Chapter 7
Geometry of 3D objects

In this chapter, you will revise the properties of prisms and pyramids, which you investigated in previous grades. This includes using nets to construct models of these objects as a further means of consolidating your understanding of polyhedra. You will also revise the properties and definitions of the five Platonic solids, which you first learnt about in Grade 8, as well as how Euler's formula describes a relation between the number of vertices, faces and edges of any polyhedron.

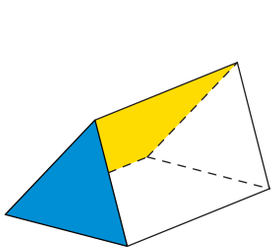
New to this grade are investigations of the properties of cylinders and spheres. Although you should be able to recognise these 3D objects by now, you will examine some of their properties in more detail, and learn how to construct a net and model of a cylinder.

7 Geometry of 3D objects

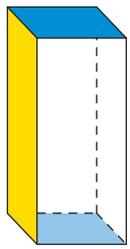
7.1 Classifying 3D objects

3D objects with flat faces which are called **polyhedra**. Prisms and pyramids are two types of polyhedra.

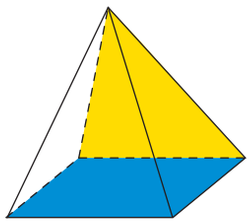
A polyhedron is a 3D object with only flat faces.



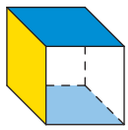
Triangular prism



Rectangular prism

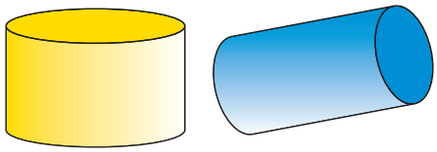


Rectangular-based pyramid

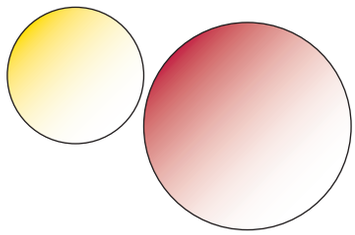


Cube

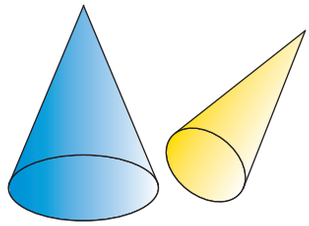
Examples of 3D objects that have at least one curved surface are **cylinders, spheres** and **cones.**



Cylinders

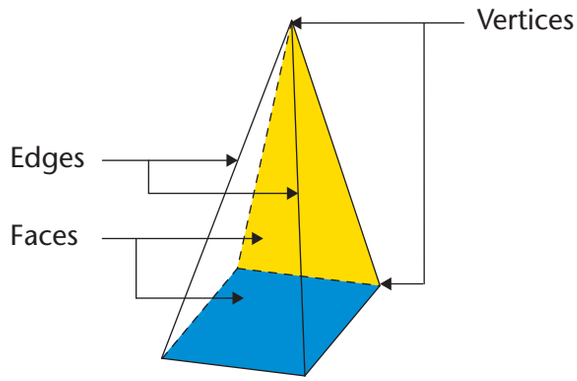


Spheres



Cones

When we study the properties of a 3D object, we investigate the shapes of its faces, its number of faces, its number of vertices and its number of edges. For example, the pyramid alongside has 1 square face and 4 triangular faces, 5 vertices and 8 edges.



Square-based pyramid

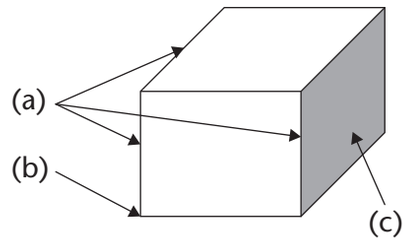
CLASSIFYING AND DESCRIBING 3D OBJECTS

1. Label parts (a) to (c) on the prism correctly.

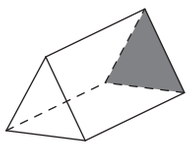
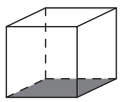
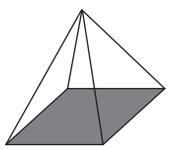
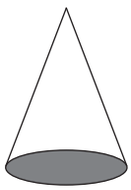
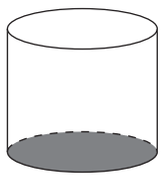
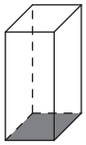
   (a) .....

   (b) .....

   (c) .....



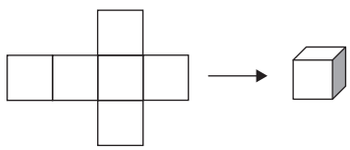
2. Complete the table.

	3D object	Name of the object	Number of faces and shape of faces	Number of vertices
(a)		Triangular prism	2 triangles and 3 rectangles	6
(b)				
(c)			6 squares	8
(d)			1 rectangle and 4 triangles	5
(e)				
(f)				
(g)				

3. Say whether each statement below is true or false.
   (a) A cylinder is a polyhedron. .....
   (b) A triangular-based pyramid has 4 triangular faces. .....
   (c) A cube is also known as a hexahedron. .....
   (d) A triangular-based pyramid has 6 vertices. .....
   (e) A pyramid is a 3D object. .....

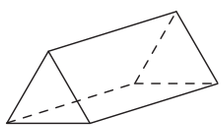
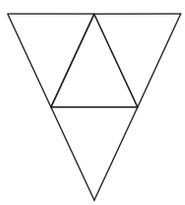
7.2 Nets and models of prisms and pyramids

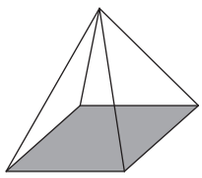
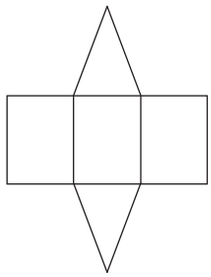
A **net** is a flat pattern that can be used to represent a 3D object. The net can be folded up to create a model of the 3D object.

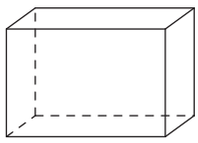
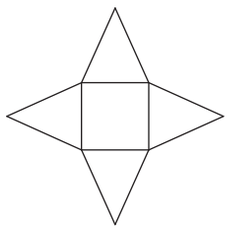


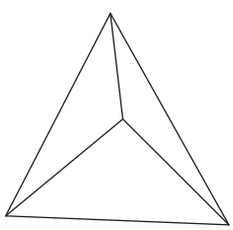
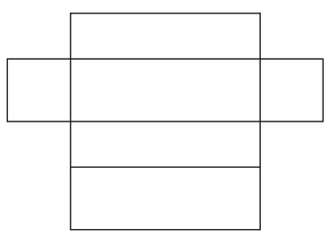
Net of a cube

1. Name each object below and draw an arrow to match it with its net.

(a)  ..... 

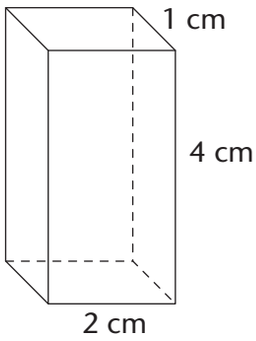
(b)  ..... 

(c)  ..... 

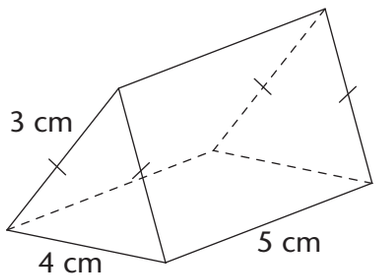
(d)  ..... 

2. Construct an accurate net for each of the following 3D objects.

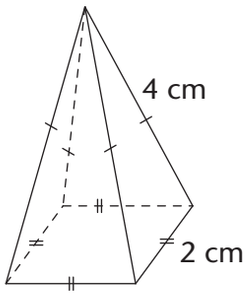
(a)



(b)



(c)



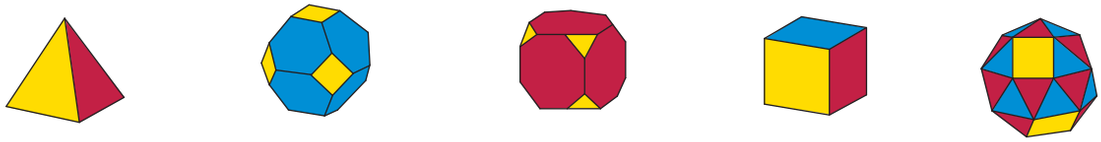
3. Construct models of the objects in question 2 but double all the measurements.

7.3 Platonic solids

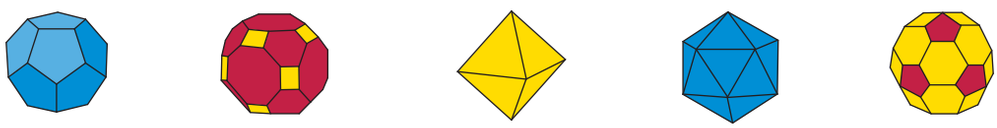
A **Platonic solid** is a 3D object which has identical faces, and all of the faces are identical regular polygons. This means that all its faces are the same shape and size and all the vertices are identical.

1. Which of the following objects are Platonic solids?

A. B. C. D. E.



F. G. H. I. J.

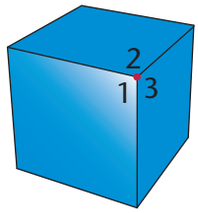


.....

2. How many Platonic solids are there in question 1? .....

ONLY FIVE PLATONIC SOLIDS?

You can use your knowledge about angles to prove that the five Platonic solids are the only 3D objects that can be made from identical regular polygons. Keep the following facts in mind:



- A 3D object has *at least* three faces that meet at each vertex.
- The sum of the angles that meet at a vertex must be less than 360°. If it is equal to 360°, it will form a flat surface. If it is greater than 360°, the faces will overlap.
- Each Platonic solid is made up of one type of regular polygon only.

What 3D objects can you make from equilateral triangles?

We use the following reasoning:

size of each interior angle = $180° \div 3 = 60°$

$\therefore$ 3 triangles = $3 \times 60° = 180°$ [< 360°]

4 triangles = $4 \times 60° = 240°$ [< 360°]

5 triangles = $5 \times 60° = 300°$ [< 360°]

6 triangles = $6 \times 60° = 360°$

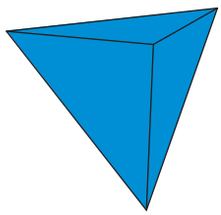
Any more than 5 triangles will be equal to or more than 360° and will therefore form a flat surface or overlap.

This means that we can make three 3D objects from equilateral triangles:

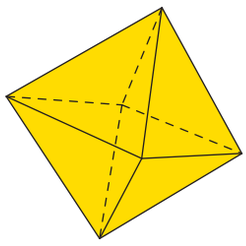
If 3 triangles are at each vertex, it will form a **tetrahedron**.

If 4 triangles are at each vertex, it will form an **octahedron**.

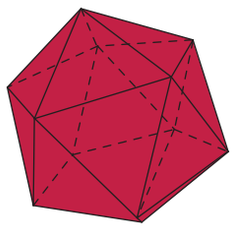
If 5 triangles are at each vertex, it will form an **icosahedron**.



Tetrahedron



Octahedron



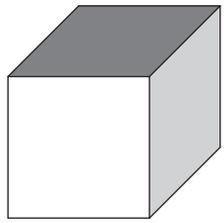
Icosahedron

What 3D objects can you make from squares?

Complete the statements: size of each interior angle .....

∴ 3 squares = 3 × .....

4 squares = 4 × .....



Hexahedron (cube)

Therefore we can make only one 3D object using squares. This 3D object is called a **hexahedron (or cube).**

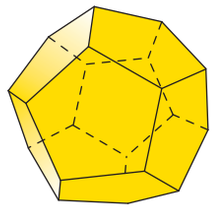
What 3D objects can you make from regular pentagons?

Complete the statements:

Size of each interior angle .....

∴ 3 pentagons = ...

4 pentagons = ...



Dodecahedron

Therefore we can make only one 3D object using regular pentagons. This 3D object is called a **dodecahedron.**

What 3D objects can you make from regular hexagons?

Complete the statements:

Size of each interior angle .....

∴ 3 hexagons = ...

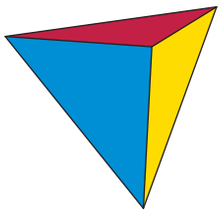
Three hexagons will already form a flat surface. Therefore it is impossible to make a 3D object from regular hexagons.

Also, the interior angles of polygons with more than 6 sides are bigger than those of a hexagon, so it is not possible to make 3D objects from any other regular polygons.

Therefore the five Platonic solids already mentioned (tetrahedron, octahedron, icosahedron, hexahedron and dodecahedron) are the only ones that can be made of identical regular polygons. Each of these solids is named after the number of faces it has.

PROPERTIES OF THE PLATONIC SOLIDS

Complete the information about each of the following Platonic solids.

1. 

Name: .....

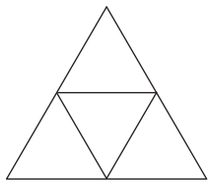
.....

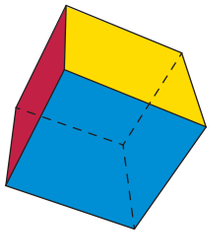
Shape of the faces: .....

Number of faces: .....

Number of edges: .....

Number of vertices: .....



2. 

Name: .....

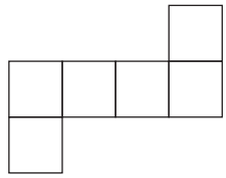
.....

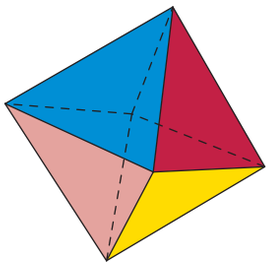
Shape of the faces: .....

Number of faces: .....

Number of edges: .....

Number of vertices: .....



3. 

Name: .....

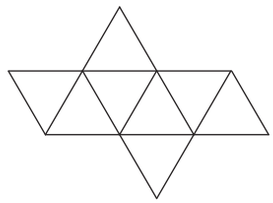
.....

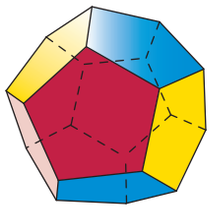
Shape of the faces: .....

Number of faces: .....

Number of edges: .....

Number of vertices: .....



4. 

Name: .....

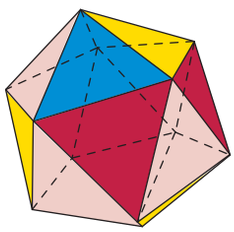
.....

Shape of the faces: .....

Number of faces: .....

Edges: .....

Vertices: .....

5. 

Name: .....

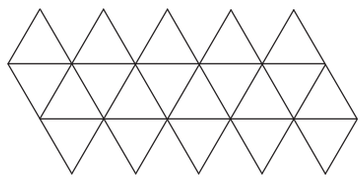
.....

Shape of the faces: .....

Number of faces: .....

Edges: .....

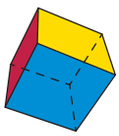
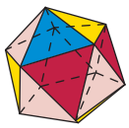
Vertices: .....



7.4 Euler's formula

EULER'S FORMULA AND PLATONIC SOLIDS

1. You learnt about Euler's formula in Grade 8. Complete the following table to investigate whether or not Euler's formula holds true for Platonic solids.

	Name	Shape of faces	No. of faces (F)	No. of vertices (V)	No. of edges (E)	F + V – E
						
						
						
						
						

2. Complete Euler's formula for polyhedra:
   F + ...

3. Apply Euler's formula to each of the following:
   (a) A polyhedron has 25 faces and 13 vertices. How many edges will it have?

   .....

   (b) A polyhedron has 11 vertices and 23 edges. How many faces does it have?

   .....

   (c) A polyhedron has 8 faces and 12 edges. How many vertices does it have?

   .....

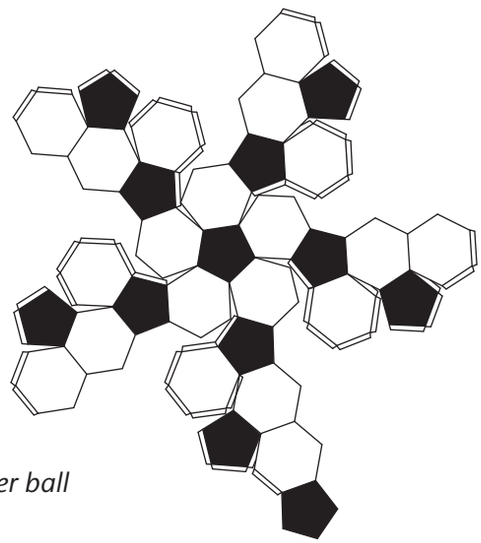
EULER'S FORMULA AND OTHER POLYHEDRA

1. Is each of the following statements true or false?
   (a) A polyhedron with 10 vertices and 15 edges must have 7 faces. .....
   (b) A polyhedron will always have more edges than either faces or vertices. .....
   (c) A polyhedron with 5 faces must have 6 edges. .....
   (d) A pyramid will always have the same number of faces and vertices. .....

2. Complete the following table.

	No. of faces (F)	No. of vertices (V)	No. of edges (E)	Name of polyhedron	Shapes of faces
(a)	6		12		Rectangles
(b)		7		Hexagonal pyramid	
(c)	4	4			
(d)	5	6	9		Triangles and rectangles

3. A soccer ball consists of pentagons and hexagons.
   (a) How many pentagons does it consist of? .....
   (b) How many hexagons does it consist of? .....
   (c) How many edges does it have? .....
   (d) How many vertices does it have? .....
   (e) Does Euler's formula apply to soccer balls too?
   .....

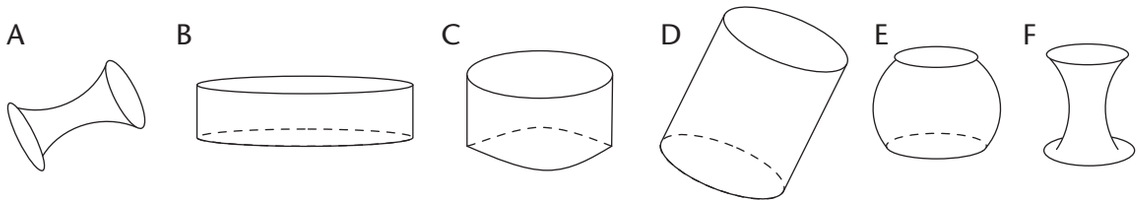


Net of a soccer ball

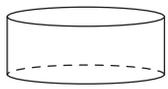
7.5 Cylinders

PROPERTIES OF CYLINDERS

1. Which of the following 3D objects are cylinders? .....

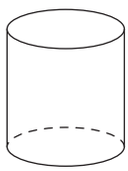


2. Tick the statement or statements below that are true only for cylinders and not for the other objects shown in question 1:



   - ☐ It is a 3D object.
   - ☐ It has a curved surface.
   - ☐ It has two circular bases that are parallel to each other.
   - ☐ It has two flat circular bases and a curved surface.
   - ☐ The radius of its curved surface is equal from the top to the bottom between the bases.
   - ☐ It has two circular bases opposite each other, joined by a curved surface whose radius is equal from the top to the bottom between the bases.

3. Look at the cylinder alongside and complete the following:



   (a) Number and shape of faces: .....

   (b) Number of vertices: .....

   (c) Number of edges: .....

NETS OF CYLINDERS

In Chapter 5, you learnt about the net of a cylinder. If you cut the curved surface of a cylinder vertically and flatten it, it will be the shape of a rectangle.

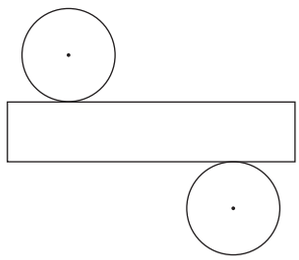
1. Explain why the length of the rectangular face is equal to the circumference of the base.

.....

.....

.....

.....

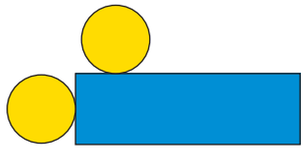


2. Will each of the following nets form a cylinder?

A.

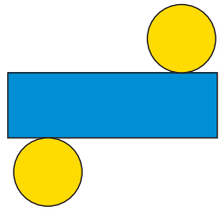
.....

B.



.....

C.



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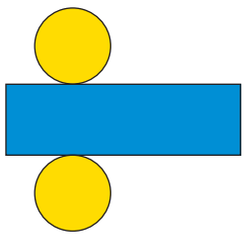
D.



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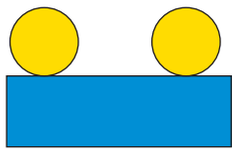
E.



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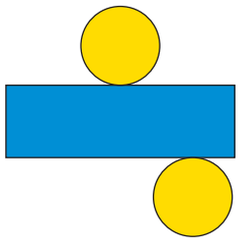
F.



.....

3. In each of the following questions, use $\pi = \frac{22}{7}$ and round off your answer to two decimal places to do the calculations.

(a) If the radius of a cylinder is 3 cm, what is the length of the rectangular surface of the cylinder?

.....

(b) If the radius of a cylinder is 5 cm, what is the length of the rectangular surface of the cylinder?

.....

(c) If the diameter of a cylinder is 8 cm, what is the length of the rectangular surface of the cylinder?

.....

(d) If the diameter of a cylinder is 9 cm, what is the length of the rectangular surface of the cylinder?

.....

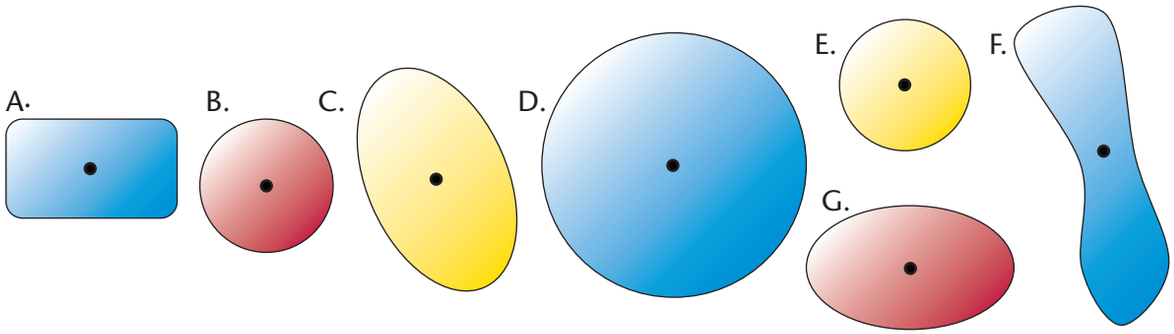
4. Use a ruler and a set of compasses to construct the following nets as accurately as possible. Show the measurements on each net.
   (a) Net of a cylinder with a radius of 1 cm and a height of 4 cm.

   (b) Net of a cylinder with a radius of 1,5 cm and a height of 3 cm

5. Construct models of the cylinders in question 5 but double the measurements.

7.6 Spheres

1. Which of the following 3D objects are spheres?



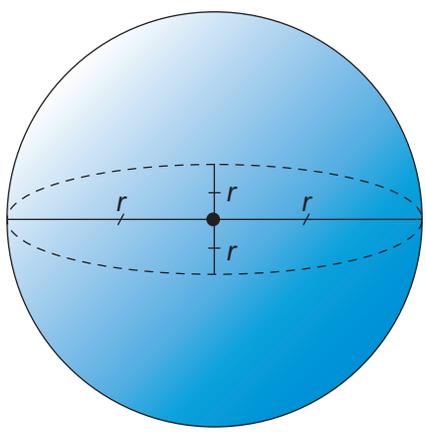
.....

2. Tick the property or properties below that are true for spheres only and not for the other objects shown in question 1.
   - ☐ It is a 3D object.
   - ☐ It has one curved surface.
   - ☐ It has no bases.
   - ☐ It has no vertices.
   - ☐ It has no edges.
   - ☐ The distance from its centre to any point on its surface is always equal.

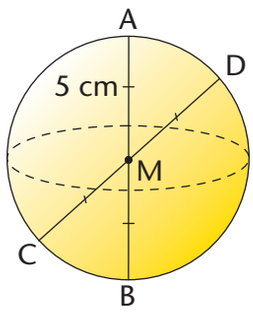
3. Complete the following information for a sphere:
   (a) Number and shape of faces: .....
   (b) Number of vertices: .....
   (c) Number of edges: .....

From your study of spheres in the above activity, you should have found the following:

A **sphere** is a round 3D object with only one curved surface and the distance from its centre to any point on its surface is always equal. It has no vertices or edges.



4. In the sphere alongside, write down the length of:



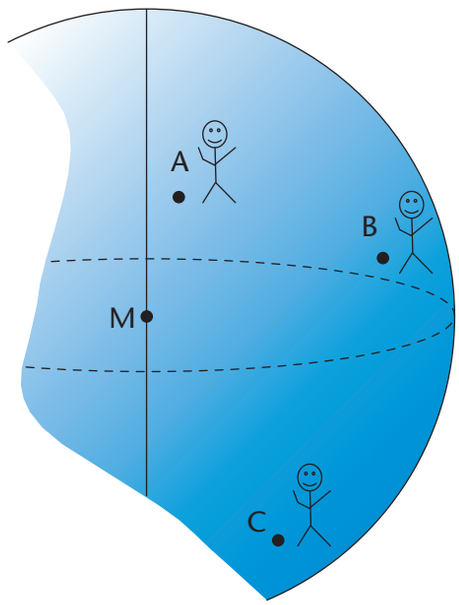
   (a) the radius: .....

   (b) the diameter: .....

   (c) MD: .....

   (d) CD: .....

5. The drawing alongside shows part of a sphere with a diameter of 100 km. Imagine that you are at point M, at the centre inside the sphere. People A, B and C are all at different places on the surface of the sphere.



   (a) Which of the people – A, B or C – is closest to you?

   .....

   .....

   .....

   (b) How far away is person C from you?

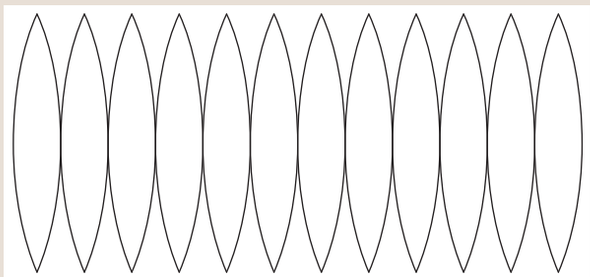
   .....

   .....

NET OF A SPHERE

It is impossible to make a perfect sphere (ball or globe) from a flat sheet of paper. Paper can curve in one direction, but cannot curve in two directions at the same time. So all spheres made from paper or card will be approximations. This is the best net we can make of a sphere.

Can you make your own paper model of a sphere?



WORKSHEET

1. Grade 9 learners were asked to represent a 3D object and give the class clues as to which polyhedron they represent. Name their objects:

   (a) Amy: I have 6 faces and they are all the same size. .....

   (b) John: I have 6 faces and 12 edges. I am not a cube. .....

   (c) Onke: I have 3 faces. I also have two edges. .....

   (d) Tessa: I have 8 edges and I have 5 vertices. .....

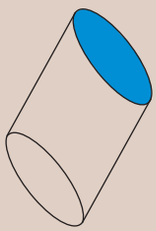
   (e) Mandlakazi: I have 6 edges and 4 vertices. .....

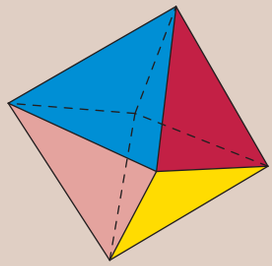
   (f) Chiquita: I have 8 faces and am a Platonic solid. .....

   (g) Seni: I do not have any edges. .....

   (h) Mpu: My faces are made only of regular pentagons. .....

2. Write down the required information about each object below.

   A. 

   B. 

	Object A	Object B
Name		
Number of faces		
Shape/s of faces		
Number of edges		
Number of vertices		
Does Euler's formula work?		
Is it a Platonic solid?		

3. (a) On a separate sheet of paper, construct a net of a cylinder with a diameter of 7 cm and a height of 10 cm.

   (b) Fold your net to make a model of the cylinder.