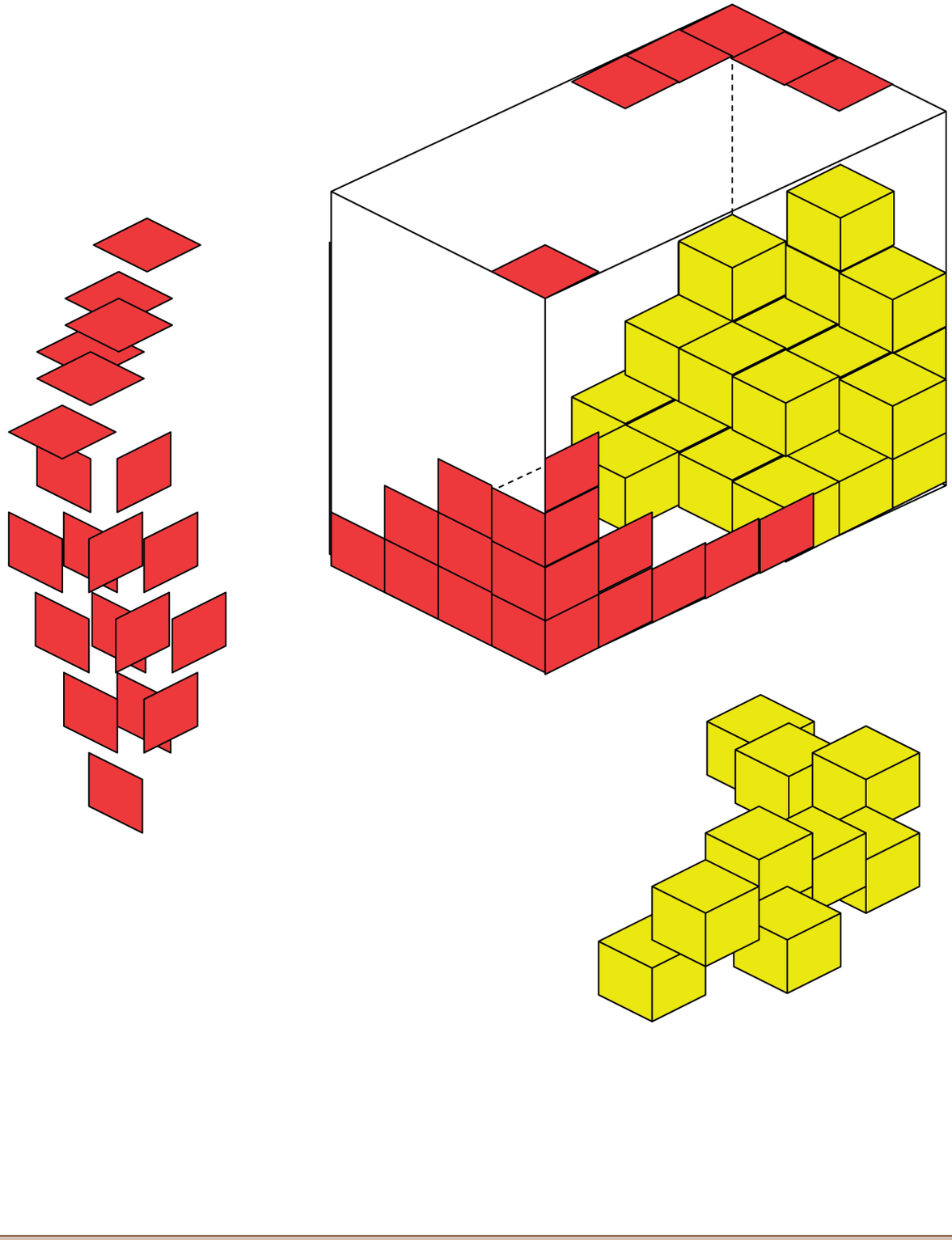
Chapter 5
Surface area, volume and capacity of 3D objects

By now you should know how to calculate the surface area and volume of cubes, rectangular prisms and triangular prisms. In this chapter, you will revise how to do this, practise converting between equivalent units used for volume, and revise the difference between volume and capacity. You will investigate how to calculate the surface area and volume of cylinders, and explore how the volumes of a prism and cylinder are affected when one or more of their dimensions is doubled.



5 Surface area, volume and capacity of 3D objects

5.1 Surface area

SURFACE AREA OF PRISMS

The **surface area** of an object is the total area of all of its faces added together. You learnt the following formula in previous grades:

- Surface area of a prism = Sum of the areas of all its faces

Calculate the surface area of the following objects to revise what you should already know.

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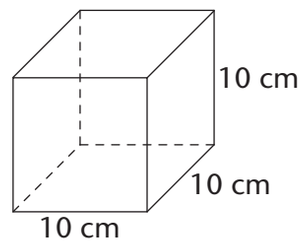

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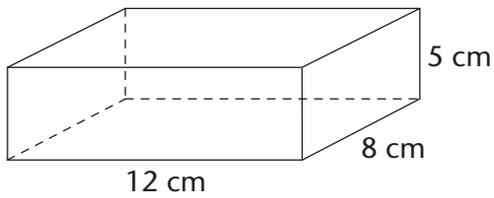

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[We use *SA* for surface area.]

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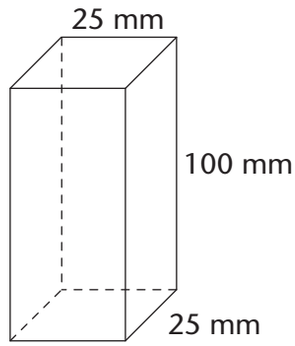

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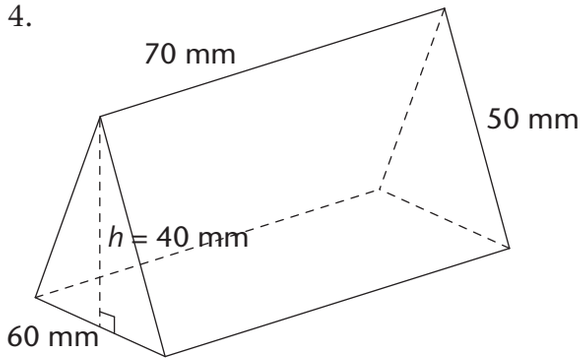

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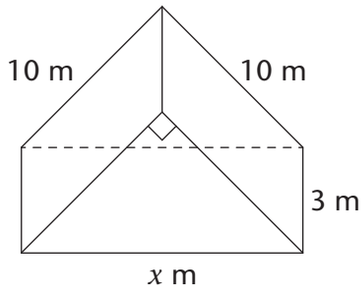

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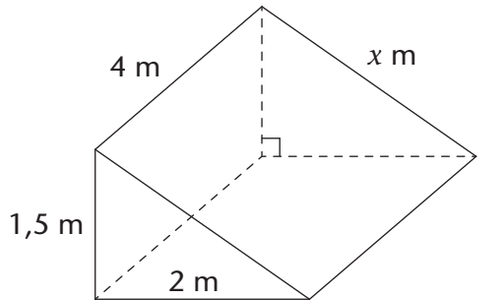

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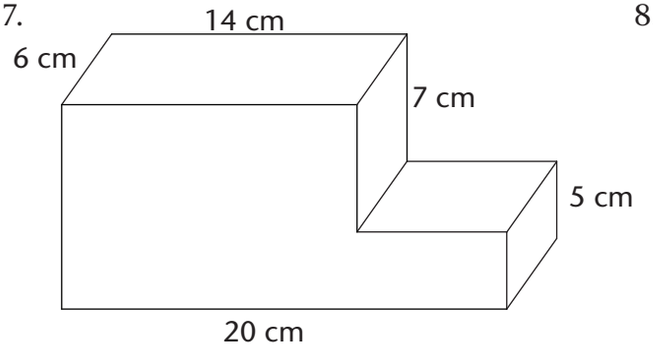

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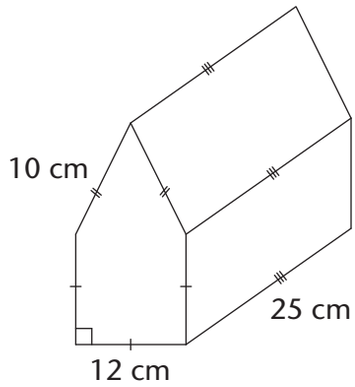
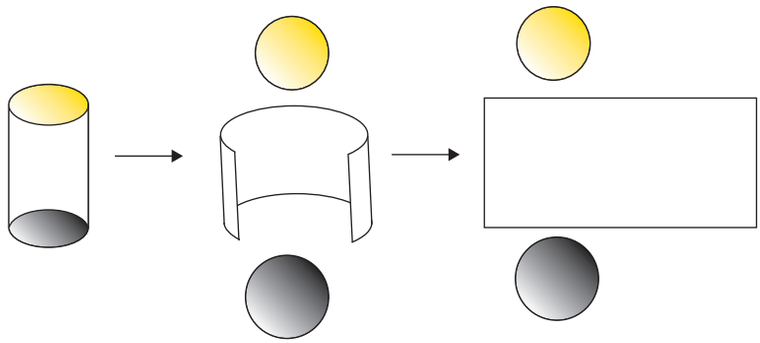


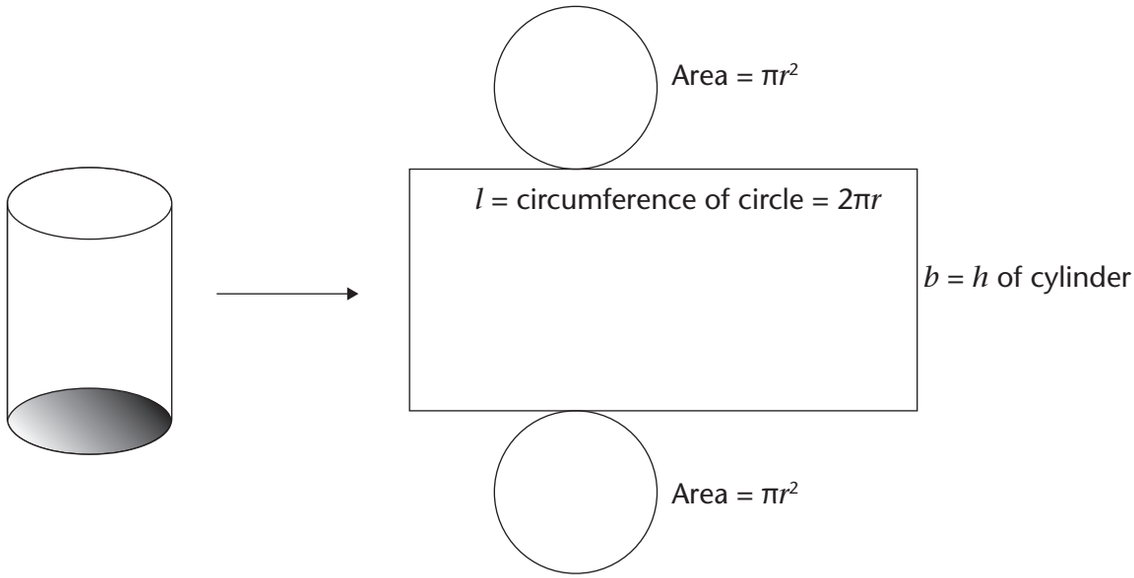
INVESTIGATING THE SURFACE AREA OF CYLINDERS

In order to calculate the surface area of a cylinder, you need to know what shape the surfaces of the cylinder are.

The surfaces of the top and base of a cylinder are made up of circles. The curved surface between the top and base of a cylinder can be unrolled to create a rectangle.



So the net of a cylinder looks like this:



Surface area of a cylinder = Area of all its surfaces

= Area of top + Area of base + Area of curved surface

= $\pi r^2 + \pi r^2 + (l \times b)$

= $2\pi r^2 + (2\pi r \times h)$

= $2\pi r(r + h)$

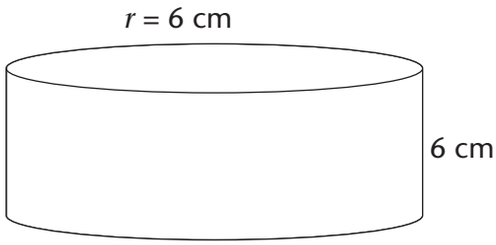
Can you explain why the length of the rectangle is equal to the circumference of the top or base of the cylinder?

CALCULATING THE SURFACE AREA OF CYLINDERS

From the formula on the previous page, you can see that we need only know the radius (r) and the height (h) of a cylinder in order to work out its surface area.

1. Calculate the surface areas of the following objects. Use $\pi = 3{,}14$ and round off all your answers to two decimal places.

A.



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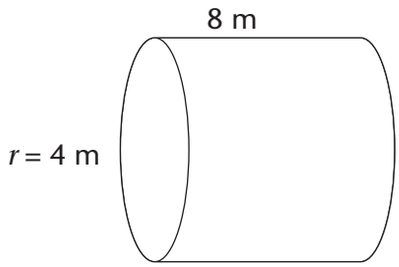

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2. Calculate the surface area of a cylinder if its height is 60 cm and the circumference of its base is 25,12 cm.

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3. Calculate the surface area of a cylinder if its height is 5 m and the circumference of its base is 12,56 m.

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4. The outside of a cylindrical structure at a factory must be painted. Its radius is 3,5 m and its height is 8 m. How many litres of paint must be bought if 1 litre covers 10 m^2? (The bottom of the structure will not be painted.)

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5.2 Volume

The **volume** of an object is the amount of space it occupies. We usually measure volume in cubic units, such as mm^3, cm^3 and m^3.

To convert between cubic units, remember:
1 cm^3 = 1 000 mm^3
1 m^3 = 1 000 000 cm^3

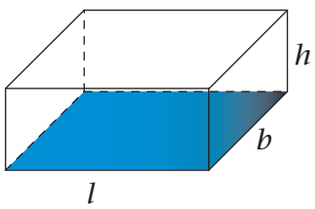
FORMULAS FOR VOLUME OF PRISMS

The general formula for the volume of a prism is:
Volume of a prism = Area of base × height.

In case of a triangular prism do not confuse the height of the base of the triangle (h_b) with the height of the prism (h_p).

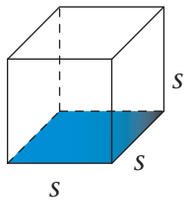
Therefore, the formulas to work out the volumes of the following prisms are:

Rectangular prism



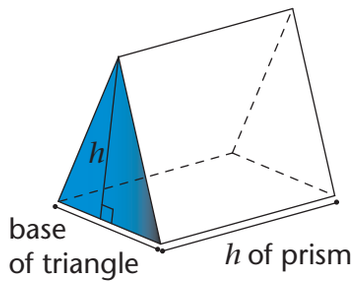
$V = (l \times b) \times h$

Cube



$V = (s \times s) \times s$
$= s^3$

Triangular prism

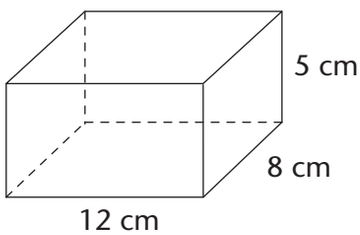


$V = (\frac{1}{2} \text{ base} \times h_b) \times h_p$

CALCULATING THE VOLUME OF PRISMS

1. Calculate the volumes of the following prisms.

A.



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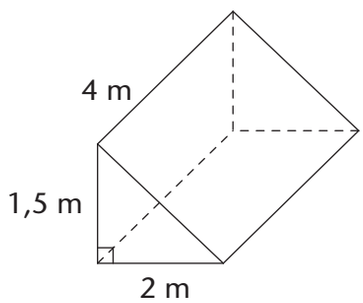

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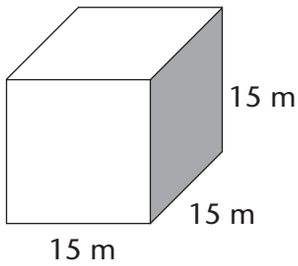

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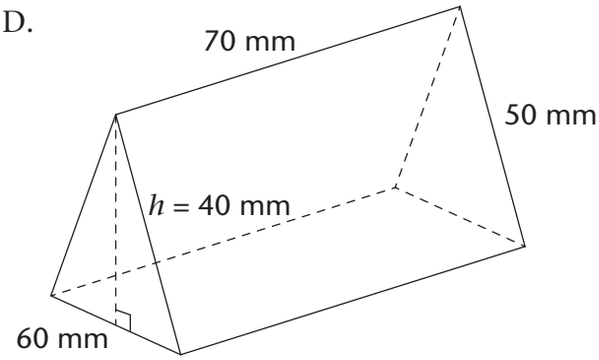

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2. (a) The area of the base of a rectangular prism is 32 m^2 and its height is 12 m. What is its volume?

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(b) The volume of a cube is 216 m^3. What is the length of one of its edges?

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3. Calculate the volume of the following objects.

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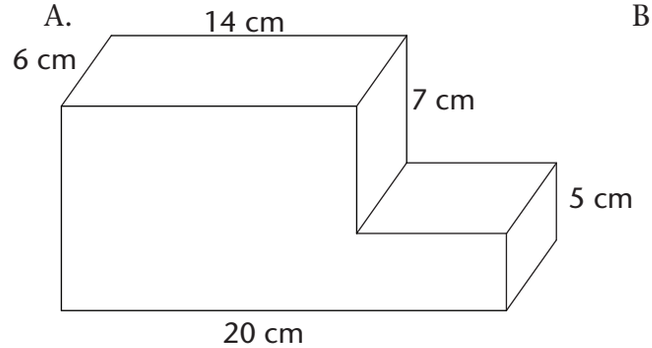

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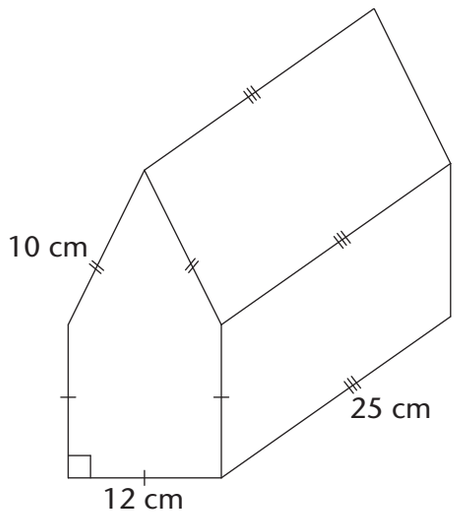

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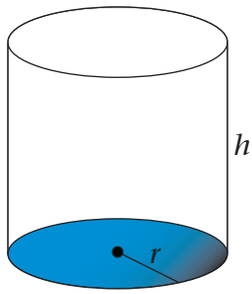
VOLUME OF CYLINDERS

You also calculate the volume of a cylinder by multiplying the area of the base by the height of the cylinder. The base of a cylinder is circular, therefore:



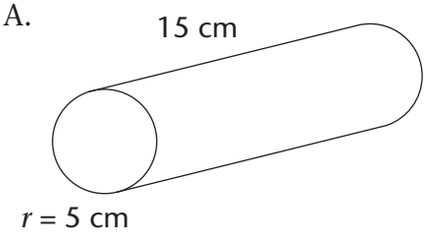
Area of circle = πr^2

Volume of a cylinder = Area of base × h

$= \pi r^2 \times h$

1. Calculate the volume of the following cylinders. Use $\pi = 3{,}14$ and round off all answers to two decimal places.

A.



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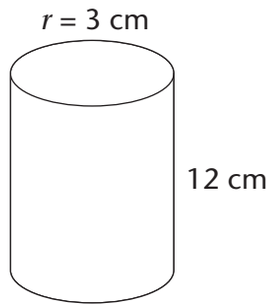

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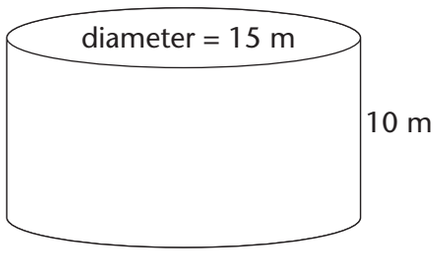

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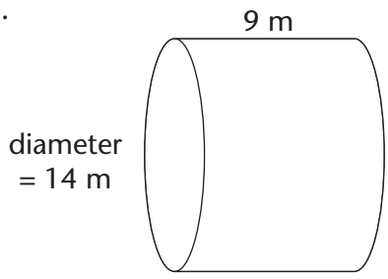

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2. Without using a calculator, calculate the volume of cylinders with the following measurements. Use $\pi = \frac{22}{7}$.

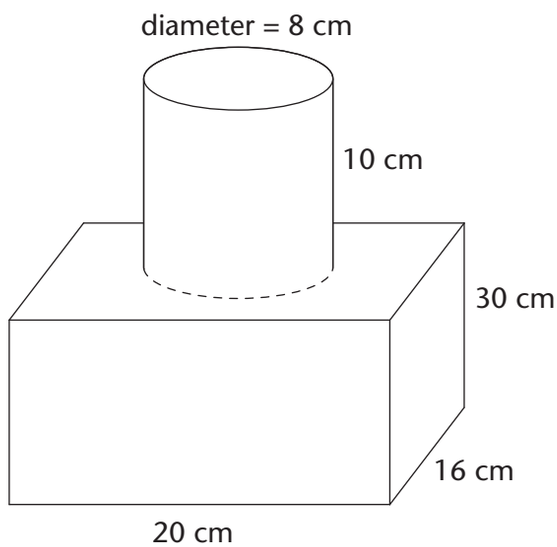
(a) $r = 14$ cm, $h = 20$ cm

(b) $r = 7$ cm, $h = 35$ cm

(c) diameter = 28 cm, $h = 50$ cm

(d) diameter = 7 cm, $h = 10$ cm

3. Calculate the volume of the following object. Use a calculator and round off all answers to two decimal places.



5.3 Capacity

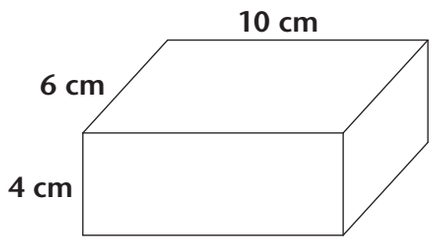
Remember that the **capacity** of an object is the amount of space *inside* the object. You can think of the capacity of an object as the amount of liquid that the object can hold.

The **volume** of an object is the amount of space that the object itself takes up.

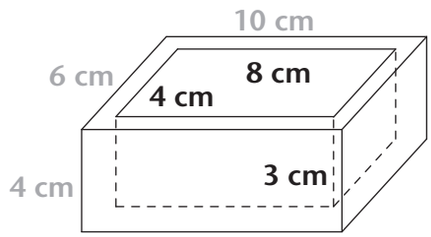
The volume of a solid block of wood is 10 cm × 6 cm × 4 cm = 240 cm^3.

The same block of wood is carved out to make a hollow container with inside measurements of 8 cm × 4 cm × 3 cm. (Its walls are 1 cm thick.) The amount of space inside the container must be calculated using the *inside* measurements. So the capacity of the container is 8 cm × 4 cm × 3 cm = 96 cm^3.

A. Solid block with outside measurements



B. Hollowed block with inside measurements



1. Write, in ml, the volume of water that would fill container B.

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Remember:
1 cm^3 = 1 ml
1 m^3 = 1 kl

2. If the walls and bottom of container B were 0,5 cm thick, what would its capacity be? Write the answer in ml.

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3. The inside measurements of a swimming pool are 9 m × 4 m × 2 m. What is the capacity of the pool in kl?

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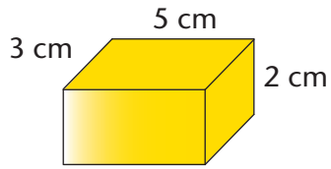
5.4 Doubling dimensions and the effect on volume

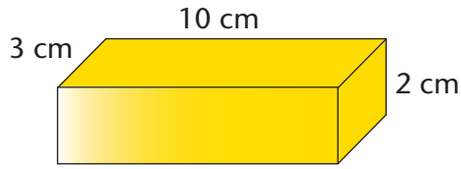
DOUBLING THE DIMENSIONS OF A PRISM

The first prism below measures 5 cm × 3 cm × 2 cm. The other diagrams show the prism with one or more of its dimensions doubled.

1. Work out the volume of each prism.



One dimension doubled



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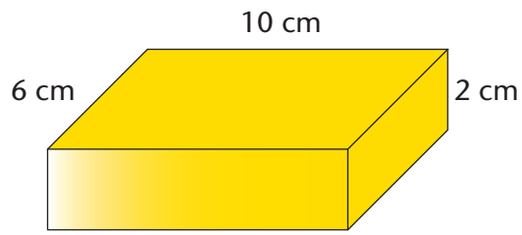
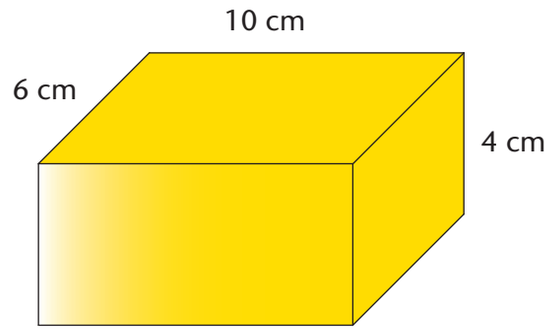
Two dimensions doubled



Three dimensions doubled



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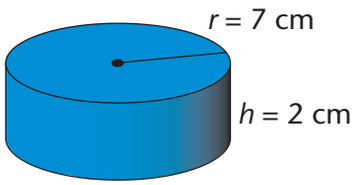
2. Complete the following:
   (a) When one dimension of a prism is doubled, the volume . . . . . .
   (b) When two dimensions of a prism are doubled, the volume increases by . . . . times.
   (c) When all three dimensions of a prism are doubled, the volume increases by . . . . times.

3. The volume of a prism is 80 cm^3. What is its volume if:
   (a) its length is doubled? .....
   (b) its length and breadth are doubled? .....
   (c) its length, breadth and height are doubled? .....

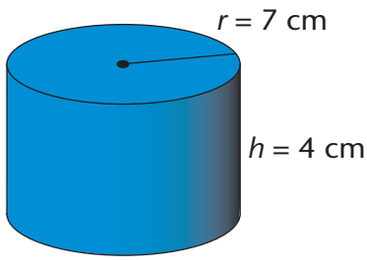
DOUBLING THE DIMENSIONS OF A CYLINDER

The first cylinder below has a radius of 7 cm and a height of 2 cm. The other diagrams show the cylinder with one or more of its dimensions doubled.

1. Work out the volume of each cylinder.



Only height doubled



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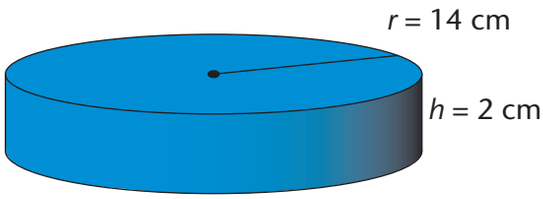
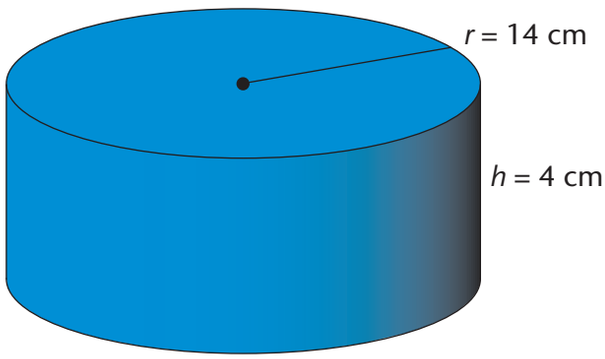
Only radius doubled



Radius and height doubled



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2. Complete the following:
   (a) When the height of a cylinder is doubled, the volume .....
   (b) When the radius of a cylinder is doubled, the volume increases by .... times.
   (c) When height and radius of cylinder are doubled, the volume increases by .... times.

3. The volume of a cylinder is 462 cm^3. What is its volume if:
   (a) its height is doubled? .....
   (b) its radius is doubled? .....
   (c) its height and radius are doubled? .....

4. (a) Study the following tables. Without using the formulas to calculate volume, complete the last column in each table. (Hint: Identify which dimensions are doubled each time, then work out the volume accordingly.)

Rectangular prism			
Length (l) in m	**Breadth (b) in m**	**Height (h) in m**	**Volume (V) in m^3**
4	2	1	
4	4	1	
8	2	1	
8	2	2	
8	4	2	

Cylinder		
Radius (r) in m	**Height (h) in m**	**Volume (V) in m^3**
3,5	4	
7	4	
3,5	8	
7	8	

   (b) Explain how you worked out the answers in the tables.

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