16}, \frac{5}{20}$$

$$2. \quad \frac{4}{10}, \frac{6}{15}, \frac{8}{20} \rightarrow \frac{2 \times 2}{2 \times 5}, \frac{3 \times 2}{3 \times 5}, \frac{4 \times 2}{4 \times 5}$$

The lowest-terms fraction is $\frac{2}{5}$.

Reducing Fractions to Lowest Terms

If we are given a fraction, we can find an equivalent lowest-terms fraction. This is called reducing the fraction to lowest terms.

We can reduce a fraction to lowest terms by **dividing out** common factors. For example, suppose that we wish to reduce $\frac{60}{84}$ to lowest terms. We can do this in two ways.

1. Divide out common factors until the fraction is in lowest terms.

$$\frac{60}{84} = \frac{60 \div 2}{84 \div 2} = \frac{30}{42} = \frac{30 \div 2}{42 \div 2} = \frac{15}{21} = \frac{15 \div 3}{21 \div 3} = \frac{5}{7}$$

OR

2. (a) Find the greatest common factor (GCF) of the numerator and the denominator.

Factors of 60: 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60

Factors of 84: 1, 2, 3, 4, 6, 7, 12, 14, 21, 28, 42, 84

GCF of 60 and 84 is 12.

- (b) Divide out the GCF.

$$\frac{60}{84} = \frac{60 \div 12}{84 \div 12} = \frac{5}{7}$$

If we know the GCF, all we need do is divide it out, as in 2 (b). If we do not know the GCF, the first method is perhaps the easier.

The following examples further illustrate the reducing of fractions to lowest terms.

$$\frac{20}{48} = \frac{20 \div 2}{48 \div 2} = \frac{10}{24} = \frac{10 \div 2}{24 \div 2} = \frac{5}{12}$$

$$\frac{9}{36} = \frac{9 \div 3}{36 \div 3} = \frac{3}{12} = \frac{3 \div 3}{12 \div 3} = \frac{1}{4}$$

Note that in each step, we must divide the numerator and the denominator by the same number.

Thus, $\frac{10}{24} = \frac{10 \div 2}{24} = \frac{5}{24}$ is incorrect, because we divided only the numerator by 2.

Also, $\frac{10}{24} = \frac{10 \div 2}{24 \div 3} = \frac{5}{8}$ is incorrect, because we divided the numerator and the denominator by different numbers.

Be very careful not to make these mistakes.

Exercises 1.3

1. (a) List the factors of 8.
(b) List the factors of 12.
(c) Is the fraction $\frac{8}{12}$ in lowest terms? Explain.

2. (a) List the factors of 15.
(b) List the factors of 28.
(c) Is the fraction $\frac{15}{28}$ in lowest terms? Explain.

3. Explain why each fraction is not in lowest terms.
(a) $\frac{6}{10}$
(b) $\frac{25}{30}$

4. Starting with lowest-term fraction, build a group of five equivalent fractions.

(a) $\frac{1}{4}$

(b) $\frac{5}{8}$

5. Reduce each fraction to lowest terms.

(a) $\frac{6}{10}$

(b) $\frac{3}{9}$

(c) $\frac{18}{20}$

(d) $\frac{5}{35}$

(e) $\frac{10}{35}$

(f) $\frac{18}{24}$

(g) $\frac{9}{21}$

(h) $\frac{15}{70}$

(i) $\frac{14}{28}$

(j) $\frac{50}{75}$

(k) $\frac{24}{30}$

(l) $\frac{16}{30}$

(m) $\frac{15}{36}$

(n) $\frac{30}{48}$

(o) $\frac{6}{52}$



Turn to the Answer Key at the end of the module to check your work.

Lesson 2

Adding and Subtracting Fractions

Learning Outcomes

By the end of this section you will be better able to:

- Add and subtract fractions with the same denominator
- Add and subtract fractions with different denominators

Now we shall consider the addition and subtraction of fractions. First, however, let's briefly review some of the main points of Part 1 in which introduced these numbers.

1. Equivalent fractions represent the same amount. For example $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$, and $\frac{4}{8}$ each represent the same amount.

$$\frac{4}{8} = \frac{1}{2} \quad \frac{3}{6} = \frac{1}{2} \quad \frac{2}{4} = \frac{1}{2}$$

2. A fraction may be reduced to lowest terms by dividing out common factors in the numerator and denominator. For example, $\frac{8}{10}$ may be reduced to lowest terms by dividing out the common factor 2.

$$\frac{8}{10} = \frac{8 \div 2}{10 \div 2} = \frac{4}{5}$$

Write the following in lowest terms:

(a) $\frac{20}{35}$

(b) $\frac{28}{42}$

(c) $\frac{40}{50}$

Did you get the following answers?

$$(a) \frac{20 \div 5}{35 \div 5} = \frac{4}{7}$$

$$(b) \frac{28 \div 14}{42 \div 14} = \frac{2}{3}$$

$$(c) \frac{40 \div 10}{50 \div 10} = \frac{4}{5}$$

3. A fraction may be changed to an equivalent fraction by multiplying the numerator and denominator by the same number. For example, $\frac{2}{3}$ may be changed to the equivalent fraction $\frac{8}{12}$ by multiplying the numerator and denominator by 4.

$$\frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$$

Now it is time to learn the operations of addition and subtraction with fractions.

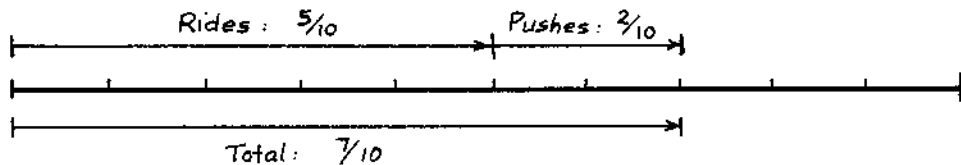
Addition of Fractions

Let us now see what we can discover about the addition of fractions. Consider the following.

Example 1

A boy rides a bicycle $\frac{5}{10}$ of a kilometre.

He then pushes the bicycle $\frac{2}{10}$ of a kilometre.

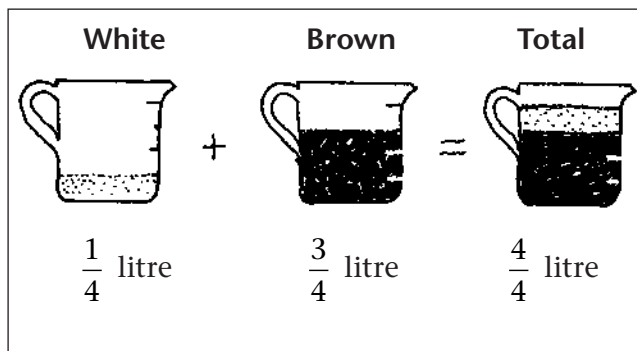


Total distance is $\frac{5}{10}$ km plus $\frac{2}{10}$ or $\frac{7}{10}$ km.

$$\frac{5}{10} + \frac{2}{10} = \frac{7}{10}$$

Example 2

A recipe calls for $\frac{1}{4}$ litre of white sugar and $\frac{3}{4}$ litre of brown sugar.



Total amount of sugar is $\frac{1}{4}$ litre plus $\frac{3}{4}$ litre or $\frac{4}{4}$ litre.

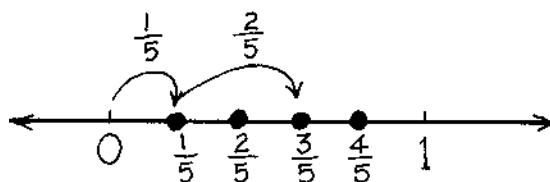
$$\frac{1}{4} + \frac{3}{4} = \frac{4}{4}$$

You will notice in each of the above examples, the denominators of the fractions are the same. We refer to this as like denominators.

Like Denominators

It is easy to add two or more fractions that have the same denominator. We simply add the numerators and keep the same denominator.

For example, let us add $\frac{1}{5}$ and $\frac{2}{5}$.



$$\frac{1}{5} + \frac{2}{5} = \frac{1+2}{5} = \frac{3}{5}$$

Let's work through a few examples together.

Examples

Add the numerators.

 (a) $\frac{2}{9} + \frac{5}{9} = \frac{2+5}{9} = \frac{7}{9}$

 Keep the same denominator.

(b) $\frac{3}{7} + \frac{4}{7} = \frac{3+4}{7} = \frac{7}{7} = 1$ $\left(\text{Reduce } \frac{7}{7} \text{ to } 1 \right)$

(c) $\frac{5}{11} + \frac{4}{11} = \frac{5+4}{11} = \frac{9}{11}$

Unlike Denominators

This is when the denominators of at least two of the fractions are different.

Examples

(a) $\frac{1}{2} + \frac{3}{8} = ?$

We need the least common multiple (LCM) of the two denominators, 2 and 8. This is called the least common denominator (LCD).

The multiples of 2 are 0, 2, 4, 6, 8, 10, ...

The multiples of 8 are 0, 8, 16, 24, 32, ...

The LCD of 2 and 8 is 8.

The least common denominator (LCD) of two numbers is the smallest nonzero common multiple. Zero is not an LCM.

From what we observed above, the common denominator is 8. We need a fraction that is equivalent to $\frac{1}{2}$ that has a denominator of 8. $\frac{1}{2} = \frac{4 \times 1}{4 \times 2} = \frac{4}{8}$

$$\frac{1}{2} + \frac{3}{8} = \frac{4}{8} + \frac{3}{8} = \frac{7}{8}$$

(b) $\frac{3}{7} + \frac{2}{5} = ?$

First, the multiples of 7 are 0, 7, 14, 21, 28, 35, ...

The multiples of 5 are 0, 5, 10, 15, 20, 25, 30, 35, ...

The LCD of 7 and 5 is 35.

$$\frac{3}{7} = \frac{5 \times 3}{5 \times 7} = \frac{15}{35} \text{ and } \frac{2}{5} = \frac{7 \times 2}{7 \times 5} = \frac{14}{35}$$

$$\frac{3}{7} + \frac{2}{5} = \frac{15}{35} + \frac{14}{35} = \frac{29}{35}$$

We need equivalent fractions that have a denominator of 35. If you want to review equivalent fractions, look at the previous lesson before continuing.

Self Test

Add the following fractions. Show your work.

a. $\frac{1}{4} + \frac{2}{4}$

b. $\frac{1}{3} + \frac{1}{2}$

c. $\frac{3}{8} + \frac{1}{3}$

Answers

a. $\frac{1}{4} + \frac{2}{4}$
 $= \frac{1+2}{4}$
 $= \frac{3}{4}$

b. $\frac{1}{3} + \frac{1}{2}$
 $= \frac{2}{6} + \frac{3}{6}$
 $= \frac{5}{6}$

c. $\frac{3}{8} + \frac{1}{3}$
 $= \frac{9}{24} + \frac{8}{24}$
 $= \frac{17}{24}$

Subtraction of Fractions

Like Denominators

This is very similar to the procedure used in addition.

Examples

Subtract the numerators.

a. $\frac{7}{9} - \frac{3}{9} = \frac{7-3}{9} = \frac{4}{9}$

Keep the same denominator.

b. $\frac{9}{10} - \frac{3}{10} = \frac{9-3}{10} = \frac{6}{10} = \frac{3}{5} \left(\text{Reduce } \frac{6}{10} \text{ to } \frac{3}{5} \right)$

c. $\frac{7}{16} - \frac{3}{16} = \frac{7-3}{16} = \frac{4}{16} = \frac{1}{4} \left(\text{Reduce } \frac{4}{16} \text{ to } \frac{1}{4} \right)$

Unlike Denominators

Once again, it is necessary to find a common denominator first.

Examples

a. $\frac{9}{12} - \frac{2}{3} = ?$

We need the LCM of the two denominators, 12 and 3.

This is called the least common denominator (LCD).

The multiples of 12 are 0, 12, 24, 36, 48, ...

The multiples of 3 are 0, 3, 6, 9, 12, 15, ...

The LCD of 12 and 3 is 12.

$$\frac{2}{3} = \frac{4 \times 2}{4 \times 3} = \frac{8}{12}$$

$$\frac{9}{12} - \frac{2}{3} = \frac{9}{12} - \frac{8}{12} = \frac{1}{12}$$

b. $\frac{4}{5} - \frac{1}{2} = ?$ The LCD of 5 and 2 is 10.

$$\frac{4}{5} = \frac{2 \times 4}{2 \times 5} = \frac{8}{10} \text{ and } \frac{1}{2} = \frac{5 \times 1}{5 \times 2} = \frac{5}{10}$$

$$\frac{4}{5} - \frac{1}{2} = \frac{8}{10} - \frac{5}{10} = \frac{3}{10}$$

Self Test

Subtract the following fractions. Show your work.

a. $\frac{5}{6} - \frac{1}{6}$

b. $\frac{7}{12} - \frac{1}{4}$

c. $\frac{11}{15} - \frac{2}{3}$

Answers

a. $\frac{5}{6} - \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$

b. $\frac{7}{12} - \frac{1}{4} = \frac{7}{12} - \frac{3}{12} = \frac{4}{12} = \frac{1}{3}$

c. $\frac{11}{15} - \frac{2}{3} = \frac{11}{15} - \frac{10}{15} = \frac{1}{15}$

Did you remember to reduce your fractions?

Exercises 2.1

1. Add. Reduce your answers to simplest terms.

a. $\frac{2}{7} + \frac{4}{7} =$

b. $\frac{3}{10} + \frac{2}{10} =$

c. $\frac{7}{9} + \frac{2}{9} =$

d. $\frac{1}{6} + \frac{3}{6} =$

e. $\frac{2}{3} + \frac{1}{6} =$

f. $\frac{1}{4} + \frac{3}{8} =$

g. $\frac{1}{5} + \frac{3}{10} =$

h. $\frac{1}{6} + \frac{1}{2} =$

i. $\frac{2}{5} + \frac{1}{10} =$

2. Subtract. Reduce your answer to simplest terms.

a. $\frac{7}{9} - \frac{2}{9} =$

b. $\frac{8}{10} - \frac{3}{10} =$

c. $\frac{7}{12} - \frac{3}{12} =$

d. $\frac{5}{6} - \frac{5}{6} =$

e. $\frac{5}{6} - \frac{3}{6} =$

f. $\frac{7}{9} - \frac{1}{3} =$

g. $\frac{8}{12} - \frac{1}{4} =$

h. $\frac{5}{10} - \frac{1}{2} =$

i. $\frac{9}{10} - \frac{2}{5} =$

j. $\frac{5}{8} - \frac{1}{4} =$



Turn to the Answer Key at the end of the module to check your work.

Summary

1. To find the sum (or difference) of two fractions that have the same denominator, we find the sum (or difference) of the numerators and leave the denominators unchanged.

$$\frac{1}{7} + \frac{2}{7} = \frac{3}{7}$$

$$\frac{5}{9} - \frac{1}{9} = \frac{4}{9}$$

2. To find the sum (or difference) of two fractions that have different denominators, we must consider two equivalent fractions that have common denominators. The smallest of these common denominators is called the least common denominator of the two fractions.

$$\frac{1}{3} + \frac{1}{2} = \frac{2}{6} + \frac{3}{6} = \frac{5}{6}$$

$$\frac{1}{3} - \frac{1}{4} = \frac{4}{12} - \frac{3}{12} = \frac{1}{12}$$

3. The least common denominator of two fractions is the LCM of the two denominators.

Lesson 3

Decimals

Learning Outcomes

By the end of this section you will be better able to:

- describe a fractional quantity using decimal numbers
- compare decimal numbers
- put a list of decimal numbers in order
- add and subtract with decimal numbers
- multiply and divide with decimal numbers

A decimal fraction is part of a whole part of a whole number just like a fraction is. A decimal fraction is another way of writing a common fraction when the denominator (the bottom number) in a fraction is a multiple of 10. The multiples of ten you will be working with are **tenths** and **hundredths**. Later on you will work in thousandths.

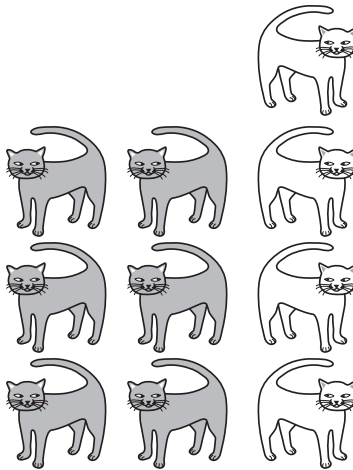
When you use money you are working with decimals.

For example: \$7.63

The decimal point separates whole objects, from parts of whole objects. Whole dollars (7 dollars) are separated from tenths of a dollar (6 dimes) and hundreds of a dollar (3 pennies).

6 dimes are $\frac{6}{10}$ of a dollar and is written 0.6.

How do you think 8/10 of a dollar would be written? (0.8)



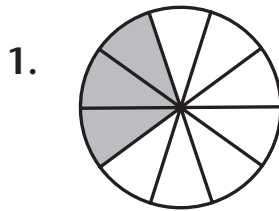
- How many cats are there in this illustration? (10)
- How many of the 10 cats are black? (6)
- What is the fraction of cats out of ten that are black? $\left(\frac{6}{10}\right)$
- If you were to write that fraction in the decimal form, how would you write it? (0.6)
- So 0.6 is the decimal form and $\frac{6}{10}$ is the fraction form.
- How many of the cats are white? (4)
- What is the fraction of cats out of ten that are white? $\left(\frac{4}{10}\right)$
- How would you write that fraction in the decimal form? (0.4)
- What does the zero (0) mean? (It means there are no whole numbers.)

It is important to remember that the decimal point follows the ones' place. If you have a number in front of the decimal, it would tell you how many whole objects you have, and the number after the decimal would tell you how many parts of the whole object you have.

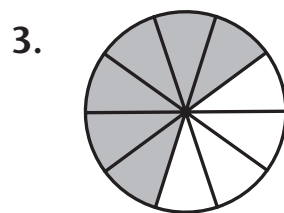
For example, 1.4 would tell you that there was one whole object and 4 parts of the whole object.

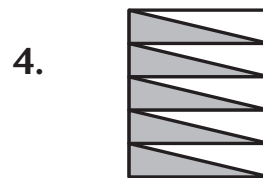
Self Test

Write a fraction and a decimal for each of the shaded parts.

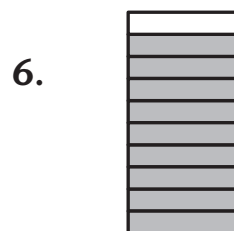












Answers

1. $\frac{3}{10} = 0.3$

2. $\frac{7}{10} = 0.7$

3. $\frac{6}{10} = 0.6$

4. $\frac{5}{10} = 0.5$

5. $\frac{1}{10} = 0.1$

6. $\frac{9}{10} = 0.9$

Exercises 3.1

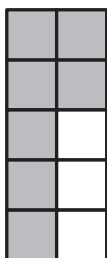
Write the number for each fraction.

1. three fifths _____
2. one half _____
3. nine tenths _____
4. one eighth _____
5. five tenths _____
6. one fifth _____
7. two thirds _____
8. two sixths _____
9. seventy hundredths _____
10. seven eighths _____

Exercises 3.2

- A. Write a common fraction and a decimal fraction for the shaded parts of each diagram.

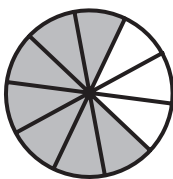
1.



2.



3.



4.



5.



6.



- B. Write each of the decimal fractions below as a common fraction.

Example: $0.5 = \frac{5}{10}$

three tenths = 0.3

1. one tenth _____
2. 0.1 _____
3. 0.7 _____
4. five tenths _____
5. nine tenths _____
6. 0.8 _____

- C. Write each common fraction below as a decimal fraction.

Example: $\frac{1}{10} = 0.1$

seven tenths = 0.7

1. eight tenths _____
2. two tenths _____
3. $\frac{3}{10}$ _____
4. $\frac{9}{10}$ _____
5. six tenths _____
6. five tenths _____

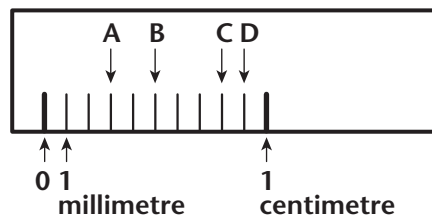
Exercises 3.3

Measuring a Tenth of a Centimetre

Metric units are always expressed (written) as a decimal.

There are 10 millimetres in 1 centimetre.

Therefore, 1 millimeter = $\frac{1}{10}$ cm.



$$1 \text{ millimetre} = 0.1 \text{ centimetre}$$

Write the following millimeters as centimeters.

1. 3 millimetres = _____ centimetres
2. 5 millimetres = _____ centimetres
3. 8 millimetres = _____ centimetres
4. 9 millimetres = _____ centimetres
5. What would 10 millimetres be equal to? _____



Turn to the Answer Key at the end of the module to check your work.

More About Decimals

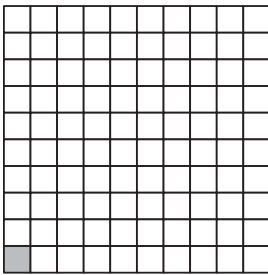
A decimal fraction is another way of writing a common fraction when the denominator—the bottom number in a fraction—is a multiple of 10. Multiples of 10 include numbers like 10, 100, 1000.

Look at these decimal fractions:

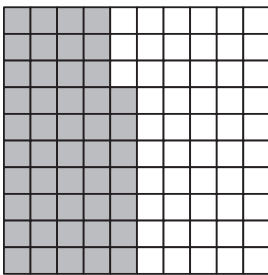
0.6 0.12 0.059

Written as common fractions they would read:

$$\frac{6}{10} \quad \frac{12}{100} \quad \frac{59}{1000}$$

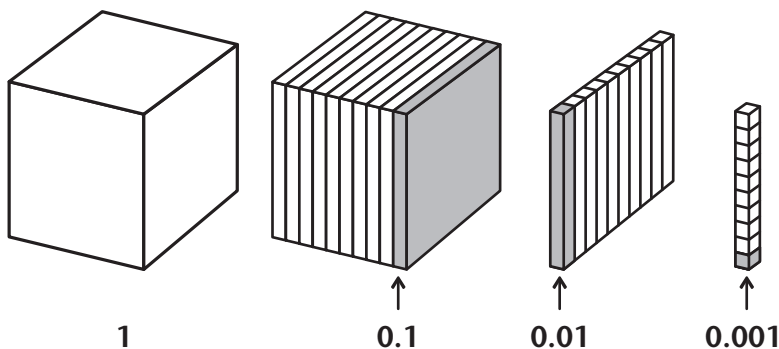


In this diagram one hundredth of the whole or $\frac{1}{100}$ is written as 0.01.



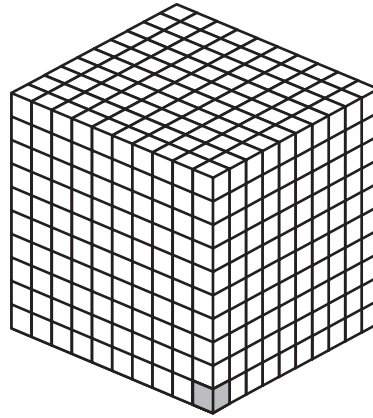
In this diagram 47 hundredths or $\frac{47}{100}$ is written as 0.47.

To understand $\frac{1}{1000}$, let's look at this cube.



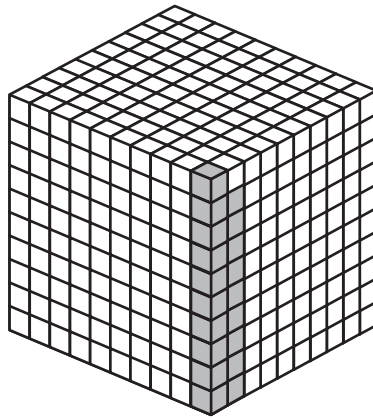
Here is what one thousandth of a whole looks like.

$\frac{1}{1000}$ is written as 0.001.



The next diagram shows you $\frac{19}{1000}$.

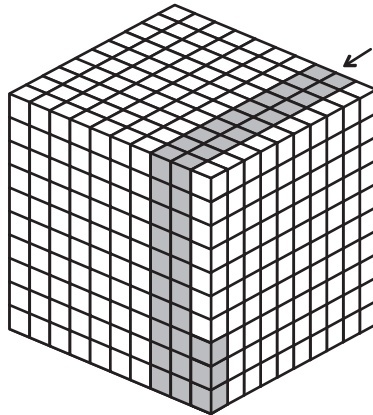
How would you write it as a decimal? (0.019)



This last diagram shows you $\frac{203}{1000}$.

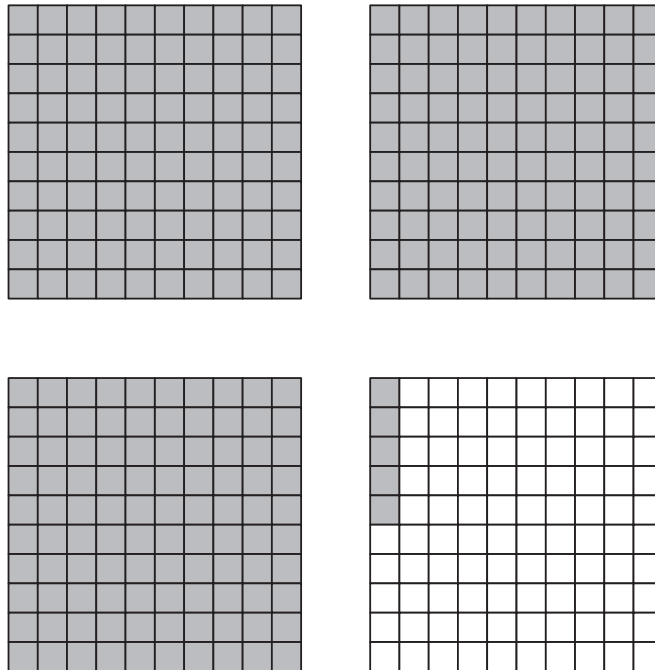
You have to use your imagination here. This shows the whole layer with 100 little cubes in each layer that has been darkened.

The decimal fraction is written as 0.203



You have learned:

- tenths—The number of parts is written on the first place to the right of the decimal point.
- hundredths—The number of parts is written on the second place to the right of the decimal point.
- thousandths—The number of parts is written on the third place to the right of the decimal point.
- whole numbers—Complete items that are not broken up into parts are recorded on the left side of the decimal point.



In the diagram above there are 3 whole and 5 hundredths of a whole which have been darkened.

You write it as: 3.05.

If you wrote 3.50, it would show that 3 whole and 50 hundredths had been darkened.

In this chart you can see whole numbers to the left of the decimal point and decimal parts to the right side.

whole numbers	•	decimal parts
hundreds tens ones	•	tenths hundredths thousandths

Exercises 3.4

A. Write the following as numerals.

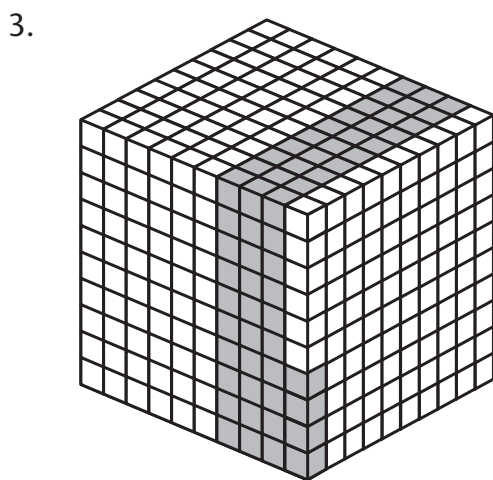
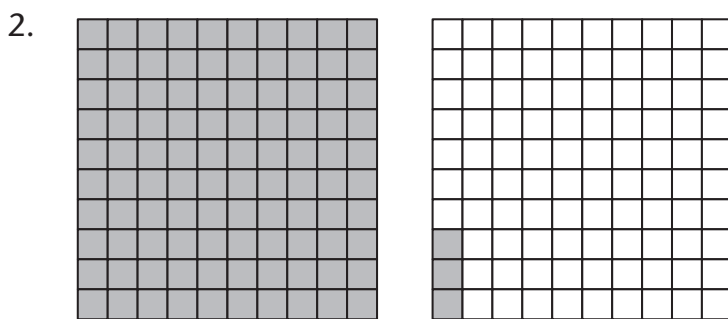
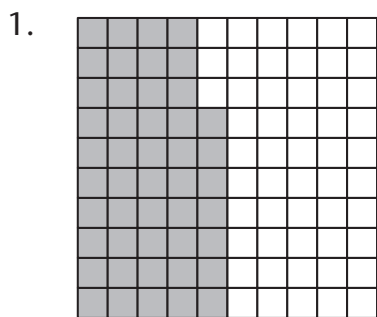
1. seven thousand six hundred five _____
2. forty thousand two hundred _____
3. nine hundred twenty-three _____
4. eight thousand one _____
5. sixty-eight thousand _____

B. Write in word form.

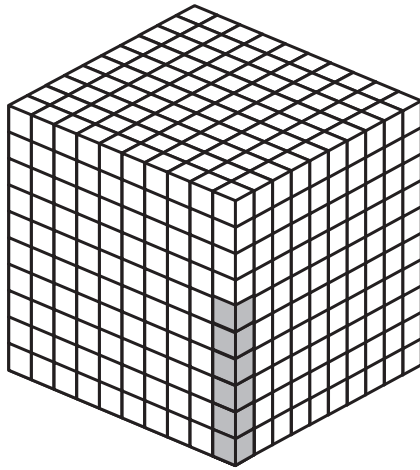
- 4200 _____
- 69 000 _____
- 507 _____
- 30 000 _____
- 406 000 _____

Exercises 3.5

- A. Write the decimal fraction that represents the shaded part of each of the following diagrams.



4.



B. Write the rule that determines which place on the right side of the decimal point a number will be written for the following:

1. tenths _____

2. hundredths _____

3. thousandths _____

C. Write the following as decimal fractions.

1. eight tenths _____
2. six hundred and nine hundredths _____
3. two thousandths _____
4. seven and fifty-one hundredths _____
5. forty-four thousandths _____
6. five hundred and seven thousandths _____

D. Write the following decimal fractions as words.

1. 0.8 _____
2. 0.03 _____
3. 0.004 _____
4. 5.36 _____
5. 2000.150 _____



Turn to the Answer Key at the end of the module to check your work.

Making Equivalent Decimals

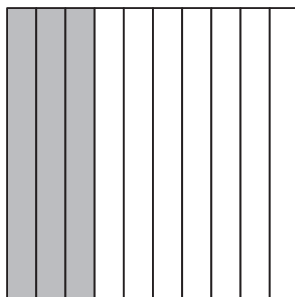
You know a decimal fraction is a part of a whole that is broken up into groups of 10, 100, 1000, and so on.

The first place after the decimal point means tenths, the second place means hundredths, and the third place means thousandths.

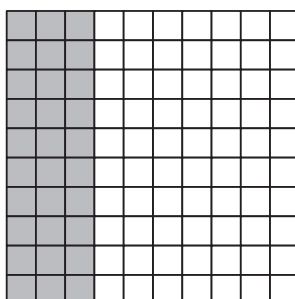
You also know that zero is used as a place holder to indicate how the number is read.

The following illustrations will help you picture equivalent decimal fractions.

In this diagram the square is divided up into tenths. Three of the tenths are darkened. In other words, 0.3 of the whole diagram is darkened.



In this next diagram the same size square is divided up into hundredths. Thirty of the hundredths or 0.30 of the whole diagram are darkened.



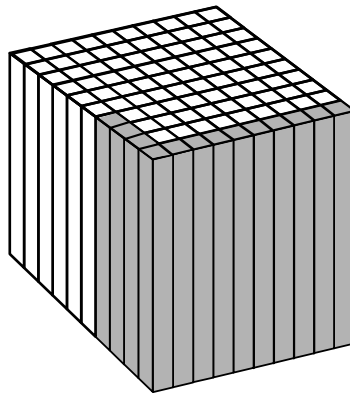
You can see from the two diagrams that the squares are the same size. Therefore 0.3 and 0.30 must show the same part of the whole.

This means they have equivalent value.

The zero at the end is written as a place holder in 0.30 to show that the whole is divided into hundredths.

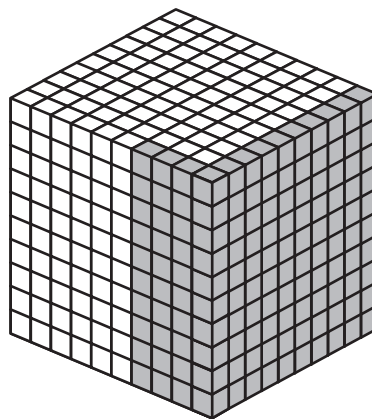
Now let's look at the following cube diagrams. The cubes are divided into hundredths and thousandths and you can see that the decimal fractions have equivalent value.

This diagram shows one whole divided into hundredths. That's 10 rods in each layer \times 10 layers = 100.



Thirteen of these hundredths or 0.13 of the diagram is darkened.

Now look at the second diagram. It shows the same cube divided into thousandths. That's 10 layers \times 10 rods in each layer \times 10 little cubes in each rod.



One hundred thirty of these thousands or 0.130 are darkened. You can see that 0.13 and 0.130 equal the same amount and are equivalent.

You can see how the final number in the decimal fractions shows how many equal pieces the whole is divided into—tenths, hundredths, or thousandths.

Exercises 3.6

Write the following numerals using zeros as place holders.

1. 3 tens _____
2. 700 hundreds _____
3. 90 thousands _____
4. 800 tens _____
5. 610 hundreds _____
6. 20 thousands _____
7. 191 thousands _____
8. 3910 tens _____

Exercises 3.7

Write equivalent decimal numbers for each of the following decimals.

A. Show as tenths:

1. 0.30 _____
2. 0.200 _____
3. 5.600 _____
4. 0.900 _____
5. 3498.00 _____
6. 679 _____

B. Show as hundredths:

1. 0.3 _____

2. 0.9 _____

3. 0.890 _____

4. 67.8 _____

5. 19.2 _____

6. 3891 _____

C. Show as thousandths:

1. 0.03 _____

2. 0.9 _____

3. 0.07 _____

4. 43.1 _____

5. 391 _____

6. 4.20 _____

Exercises 3.8

A. Circle the greater decimal in each pair.

1. 0.46 or 0.37

4. 2.70 or 2.07

2. 3.06 or 4.02

5. 0.22 or 0.06

3. 0.18 or 0.72

6. 7.60 or 7.51

B. John said that 1.40 is greater than 1.4 because 40 is greater than 4. Is he correct? Use pictures to support your answer.



Turn to the Answer Key at the end of the module to check your work.

Comparing Decimal Fractions Using Place Value

Look at these two decimal fractions: 2.1 and 1.7.

Which one is the greater? Why?

Bob jogged 2.9 km (kilometers) while Elsa jogged 3.3. km.

Who jogged the greatest distance?



A place value chart is a handy tool to use when you are working with equivalent numbers. This is what it would look like if we placed 24.57 on the place chart below.

Tens		Ones		Tenths		Hundredths	
2		4		5		7	
↓		↓		↓		↓	
value		value		value		value	
20	+	4	+	0.5	+	0.07	
OR							
20	+	4	+	$\frac{5}{10}$	+	$\frac{7}{100}$	
24.57							

Let’s see what 6.09 looks like on a place value chart.

Tens	Ones	Tenths	Hundredths		
	6	0	9		
↓ value	↓ value	↓ value	↓ value		
0	6	+	0.0	+	0.09
OR					
	6	+	$\frac{0}{10}$	+	$\frac{9}{100}$
6.09					

Now we are going to find which is greater—1.6 or 1.60. To do this we will place each number on the place value chart.

Place 1.6 on this chart.

Tens	Ones	Tenths	Hundredths
	1	.	6
	↓		↓
	value		value
	1	+	.6
	1.6		

Place 1.60 on this chart.

Tens	Ones	Tenths	Hundredths		
	1	.	6	0	
	↓		↓	↓	
	value		value	value	
	1	+	.6	+	0
	1.60				

Zeros on the end of the decimal do not increase value. They just rename a decimal. Look at these examples:

$$\begin{array}{ccc}
 0.4 & \rightarrow & 0.40 \\
 4 \text{ tenths} & & 40 \text{ hundredths} \\
 0.4 = 0.40
 \end{array}$$

7.8 is seven and eight tenths.

7.80 is seven and eighty hundredths.

$$7.8 = 7.80$$

By using this rule you can compare numbers up to the thousandths place or even higher.

Compare these two numbers.

61.090 61.009

Look at the number on the left first (the greatest place value) and then compare the remaining numbers. This is just the same as comparing whole numbers.

The numbers on the left are the same.

61.090 61.009

Look at the next place to the right of the first number and compare.

Again the numbers are the same.

61.090 61.009

Look to the next place to the right and compare.

Once more the numbers are the same.

61.090 61.009

Look to the next place to the right and compare.

The numbers are different. 9 hundredths are more than 0 hundredths. Therefore 61.090 is greater than 61.009.

To sum up: **The first digits compared from the left to the right that are found to be different tell which number, as a whole, has the greater or lesser value.**

Use < (less than), > (greater than) or = to describe the relationship between the following pairs of numbers.

a. 1.70 1.71 b. 9.8 9.08

c. 6.6 6.60 d. 0.8 0.80

e. 6.50 6.45 f. 0.22 0.220

(Answers: a. <, b. >, c. =, d. =, e. >, f. =)

Exercises 3.9

Write the value of each of the underlined digits.

Example: 417 623 7000

1. 14 235 _____

2. 86 301 _____

3. 114 265 _____

4. 7065 _____

5. 178 290 _____

6. 454 154 _____

7. 1 001 321 _____

8. 2 751 345 _____

Exercises 3.10

A. Write each number as a decimal in standard form.

Example: two and sixty hundredths **2.60**

1. four and seven tenths _____
2. eleven and forty-two hundredths _____
3. seven hundredths _____
4. nine tenths _____

B. Write the value of the underlined digit as a decimal and as a fraction.

Example: 7.15

$$0.1 = \frac{1}{10}$$

1. 12.16 _____
2. 8.27 _____
3. 19.75 _____
4. 61.05 _____

- C. Use the place value chart for each number and put each number into its correct position on the chart.

Example: 16.45

Tens	Ones	Tenths	Hundredths
1	6	.	45

place the decimal point on the chart

1. 8.45

Tens	Ones	Tenths	Hundredths
------	------	--------	------------

2. 12.07

Tens	Ones	Tenths	Hundredths
------	------	--------	------------

D. Write the value of each underlined digit.

1. 17.42 _____

2. 11.75 _____

3. 6.09 _____

4. 14.06 _____

5. 20.70 _____

6. 46.41 _____

E. Write < (less than) or > (greater than) to complete each number statement below.

1. 0.21 ____ 0.12

6. 99.002 ____ 98.763

2. 3.16 ____ 4.99

7. 11.310 ____ 11.301

3. 17.21 ____ 7.89

8. 5.005 ____ 5.050

4. 13.01 ____ 13.00

9. 783.90 ____ 784.90

5. 619.444 ____ 691.444

10. 20.016 ____ 20.106

Exercises 3.11

- A. Write an equivalent decimal for each of the given decimals below.

Example: $1.70 = 1.7$

1. 7.70 _____

2. 3.4 _____

3. 17.8 _____

4. 2.3 _____

5. 19.90 _____

6. 25.60 _____

- B. Order from least to greatest.

1. 0.034 0.043 0.039 0.304 0.344

2. 392.01 391.02 390.99 392.21 391.22

- C. Order from greatest to least.

1. 79.41 178.41 77.04 79.14 79.07

2. 0.002 0.012 0.200 0.120 0.001



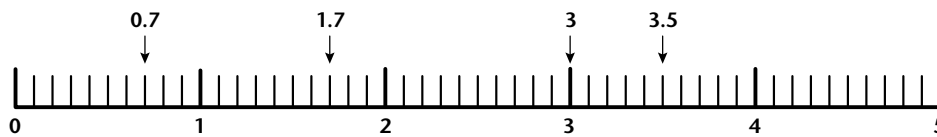
Turn to the Answer Key at the end of the module to check your work.

Ordering Decimals

In the last lesson we compared decimals. In this lesson you will put the decimals in order. You will order them from the greatest to the least, and from the least to the greatest.

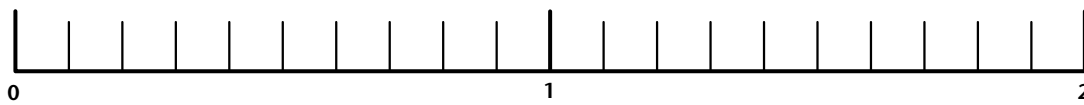
Look at this number line.

The numbers at the bottom of the line are the whole numbers.



The spaces between the whole numbers are divided into ten equal pieces. Each of those pieces is a tenth.

The number 1.7 describes the spot that is seven tenths past the number 1.



Self Test

- Place each of the following decimals on the number line below. Write the decimal numbers in order from the least to the greatest.

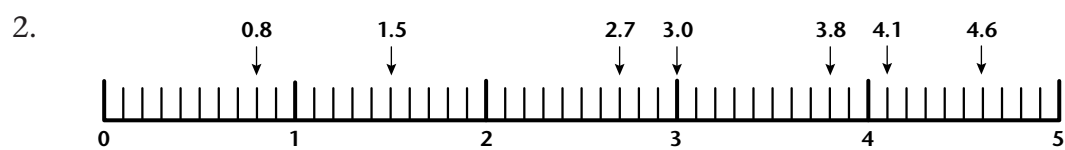
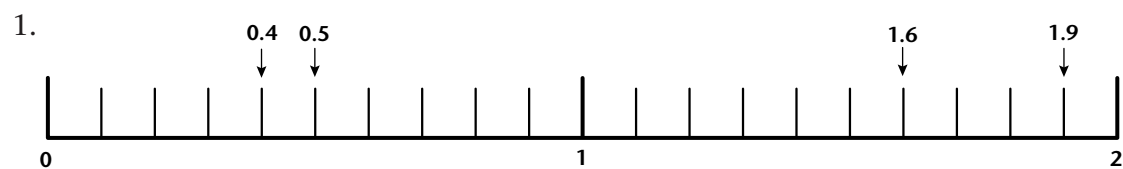
0.5, 1.9, 0.4, 1.6



- Use this number line below to help you write the list of decimals in order from the greatest to the least.

4.1, 0.8, 2.7, 4.6, 1.5, 3.8, and 3.0



Answers

Exercises 3.12

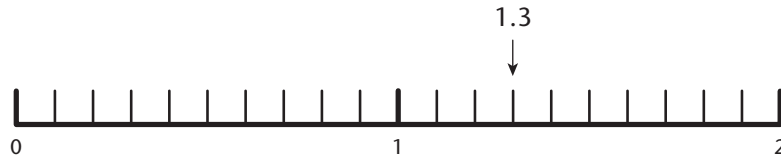
A. Complete the chart below by filling in the common fraction and decimal fraction for each of the numbers given.

Number	Common Fraction	Decimal Fraction
1. $\frac{12}{10}$	_____	_____
2. $\frac{23}{10}$	_____	_____
3. $\frac{38}{10}$	_____	_____
4. $\frac{26}{10}$	_____	_____
5. $\frac{17}{10}$	_____	_____
6. $\frac{10}{10}$	_____	_____

Exercises 3.13

A. Place each of the following decimals on the number line.

Example: 1.3



1. 3.9

2. 0.6

3. 4.7

4. 1.4

5. 2.8

6. 4.2



B. Order each group of numbers from least to greatest.

1. 4.2, 0.4, 2.1

2. 7.9, 7.1, 8.2

3. 5.3, 6, 5.5, 6.1

4. 6.8, 7, 7.3, 7.1

5. 2.3, 1, 0.7, 1.6, 1.9

6. 3.0, 4.1, 3.9, 4.0

Exercises 3.14

Order each group of numbers from the greatest to the least.

1. 7.2, 2.7, 0.7, 7.8, 7.1

2. 0.9, 9.0, 9.9, 9.1, 9.6

3. 10.1, 10.6, 10, 10.9, 11.2

4. 5.1, 7.3, 8.3, 0.9, 4.8, 3.7

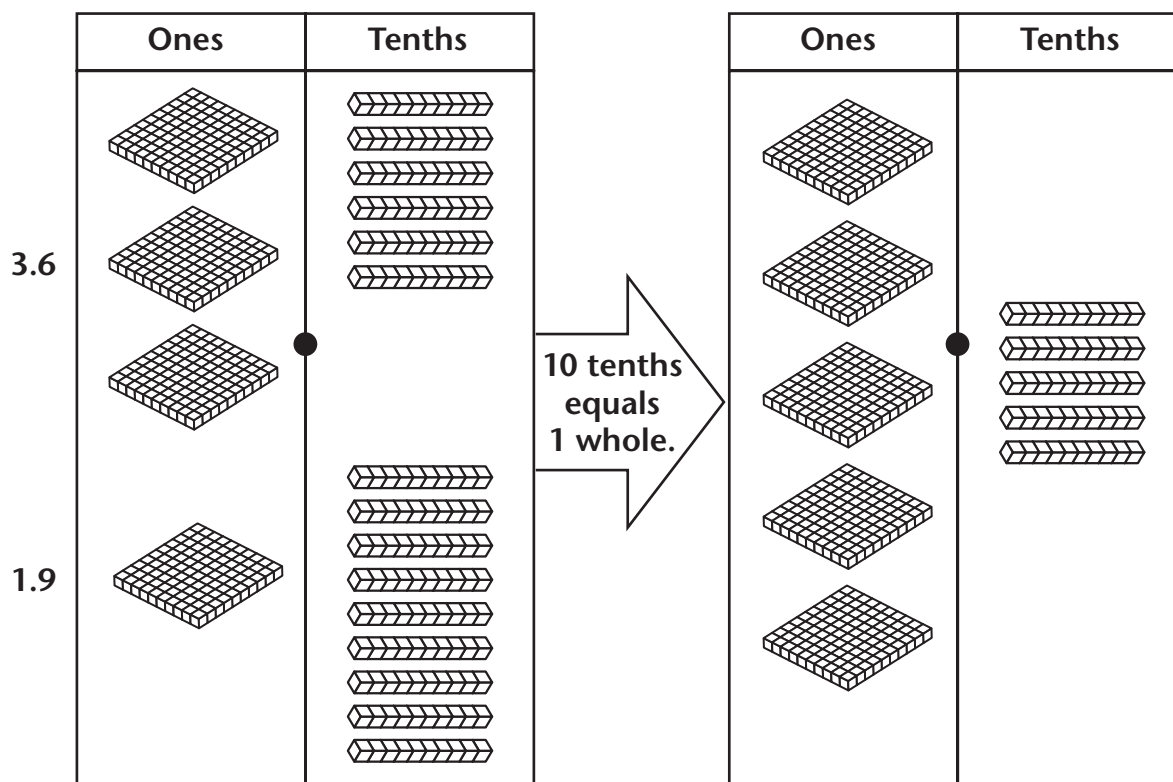


Turn to the Answer Key at the end of the module to check your work.

Adding Decimal Fractions

You know how to add whole numbers. You can use the same strategies to add decimals.

Look at the pictures of Base 10 blocks. They show you what happens when you add $3.6 + 1.9$



The sum of $3.6 + 1.9 = 5.5$

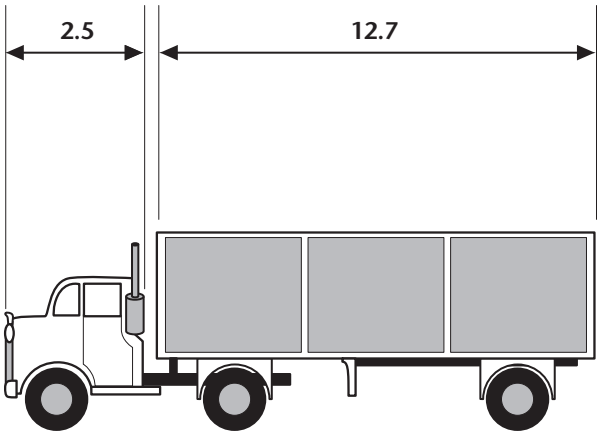
To add without pictures, use place value. Look at this example.

Add the tenths: 12 tenths $\begin{array}{r} 2.8 \\ + 3.4 \\ \hline \end{array}$	10 tenths equals 1 whole. That's 1 and 2 tenths. $\begin{array}{r} 1 \\ 2.8 \\ + 3.4 \\ \hline 2 \end{array}$	Add the ones: $\begin{array}{r} 1 \\ 2.8 \\ + 3.4 \\ \hline 6.2 \end{array}$
---	--	---

The length of the tractor (or cab) is 2.5 m (metres) and the length of the trailer is 12.7 m. How long is the entire tractor trailer?

Think: $12.7\text{ m} + 2.5\text{ m} = ?$

Write:



	tens	ones	tenths
	1	2 ¹	7
+		2	5
Answer	1	5	2

The tenths have been regrouped into 1 ones and 2 tenths.

The tractor trailer is 15.2 long.

Mrs. Jones bought 0.5 kg of peaches. The peaches went in a fruit salad with 1.2 kg of pears. How much fruit was in the fruit salad?

Think: " $0.5\text{ kg} + 1.2\text{ kg} = ?$ "

Write:

	ones	tenths
	0	5
	1	2
Answer	1	7

Mrs. Jones has 1.7 kg of fruit in the salad.

Self Test

Try these addition questions.

$$\begin{array}{r} 4.2 \\ + 2.3 \\ \hline \end{array} \quad \begin{array}{r} 1.7 \\ + 5.6 \\ \hline \end{array} \quad \begin{array}{r} 5.7 \\ + 6.7 \\ \hline \end{array} \quad \begin{array}{r} 3.3 \\ + 9.8 \\ \hline \end{array}$$

Answers

6.5 7.3 12.4 13.1

You know that decimal fractions must be lined up correctly before you can add. Study the following problem.

Jake was training for the city cross-country run. He ran 6.1 km on Friday, 5 km on Saturday, 5.2 km on Sunday and 6 km on Monday. How many kilometers did Jake run altogether during those 4 days?

Let's review what we already know about adding decimals.

To add decimal numbers you must follow two simple steps:

- Align (line up) the decimal points, which will line up the place values
- Add the decimal numbers exactly as you would add whole numbers.

To answer this problem, you would write the decimal numbers vertically and then add. It looks like this:

$$\begin{array}{r} 6.1 \\ 5.0 \\ 5.2 \\ + 6.0 \\ \hline 22.3 \end{array}$$

Mr. Beaumont mailed three parcels at the post office. The first parcel weighed 0.823 kg, the second 1.3 kg, and the third 0.68 kg. What was the total weight of Mr. Beaumont's three parcels?

You line up the decimal points:

$$\begin{array}{r} .823 \\ 1.3 \\ + .68 \\ \hline \end{array}$$

Then you add using regrouping as you would with whole numbers.

$$\begin{array}{r} \overset{1}{\overset{1}}{.823} \\ 1.3 \\ + .68 \\ \hline 2.803 \end{array}$$

Exercises 3.15

Line up the following sets of decimal fractions so the decimal points are in a vertical (straight up and down) line. Use zeros as place holders if necessary.

1. 34.15 600 0.051 6.18 9.136

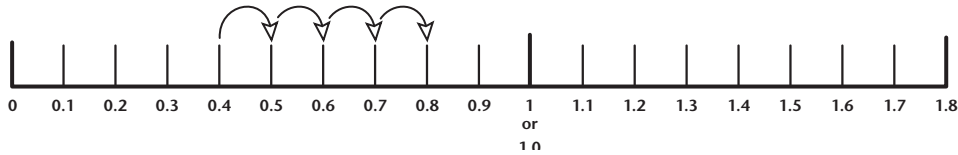
2. 481.2 13 619.51 0.002 1732

3. 14.900 6.84 0.05 182.13 72

Exercises 3.16

A. Use the number line to help you find the sums.

Example: $0.4 + 0.4 = 0.8$



1. $0.2 + 0.4 =$ _____ 2. $0.7 + 0.3 =$ _____

3. $0.6 + 0.5 =$ _____ 4. $0.8 + 0.9 =$ _____

B. Add these questions.

1. $\begin{array}{r} 0.2 \\ +0.7 \\ \hline \end{array}$	2. $\begin{array}{r} 2.9 \\ +3.5 \\ \hline \end{array}$	3. $\begin{array}{r} 27.2 \\ +47.9 \\ \hline \end{array}$	4. $\begin{array}{r} 21.6 \\ 75.2 \\ +49.2 \\ \hline \end{array}$
---	---	---	---

C. Line up (align) the decimal points and then add each of the following questions.

1. $14.5 + 5.3$

2. $\$12.38 + \$1.89 + \$43.98$


3. $6.2 + 14.6$

4. $8.403 + 12 + 3.98$

5. $33.9 + 41.2$

D. Solve each problem. Show all your work and then write a statement to answer each question.

1. Jenny jogged 6.2 km (kilometers) on Saturday and 4.8 km on Sunday. How far did she jog on the weekend?



Statement: _____

2. In a jumping contest the judges combined the best three jumps of each contestant. If Laurie had jumps of 1.3m, 1.6m, 1.7m, and 1.6m, what was her combined score?



Statement: _____

Exercises 3.17

Take out your calculator. Here are some activities to help you discover how decimal fractions are recorded on it.

When you enter numbers on a calculator, it is not necessary to press the 0 to the left of the decimal unless it is a whole number place holder (such as 40). The calculator does it for you.

- Press **.96** on your calculator. Remember, you don't need to press 0.96, just .96.
- Now add **one hundredth (0.01)** to .96. Press the equal sign.
- Your calculator should read **0.97**. Continue adding 0.01. Don't forget to press = and then + each time.

What do you notice when you reach **1.00**?

You will notice that your calculator probably reads 1. with no zeros.

Many calculators do not display end zeros in a decimal answer.

Try another number:

Press 23.248 on your calculator.

Now add one thousandth (.001)

Your calculator should show 23.249.

Continue adding .001.

What happens when you reach 23.250?

Your calculator should read 23.25 instead of 23.250.

Now use your calculator to add the following sets of decimal numbers.

1. $19.35 + 18.645 =$ _____

2. $84 + 0.03 + 16.81 =$ _____

3. $93.186 + 4 + 3.027 + 91.5 =$ _____

4. $86.3 + 19.85 + 4.27 + 5.007 =$ _____

Solve this problem.

Karli and her brother Barry entered their pumpkins in the Fall Fair. Karli's pumpkin weighed 13.512 kg. Barry's pumpkin weighed 9.7 kg. What were the combined weights of the 2 pumpkins?

Statement: _____



Turn to the Answer Key at the end of the module to check your work.

Subtracting Decimal Fractions

Subtracting decimal numbers is very similar to the process you just learned for adding decimal numbers. Line up the decimal points, then subtract just like you would for whole numbers.

*When Baby Jamie was born his mass was 3.2 kg.
When he was weighed the next day he had lost 0.4 kg.
What was his mass on the second day?*



Think: $3.2 \text{ kg} - 0.4 \text{ kg}$

Write:

ones		tenths
² 3		¹² 2
- 0		4
2		8 kg

The baby's mass was 2.8 kg on the second day.

Jim is 151.8 cm tall. Cheryl is 3.91 cm shorter than Jim. How tall is Cheryl?

You know the first step is to align the numbers by the decimal points. Because Jim's height is written to the tenth decimal place, and Cheryl's is written to the hundredth place, it is easier to find the difference if you use a zero as a place holder. Look at this example.

$$\begin{array}{r} 151.80 \\ - 3.91 \\ \hline \end{array}$$

The next step is to subtract the same way you would with whole numbers. It's a good idea to write down the decimal point on the answer line before you calculate the answer.

Look at this example.

$$\begin{array}{r} 151.80 \\ - 3.91 \\ \hline . \end{array}$$

The final step is to complete the subtraction, borrowing (regrouping) if it's necessary. You'll see in this question, you need to regroup.

$$\begin{array}{r}
 \overset{4}{1} \overset{10}{5} \overset{17}{1} \overset{1}{.} 80 \\
 - 3.91 \\
 \hline
 147.89
 \end{array}$$

Cheryl is 147.89 cm tall.

Check: You can add the answer to the piece you subtracted. If you get the number you started with, you know you did the work correctly. *Align the decimal points.*

$$\begin{array}{r}
 147.89 \\
 + 3.91 \\
 \hline
 151.80
 \end{array}$$

Self Test

Try these subtraction questions. Copy the problem onto a piece of paper and line up the decimal points. You should check your work by adding.

1. $8.7 - 7.2 =$
2. $100.2 - 84.7 =$
3. $7.509 - 0.29 =$

Answers

1. 1.5
2. 15.5
3. 7.219

Exercises 3.18

Review your regrouping skills by subtracting these whole numbers.

$$\begin{array}{r} 1. \quad 459 \\ - 389 \\ \hline \end{array}$$

$$\begin{array}{r} 2. \quad 3964 \\ - 1892 \\ \hline \end{array}$$

$$\begin{array}{r} 3. \quad 9733 \\ - 4819 \\ \hline \end{array}$$

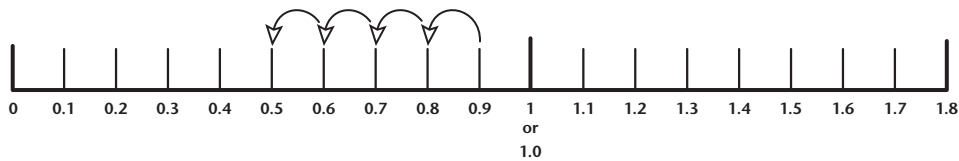
$$\begin{array}{r} 4. \quad 6051 \\ - 3944 \\ \hline \end{array}$$

$$\begin{array}{r} 5. \quad 9205 \\ - 3856 \\ \hline \end{array}$$

Exercises 3.19

A. Use the number line to help you find the differences.

Example: $0.9 - 0.4 = 0.5$



$$1. \quad 0.5 - 0.2 = \underline{\hspace{2cm}} \quad 2. \quad 0.8 - 0.6 = \underline{\hspace{2cm}}$$

$$3. \quad 1.0 - 0.3 = \underline{\hspace{2cm}} \quad 4. \quad 1.4 - 0.9 = \underline{\hspace{2cm}}$$

$$5. \quad 1.7 - 1.2 = \underline{\hspace{2cm}} \quad 6. \quad 1.3 - 1.2 = \underline{\hspace{2cm}}$$

B. Align these numbers and find the differences.

1. $\$284.16 - \103.79

2. $423.1 - 16.5$

3. $18 - 9.37$

C. Round each decimal to the nearest one and estimate the differences. Find the exact differences to check your estimates. Show your estimation.

1.
$$\begin{array}{r} 34.5 \\ -23.6 \\ \hline \end{array}$$

2.
$$\begin{array}{r} 17.4 \\ -10.7 \\ \hline \end{array}$$

3.
$$\begin{array}{r} 23.50 \\ -19.78 \\ \hline \end{array}$$

4.
$$\begin{array}{r} 431.25 \\ -330.55 \\ \hline \end{array}$$

Exercises 3.20

A. Complete these questions using your calculator.

1. $6.902 - 4.500 =$ _____

2. $5.120 - 5.034 =$ _____

3. $\$954.89 - \$132.45 =$ _____

4. $98.756 - 0.510 =$ _____

B. Use your calculator to help solve these problems.

1. Al completed a 10 km run in 45.61 min. His brother Chris completed the same run in 43.19 min. Which brother won the race and by how much?



Statement: _____

2. Janice bought a sweater priced at \$37.89. She received a sale discount of \$6.78 and then was charged \$1.87 tax. What did Janice pay for the sweater?

Statement: _____



Turn to the Answer Key at the end of the module to check your work.