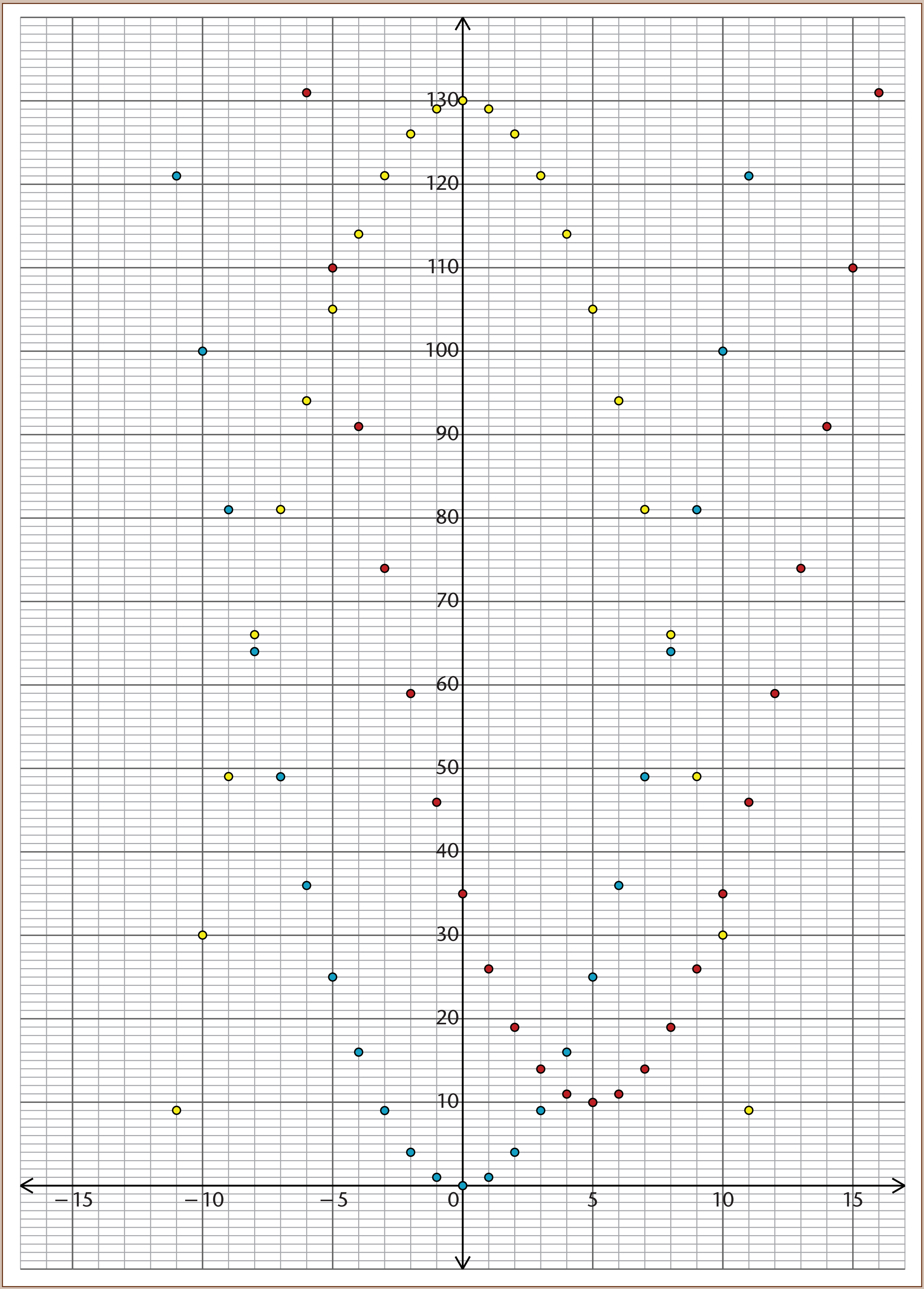
Chapter 4
Graphs

In this chapter you will learn more about making graphs to show how quantities change, and about interpreting graphs. Graphs can show how quantities increase and decrease, how rapidly they increase and decrease, and where they have maximum and minimum values. You will pay special attention to graphs of quantities which change at a constant rates. These graphs are straight lines.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4 Graphs

4.1 Global graphs

DISCRETE AND CONTINUOUS VARIABLES

Sibongile collects honey on his farm and puts it in large jars to sell. His business is doing so well that he can no longer do all the work himself. He needs to get some help. Sibongile knows that one person can normally fill 2 jars in 3 days. He sets up this table to help him determine how many full-time workers he should employ to fill different numbers of jars in a five-day week.

Jars per week	$3\frac{1}{3}$	$6\frac{2}{3}$	10	$13\frac{1}{3}$	$16\frac{2}{3}$	20	$23\frac{1}{3}$
Workers	1	2	3	4	5	6	7

1. (a) If Sibongile needs to produce 40 jars a week, how many workers does he need?

   ....

   (b) How many jars can 9 workers fill in a week? ....

   (c) How many workers does Sibongile need to produce 15 jars per week?

   ....

   (d) What are the two variables in the above situation?

   ....

In a situation like the above, one can have any number of jars, as well as fractions of a jar. One can have a whole number of jars (for example 4 jars) or a fractional quantity of jars (for example $6\frac{2}{3}$ or 4,45 jars). The other variable in the above situation, the number of full-time employees, is different. Only whole numbers of people are possible.

Quantities like the quantity of jars of honey, which can include any fraction, are sometimes called "continuous quantities" or "continuous variables". Quantities that can be counted, like a number of people or a number of motor cars or rivers or towns, are sometimes called "discrete quantities" or "discrete variables".

When a graph of a discrete variable is drawn, it does not normally make sense to join the dots with a line, but for some purposes it may be useful.

2. Can you use the second graph on the next page to find out how many workers are needed to fill 30 jars in a week, and how many to fill 40 jars? Check your answers by doing calculations.

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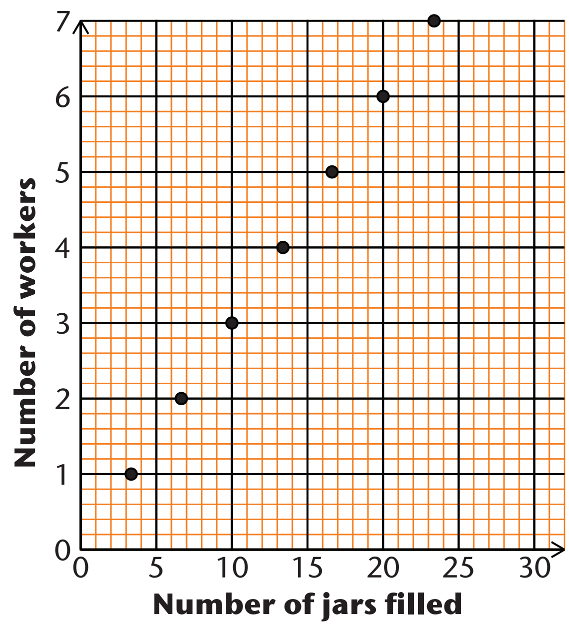
Here is a graph of the information in Sibongile's table.



Here is another graph of the same information.

3. In what way are these two graphs different?

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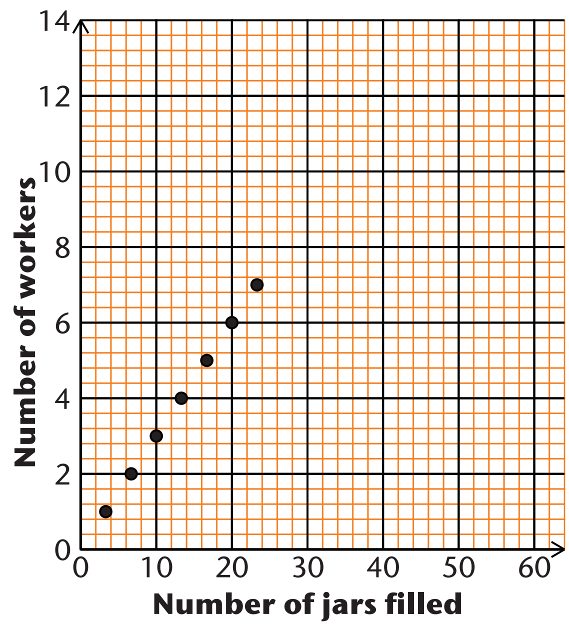

4. In each case say whether the variables are “discrete” or “continuous”.
   (a) You order pizzas for a class party and you need 1 pizza for every 3 learners.

   ....

   (b) Your height measured at different stages as you grew up.

   ....

   (c) The speed the car is travelling as you drive to town.

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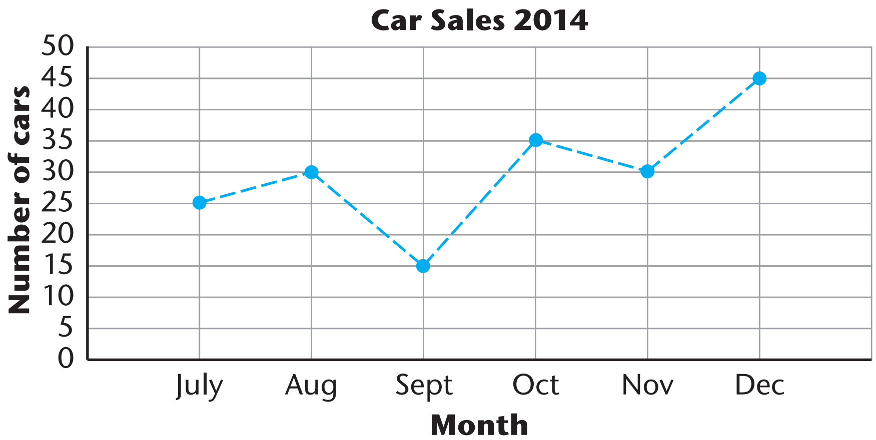
5. The line graph shows the number of cars that a company sold between July and December of 2014.



   (a) Is the data shown in the graph discrete or continuous? Explain your answer

   ....

   (b) How many cars were sold in August? ....

   (c) During which months were the maximum and minimum number of cars sold?

   ....

   (d) How many more cars were sold in November than in July? ....

   (e) During which months did the car sales decrease?

   ....

   (f) Would you say that the car sales generally improved over the 6 months? Explain your answer.

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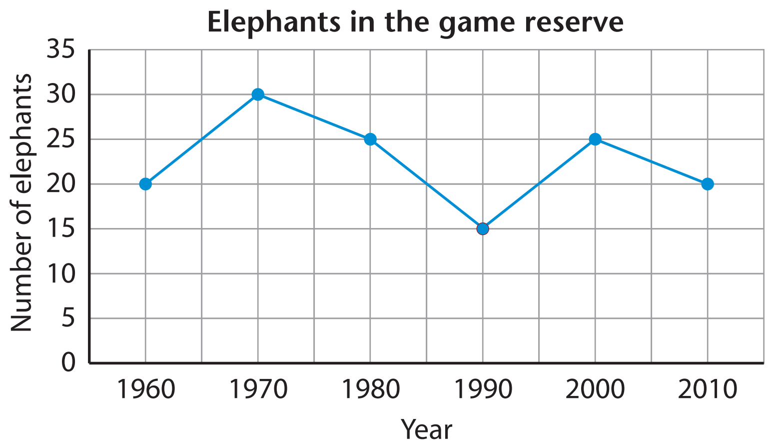
6. The graph below shows the population of elephants at a game reserve in South Africa between 1960 and 2010. Study the graph and answer the questions that follow.



(a) Did the elephant population increase or decrease between 1970 and 1990?

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(b) Between which years did the elephant population increase?

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(c) In which year were there the most elephants on the game farm?

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(d) Is the data in this graph discrete or continuous?

....

(e) How many elephants do you think there were on the game reserve in 1995?

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(f) The following data shows the number of elephants at a different game reserve. Plot this information on the grid above.

Year	1960	1970	1980	1990	2000	2010
Elephants	30	25	20	15	20	35

(g) Would you say that the second game reserve had more elephants than the first game reserve between 1960 and 2010? Explain your answer.

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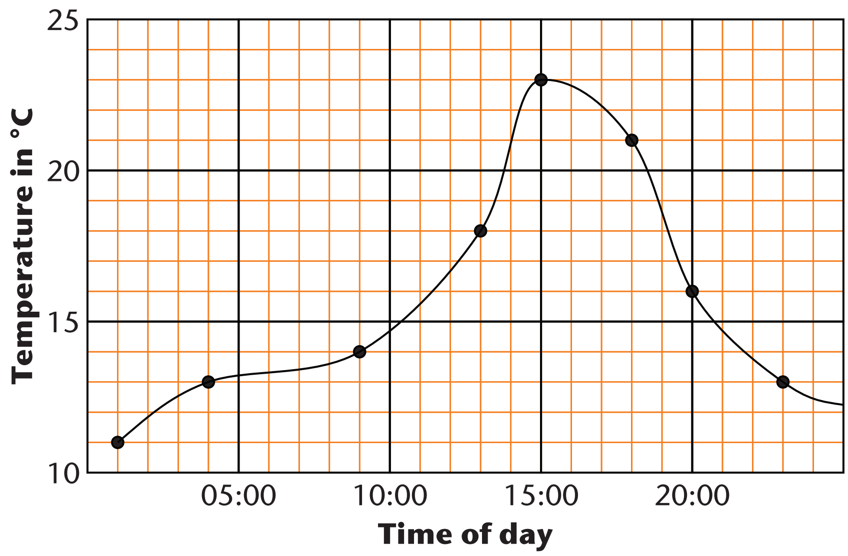
SHOWING INCREASE AND DECREASE ON GRAPHS

The graph below shows the temperature over a 24-hour period in a town in the Free State. The graph was drawn by connecting the points that show actual temperature readings.



1. (a) Do you think the above temperatures were recorded on a summer day or a winter day?

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(b) At what time of the day was the highest temperature recorded, and what was this temperature?

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(c) During what part of the day did the temperature rise, and during what part did the temperature drop?

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(d) During what part of the period when the temperature was rising did it rise most rapidly?

....

(e) During what part of the day did the temperature drop most rapidly?

....

....

....

2. Here are descriptions of the temperature changes on five different days.

Day A: It is already warm early in the morning. The temperature does not change much during the day but late in the afternoon a breeze causes the temperature to drop quite sharply.

Day B: It is very cold early in the morning but it gets quite hot soon after the sun gets up. By midday a cold wind comes up and the temperature drops till late in the afternoon. The wind then stops and it gets warmer again into the evening.

Day C: It is warm in the early morning and the temperature remains about the same till midday. then the temperature drops slowly during the afternoon.

Day D: It is cold in the early morning and it remains cold for the whole day, except for a short time after lunch when the sun comes out for a while.

Day E: It is warm early in the morning, but the temperature drops sharply soon after sunrise and remains low until mid-afternoon, when it slowly warms up a little.

The shapes of some temperature graphs for 24-hour periods, starting early in the morning, are given below. Below each graph, write which of the above days is possibly represented by the graph.

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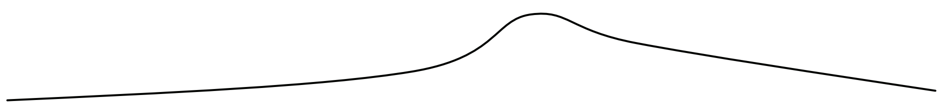

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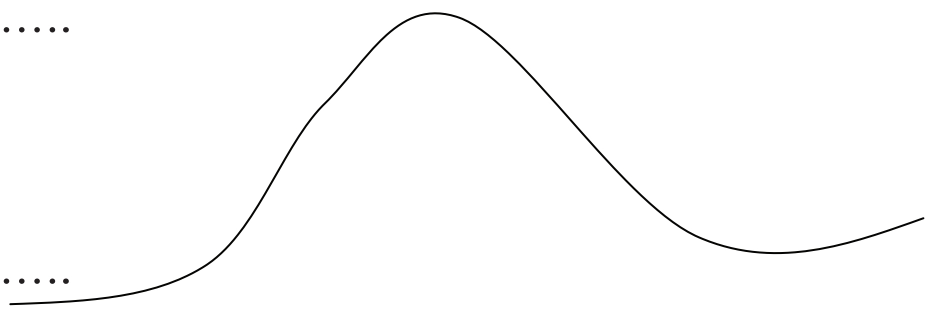

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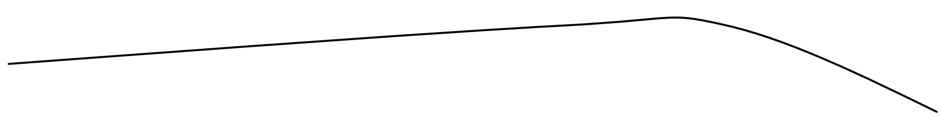

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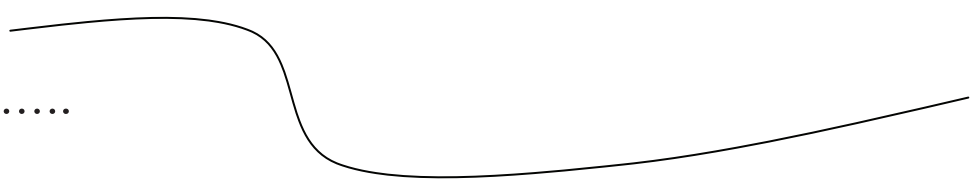

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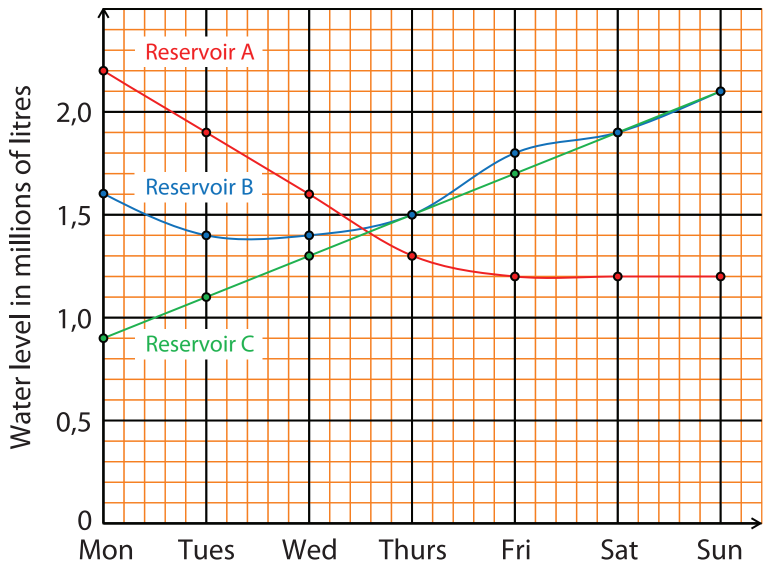
Water is supplied to a township from three reservoirs. The amount of water in each reservoir is measured each day at 08:00 am. The water level in reservoir A is represented in red on the graph below, and the water levels in reservoirs B and C are represented in blue and green respectively.



The daily water levels in the three reservoirs, in millions of litres, are also given in the table below.

	Mon.	Tue.	Wed.	Thu.	Fri.	Sat.	Sun.
Reservoir A	2,2	1,9	1,6	1,3	1,2	1,2	1,2
Reservoir B	1,6	1,4	1,4	1,5	1,8	1,9	2,1
Reservoir C	0,9	1,1	1,3	1,5	1,7	1,9	2,1

3. You may use the graph or the table, or both, to find the answers to the questions below.
   (a) On which days does the water level in reservoir B increase from one day to the next?

   ....

   (b) On which of these days does the water level in reservoir B increase most, and by how much does it increase from that day to the next?

   ....

   (c) By how much does the water level in reservoir B change each day?

   ....

   (d) By how much does the water level in reservoir C change each day?

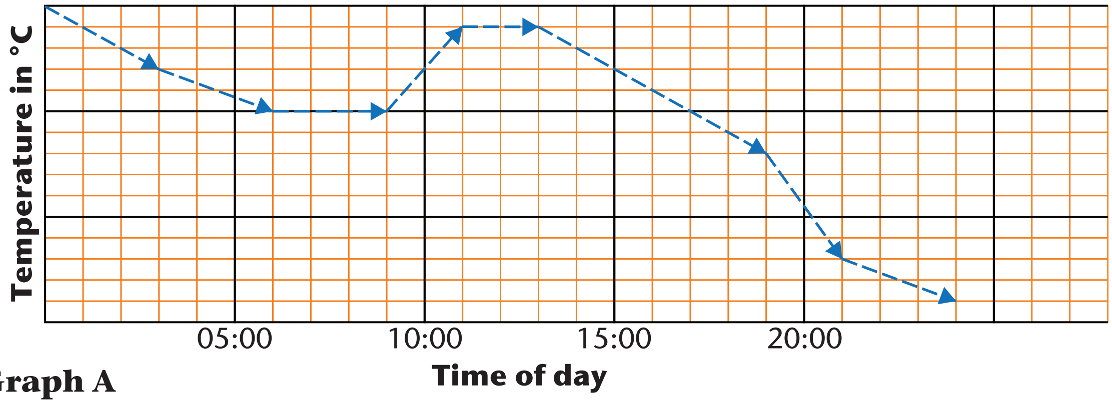
   ....

   (e) Describe the water level situation from Friday to Sunday, in reservoir A.

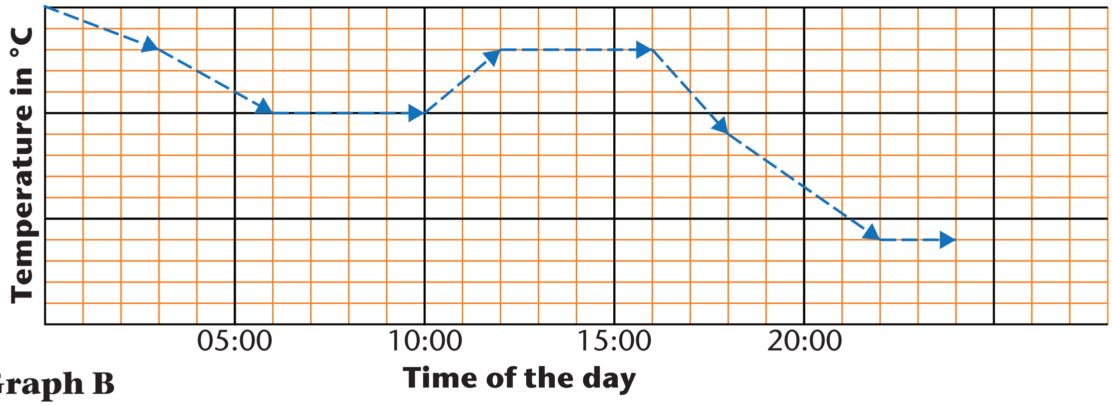
   ....

4. During a certain day, these changes occur in the temperature at a certain place.
   Between 00:00 and 03:00, the temperature drops by 2°C.
   Between 03:00 and 06:00, the temperature drops by 3°C.
   Between 06:00 and 10:00, the temperature remains constant.
   Between 10:00 and 12:00, the temperature rises by 3°C.
   Between 12:00 and 16:00, the temperature remains constant.
   Between 16:00 and 18:00, the temperature drops by 4°C.
   Between 18:00 and 22:00, the temperature drops by 5°C.
   Between 22:00 and 24:00, the temperature remains constant.

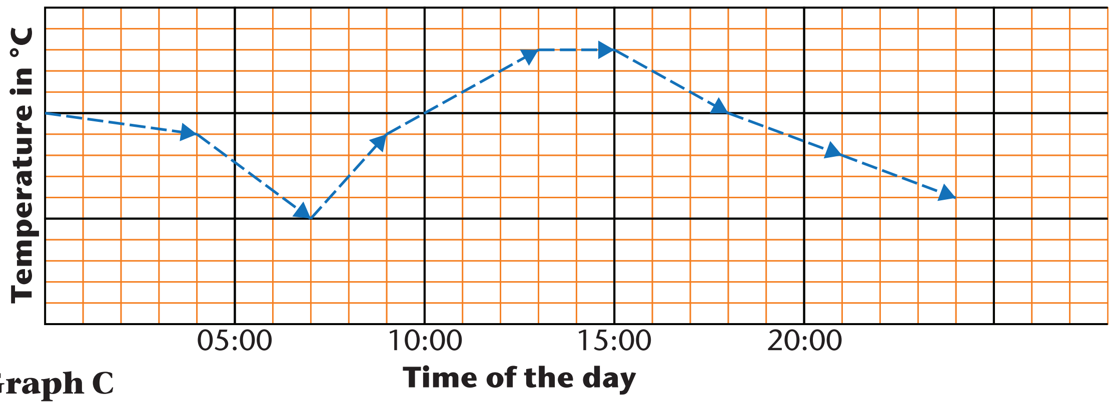
   Which of the graphs below show the above temperature changes? ....



Graph A



Graph B



Graph C

5. Write a verbal description, like in question 4, of the temperature changes shown in graph A in question 4.

Between .... and ...., the temperature ....

Between .... and ...., the temperature ....

Between .... and ...., the temperature ....

Between .... and ...., the temperature ....

Between .... and ...., the temperature ....

Between .... and ...., the temperature ....

Between .... and ...., the temperature ....

Between .... and ...., the temperature ....

6. Write a verbal description, like in question 4, of the temperature changes shown in graph C in question 4.

Between .... and ...., the temperature ....

Between .... and ...., the temperature ....

Between .... and ...., the temperature ....

Between .... and ...., the temperature ....

Between .... and ...., the temperature ....

Between .... and ...., the temperature ....

Between .... and ...., the temperature ....

Between .... and ...., the temperature ....

7. Look at graph A in question 4.
   (a) By how much does the temperature drop from 13:00 to 19:00? ....
   (b) By how much does the temperature drop from 19:00 to 21:00? ....
   (c) When does the temperature drop most rapidly, from 13:00 to 19:00 or from 19:00 to 21:00? Explain your answer.

....

....

....

8. Look at graph C in question 4.
   (a) By how much does the temperature increase from 07:00 to 09:00? ....
   (b) By how much does the temperature increase from 09:00 to 13:00? ....
   (c) When does the temperature increase more rapidly, from 07:00 to 09:00 or from 09:00 to 13:00? Explain your answer.

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9. Look at graph B in question 4.
   (a) By how much does the temperature drop from 16:00 to 18:00? ....
   (b) By how much does the temperature drop from 18:00 to 22:00? ....
   (c) When does the temperature drop more rapidly, from 16:00 to 18:00 or from 18:00 to 22:00? Explain your answer.

   ....

   ....

4.2 Change at different rates

The water levels in kilolitres (kl) in different water storage tanks over a period of 30 hours are represented on the graphs below and on the next page.

1 kilolitre = 1 000 litre

1. (a) In which tanks does the water level rise during the 30-hour period? ....
   (b) In which tanks does the water level drop during the 30-hour period? ....
2. How much water is there at the start of the 30-hour period, in each of the tanks?

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   ....

3. (a) Which tank is losing water most rapidly? Explain your answer.

   ....

   (b) Which tank is gaining water most slowly? Explain your answer.

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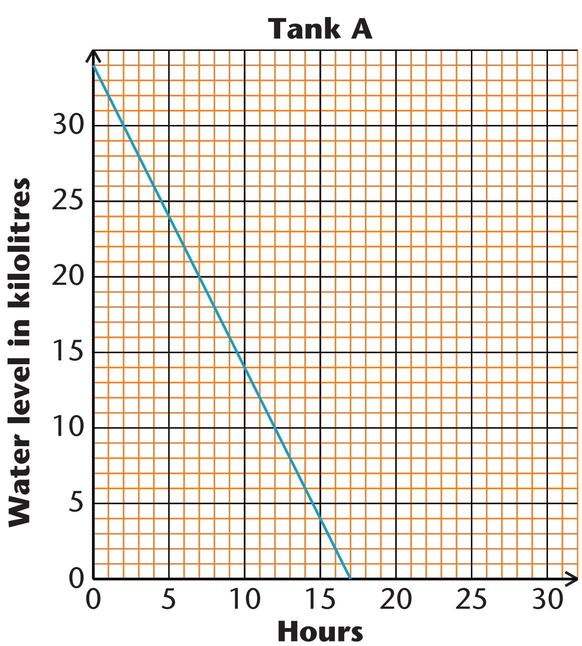
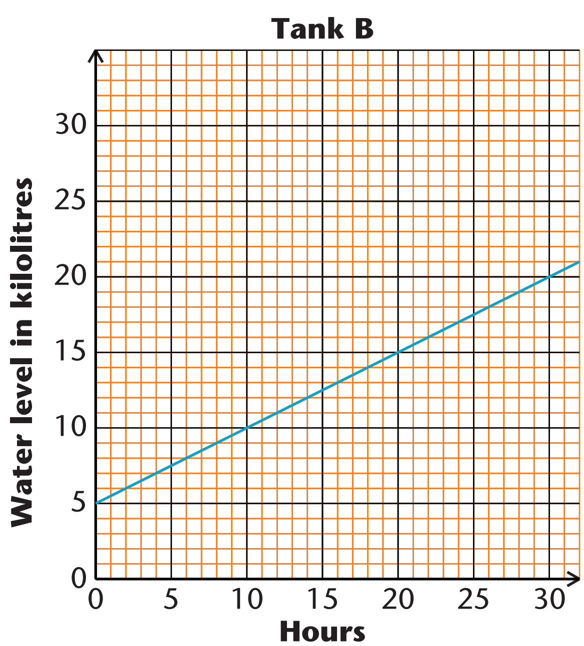
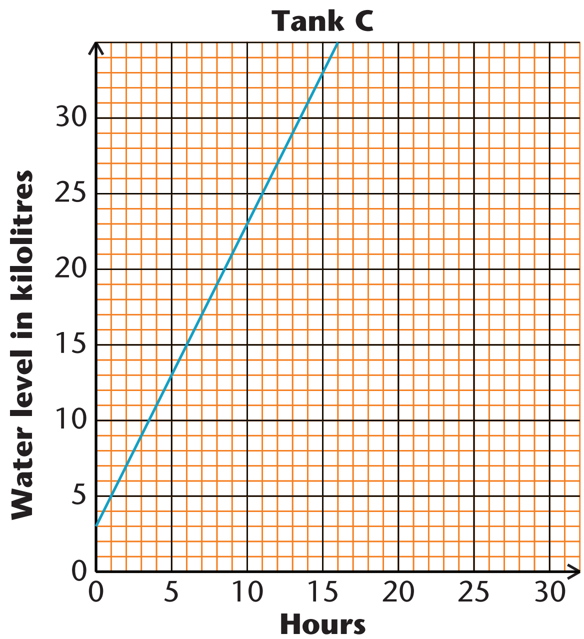
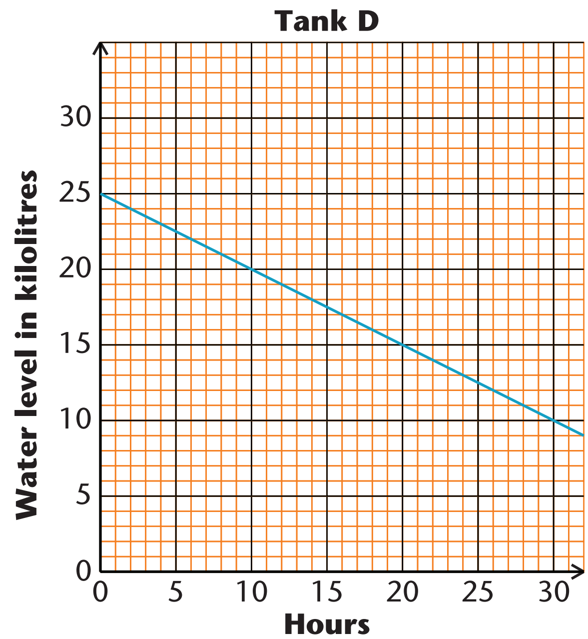
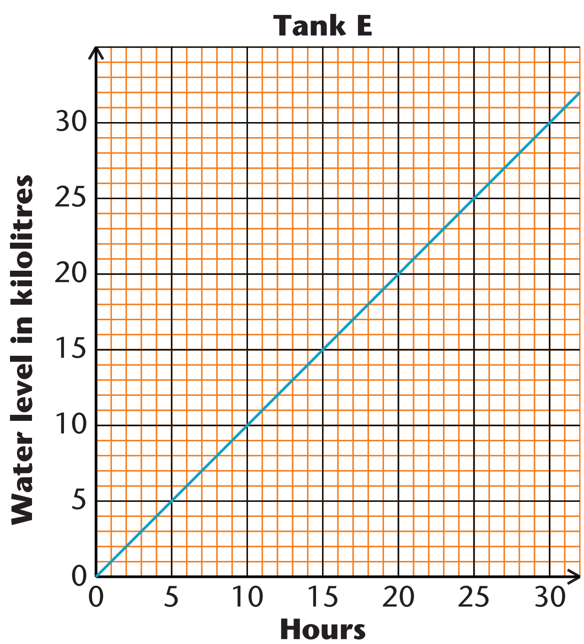
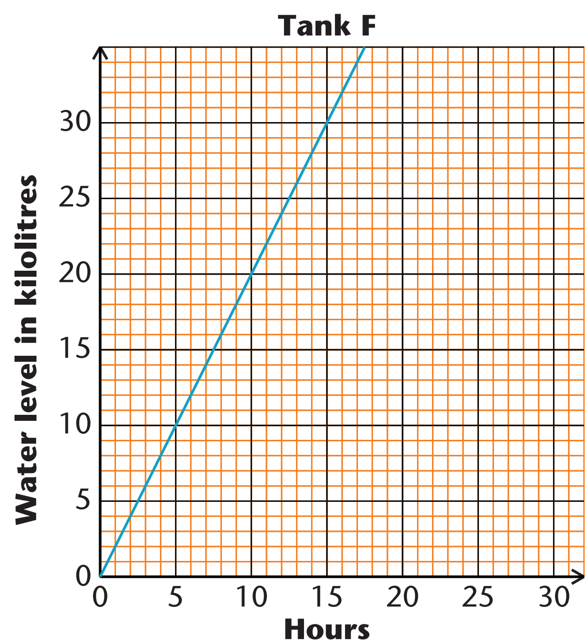












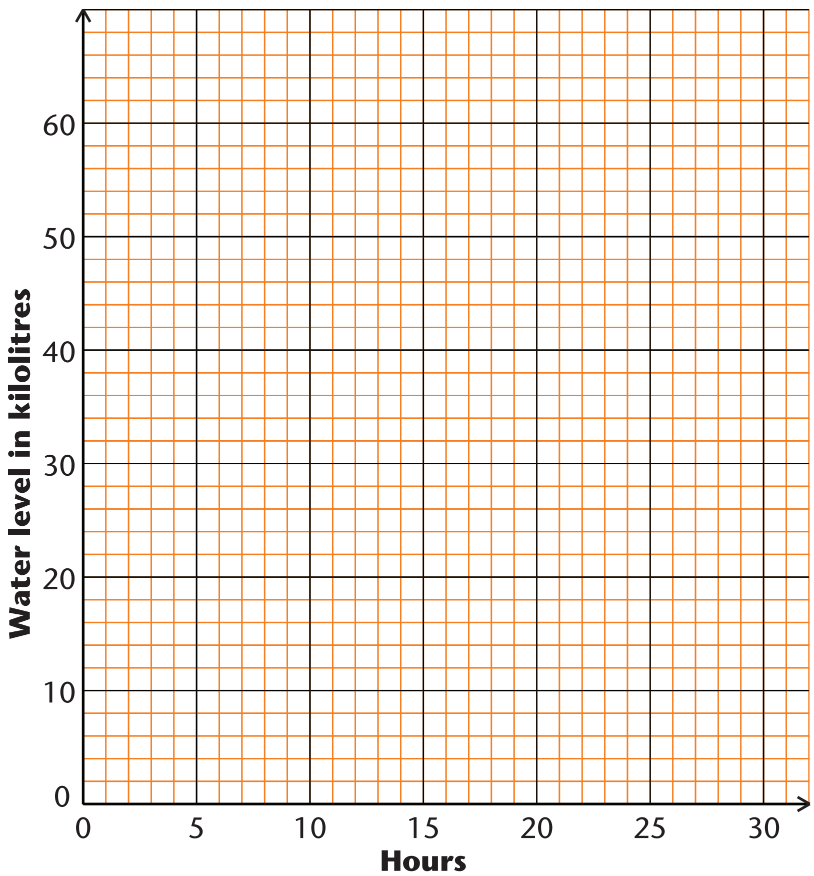
3. Complete the table. Use negative numbers for decreases.

	Change over each hour	Change over any period of 5 hours
Tank A		
Tank B		
Tank C		
Tank D		
Tank E		
Tank F		

If a constant stream of water is pumped into a tank so that the water level is increased by 3 kilolitre in each hour, we say:

Water is pumped into the tank at a **constant rate** of **3 kilolitres per hour**.

4. (a) Tank G contains 12 kilolitres at the beginning of a 30-hour period. Water is then pumped into it at a constant rate of 3 kilolitres per hour. Draw a dotted line graph to show the water level in Tank G on the graph sheet below.
   (b) Tank H also contains 12 kilolitres at the beginning of a 30-hour period. Water is then pumped into at a constant rate of 1,5 kilolitres per hour. Draw a solid line graph to show the water level in Tank H on the graph sheet below.



5. Complete the table for tanks G and H over the 30-hour period.

Hours	0	5	10	15	20	25	30
Kilolitres in tank G	12						
Kilolitres in tank H	12						

4.3 Draw graphs from tables of ordered pairs

A "coordinate" graph shows the relationship between two variables, the dependent and independent variable in a function. The value of the dependent variable depends on the value given to the independent variable, hence its name. Sometimes there is no pattern to the relationship between the two variables and sometimes there is. In Grade 9 we will focus on graphs where there is a pattern to the relationship. Specifically, we will focus on graphs of linear functions. The graph of a linear function is a straight line.

GRAPHS OF FUNCTIONS WITH CONSTANT DIFFERENCES

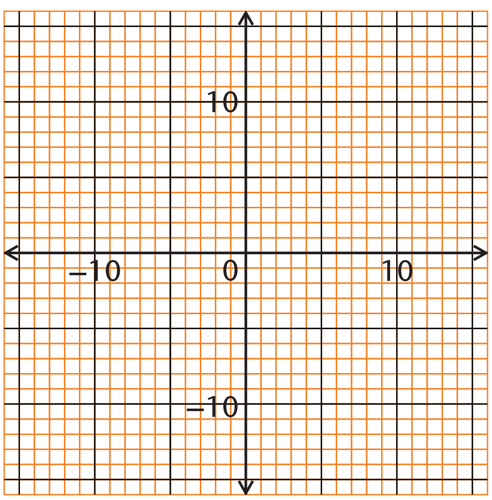
1. Complete the table.

x	0	1	2	3	4	5	6	7	8	9
Function A	8	$8\frac{1}{2}$	9	$9\frac{1}{2}$						
Function B	4	5	6	7	8	9	10	11	12	13
Function C	0	$1\frac{1}{2}$	3	$4\frac{1}{2}$						
Function D	–4	–2	0	2						

2. Represent each of the functions in question 1 with a graph by plotting the points on the grids below. You may join the points in each case and write down the constant difference between the function values.

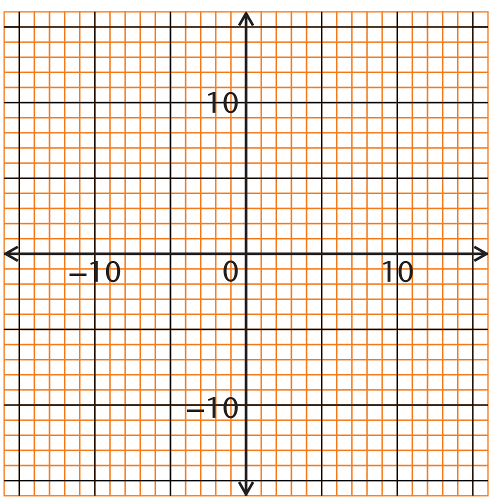
Function A

Constant difference = ...



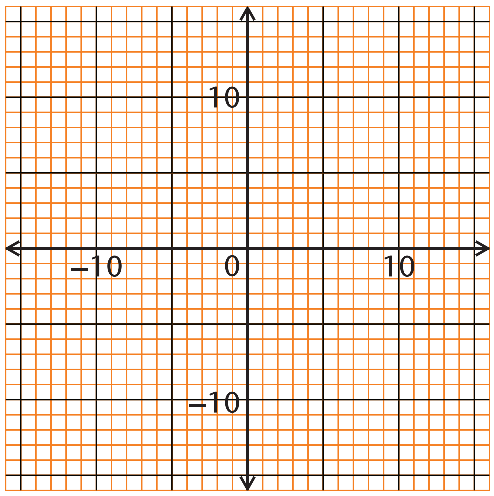
Function B

Constant difference = ...



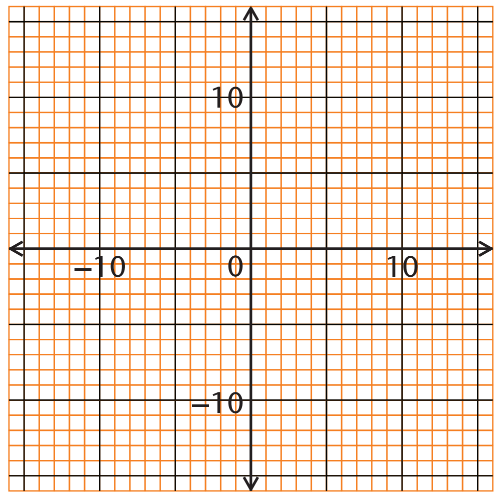
Function C

Constant difference = ...



Function D

Constant difference = ...



3. Some of the graphs you have drawn "go upwards" (or downwards) quickly, like a steep hill or mountain; others "go up" (or down) slowly.
   (a) Is there a link between the constant difference and the "steepness" of the graph?

   . . . .

   (b) Try to explain why this is the case.

   . . . .

   . . . .

4. (a) Complete the following tables.

x	1	2	3	4	5	6	7	8	9	10
$2x + 3$										
$5x + 4$										
$3x + 3$										

   (b) Determine the difference between consecutive terms in each of the above three number sequences. What do you notice about this difference?

   . . . .

   . . . .

   (c) What difference between consecutive terms would you expect in the output numbers for $4x + 5$, if the input numbers are the natural numbers 1; 2; 3; . . . . . ?

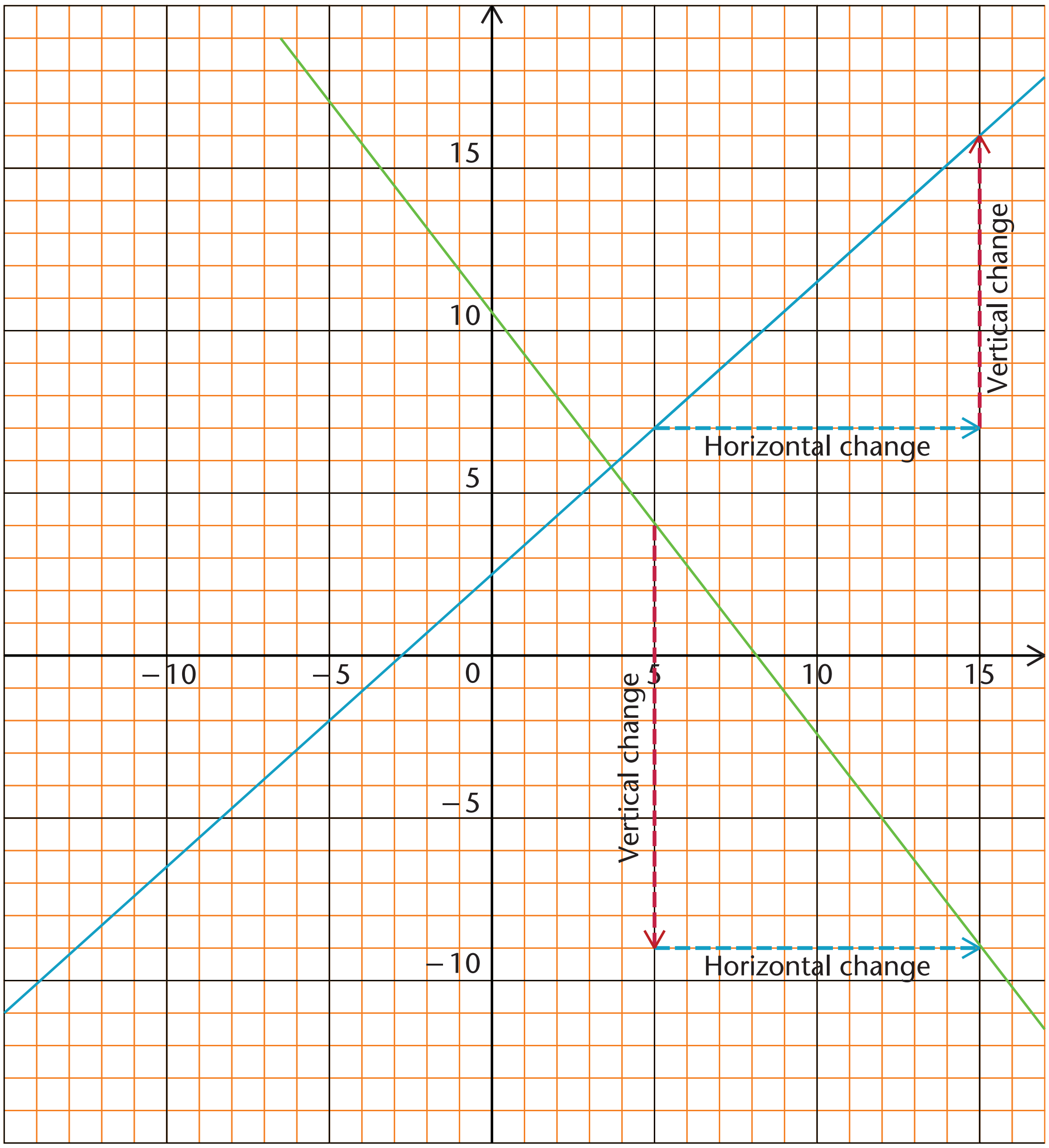
   . . . .

4.4 Gradient

The “steepness” or **slope** of a line can be indicated by a number, as described below. This number is called the **gradient** of the line.

The gradient is the vertical change divided by the horizontal change as you move from left to right on the line.

$$\text{Gradient} = \frac{\text{vertical change}}{\text{horizontal change}}$$



The gradient of the blue line above is $\frac{9}{10} = 0{,}9$.

The gradient of the green line is $\frac{-13}{10} = -1{,}3$.

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Note that the horizontal change is always taken to be positive (moving to the right), but the vertical change can be positive (if it is upwards) or negative (if it is downwards).

1. A certain line passes through the points (2; 3) and (8; 15). A straight line is drawn through the two points.
   (a) Try to think of a way in which you can work out the gradient of the line that passes through the two points.

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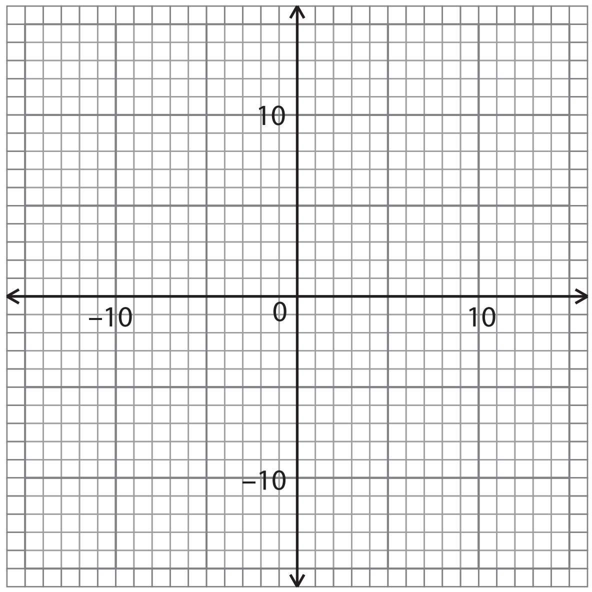
   ....

   ....

   ....



   (b) Plot the two points on the graph sheet on the right.
   (c) What horizontal change and vertical change is needed to move from the point (2; 3) to the point (8; 15)? You may draw arrows on your graph to help you to think clearly about this.

   ....

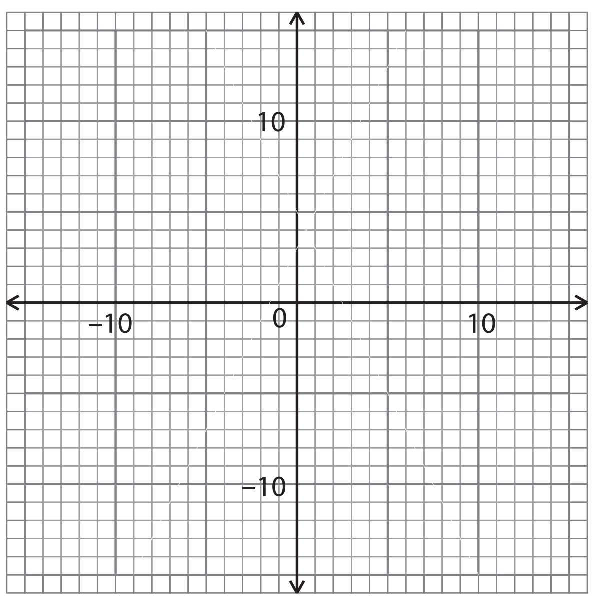
   ....

   (d) Work out the gradient of the line that passes through the two points.

   ....

2. Complete the table and plot graphs of $y = 2x + 3$ and $y = -2x + 5$ on the given graph sheet.

x	–3	1	3	5
$2x + 3$				
$-2x + 5$				



3. Work out the gradients of the graphs of $y = 2x + 3$ and $y = -2x + 5$. You may use the coordinates of any of the points you have plotted.

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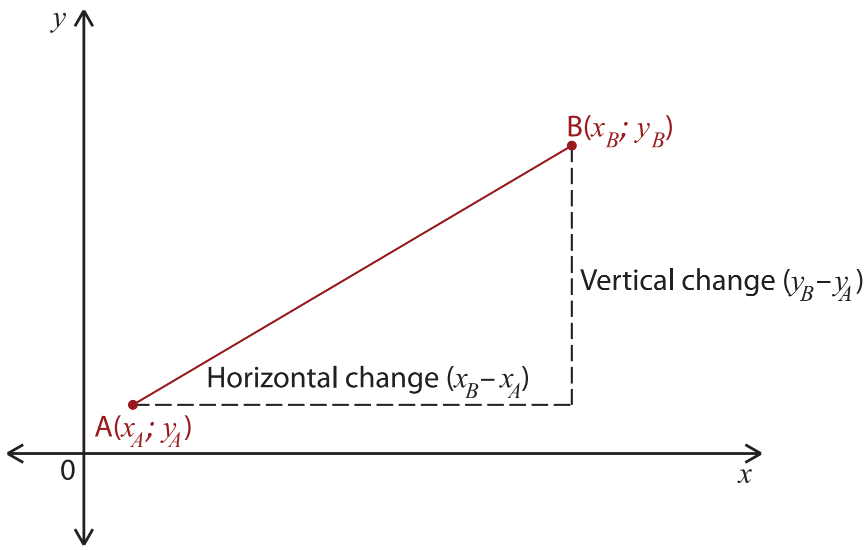
Suppose the coordinates of point A are $(x_A; y_A)$ and the coordinates of B are $(x_B; y_B)$.



The gradient of line AB is: $m_{AB} = \dfrac{\textit{Vertical change}}{\textit{Horizontal change}} = \dfrac{y_A - y_B}{x_A - x_B}$

In summary:
If you have two points A $(x_A; y_A)$ and B $(x_B; y_B)$ then the formula for the gradient is: $m = \dfrac{y_A - y_B}{x_A - x_B}$

Examples of finding the gradient between two points:

Calculate the gradient of the line that goes through the points:

(a) A(2; 5) and B(4; 1)

$$m = \frac{y_A - y_B}{x_A - x_B} = \frac{5-1}{2-4} = \frac{4}{-2} = -2$$

(b) C(2; 2) and D(–6; 0)

$$m = \frac{y_C - y_D}{x_C - x_D} = \frac{2-0}{2-(-6)} = \frac{2}{8} = \frac{1}{4}$$

(c) A(0; –1) and B(1; 1)

$$m = \frac{y_A - y_B}{x_A - x_B} = \frac{-1-1}{0-1} = \frac{-2}{-1} = 2$$

The gradient of a straight line is the same everywhere, so it doesn't matter which 2 points you use to determine the gradient.

DETERMINE THE GRADIENT

Do the following task in your exercise book.

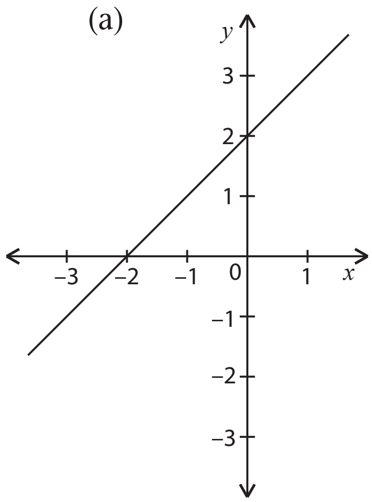
1. Determine the gradient of the lines that go through the following points:
   (a) A(2; 10) and B(6; 12) (b) C(1; 3) and D(–2; –3) (c) E(0; 3) and F(4; –1)
   (d) G(5; 2), H(4; 4) and I(2; 8)

   ....

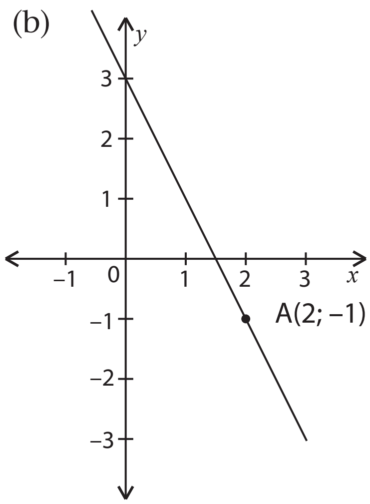
   ....

2. Determine the gradient of the following lines:

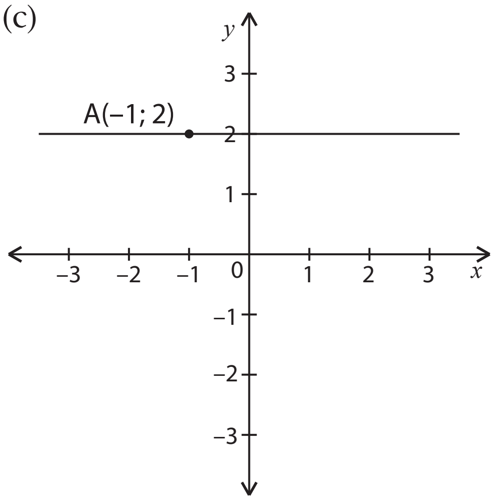
(a)



(b)



(c)



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4.5 Finding the formula for a graph

TABLES AND FORMULAS

1. Each table on the next page shows values for a relationship represented by one of these rules:

$y = -2x + 3$	$y = 2x - 5$	$y = -3x + 5$	$y = -3(x + 2)$
$y = 3x + 2$	$y = 5(x - 2)$	$y = 2x + 3$	$y = 2x + 5$
$y = -3x + 6$	$y = 5x + 10$	$y = 5x - 10$	$y = -x + 3$

(a) Complete the tables below by extending the patterns in the output values.

A.

x	0	1	2	3	4	5	6	7
y	2	5	8					

B.

x	0	1	2	3	4	5	6	7
y	3	1	–1	–3				

C.

x	0	1	2	3	4	5	6	7
y	–10	–5	0	5				

D.

x	0	1	2	3	4	5	6	7
y	–5	–3	–1					

E.

x	0	1	2	3	4	5	6	7
y	6	3	0					

F.

x	0	1	2	3	4	5	6	7
y	3	2	1	0				

G.

x	0	1	2	3	4	5	6	7
y	3	5	7					

(b) For each table, describe what you did to produce more output values. Also write down the rule (formula) that corresponds to the table.

....

....

....

....

....

....

....

You may have noticed that the equations of straight lines look similar.

The equation of a straight line is: $y = mx + c$

m tells us the **gradient** of the line.

c tells us where the line crosses the y-axis.

This is called the ***y*-intercept** and it has the coordinates (0; c).

Gradient

Gradient means the steepness or slope of the line.

Intercept

The point where a line crosses one of the axes.

- the line $y = 3x + 4$ has a **gradient of 3** and the y-intercept is **(0; 4)**.
- the equation of a line with a **gradient of −2** and y-intercept of **(0;10)** is $y = -2x + 10$.
- the line $y = 2x$ has a **gradient of 2** and the y-intercept is **(0; 0)**.
- the line $y = 5$ has a **gradient of 0** and the y-intercept is **(0; 5)**.
- What is the gradient and y-intercept of the line $2y = 6x + 10$?

If you said $m = 6$ and $c = 10$ you would be wrong. The equation is not in standard form. The equation must be written in standard form before you can read off the values of the gradient and the y-intercept.

$$2y = 6x + 10 \quad \rightarrow \text{Divide both sides by 2}$$
$$y = 3x + 5$$

Therefore the **gradient is 3** and the y-intercept is **(0; 5)**.

Standard form
The standard form of a straight line graph is $\boldsymbol{y = mx + c}$.
On one side there should only be a "y" (with a coefficient of 1).

If $m > 0$ the line will be increasing.
If $m < 0$ the line will be decreasing.
If the line is horizontal $m = 0$.
If the line is vertical m is undefined.

2. Complete the following table:

Equation	Gradient	y-intercept
$y = 3x + 5$		
$y = \frac{x}{2} - 7$		
$y = 2 - 3x$		
$-y = 5x - 10$		
$y = 3$		
	1	(0; 0)
	−2	(0; −7)

3. Write each of the following equations in standard form and then determine the gradient and y-intercept.
   (a) $2y + 4x = 10$ (b) $-3x = y + 4$ (c) $3x - 4 = y$

   ....

   ....

   ....

(d) $3y + 6 = x$

(e) $y = -3x + 4y - 12$

(f) $y = 3x - 2$

....

....

....

(g) $y = \frac{1}{4}x + 6$

(h) $y = -12$

(i) $x = 15$

....

....

....

DETERMINE THE EQUATION OF A STRAIGHT LINE

The equation of a straight line is $y = mx + c$. If you need to determine the equation of a straight line then all you need to know are the values of m and c.

If you know the values of two points on the graph then you can determine the gradient using the formula: $m = \frac{y_A - y_B}{x_A - x_B}$

Once you know the gradient you can calculate the value of the y-intercept using substitution.

Example 1: Determine the equation of the straight line that goes through (1; 1) and (5; 13).

Step 1: Calculate the gradient.

$$m = \frac{y_A - y_B}{x_A - x_B} = \frac{1 - 13}{1 - 5} = \frac{-12}{-4} = 3$$

Step 2: Since you now know $m = 3$ you can substitute it into the equation $y = mx + c$.

Therefore $y = 3x + c$.

Step 3: To determine c you need to substitute the coordinates of a point on the line into the equation. (It can be either of the points that were given, so choose the easier one.)

Substitute (5; 13) into $y = 3x + c$

$$\begin{aligned}(13) &= 3(5) + c\\ 13 &= 15 + c\\ 13 - 15 &= c\\ -2 &= c\end{aligned}$$

Step 4: Write down the equation: $y = 3x - 2$

Example 2: Determine the equation of the line that passes through (4; –1) and (7; 5).

Information	m (Gradient)	c (y-intercept)	$y = mx + c$ (Equation)
(4; –1) **(7; 5)**	$m = \frac{y_A - y_B}{x_A - x_B}$ $= \frac{-1-5}{4-7}$ $= \frac{-6}{-3}$ $= 2$	Substitute $m = 2$ and (7; 5) $y = mx + c$ $y = 2x + c$ $(5) = 2(7) + c$ $5 = 14 + c$ $-9 = c$	$y = 2x - 9$

Example 3: Determine the equation of the line with a gradient of 4 passing through (2; 6).

Information	m (Gradient)	c (y-intercept)	$y = mx + c$ (Equation)
$\boldsymbol{m = 4}$ **(2; 6)**	$m = 4$	Substitute $m = 4$ and (2; 6) $y = mx + c$ $y = 4x + c$ $6 = 4(2) + c$ $-2 = c$	$y = 4x - 2$

You may want to set your work out as shown in Examples 2 and 3 above.

1. Determine the equation of the each of the straight lines passing through the points given.

   (a) (3; 10) and (2; 5) (b) (–4; 5) and (2; 5) (c) (0; 0) and (4; –8)

   ....

   (d) $(1\frac{1}{2}; 4)$ and $(-\frac{1}{2}; 12)$ (e) (3; 4) and (–7; –1) (f) (0; 3) and (–14; –4)

   ....

2. Determine the equation of the straight line with:

   (a) a gradient of 5 and passing through the point (1; –3)

   ....

   (b) a gradient of –2 passing through the point (0; 0)

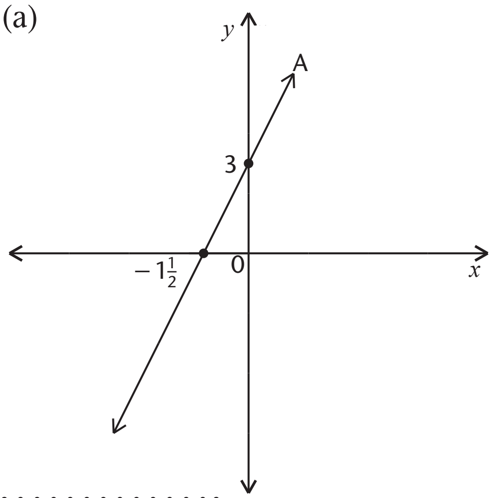
   ....

   (c) a y-intercept of 7 passing through the point (1; –3)

   ....

3. Determine the equations of the straight lines. Question (d) is a challenge.

(a)



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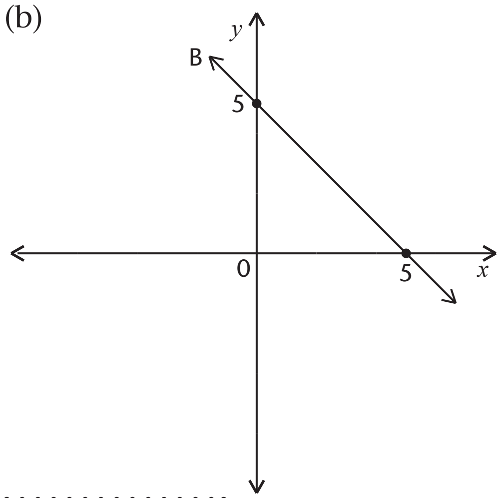
(b)



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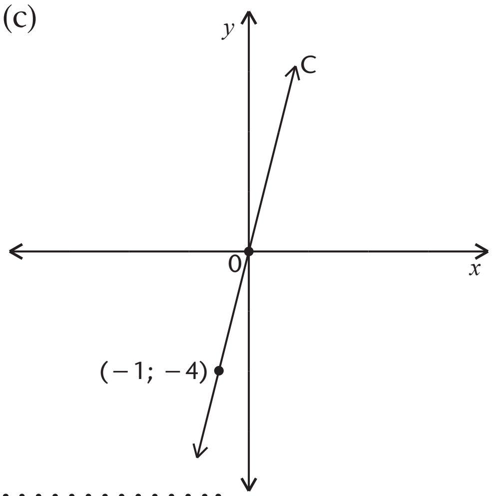
(c)



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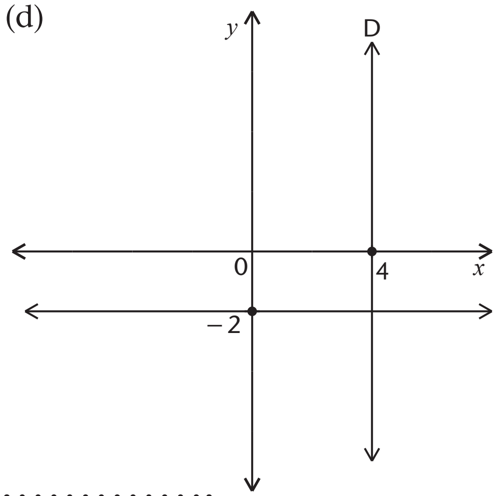
(d)



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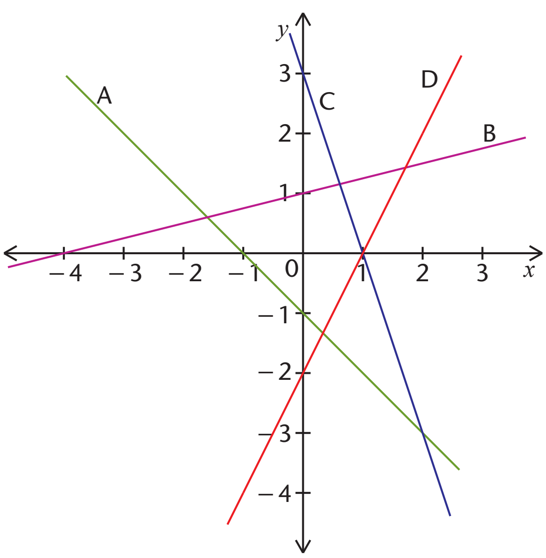
4.6 *x*- and *y*-intercepts



1. Write down the coordinates of the points where each line cuts the 2 axes:

	x-intercept	*y*-intercept
A		
B		
C		
D		

2. What do all the *x*-intercepts have in common?

....

3. What do all the y-intercepts have in common?

....

4. Determine the coordinates of the intercepts of the following straight line graphs.
   (a) $y = 3x + 12$ (b) $y = x - 3$

   ....

   (c) $y = -2x - 4$ (d) $2y = 6x + 12$

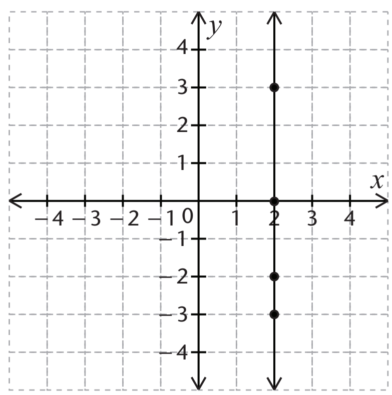
   ....

   (e) $4x + 2y = 20$ (f) $13 - y = -26x$

   ....

VERTICAL AND HORIZONTAL LINES

Some special lines are so easy that you don't need any fancy methods to draw them or get their equation; you can just look at them.



1. What do the following coordinate pairs have in common?
   (2; 3), (2; –2), (2; 0) and (2; –3)

   ....

2. Write down two more points that have an x-coordinate of 2.

   ....

If you plot these points on a set of axes you will see that they form a **vertical line**. The equation of the line is $x = 2$.

3. Will the two extra points you wrote down (question 2) also be on the line?

   ....

4. Write down five coordinate pairs with $x = -1$.

   ....

4.7 Graphs of non-linear functions

Some of the following relationships are represented by graphs on the next page. Identify which of the relationships are represented by which set of points on the graph. You may use the tables below to help you to answer this question. For example, you may calculate some output numbers by using the formulas and record this in the tables.

$y = -x^2$ $\quad$ $y = (-x)^2$ $\quad$ $y = x^2 + 130$ $\quad$ $y = (x - 5)^2 + 10$

$y = x^2$ $\quad$ $y = -x^2 + 130$ $\quad$ $y = 130 - x^2$ $\quad$ $y = x^2 - 10x + 35$

Write your answers here:

Set of points in yellow ....

Set of points in blue ....

Set of points in red ....

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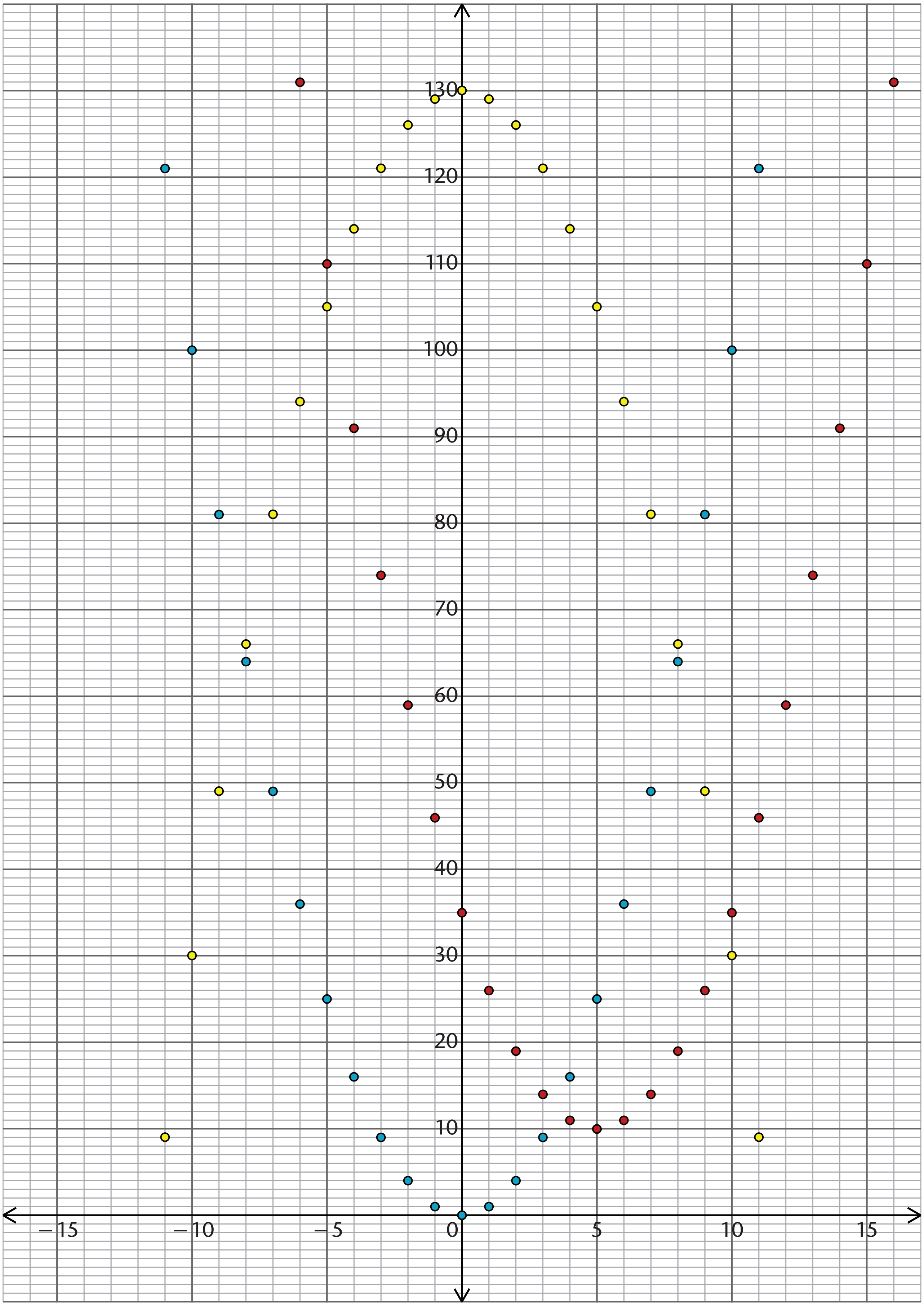
130
120
110
100
90
80
70
60
50
40
30
20
10
– 15
– 10
– 5
0
5
10
15