Chapter 8
Algebraic expressions

An algebraic expression is a description of a set of operations that are to be done in a certain order. In this chapter, you will learn to specify a different set of operations that will produce the same results as a given set of operations. Two different expressions that produce the same results are called equivalent expressions.

8 Algebraic expressions

8.1 Algebraic language

WORDS, DIAGRAMS AND EXPRESSIONS

1. Complete this table.

	Words	Flow diagram	Expression
	Multiply a number by 5 and then subtract 3 from the answer.	$\longrightarrow \boxed{\times 5} \longrightarrow \boxed{-3} \longrightarrow$	$5x - 3$
(a)	Add 5 to a number and then multiply the answer by 3.	$\longrightarrow \boxed{} \longrightarrow \boxed{} \longrightarrow$	
(b)		$\longrightarrow \boxed{-3} \longrightarrow \boxed{\times 5} \longrightarrow$	
(c)			$3(2x + 3)$

An **algebraic expression** indicates **a sequence of operations** that can also be described in words. In some cases they can be described with flow diagrams.

Expressions in brackets should always be calculated first. If there are no brackets in an algebraic expression, it means that multiplication and division must be done first, and addition and subtraction afterwards.

For example, if $x = 5$ the expression $12 + 3x$ means "multiply 5 by 3, then add 12". It does **not** mean "add 12 and 3, then multiply by 5".

If you wish to say "add 12 and 3, then multiply by 5", the numerical expression should be $5 \times (12 + 3)$ or $(12 + 3) \times 5$.

2. Describe each of these sequences of calculations with an algebraic expression:
   (a) Multiply a number by 10, subtract 5 from the answer, and multiply the answer by 3.

   ....

   (b) Subtract 5 from a number, multiply the answer by 10, and multiply this answer by 3.

   ....

3. Evaluate each of these expressions for $x = 10$:

(a) $200 - 5x$ ....

(b) $(200 - 5)x$ ....

(c) $5x + 40$ ....

(d) $5(x + 40)$ ....

(e) $40 + 5x$ ....

(f) $5x + 5 \times 40$ ....

SOME WORDS WE USE IN ALGEBRA

An expression with one term only, like $3x^2$, is a **monomial**.
An expression which is a sum of two terms, like $5x + 4$, is called a **binomial**.
An expression which is a sum of three terms, like $3x^3 + 2x + 9$, is called a **trinomial**.

The symbol x is often used to represent the **variable** in an algebraic expression, but other letter symbols may also be used.

In the monomial $3x^2$, the 3 is called the **coefficient** of x^2.

In the binomial $5x + 4$, and the trinomial $3x^2 + 2x + 9$ the numbers 4 and 9 are called **constants**.

1. Complete the table, using the completed first row as an example.

Expression	Type of expression	Symbol used to represent the variable	Constant	Coefficient of
$x^2 + 6x + 10$	Trinomial	x	10	the second term is 6
$6s^3 + s^2 + 5$				s^2 is ......
$\frac{k}{3} + 12$				the first term is
$4p^{10}$				p^{10} is ......

2. Consider the pattern-polynomial starting with $7x^5 + 5x^4 + 3x^3 + x^2 + ...$

(a) What is the coefficient of the fourth term? ....

(b) What is the exponent value of the fifth term? ....

(c) Do you think the sixth term will be a constant? Why? ....

....

EQUIVALENT ALGEBRAIC EXPRESSIONS

1. Calculate the numerical value of the expressions for the various values of x. Do the calculations in your exercise book. Then fill in your answers.

	x	-2	-1	0	1	2
(a)	$3x + 2$					
(b)	$2x - 3$					
(c)	$3x + 2 + 2x - 3$					
(d)	$2x - 3 + 3x + 2$					
(e)	$5x - 1$					
(f)	$(3x + 2)(2x - 3)$					
(g)	$3x(2x - 3) + 2(2x - 3)$					
(h)	$6x^2 - 5x - 6$					
(i)	$\frac{(3x+2)(2x-3)}{3x+2}$					
(j)	$\frac{6x^2-5x-6}{3x+2}$					

2. Make a list of all the algebraic expressions above which have the same numerical value for the same value of x, although they may look different:

....

....

....

Equivalent expressions are algebraic expressions that have different sequences of operations, but have the same numerical value for any given value of x.

It is often convenient not to work with a given expression, but to **replace** it with an equivalent expression.

3. Complete this table.

x	2	3	5	10	–5	–10
$12x - 7 + 3x + 10 - 5x$						

4. Complete this table.

x	2	3	5	10	–5	–10
$10x + 3$						

5. (a) Is $10x + 3$ equivalent to $12x - 7 + 3x + 10 - 5x$? Explain your answer.

....

....

....

(b) Suppose you need to know how much $12x - 7 + 3x + 10 - 5x$ is for $x = 37$ and $x = -43$. What do you think is the easiest way to find out? ....

....

CONVENTIONS FOR WRITING ALGEBRAIC EXPRESSIONS

Here are some things that mathematicians have agreed upon, and it makes mathematical work much easier if all people stick to these agreements.

A **convention** is something that people have agreed to do in the same way.

The multiplication sign is often omitted in algebraic expressions: We normally write $4x$ instead of $4 \times x$ and $4(x - 5)$ instead of $4 \times (x - 5)$.

It is a convention to write a known number first in a product, i.e. we write $3 \times x$ rather than $x \times 3$, and we write $3x$ **but not** $x3$.

1. Rewrite each of the following in the way in which it is normally written in algebraic expressions.

(a) $x \times 4 + x \times y - y \times 3$

(b) $7 \times (10 - x) + (5 \times x + 3)10$

....

People all over the world have agreed that, in expressions that do not contain brackets, addition and subtraction should be performed as they appear from left to right in the expression.

According to this convention, $x - y + z$ means that you first have to subtract y from x, then add z. For example if $x = 10$, $y = 5$ and $z = 3$, $x - y + z$ is $10 - 5 + 3$ and it means $10 - 5 = 5$, then $5 + 3 = 8$. It does not mean $5 + 3 = 8$, then $10 - 8 = 2$.

2. Calculate $50 - 20 + 30$ and $50 + 30 - 20$ and $50 - 30 + 20$

....

3. Evaluate each of the following expressions for $x = 10$, $y = 5$ and $z = 2$.

(a) $x + y - z$

....

(b) $x - z + y$

....

(c) $10y - 3x + 5z - 4y$

....

(d) $10y - 3x - 5z + 4y + 3x$

....

People have also agreed that, in expressions that do not contain brackets, we should do **multiplication** (and division) **before addition and subtraction**.

Hence $5 + 3 \times 4$ should be understood as "multiply 4 by 3, then add the answer to 5" and not as "add 5 and 3 then multiply the answer by 4".

Also, $3 \times 4 + 5$ should be understood to mean "multiply 4 by 3, then add 5 to the answer", and not as "add 4 and 5 then multiply the answer by 3".

4. Do each of the following calculations.

(a) multiply 4 by 3, then add 5 to the answer

....

(b) add 4 and 5 then multiply the answer by 3

....

(c) multiply 4 by 3, then add the answer to 5

....

(d) add 5 and 3 then multiply the answer by 4

....

5. Rewrite the instructions in 4(a) and 4(c) without using words.

....

....

6. Calculate each of the following.

(a) $10 \times 5 + 30$ (b) $30 + 10 \times 5$

....

(c) $10 \times 5 - 30$ (d) $30 - 10 \times 5$

....

7. (a) Add 4 and 5, then subtract the answer from 20.

....

(b) Subtract 4 from 20 and then add 5.

....

(c) Add 4 and 5, then multiply the answer by 3.

....

(d) Multiply 3 by 5 and then add the answer to 4.

....

If we want to specify the calculations in 7(a) and 7(c) without using words we face challenges.

We cannot write $20 - 4 + 5$ for "*add 4 and 5 then subtract the answer from 20*", because that would mean "*subtract 4 from 20 then add 5*". We need a way to indicate, without using words, that we want the addition to be performed before the subtraction in this case.

Similarly we cannot write $4 + 5 \times 3$ for "*add 4 and 5 then multiply the answer by 3*", because that would mean "*multiply 3 by 5 and then add the answer to 4*". We need a way to indicate, without using words, that we want the addition to be performed before the multiplication in this case.

Mathematicians have agreed to use brackets to address the above challenges. The following convention is used all over the world:

> Whenever there are brackets in an expression, the calculations within the brackets should be performed first.

Hence $20 - (4 + 5)$ means *add 4 and 5 then subtract the answer from 20*, but $20 - 4 + 5$ means *subtract 4 from 20 then add 5*.

$(4 + 5) \times 3$ or $3 \times (4 + 5)$ means *add 4 and 5 then multiply the answer by 3*, but $4 + 5 \times 3$ means *multiply 3 by 5 then add the answer to 4*.

$10 + 2(5 + 9)$ means *add 5 and 9, multiply the answer by 2, then add this answer to 10*:

$5 + 9 = 14$ $\quad$ $14 \times 2 = 28$ $\quad$ $28 + 10 = 38$

8. Calculate each of the following.

(a) $100 + 50 - 30$ (b) $100 + (50 - 30)$

....

(c) $100 - 50 + 30$ (d) $100 - (50 + 30)$

....

(e) $3(10 - 4) + 2$ (f) $10(5 + 7) + 3(18 - 8)$

....

(g) $250 - 10 \times (18 + 2) + 35$ (h) $(20 + 20) \times (20 - 10)$

....

(i) $(250 - 10) \times (18 + 2) + 35$ (j) $20 + 20 \times (20 - 10)$

....

(k) $200 + (100 \times 2(15 + 5))$ (l) $(200 + 100) \times 2 \times 15 + 5$

....

In algebra, we normally write $3(x + 2y)$ instead of $(x + 2y) \times 3$, and we write $3(x - 2y)$ instead of $(x - 2y) \times 3$. Don't let this conventional way of writing in algebra confuse you. The expression $3(x + 2y)$ does not mean that multiplication by 3 is the first thing you should do when you evaluate the expression for certain values of x and y. The first thing you should do is to add the values of x and y. That is what the brackets tell you!

However, performing the instructions $3(x + 2y)$ is not the only way in which you can find out how much $3(x + 2y)$ is for any given values of x and y. Instead of working out $3(x + 2y)$, you may work out $3x + 6y$. In this case you will multiply each term before you add them together.

9. Evaluate each of the following expressions for $x = 10$, $y = 5$ and $z = 2$.

(a) $xy + z$ (b) $x(y + z)$

....

(c) $x + yz$

(d) $xy + xz$

....

....

(e) $xy - z$

(f) $x(y - z)$

....

....

(g) $x - yz$

(h) $xy - yz$

....

....

(i) $x + (y - z)$

(j) $x - (y - z)$

....

....

(k) $x - (y + z)$

(l) $x - y - z$

....

....

(m) $x + y - z$

(n) $x - y + z$

....

....

8.2 Properties of operations

1. Calculate the following:

(a) $5(3 + 4)$

(b) $5 \times 3 + 5 \times 4$

....

....

(c) $6 \times 3 + (4 + 6)$

(d) $(6 + 4) + 3 \times 6$

....

....

(e) $3 \times (4 \times 5)$

(f) $(3 \times 4) \times 5$

....

....

You should have noticed that for each row the results are the same. This is because operations with numbers have certain properties, namely the **distributive**, **commutative** and **associative** properties.

The **distributive** property is used each time you multiply a number in parts. For example:

The number thirty-four is actually 30 + 4. You may calculate 5 × 34 by calculating 5 × 30 and 5 × 4, and then adding the two answers:

$$5 \times 34 = 5 \times 30 + 5 \times 4$$

The word "distribute" means to spread out. The distributive property may be described as follows:

$a(b + c) = ab + ac$

where a, b and c can be any numbers.

We may say: "multiplication distributes over addition"

2. Calculate each of the following:

(a) $5(x - y)$ for $x = 10$ and $y = 8$

. . . .

(b) $5x - 5y$ for $x = 10$ and $y = 8$

. . . .

(c) $5(x - y)$ for $x = 100$ and $y = 30$

. . . .

(d) $5x - 5y$ for $x = 100$ and $y = 30$

. . . .

(e) $5(x - y + z)$ for $x = 10$, $y = 3$, $z = 2$

. . . .

(f) $5x - 5y + 5z$ for $x = 10$, $y = 3$, $z = 2$

. . . .

3. We say "multiplication distributes over addition".
Does multiplication also distribute over subtraction?
Give examples to support your answer.

. . . .

. . . .

For any values of x and y,

- $x + y$ and $y + x$ give the same answers, and
- xy and yx give the same answers.

This is called the **commutative property** of addition, and multiplication.

4. We say "addition is commutative" and "multiplication is commutative".
Is subtraction also commutative? Demonstrate your answer with an example.

. . . .

The **associative property** allows you to arrange three or more numbers in any sequence when adding or multiplying. For any values of x, y and z, the following expressions all have the same answer:

$x + y + z$ $\quad\quad$ $y + x + z$ $\quad\quad$ $z + y + x$

5. Calculate 16 + 33 + 14 + 17 in the easiest possible way.

....

The associative property of multiplication allows you to simplify something like the following.

$abc + bca + cba$

Because the order of multiplication does not change the result we can rewrite this expression as: $abc + abc + abc$.

This then can be simplified by adding like terms to be $3abc$. You will be able to use these properties throughout this chapter and when you do algebraic manipulations.

When you form an expression that is equivalent to a given expression you say that you *manipulate* the expression.

6. Replace each of the following expressions with a simpler expression that will give the same answer. **Do not do any calculations now.** In each case state why your replacement will be easier to do.
   (a) $17 \times 43 + 17 \times 57$

   ....

   ....

   (b) $7 \times 5 \times 8 \times 4 + 12 \times 8 \times 4 \times 7 - 9 \times 4 \times 5 \times 8$

   ....

   ....

   (c) $43 \times 17 + 57 \times 17$

   ....

   ....

   (d) $43x + 57x$ (for $x = 213$ or any other value)

   ....

   ....

7. Which properties of operations did you use in each part of question 6?

....

....

8.3 Combining like terms in algebraic expressions

REARRANGE TERMS, THEN COMBINE LIKE TERMS

To check whether two expressions are possibly equivalent, you can evaluate both expressions for several different values of the variable.

1. In each case below, first predict whether the two expressions are equivalent and then check by evaluating both for $x = 1$, $x = 10$, $x = 2$ and $x = -2$ in the tables.

   (a) $x(x + 3)$ and $x^2 + 3$

   ....

   ....

   (b) $x(x + 3)$ and $x^2 + 3x$

   ....

   ....

Some expressions can be simplified by rearranging the terms and combining "like terms".

In the expression $5x^2 + 13x + 7 + 2x^2 - 8x - 12$, the terms $5x^2$ and $2x^2$ are like terms.

Two or more like terms can be combined to form a single term.

$5x^2 + 2x^2$ can be replaced by $7x^2$ because for any value of x, for example $x = 2$ or $x = 10$, calculating $5x^2 + 2x^2$ and $7x^2$ will produce the same output value (try it!).

2. Complete the table.

x	10	2	5	1
$5x^2 + 2x^2$				
$7x^2$				
$13x - 8x$				
$5x$				

It is difficult to see the like terms in a long expression like $3x^2 + 13x + 7 + 2x^2 - 8x - 12$. Fortunately, you can rearrange the terms in an expression so that the like terms are next to each other.

3. (a) Complete the second and third rows of the table below. You will complete the next two rows when you do question (g).

x	10	2	5	1
$3x^2 + 13x + 7 + 2x^2 - 8x - 12$				
$3x^2 + 2x^2 + 13x - 8x + 7 - 12$				

(b) What do you observe? ....

(c) How does the one expression in the above table differ from the other one?

....

(d) Combine like terms in $3x^2 + 2x^2 + 13x - 8x + 7 - 12$ to make a shorter equivalent expression.

....

(e) Evaluate your shorter expression for $x = 10$, $x = 2$ and $x = 5$.

....

....

(f) Is your shorter expression equivalent to $3x^2 + 13x + 7 + 2x^2 - 8x - 12$? Explain how you know whether it is or is not.

....

....

....

(g) Evaluate $5x^2 + 5x - 5$ and $5(x^2 + x - 1)$ for $x = 10$, $x = 2$, $x = 5$ and $x = 1$, and write your answers in the last two rows of the above table.

4. Simplify each expression:

(a) $(3x^2 + 5x + 8) + (5x^2 + x + 4)$

....

(b) $(7x^2 + 3x + 5) + (2x^2 - x - 2)$

....

(c) $(6x^2 - 7x - 4) + (4x^2 + 5x + 5)$

....

(d) $(2x^2 - 5x - 9) - (5x^2 - 2x - 1)$

....

(e) $(-2x^2 + 5x - 3) + (-3x^2 - 9x + 5)$

....

(f) $(y^2 + y + 1) + (y^2 - y - 1)$

....

5. Complete the table. (Hint: Save yourself some work by simplifying first!)

....

....

....

x	2,5	3,7	6,4	12,9	35	−4,7	−0,04
$(3x + 6{,}5) + (7x + 3{,}5)$							
$(13x - 6) + (26 - 12x)$							

6. Simplify:

(a) $(2r^2 + 3r - 5) + (7r^2 - 8r - 12)$

....

(b) $(2r^2 + 3r - 5) - (7r^2 - 8r - 12)$

....

(c) $(2x + 5xy + 3y) - (12x - 2xy - 5y)$

....

(d) $(2x + 5xy + 3y) + (12x - 2xy - 5y)$

....

7. Evaluate the following expressions for $x = 3$, $x = -2$, $x = 5$ and $x = -3$.
   (a) $2x(x^2 - x - 1) + 5x(2x^2 + 3x - 5) - 3x(x^2 + 2x + 1)$

   ....

   ....

   ....

   (b) $(3x^2 - 5x + 7) - (7x^2 + 3x - 5) + (5x^2 - 2x + 8)$

   ....

   ....

   ....

8. Write equivalent expressions without brackets.
   (a) $3x^4 - (x^2 + 2x)$ ....
   (b) $3x^4 - (x^2 - 2x)$ ....
   (c) $3x^4 + (x^2 - 2x)$ ....
   (d) $x - (y + z - t)$ ....

9. Write equivalent expressions without brackets, rearrange so that like terms are grouped together, and then combine the like terms.
   (a) $2y^2 + (y^2 - 3y)$ ....
   (b) $3x^2 + (5x + x^2)$ ....
   (c) $6x^2 - (x^4 + 3x^2)$ ....
   (d) $2t^2 - (3t^2 - 5t^3)$ ....
   (e) $6x^2 + 3x - (4x^2 + 5x)$ ....
   (f) $2r^2 - 5r + 7 + (3r^2 - 7r - 8)$ ....
   (g) $5(x^2 + x) + 2(x^2 + 3x)$ ....
   (h) $2x(x - 3) + 5x(x + 2)$ ....

10. Write equivalent expressions without brackets and simplify these expressions as far as possible.

    Example $5r^2 - 2r(r + 5) = 5r^2 - 2r^2 - 10r$
    $= 3r^2 - 10r$

    (a) $3x^2 + x(x + 3)$ ....
    (b) $5x + x(7 - 2x)$ ....
    (c) $6r^2 - 2r(r - 5)$ ....
    (d) $2a(a + 3) + 5a(a - 2)$ ....

(e) $6y(y + 1) - 3y(y + 2)$

(f) $4x(2x - 3) - 3x(x + 2)$

....

(g) $2x^2(x - 5) - x(3x^2 - 2)$

(h) $x(x - 1) + x(2x + 3) - 2x(3x + 1)$

....

8.4 Multiplication of algebraic expressions

MULTIPLY POLYNOMIALS BY MONOMIALS

1. (a) Calculate 3×38 and 3×62 and add the two answers.

    ....

    ....

    (b) Add 38 and 62, then multiply the answer by 3.

    ....

    (c) If you do not get the same answer for (a) and (b), you have made a mistake. Rework until you get it right.

The fact that if you work correctly, you get the same answer in questions 1(a) and 1(b), is a demonstration of the **distributive property**.

> The distributive property may be described as follows:
> $a(b + c) = ab + ac$ and
> $a(b - c) = ab - ac$,
> where a, b and c can be any numbers.

What you saw in question 1 was that
$3 \times 100 = 3 \times 38 + 3 \times 62$.

This can also be expressed by writing $3(38 + 62) = 3 \times 38 + 3 \times 62$.

2. (a) Calculate 10×56 ....

    (b) Calculate $10 \times 16 + 10 \times 40$ ....

3. (a) Write down any two numbers smaller than 100. Let us call them x and y. Add your two numbers, and multiply the answer by 3.

    ....

    (b) Calculate $3 \times x$ and $3 \times y$ and add the two answers.

    ....

    (c) If you do not get the same answers for (a) and (b) you have made a mistake somewhere. Correct your work.

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4. Complete the table.

x	12	50	5
y	4	30	10
$5x - 5y$			
$5(x - y)$			
$5x + 5y$			
$5(x + y)$			

Performing the instructions $5(x + y)$ is not the only way in which you can find out how much $5(x + y)$ is for any given values of x and y. Instead of doing $5(x + y)$ you may do $5x + 5y$. In this case you will multiply first, and again, before you add.

5. (a) For $x = 10$ and $y = 20$, evaluate $8(x + y)$ by first adding 10 and 20, and then multiplying by 8.

   ....

   (b) Now evaluate $8(x + y)$ by doing $8x + 8y$, in other words first calculate 8×10 and 8×20.

   ....

6. In question 5 you evaluated $8(x + y)$ in two different ways for the given values of x and y. Now also evaluate $20(x - y)$ in two different ways, for $x = 5$ and $y = 3$.

   ....

   ....

7. Use the distributive property in each of the following cases to make a different expression that is equivalent to the given expression.

   (a) $a(b + c)$ (b) $a(b + c + d)$

   ....

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(c) $x(x + 1)$

(d) $x(x^2 + x + 1)$

(e) $x(x^3 + x^2 + x + 1)$

(f) $x^2(x^2 - x + 3)$

What you do in this question is sometimes called "multiplication of a polynomial by a monomial". One may also say that in each case you **expand** the expression, or you write an equivalent expression in **expanded form**.

(g) $2x^2(3x^2 + 2)$

(h) $3x^3(2x^2 + 4x - 5)$

(i) $-2x^4(x^3 - 2x^2 - 4x + 5)$

(j) $a^2b(a^3 - a^2 + a + 1)$

(k) $x^2y^3(3x^2y + xy^2 - y)$

(l) $-2x(x^3 - y^3)$

(m) $2a^2b(3a^2 + 2a^2b^2 + 4b^2)$

(n) $2ab^2(3a^3 - 1)$

8. Expand the parts of each expression and simplify.
   Then evaluate the expression for $x = 5$.

   (a) $5(x - 2) + 3(x + 4)$

   (b) $x(x + 4) - 4 (x + 4)$

   (c) $x(x - 4) + 4(x - 4)$

   (d) $x(x^2 + 3x + 9) - 3(x^2 + 3x + 9)$

   (e) $x(x^2 - 3x + 9) + 3(x^2 - 3x + 9)$

   (f) $x^2(x^2 - 3x + 4) - x(x^3 + 4x^2 + 2x + 3)$

9. Write in expanded form.

   (a) $x(x^2 + 2xy + y^2) + y(x^2 + 2xy + y^2)$

(b) $x^2y(x^2 - 2xy + y^2) - xy^2(2x^2 - 3xy - y^2)$

....

(c) $ab^2c(b^2c^2 - ac) + b^2c^4(a^2 + abc^2)$

....

(d) $p^2q(pq^2 + p + q) + pq(p - q^2)$

....

SQUARES AND CUBES AND ROOTS OF MONOMIALS

1. Evaluate each of the following expressions for $x = 2$, $x = 5$ and $x = 10$.

(a) $(3x)^2$

....

(b) $9x^2$

....

(c) $(2x)^2$

....

(d) $4x^2$

....

(e) $(2x)^3$

....

(f) $8x^3$

....

(g) $(2x + 3x)^2$

....

(h) $(10x - 7x)^2$

....

2. In each case, write an equivalent monomial without brackets.

(a) $(5x)^2$

....

(b) $(5x)^3$

....

(c) $(20x)^2$

....

(d) $(10x)^3$

....

(e) $(2x + 7x)^2$

....

(f) $(20x - 13x)^3$

....

The square root of $16x^2$ is $4x$, because $(4x)^2 = 16x^2$.

3. Write down the square root of each of the following expressions.

(a) $\sqrt{(7x)^2}$

....

(b) $\sqrt{9x^2}$

....

(c) $\sqrt{(20x)^2}$

....

(d) $\sqrt{100x^2}$

....

(e) $\sqrt{(20x - 15x)^2}$

(f) $\sqrt{25x^2}$

....

(g) $\sqrt{(21x - 16x)^2}$

(h) $\sqrt{(5x)^2}$

....

The cube root of $64x^3$ is $4x$, because $(4x)^3 = 64x^3$

4. Write down the cube root of each of the following expressions.

(a) $\sqrt[3]{(7x)^3}$

(b) $\sqrt[3]{27x^3}$

....

(c) $\sqrt[3]{(20x)^3}$

(d) $\sqrt[3]{1\,000x^3}$

....

(e) $\sqrt[3]{(20x - 15x)^3}$

(f) $\sqrt[3]{125x^3}$

....

8.5 Dividing polynomials by integers and monomials

1. Complete the table.

x	20	10	5	–5	–10	–20
$(100x - 5x^2) \div 5x$						
$20 - x$						

Can you explain your observations?

....

....

2. (a) R240 prize money must be shared equally between 20 netball players. How much should each one get?

....

(b) Mpho decided to do the calculations below. Do not do Mpho's calculations, but think about this: Will Mpho get the same answer that you got for question (a)?

$(140 \div 20) + (100 \div 20)$ ....

(c) Gert decided to do the calculations below. Without doing the calculations, say whether Gert will get the same answer that you got for question (a).

$(240 \div 12) + (240 \div 8)$ ....

3. Do the necessary calculations to find out whether the following statement are true or false:
   (a) $(140 + 100) \div 20 = (140 \div 20) + (100 \div 20)$

   ....
   (b) $240 \div (12 + 8) = (240 \div 12) + (240 \div 8)$

   ....
   (c) $(300 - 60) \div 20 = (300 \div 20) - (60 \div 20)$

   ....

> Division is **right-distributive** over addition and subtraction, for example, $(2 + 3) \div 5 = (2 \div 5) + (3 \div 5)$.
> The division symbol is to the right of the brackets.
> But it is not left-distributive, for example, $10 \div (2 + 4) \neq (10 \div 2) + (10 \div 4)$.

For example $(200 + 40) \div 20 = (200 \div 20) + (40 \div 20) = 10 + 2 = 12$, and
$(500 + 200 - 300) \div 50 = (500 \div 50) + (200 \div 50) - (300 \div 50)$

4. Evaluate each expression for $x = 2$ and $x = 10$
   (a) $(10x^2 + 5x) \div 5$ ....
   ....
   (b) $(10x^2 \div 5) + (5x \div 5)$ ....
   ....
   (c) $2x^2 + x$
   ....
   (d) $(10x^2 + 5x) \div 5x$ ....
   ....
   (e) $(10x^2 \div 5x) + (5x \div 5x)$ ....
   ....
   (f) $2x + 1$
   ....

The distributive property of division can be expressed like this:

> $(x + y) \div z = (x \div z) + (y \div z)$
> $(x - y) \div z = (x \div z) - (y \div z)$

5. (a) Do not do any calculations. Which of the following expressions do you *think* will have the same value as $(10x^2 + 20x - 15) \div 5$, for $x = 10$ as well as $x = 2$?

   $2x^2 + 20x - 15$ $\quad$ $10x^2 + 20x - 3$ $\quad$ $2x^2 + 4x - 3$ ....
   (b) Do the necessary calculations to check your answer.

   ....

6. Simplify:

(a) $(2x + 2y) \div 2$

(b) $(4x + 8y) \div 4$

(c) $(20xy + 16x) \div 4x$

(d) $(42x - 6) \div 6$

(e) $(28x^4 - 7x^3 + x^2) \div x^2$

(f) $(24x^2 + 16x) \div 8x$

(g) $(30x^2 - 24x) \div 3x$

7. Simplify:

(a) $(9x^2 + xy) \div xy$

(b) $(48a - 30ab + 16ab^2) \div 2a$

(c) $(3a^3 + a^2) \div a^2$

(d) $(13a - 17ab) \div a$

(e) $(3a^2 + 5a^3) \div a$

(f) $(39a^2b + 13ab + ab^2) \div ab$

The instruction $72 \div 6$ may also be written as $\frac{72}{6}$.

This notation, which looks just like the common fraction notation, is often used to indicate division.

Hence, instead of $(10x^2 + 20x - 15) \div 5$ we may write $\frac{10x^2 + 20x - 15}{5}$.

Since $(10x^2 + 20x - 15) \div 5$ is equivalent to $(10x^2 \div 5) + (20x \div 5) - (15 \div 5)$, $\frac{10x^2 + 20x - 15}{5}$ is equivalent to $\frac{10x^2}{5} + \frac{20x}{5} - \frac{15}{5}$.

8. Find a simpler equivalent expression for each of the following expressions (clearly, these expressions do not make sense if $x = 0$).

(a) $\frac{16x^2 - 12x}{4x}$

(b) $\frac{16x^3 - 12x}{4x}$

(c) $\frac{16x^3 - 12x^2}{4x}$

(d) $\frac{16x^3 - 12x^2}{4x^2}$

(e) $\frac{16x^3 - 12x^2}{2x}$

(f) $\frac{16x^3 - 12x}{8x}$

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9. In each case check whether the statement is true for $x = 10$; $x = 100$; $x = 5$; $x = 1$ and $x = -2$.

(a) $\frac{x^2}{x} = x$ ....

(b) $\frac{x^3}{x} = x^2$ ....

(c) $\frac{x^3}{x^2} = x$ ....

(d) $\frac{5x^3}{x} = 5x^2$ ....

(e) $\frac{5x^3}{x} = 5^3$ ....

(f) $\frac{5x}{x^2} = \frac{5}{x}$ ....

10. Explain why the equations below are true:

(a) $\frac{100x - 5x^2}{5x} = 20 - x$ for all values of x except $x = 0$

....

(b) $\frac{15x^2 - 10x}{5x}$ is equivalent to $3x - 2$, excluding $x = 0$.

....

11. Complete the table:

x	1,5	2,8	–3,1	0,72
$\frac{3x+12}{3}$				
$\frac{18x^2+6}{6}$				
$\frac{5x^2+7x}{x}$				

(Hint: Simplify the expressions first to save yourself some work!)

12. Simplify each expression to the equivalent form requiring the fewest operations.

(a) $\frac{3a + a^2}{a}$ ....

(b) $\frac{x^3 + 2x^2 - x}{x}$ ....

(c) $\frac{2a + 12ab}{2a}$ ....

(d) $\frac{12x^2 + 10x}{2x}$ ....

(e) $\frac{21ab - 14a^2}{7a}$ ....

(f) $\frac{15a^2b + 30ab^2}{5ab}$ ....

(g) $\frac{7x^3 + 21x^2}{7x^2}$ ....

(h) $\frac{3x^2 + 9x}{3x}$ ....

13. Solve the equations.

(a) $\frac{3x^2 + 15x}{3x} = 20$

....

....

(b) $\frac{30x - 18x^2}{6x} = 2$

....

....

14. Complete the table.

	x	1,1	1,2	1,3	1,4	1,5
(a)	$\frac{x^3+2x^2-x}{x}$					
(b)	$\frac{7x^3+21x^2}{7x^2}$					
(c)	$\frac{50x^2+5x}{5x}$					

15. Simplify the following expressions.

(a) $\frac{3x(5x+4)+6x(5x+3)}{5x}$

(b) $\frac{14x^2-28x}{7x}+\frac{24x-18x^2}{3x}$

....

....

....

8.6 Products and squares of binomials

How can we obtain the expanded form of $(x + 2)(x + 3)$?

In order to expand $(x + 2)(x + 3)$, you can first keep $(x + 2)$ it is, and apply the distributive property:

$(\mathbf{x + 2})(x + 3)$
$= (\mathbf{x + 2})x + (\mathbf{x + 2})3$
$= x^2 + 2x + 3x + 6$
$= x^2 + 5x + 6$

1. Describe how can you check whether $(x + 2)(x + 3)$ is actually equivalent to $x^2 + 5x + 6$.

....

....

To expand $(x - y)(x + 3y)$ it can be written as $(x - y)x + (x - y)3y$ and the two parts can then be expanded.

$(x - y)(x + 3y)$
$= (x - y)x + (x - y)3y$
$= x^2 - xy + 3xy - 3y^2$
$= x^2 + 2xy - 3y^2$

2. Do some calculations to check whether $(x - y)(x + 3y)$ and $x^2 + 2xy - 3y^2$ are equivalent. Write the results of your calculations in the table below.

x					
y					

3. Expand each of these expressions.

(a) $(x + 3)(x + 4)$

....

....

....

(b) $(x + 3)(4 - x)$

....

....

....

(c) $(x + 3)(x - 5)$

....

....

....

(d) $(2x^2 + 1)(3x - 4)$

....

....

....

(e) $(x + y)(x + 2y)$

....

....

....

(f) $(a - b)(2a + 3b)$

....

....

....

(g) $(k^2 + m)(k^2 + 2m)$

....

....

....

(h) $(2x + 3)(2x - 3)$

....

....

....

(i) $(5x + 2)(5x - 2)$

....

....

....

(j) $(ax - by)(ax + by)$

....

....

....

4. Expand each of these expressions.

(a) $(a + b)(a + b)$

....

(b) $(a - b)(a - b)$

....

(c) $(x + y)(x + y)$

(d) $(x - y)(x - y)$

(e) $(2a + 3b)(2a + 3b)$

(f) $(2a - 3b)(2a - 3b)$

(g) $(5x + 2y)(5x + 2y)$

(h) $(5x - 2y)(5x - 2y)$

(i) $(ax + b)(ax + b)$

(j) $(ax - b)(ax - b)$

5. Can you guess the answer to each of the following questions without working it out as you did in question 3? Try them out and then check your answers.
Expand these expressions:

(a) $(m + n)(m + n)$

(b) $(m - n)(m - n)$

(c) $(3x + 2y)(3x + 2y)$

(d) $(3x - 2y)(3x - 2y)$

All the expressions in questions 4 and 5 are **squares of binomials**, for example $(ax + b)^2$ and $(ax - b)^2$

6. Expand:

(a) $(ax + b)^2$

(b) $(ax - b)^2$

(c) $(2s + 5)^2$

(d) $(2s - 5)^2$

(e) $(ax + by)^2$

(f) $(ax - by)^2$

(g) $(2s + 5r)^2$

(h) $(2s - 5r)^2$

7. Expand and simplify.

(a) $(4x + 3)(6x + 4) + (3x + 2)(8x + 5)$

(b) $(4x + 3)(6x + 4) - (3x + 2)(8x + 5)$

8.7 Substitution into algebraic expressions

1. In question 2 you have to find the values of different expressions, for some given values of x. Look carefully at the different expressions in the table. Do you think some of them may be equivalent?

   Simplify the longer expression to check whether you end up with the shorter expression.

2. Complete the table.

	x	13	–13	2,5	10
(a)	$(2x + 3)(3x - 5)$				
(b)	$10x^2 + 5x - 7 + 3x^2 - 4x - 3$				
(c)	$3(10x^2 - 5x + 2) - 5x(6x - 4)$				
(d)	$13x^2 + x - 10$				
(e)	$6x^2 - x - 15$				
(f)	$5x + 6$				

3. Complete this table.

	x	1	2	3	4
(a)	$(2x + 3)(5x - 3) + (10x + 9)(1 - x)$				
(b)	$\frac{9x^2 + 30x}{3x}$				
(c)	$3x(10x - 5) - 5x(6x - 4)$				
(d)	$5x(4x + 3) - 2x(7 + 13x) + 2x(3x + 2)$				

4. Describe any patterns that you observe in your answers for question 3.

....

....

....

5. Complete this table.

	x	1,5	2,5	3,5	4,5
(a)	$(2x + 3)(5x - 3) + (10x + 9)(1 - x)$				
(b)	$\frac{9x^2 + 30x}{3x}$				
(c)	$3x(10x - 5) - 5x(6x - 4)$				
(d)	$5x(4x + 3) - 2x(7 + 13x) + 2x(3x + 2)$				