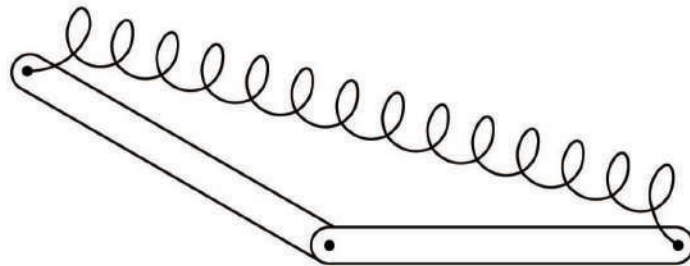
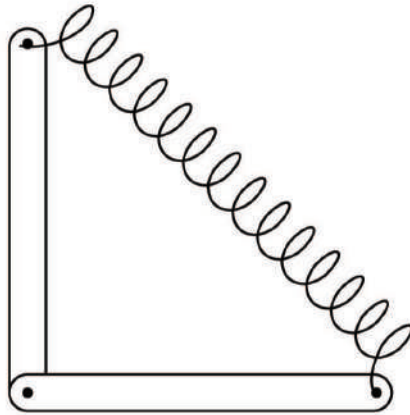
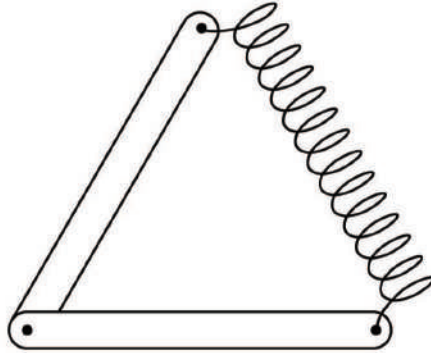
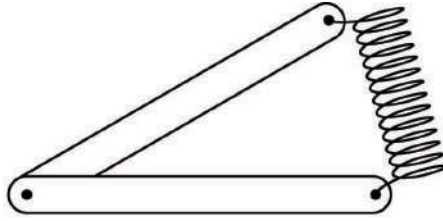


CHAPTER 1

Functions

In this chapter you will work with relationships between variables. You will find values of the output variable by doing calculations described in algebraic language. You will represent relations between variables in different ways, by describing them in words, in flow diagrams, in algebraic language and by means of tables and graphs. You will learn to recognise how the same relationship can be represented in different ways.

1.1	From formulas to words, tables and graphs.....	3
1.2	Tables and graphs	6



1 Functions

1.1 From formulas to words, tables and graphs

THE SAME INSTRUCTIONS IN WORDS AND IN SYMBOLS

1. Each of the formulas below indicates a relationship between two sets of numbers that may be called the *input numbers* and the *output numbers*. For each formula, calculate the output numbers that correspond to the input numbers 0; 1; 2 and 10.

(a) $y = 3x + 5$

.....

(b) $y = 3(x + 5)$

.....

(c) $y = 3x + 5x$

.....

(d) $y = 3x^2 + 5$

.....

(e) $y = 3x^2 + 5x$

.....

(f) $y = 3x(x + 5)$

.....

2. The information provided in the formula $y = 5x^2 - 3x$ can also be represented in words, for example: *To get the output number, you have to subtract 3 times the input number from 5 times the square of the input number.*

Represent each of the formulas in question 1 in words:

(a) $y = 3x + 5$

.....

(b) $y = 3(x + 5)$

.....

(c) $y = 3x + 5x$

.....

.....

.....

(d) $y = 3x^2 + 5$

.....

.....

.....

(e) $y = 3x^2 + 5x$

.....

.....

.....

(f) $y = 3x(x + 5)$

.....

.....

3. For each set of instructions write a formula that provides the same information:

- (a) *multiply the input number by 10, then subtract 3 to get the output number*
- (b) *subtract 3 from the square of the input number, then multiply by 10 to get the output number*
- (c) *multiply the square of the input number by 10, then add 5 times the input number to get the output number*
- (d) *subtract 7 times the square of the input number from 100, then multiply by 3 to get the output number*
- (e) *add 4 to the input number, then subtract the answer from 50 to get the output number*
- (f) *multiply the input number by 3, then subtract the answer from 15 to get the output number*

4. To check your answers for question 3, use the table below. First apply the verbal instructions for the input numbers 1, 5 and 10 in each case. Then choose another input number and do the same thing. Next use the formula you have written to calculate the output numbers. Do corrections where there are differences.

		1	5	10	
(a)	verbal description				
	formula				
(b)	verbal description				
	formula				
(c)	verbal description				
	formula				
(d)	verbal description				
	formula				
(e)	verbal description				
	formula				
(f)	verbal description				
	formula				

5. In certain cases, flow diagrams can be used to provide instructions on how output numbers can be calculated. For each flow diagram below, represent the information in a formula and also in words.

- (a) $\boxed{\times 3} \rightarrow \boxed{+ 17} \rightarrow \dots\dots\dots$
- (b) $\boxed{+ 5} \rightarrow \boxed{\times 3} \rightarrow \boxed{+ 2} \rightarrow \dots\dots\dots$
- (c) $\boxed{- 2} \rightarrow \boxed{\times 3} \rightarrow \boxed{+ 23} \rightarrow \dots\dots\dots$
- (d) $\boxed{\times 2} \rightarrow \boxed{+ 3} \rightarrow \boxed{\times 5} \rightarrow \boxed{+ 4} \rightarrow \dots\dots\dots$
- (e) $\boxed{+ 3} \rightarrow \boxed{\times 2} \rightarrow \boxed{+ 4} \rightarrow \boxed{\times 5} \rightarrow \dots\dots\dots$
- (f) $\boxed{\times 10} \rightarrow \boxed{+ 19} \rightarrow \dots\dots\dots$
- (g) $\boxed{+ 5} \rightarrow \boxed{\times 10} \rightarrow \dots\dots\dots$

6. (a) Complete the following table.

x	0	1	2	3
y according to your formula for 5(a)				
y according to your formula for 5(b)				
y according to your formula for 5(c)				

(b) If your output numbers for 5(a), 5(b) and 5(c) are not the same, you have made a mistake somewhere. If this is the case, find your mistake and correct it.

7. (a) Complete the following table.

x	-3	-2	-1	0
y according to your formula for 5(d)				
y according to your formula for 5(e)				
y according to your formula for 5(f)				
y according to your formula for 5(g)				

- (b) If your output numbers for 5(d) and 5(f) are not the same, you have made a mistake somewhere. If this is the case, find your mistake and correct it.
- (c) If your output numbers for 5(e) and 5(g) are not the same, you have made a mistake somewhere. If this is the case, find your mistake and correct it.

8. Explain why the output numbers in 5(a), 5(b) and 5(c) are the same.

.....

.....

1.2 Tables and graphs

1. Complete the table to show some of the input and output numbers of the relationship described by the formula $y = 2x - 3$.

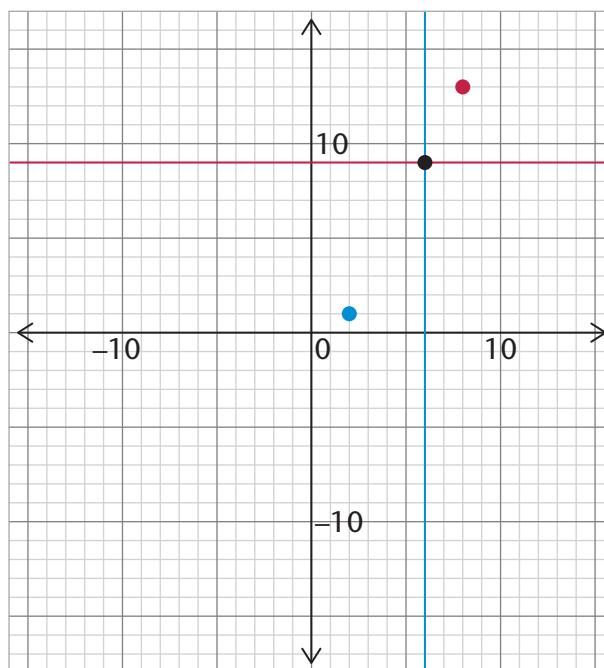
input numbers	-5	0	2	4	6	8
output numbers						

The vertical blue line on this graph represents the input number 6.

The heavy horizontal red line represents the output number 9.

The black point where the blue and red lines intersect indicates that the input number 6 is associated with the output number 9.

We also say the black point represents the **ordered number pair** (6; 9).

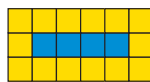
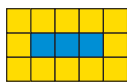
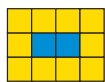


2. (a) Which ordered number pair does the red point on the graph represent?

.....

(b) Which ordered number pair does the blue point on the graph represent?

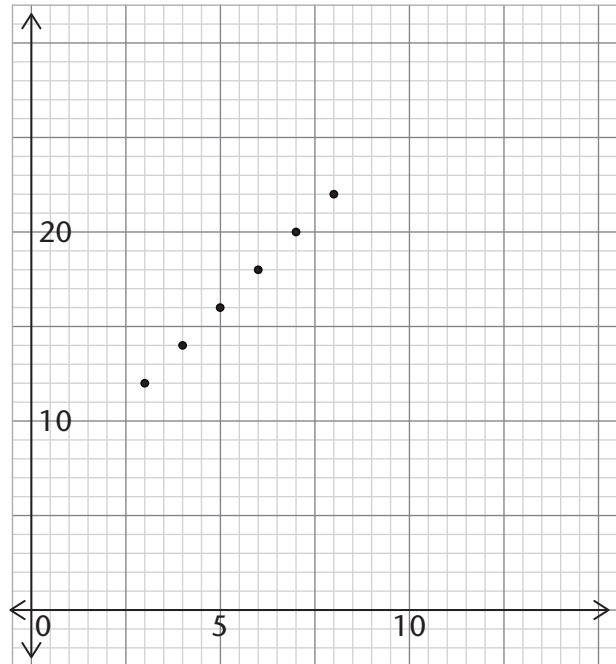
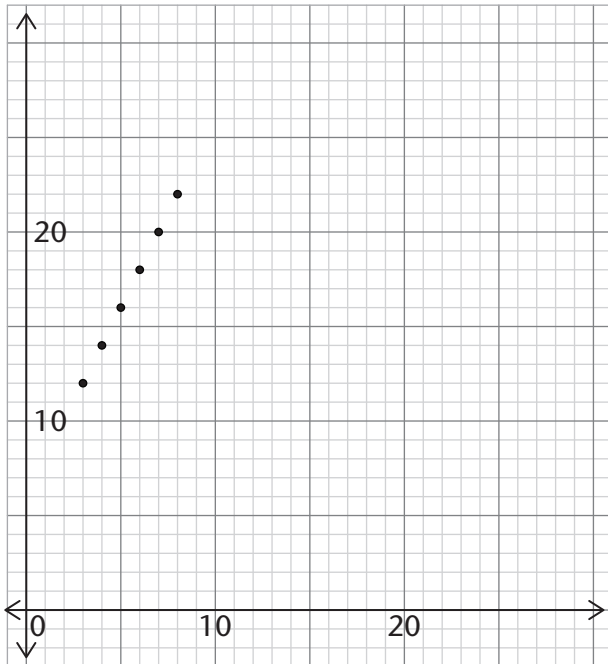
.....



A relationship between two variables can be represented by a table of some values of the independent and dependent variables (input and output numbers):

values of the independent variable	3	4	5	6	7	8
values of the dependent variable	12	14	16	18	20	22

The same information can also be shown on a graph:



3. Do the two graphs show the same relationship, or different relationships between two variables?

.....

4. How do the two graphs differ?

.....

5. Use one of the graphs to find out how many yellow squares there will be, in an arrangement like those at the top, with 12 blue squares.

6. Does the table below represent the same relationship as the table at the top of the page? Explain your answer.

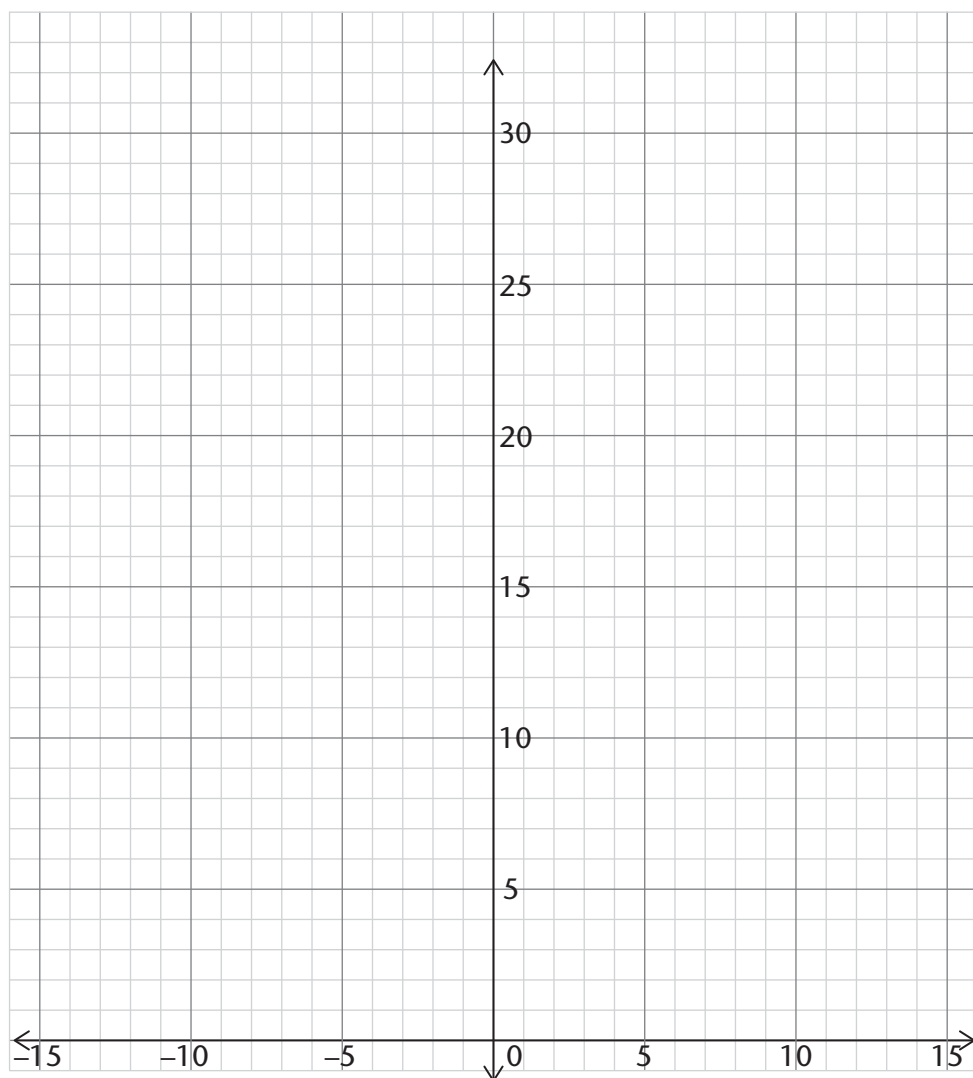
values of the independent variable	0	5	10	15	20	25
values of the dependent variable	8	18	28	38	48	58

.....

7. (a) Complete the following table for the relationship described by $y = x^2$.

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
y											

- (b) Represent the ordered number pairs in the table on the graph sheet below.



8. Complete the table for the relationship $y = 15 + x$. Represent the ordered number pairs on the graph sheet above.

x	-15	-10	-5	0	5	10	15
$15 + x$							

9. Complete the table for the relationship $y = 15 - x$. Represent the ordered number pairs on the graph sheet above.

x	-15	-10	-5	0	5	10	15
$15 - x$							

10. (a) The output values for $y = x^2$ and $y = 15 + x$ show patterns. Describe in words how the patterns differ. Use the words *increase* and *decrease* in your description.

.....

(b) Describe in words how the graphs of $y = x^2$ and $y = 15 + x$ differ.

.....

11. (a) Describe in words how the patterns in the output values for $y = 15 + x$ and $y = 15 - x$ differ. Use the words *increase* and *decrease* in your description.

.....

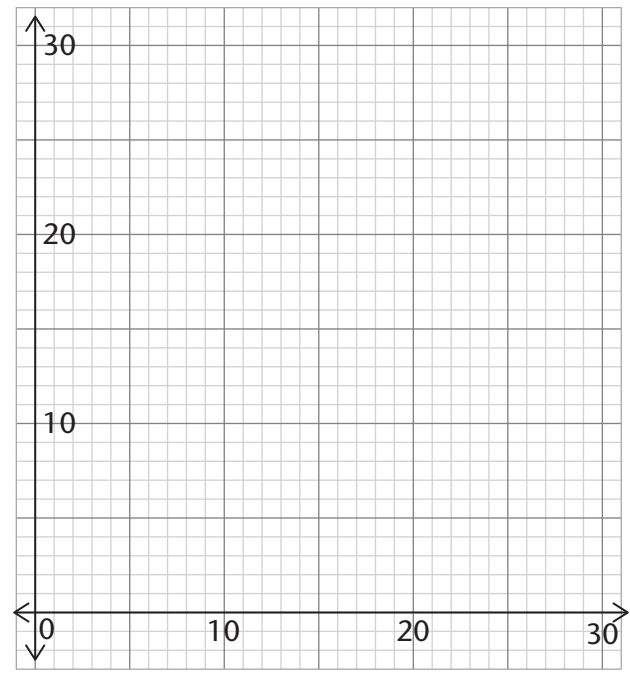
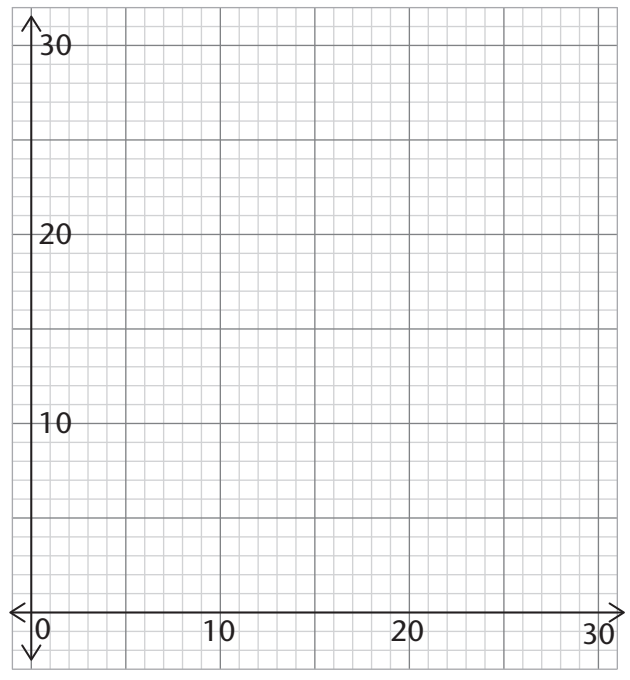
(b) Describe in words how the graphs of $y = 15 + x$ and $y = 15 - x$ differ.

.....

12. Complete each of the following tables by extending the pattern in the output numbers. Also represent the relationship on the graph sheets below.

(a)	input numbers	0	5	10	15	20	25	30
	output numbers	0	4	8	12			

(b)	input numbers	0	5	10	15	20	25	30
	output numbers	0	2	4	6			



13. How do the patterns in 12(a) and (b) differ, and how do the graphs differ?

.....

14. Each table below shows some values for a relationship represented by one of these rules:

$$y = -2x + 3$$

$$y = 2x - 5$$

$$y = -3x + 5$$

$$y = -3(x + 2)$$

$$y = 3x + 2$$

$$y = 5(x - 2)$$

$$y = 2x + 3$$

$$y = 2x + 5$$

$$y = -3x + 6$$

$$y = 5x + 10$$

$$y = 5x - 10$$

$$y = -x + 3$$

- (a) Complete the tables below by extending the patterns in the output values.
 (b) For each table, describe what you did to produce more output values. Also write down the rule (formula) that corresponds to the table.

- A.
 B.
 C.
 D.
 E.
 F.
 G.

A.

x	0	1	2	3	4	5	6	7
y	2	5	8					

B.

x	0	1	2	3	4	5	6	7
y	3	1	-1	-3				

C.

x	0	1	2	3	4	5	6	7
y	-10	-5	0	5				

D.

x	0	1	2	3	4	5	6	7
y	-5	-3	-1					

E.

x	0	1	2	3	4	5	6	7
y	6	3	0					

F.

x	0	1	2	3	4	5	6	7
y	3	2	1	0				

G.

x	0	1	2	3	4	5	6	7
y	3	5	7					

AN INVESTIGATION: PATTERNS IN DIFFERENCES

1. Complete the tables for $y = x^2$, $z = x^2 + 1^2$, $w = x^2 + 2^2$ and $s = x^2 + 3^2$.

x	1	2	3	4	5	6	7	8	9	10
y										
z										
w										
s										

2. Complete the tables for $y = x^2$, $p = (x + 1)^2$, $q = (x + 2)^2$ and $r = (x + 3)^2$.

(a)

x	1	2	3	4	5	6	7	8	9	10
p										
y										
$p - y$										

(b)

x	1	2	3	4	5	6	7	8	9	10
q										
y										
$q - y$										

(c)

x	1	2	3	4	5	6	7	8	9	10
r										
y										
$r - y$										

3. In each of the following cases, you should have different output values for the two relationships. If your output values are the same, find your mistakes and correct your work.

(a) $z = x^2 + 1^2$ and $p = (x + 1)^2$

(b) $w = x^2 + 2^2$ and $q = (x + 2)^2$

(c) $s = x^2 + 3^2$ and $r = (x + 3)^2$

4. Complete the tables, for $y = x^2$, $p = (x + 1)^2$, $q = (x + 2)^2$ and $r = (x + 3)^2$.

(a)

x	1	2	3	4	5	6	7	8	9	10
$p - y$										
$q - y$										
$r - y$										

(b)

x	10	11	12	13	14	15	16	17
$p - y$								
$q - y$								
$r - y$								

5. (a) Complete the table.

x	1	2	3	4	5	6	7	8	9	10
$2x + 1$										
$4x + 4$										
$6x + 9$										

- (b) What are the constant differences in the sequences of values of $2x + 1$, $4x + 4$ and $6x + 9$, for $x = 1; 2; 3; 4 \dots$?

- (c) Do you have an idea whether the corresponding sequence for $12x + 36$ will also have a constant difference and what the constant difference may be?

- (d) There are certain patterns in the coefficients and constant terms in the expressions in the above table. Continue the patterns to make some more similar expressions and complete the table below for your expressions.

x	1	2	3	4	5	6	7	8	9	10

6. (a) If your answers for the tables in 4(a) and 5(a) are correct, they will be the same. Try to explain why they are the same.

- (b) What expressions, similar to those in question 5(a), may have the same values as $(x + 4)^2 - x^2$ and $(x + 5)^2 - x^2$ respectively?