Chapter 11
Probability

In this chapter you will learn about the idea of probability, and what information probabilities provide about what may happen in future. You will also learn about compound events.

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11 Probability

11.1 Simple events

REVISION

yellow	green	pink	blue	red	brown	grey	black

1. (a) Suppose the 8 coloured buttons above are in a bag and you draw one button from the bag without looking. Can you tell what colour you will draw? .....

   (b) Suppose you repeatedly draw a button from the bag, note its colour, then put it back. Can you tell in approximately what fraction of all the trials the button will be yellow?

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Archie has a theory. Because the 8 possible outcomes are equally likely, he believes that if you perform 8 trials in a situation like the above you will draw each colour once.

2. If Archie's theory is correct, how many times will each colour be drawn if 40 trials are performed?

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3. If Archie's theory is correct, in what fraction of the total number of trials will each colour be drawn?

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Each time you draw a button from the bag without looking you perform a **trial**. If you do this and put the button back, and repeat the same actions 8 times, you have performed 8 trials.

The number of times an event occurs during a set of trials is called the **frequency** of the event.

When the frequency of an event is expressed as a fraction of the total number of trials, it is called the **relative frequency**.

4. If Archie's theory is correct, how many times will each of the colours be drawn if a total of 40 trials is performed? Write your answers in the second row of the table below. Write the predicted relative frequencies in row 3 as fortieths, and in row 4 as twohundredths.

colour	yellow	green	pink	blue	red	brown	grey	black
frequencies predicted by Archie								
relative frequencies predicted by Archie expressed in 40ths								
relative frequencies predicted by Archie expressed in 200ths								

The relative frequency for each colour that Archie predicted is called the **probability** of drawing that colour. If all the outcomes are equally likely, then

$$\textbf{probability of an outcome} = \frac{\textbf{1}}{\textbf{the total number of equally-likely outcomes}}$$

You will now investigate whether Archie's theory is correct.

5. (a) Make 8 small cards and write the name of one of the above colours on each card, so that you have cards with the eight colour names. Perform 8 trials to check whether Archie's theory is correct. Record your results (your tally marks 1 and your frequencies 1) in the relevant row of the table below.
   (b) Find out what any four of your classmates found when they did the experiment. Enter their results in your table too (Friend 1, 2, 3, 4 frequencies).

Table for the results of the experiments

colour	yellow	green	pink	blue	red	brown	grey	black
your tally marks (1)								
your frequencies (1)								
Friend 1 frequencies								
Friend 2 frequencies								
Friend 3 frequencies								
Friend 4 frequencies								
Total frequencies for 5 experiments								

6. (a) What was the total number of trials in the five experiments you recorded in the above table? .....

   (b) What is the total of the frequencies for the different colours, in the last row of your table? .....

7. Is Archie's theory correct? .....

Bettina has a different theory to Archie's. She believes that if one does many trials with the eight buttons in a bag, each colour will be drawn in **approximately** one-eighth of the cases. In other words Bettina believes that the relative frequency of each outcome will be close to the probability of that outcome, but may not be equal to it.

8. (a) You and your four classmates performed 40 trials in total. Enter the results in the second row of the table below. Also express each frequency as a fraction of 40, in fortieths and in twohundredths.

colour	yellow	green	pink	blue	red	brown	grey	black
actual frequencies obtained in your experiments (40 trials)								
relative frequencies as 40ths								
relative frequencies as 200ths								
probability as 200ths								

   (b) Do your experiments show that Bettina's theory is correct or not?

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Jayden believes that **when more trials are performed, the relative frequencies will get closer to the probabilities**.

You will now do an investigation to investigate whether Jayden's theory is true.

INVESTIGATE WHAT HAPPENS WHEN MORE TRIALS ARE DONE

1. Perform 40 trials by drawing one card at a time from eight small cards with the names of the colours written on them, and enter your results in the second and third rows of the table below.

colour	yellow	green	pink	blue	red	brown	grey	black
tally marks								
frequencies								
relative frequencies as 40ths								
relative frequencies as 200ths								
probabilities as 200ths								

2. Make a copy of the above table, without the row for tally marks, and without the row for the relative frequencies as fourtieths and the row for the probabilities, on a loose sheet of paper. Exchange it with a classmate. Enter the results of your classmate on table 1 and 2 on the next page. Also enter your own results for question 1 on the tables.

3. Get hold of the data reports of three other classmates, and enter these on the tables on the next page too.

4. Add the frequencies of the various colours in the five sets of data for 40 trials each, and calculate the relative frequencies expressed as twohundredths.

5. Is the range of relative frequencies for 200 trials smaller than the ranges for the five different sets of 40 trials each? What does this indicate with respect to Jayden's theory?

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When only a small number of trials are done, the actual relative frequencies for different outcomes may differ a lot from the probabilities of the outcomes.

When many trials are done, the actual relative frequencies of the different outcomes are quite close to the probabilities of the outcomes.

Table 1: Frequencies for 5 sets of 40 trials each

colour	yellow	green	pink	blue	red	brown	grey	black
frequencies for your own 40 trials in question 1								
frequencies for 40 trials by classmate 1								
frequencies for 40 trials by classmate 2								
frequencies for 40 trials by classmate 3								
frequencies for 40 trials by classmate 4								
total frequencies for 200 trials								
relative frequencies for 200 trials as 200ths								

Table 2: Relative frequencies for each of the 5 sets of 40 trials each

(expressed as 200ths)

colour	yellow	green	pink	blue	red	brown	grey	black
relative frequencies for your own 40 trials								
relative frequencies for 40 trials by classmate 1								
relative frequencies for 40 trials by classmate 2								
relative frequencies for 40 trials by classmate 3								
relative frequencies for 40 trials by classmate 4								

6. How many different three-digit numbers can be formed with the symbols 3 and 5, if no other symbols are used? You may use one, two or three of the symbols in each number, and you may repeat the same symbol.

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11.2 Compound events

TOSSING A COIN AND GIVING BIRTH

1. Simon threw a coin and the outcome was heads. He will now throw the coin again.
   (a) What are the possible outcomes? .....
   (b) What is the probability of each of the possible outcomes? .....
   (c) What are the possible outcomes if Simon throws the coin for a third time?
   .....
   (d) What is the probability of each of the possible outcomes for the third throw?
   .....

What happens when a coin is thrown for a second time has nothing to do with what happened when it was thrown the first time.

The first throw and the second throws are called **independent events**: what happened on the first throw cannot influence what will happen on the second throw.

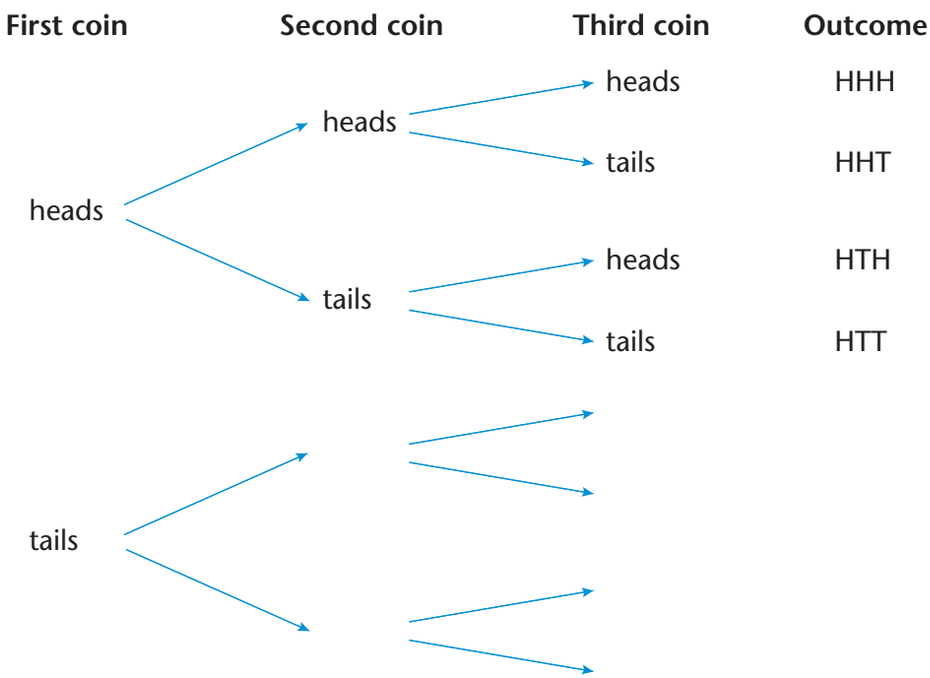
2. (a) If an event has four different equally-likely outcomes, what is the probability of each of the four outcomes? .....
   (b) Does that mean that if the event is repeated 4 times, each of the four outcomes will happen once? .....
   (c) Does your answer in (a) means that if the event is repeated 100 times, each of the four outcomes will happen 25 times? .....
   .....

3. (a) What are the possible outcomes when two coins are thrown? Use the **two-way table** below to answer this question. One possible outcome is already given.

	Heads	**Tails**
Heads		H T
Tails		

   (b) Do you think these four outcomes are equally likely? .....
   (c) What is the probability of each of the four outcomes? .....
   (d) What is the probability of getting a head and a tail? .....

4. Let us consider the possible outcomes if three coins are thrown.
   Below is a tree diagram that can help you figure out what the different possible outcomes are. Complete the diagram by filling in the missing information.



5. (a) Do you think the eight different outcomes in question 4 are equally likely?

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   (b) What is the probability of each of the eight outcomes?

   .....

   (c) What is the probability of throwing two heads and one tail?

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6. In question 6 on page 183 you were asked to write down the various numbers that can be formed by using symbols 3 and 5. Think of all the four-letter codes that you can form by using only two letters, P and Q. Any letter can be used more than once in one code. First think about how you will go about finding all the possibilities in a systematic way and then try to set up a tree diagram to help you.

   (a) Draw a tree diagram in your exercise book to help you to solve this problem. List all the outcomes.

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(b) If the codes are formed by randomly choosing the letters, what is the probability that the code will consist of the using the same letter four times?

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(c) What is the probability that the code will consist of two P's and two Q's?

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When a woman is pregnant, the baby can be a boy or a girl. Suppose we make the assumption that the two possibilities are equally likely, so the probability of a boy is $\frac{1}{2}$ and the probability of a girl is $\frac{1}{2}$.

7. (a) Complete this two-way table to show the possible outcomes of the gender of the two children in a family

	Boy	Girl

(b) List the possible outcomes.

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(c) What is the probability that the two children in the family will be of the same gender?

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(d) What is the probability that the eldest child will be a boy and then they will have a girl?

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8. A certain woman already has one child, which is a boy. She now expects a second child. What is the probability of it being a boy again, if we make the assumption that a baby being a boy or a girl are equally likely events?

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The assumption that a boy or a girl being born are equally likely events may not actually be true. However, probabilities can only be calculated and used to make predictions if it is assumed that outcomes are equally likely.

9. (a) A woman gets married and plans to have a baby in one year and another baby in the next year. What is the probability that both babies will be girls?

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(b) A woman gets married and plans to have a baby in each of the first three years of the marriage. What is the probability that she will have a boy in the first year, and girls in the second and third years?

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