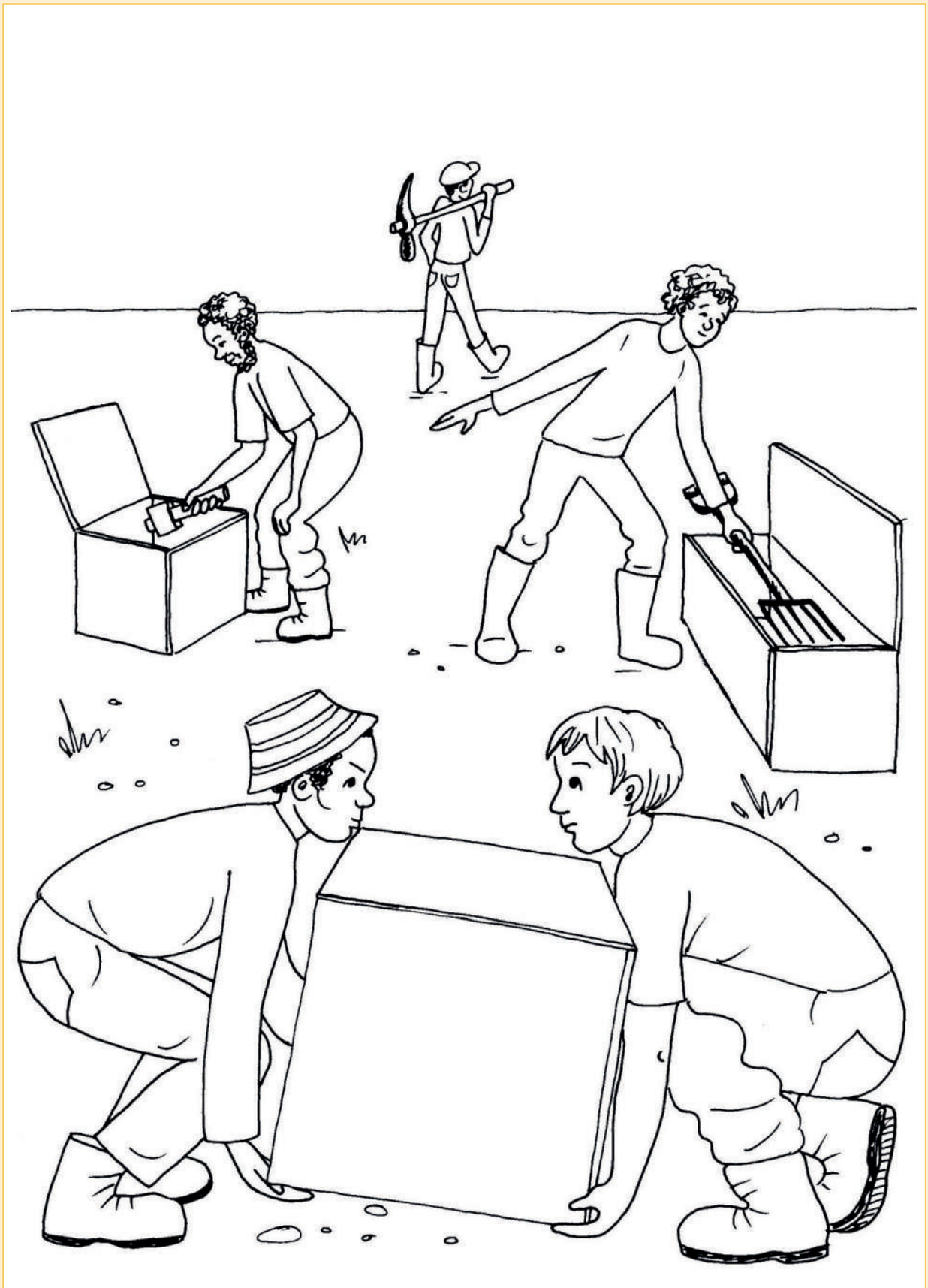
Chapter 5
Surface area and volume of 3D objects

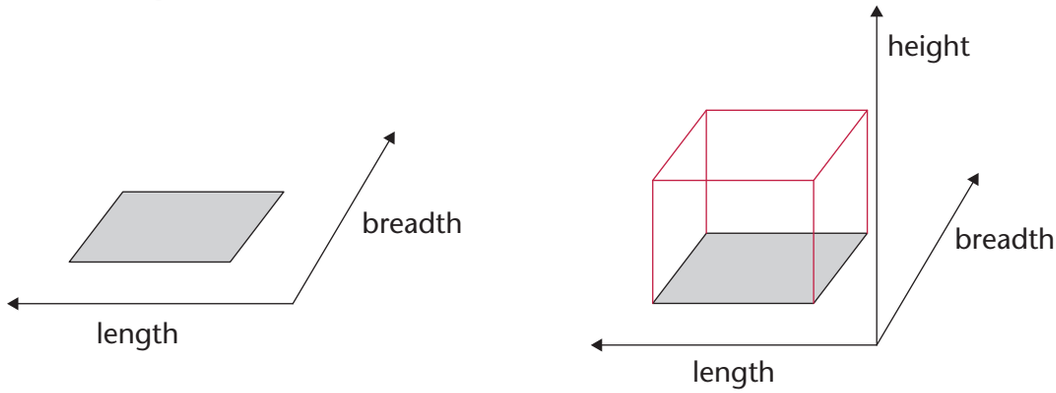
The surface area of an object is the size of the flat surfaces all around the object. The volume of an object is the amount of space that the object takes up. In this chapter, you will use formulae to calculate the volumes and surface areas of cubes, rectangular prisms and triangular prisms. You will also investigate the relationship between surface area and volume, as well as revise how to convert between the different units used to measure volume.



5 Surface area and volume of 3D objects

5.1 From 2D to 3D measurements

Remember that 2D shapes have only length and breadth, while 3D objects have length, breadth and height.



2D shape *3D object*

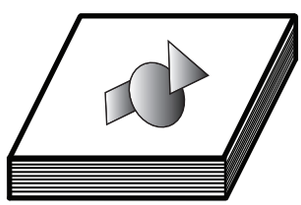
A 2D shape has only one surface. We call the size of this flat surface the **area** of the shape.

A 3D object has more than one surface. For example, a cube has 6 surfaces, or faces. The sizes of these surfaces on the outside of the 3D object are called its **surface area**.

A 2D shape is flat, so it takes up space in only two directions. But a 3D object has height as well, so it takes up space in a third direction also. The space that a 3D object takes up is called its **volume**.

INVESTIGATING THE SURFACE AREA AND VOLUME OF A BOOK

Work with a partner. Choose a book each. The books must be different sizes.



1. Run your hand over all the outside surfaces of your book. How many surfaces (or faces) does your book have?

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2. Estimate whether the surface area of your book is bigger or smaller than that of your partner's book.

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3. If you were to cover the book with wrapping paper, explain how you would calculate the minimum size of paper you would need.

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4. Estimate whose book takes up the most space. How could you calculate which book really takes up the most space?

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5.2 Surface area of 3D objects

USING NETS TO EXPLORE SURFACE AREA

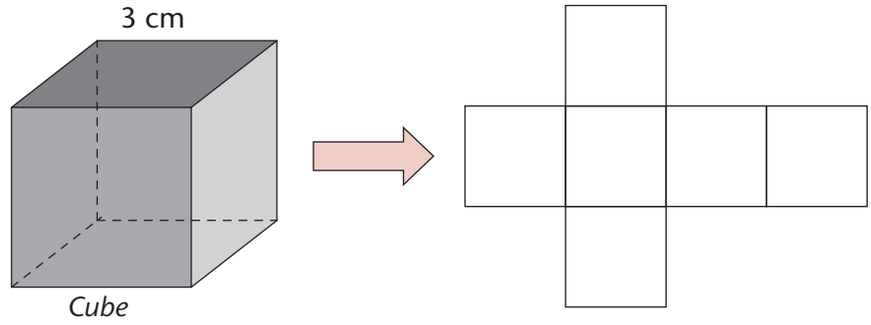
The **surface area** of an object is equal to the sum of the areas of all its faces. So we can use the net of an object to investigate its surface area.

A net is a flat shape that can be folded to make a 3D object.

The diagrams below show 3D objects with their matching nets.

1. Use the measurements given to calculate the area of each face shown by the net. (Use your calculator if necessary and round off to two decimal places.)
2. Add up the areas to calculate the surface area of each object.

A



Cube

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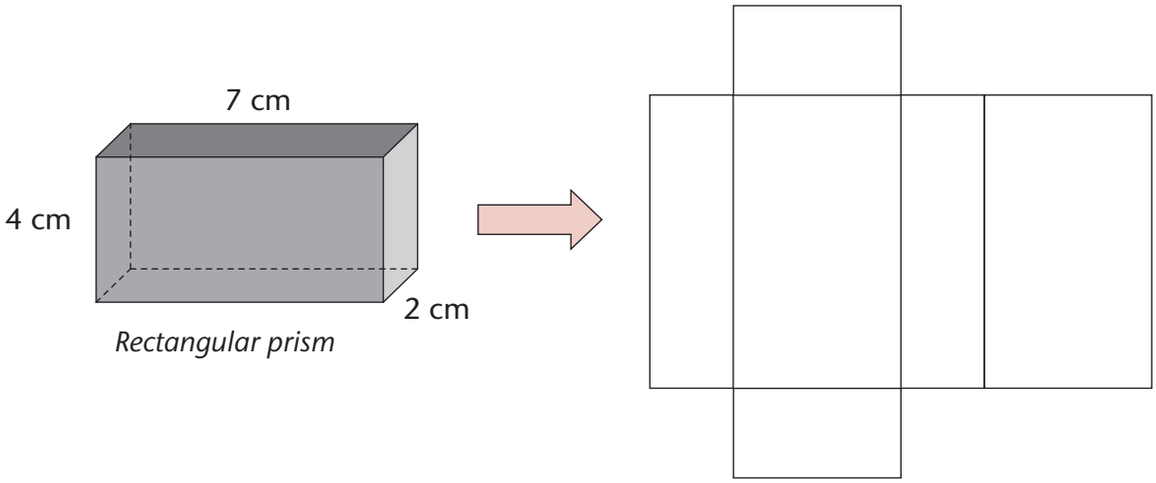
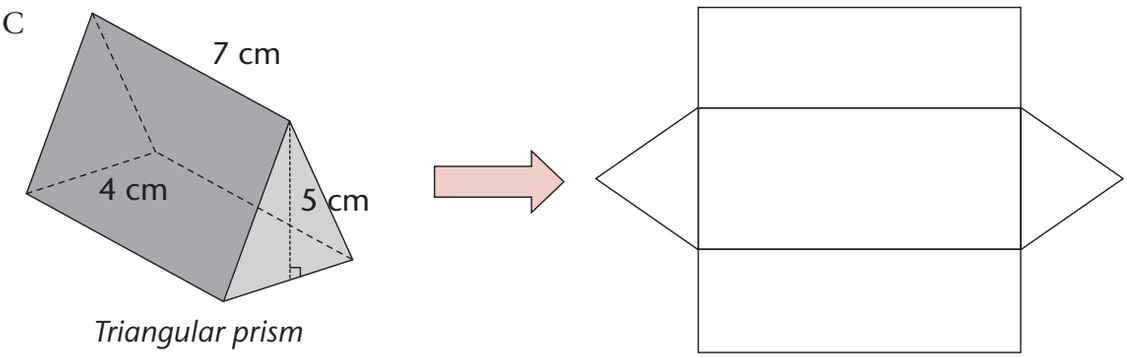


Rectangular prism

C

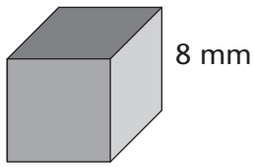


Triangular prism

DEDUCING FORMULAE FOR SURFACE AREAS

The surface area of a prism = the sum of the areas of all its faces

1. (a) Use the general formula above and the work you did on the cube on page 74 to determine which of the following formulae are correct. Tick the correct one(s).



- ☐ Surface area of a cube = $4 \times s$
- ☐ Surface area of a cube = $s \times s \times s \times s$
- ☐ Surface area of a cube = $6 \times s^2$
- ☐ Surface area of a cube = s^6

(b) Explain your choice above.

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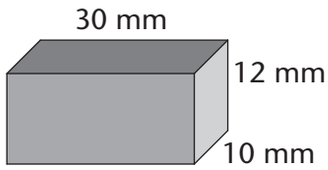
2. (a) Write a formula for the surface area of any rectangular prism.



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(b) Explain your formula.

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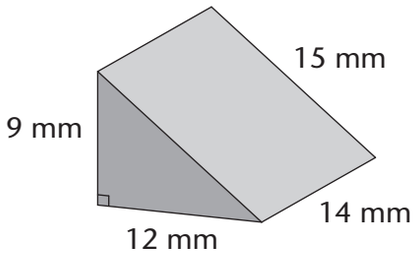
3. (a) Write a formula for the surface area of any triangular prism.



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(b) Explain your formula.

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4. Use the formulae in questions 1 to 3 to calculate the surface areas of the cube, rectangular prism and triangular prism shown in questions 1 to 3.

Surface area of cube:

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Surface area of rectangular prism:

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Surface area of triangular prism:

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SURFACE AREA CALCULATIONS

Work out the surface areas of the following four objects.
Give all answers in cm^2.

Remember:

1 cm^2 = 100 mm^2	1 mm^2 = 0,01 cm^2
1 m^2 = 10 000 cm^2	1 cm^2 = 0,0001 m^2
1 km^2 = 1 000 000 m^2	1 m^2 = 0,000001 km^2

It may be a good idea to sketch the net for each object before doing the calculations.

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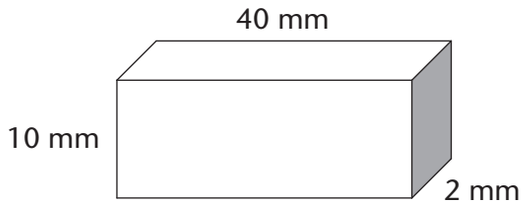

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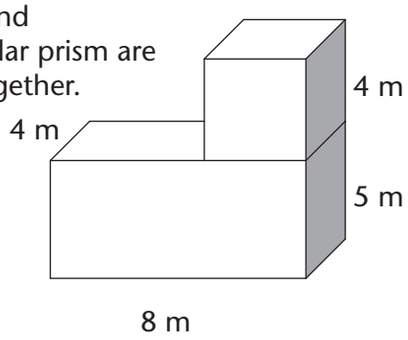
A cube and rectangular prism are glued together.



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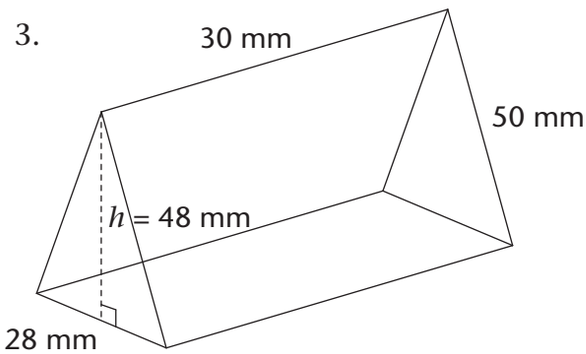

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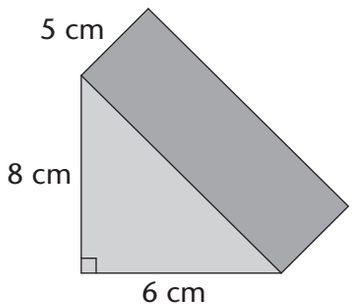

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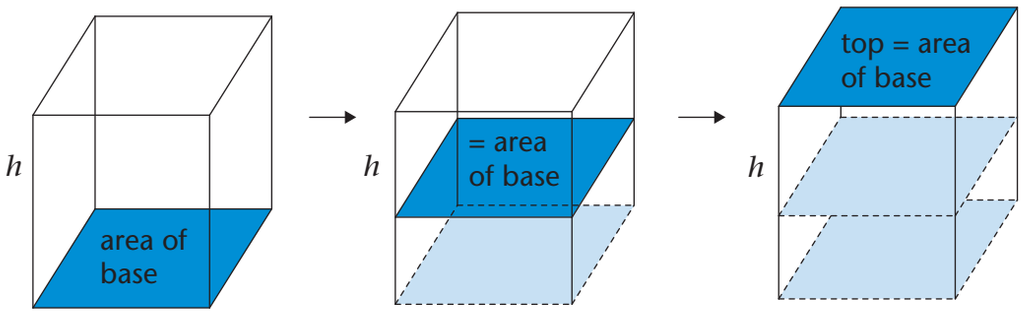
5.3 Volume of 3D objects

DERIVING FORMULAE TO CALCULATE VOLUME

Think of a prism and its base. If you were to move the base up to the top, between the lateral faces of the prism, the area of the base would remain exactly the same.

Lateral faces are faces that aren't bases.

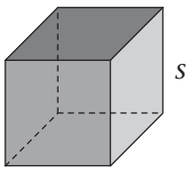


The volume of a prism = Area of base × height

Volume is the amount of space that an object takes up.

Use this general formula above to write the formula for the volume of a cube, a rectangular prism and a triangular prism.

A. Cube



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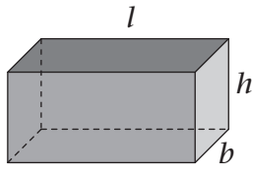
B. Rectangular prism



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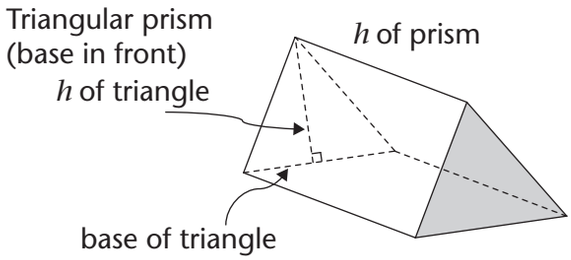
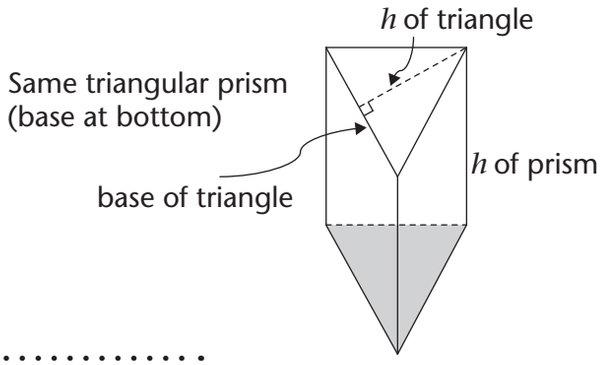
Note about triangular prism

Do not get confused between:

- the base of the prism and the base of the triangular face of the prism
- the height of the prism and the height of the triangular face of the prism.

C. Triangular prism





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You should have found the following volume formulae:

Volume of a cube = s^3 or $s \times s \times s$

Volume of a rectangular prism = $l \times b \times h$

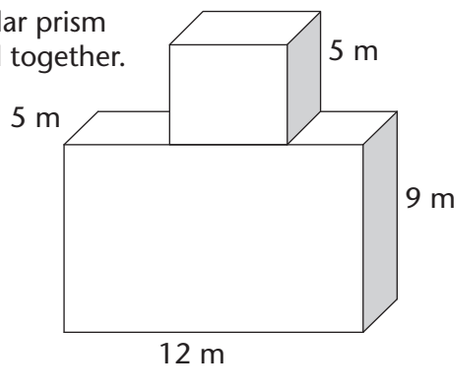
Volume of a triangular prism = $\frac{1}{2}$ (base × h) × height of prism

Because we multiply three dimensions, the units used are cubic units, such as mm^3, cm^3 or m^3.

VOLUME CALCULATIONS

Calculate the volume of the following objects using the formulae given above.

1. A cube and rectangular prism are glued together.



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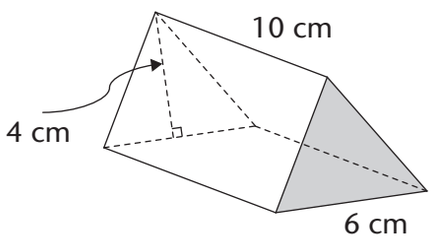

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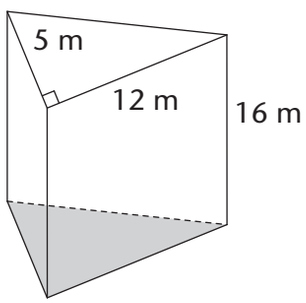

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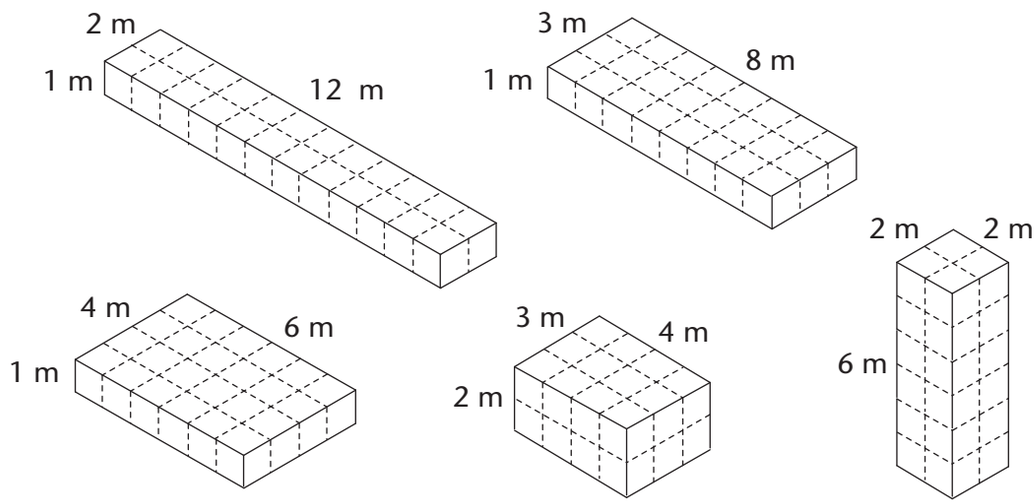
5.4 Relationship between surface area and volume

Do objects with the same volume always have the same surface area? Do the investigation below in order to find out.

1. (a) Calculate the surface area and volume of the following five rectangular prisms by completing the table below.



Length (m)	Breadth (m)	Height (m)	Surface area (m^2)	Volume (m^3)
12	2	1		
8	3	1		
6	4	1		
4	3	2		
2	2	6		

(b) In the last row of the table, write another set of dimensions (l, b and h) that will give the same volume but a different surface area as the ones already recorded.

2. Look at the completed table. What can you conclude about the surface area and volume of objects?

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3. A rectangular prism has a volume of 8 m^3. Write down two possible sets of dimensions. Draw the prisms below with their dimensions written on the drawings.

4. The following table shows surface area and volume calculations for cubes with different side lengths.

Side length of cube (m)	Surface area (m^2)	Volume (m^3)
1	6	1
2	24	8
3	54	27
5	150	125
8	384	512
10	600	1 000

(a) Look at the surface area column. Does the surface area increase or decrease as the side length of the cube increases?

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(b) Look at the volume column. Does the volume increase or decrease as the side length of the cube increases?

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(c) Does *volume* or *surface area* increase more rapidly when the side length of the cube increases?

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(d) Sketch a global graph of the volume of a cube versus its surface area.

5.5 Converting between cubic units

HOW MANY CUBES?

1. The small cube below has the dimensions 1 cm × 1 cm × 1 cm and a volume of 1 cm^3. How many 1 cm^3 cubes will you need to form a large cube with dimensions 10 cm × 10 cm × 10 cm like the one shown below?

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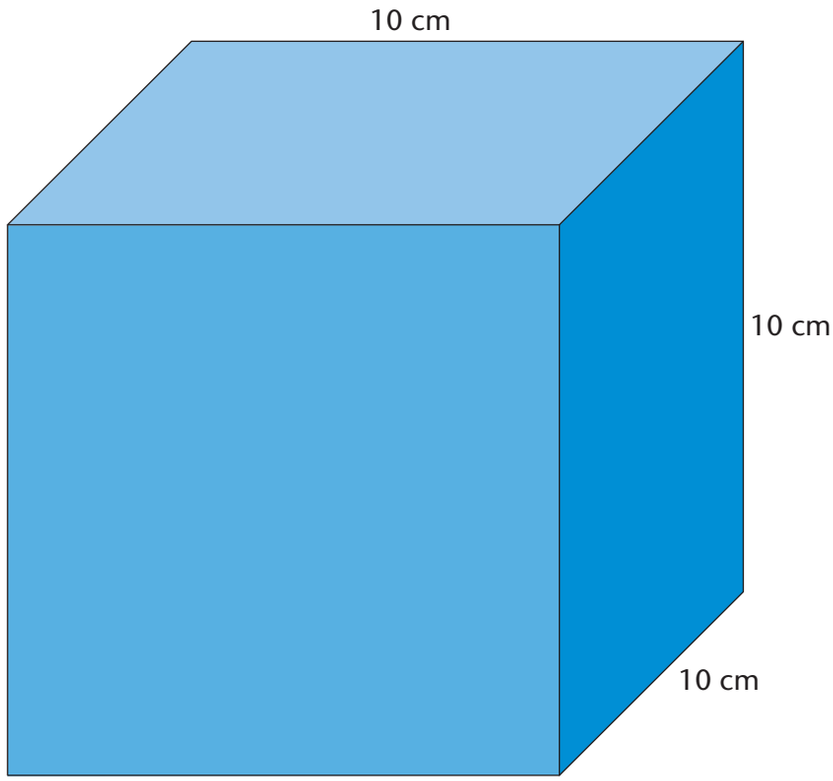

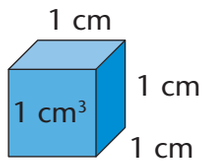

2. How many 10 cm × 10 cm × 10 cm cubes will form a 100 cm × 100 cm × 100 cm cube?

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3. (a) To form a 1 000 cm^3 cube you need 1 000 cubes with a volume of 1 cm^3. If cubes of 1 000 cm^3 (10 cm × 10 cm × 10 cm) are then used to form a cube of 100 cm × 100 cm × 100 cm, how many 1 000 cm^3 cubes will there be?

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(b) What is the volume of this new cube? .....

(c) How many cubes of 1 cm^3 will form a cube with a volume of 1 000 000 cm^3?

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4. Which of the cubes given below has a bigger volume? Explain.
   A. A cube with a volume of 1 m^3
   B. A cube with a volume of 1 000 000 cm^3

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5. (a) How many 1 mm × 1 mm × 1 mm cubes (1 mm^3) are needed to form a 1 cm × 1 cm × 1 cm cube?

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   (b) What is the total volume of the 1 mm^3 cubes forming the 1 cm^3 cube?

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PRACTISE CONVERTING BETWEEN UNITS

When working with volume, you often have to convert between different cubic units. Here are two examples of how you can work out equivalent units.

Converting cm^3 to mm^3:

$1\ cm^3 = 1\ cm \times 1\ cm \times 1\ cm$
$= 10\ mm \times 10\ mm \times 10\ mm$
$= 1\ 000\ mm^3$

$\therefore$ multiply by 1 000

Converting cm^3 to m^3:

$1\ cm^3 = 1\ cm \times 1\ cm \times 1\ cm$
$= 0{,}01\ m \times 0{,}01\ m \times 0{,}01\ m$
$= 0{,}000001\ m^3$

$\therefore$ multiply by 0,000001 or divide by 1 000 000

1. Write the following volumes in cm^3.
   (a) 3 mm^3 (b) 45 mm^3

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   (c) 0,6 m^3 (d) 1,22 m^3

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2. Write the following volumes in mm^3.
   (a) 20 cm^3 (b) 151 cm^3

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   (c) 4,7 cm^3 (d) 89,5 cm^3

....

3. Write the following volumes in m^3.
   (a) 9 cm^3 (b) 50 cm^3

   . . . . . .

   (c) 643 cm^3 (d) 1 967 cm^3

   . . . . . .

4. Write the following answers in cm^3.
   (a) 4 m^3 + 68 cm^3 (b) 12 m^3 + 143 cm^3

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5.6 Capacity of 3D objects

DIFFERENCE BETWEEN CAPACITY AND VOLUME

Capacity is the amount of space available *inside* an object.

Volume is the amount of space that the object itself takes up.

1. A solid block of wood measures 30 cm × 20 cm × 10 cm.
   (a) What is its volume?

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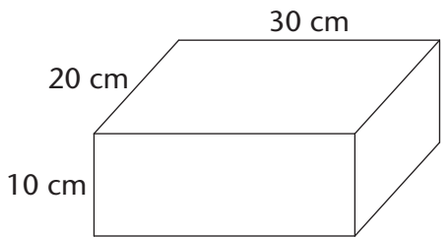

   The same solid block of wood is carved out to make a hollow container. The measurements inside the container are 25 cm × 15 cm × 8 cm.
   (b) How thick are the walls of the container?

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   (c) What is the capacity of the container?

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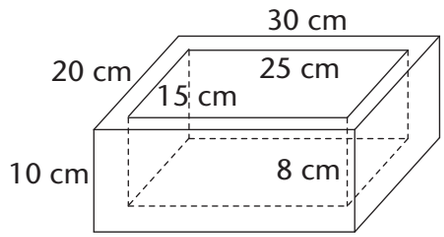

   (d) If you filled the container with water, what volume of water would the container hold?

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2. More of the wood is carved out of the container to make walls 1 cm thick at the sides and the bottom. Calculate the capacity of the container in litres.

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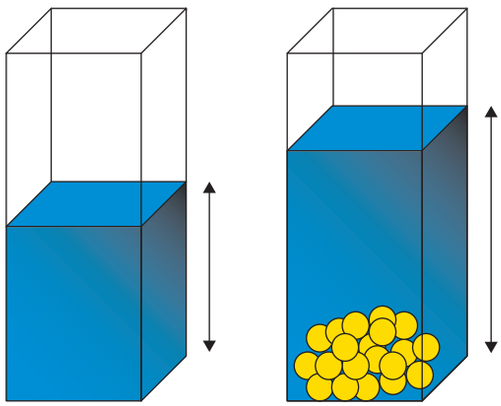
DISPLACEMENT AND MORE CAPACITY CALCULATIONS

Consider a glass vase half full of water. As soon as you place marbles into the water, the level of the water rises. This is not because the amount of water has changed, but rather because the marbles have taken the place of the water and have pushed the water higher up in the vase.



If one of the marbles has a volume of 1 cm^3, it would displace 1 ml of water.

$\therefore$ We know that:

$1\ cm^3 = 1\ ml$

$1\ m^3 = 1\ kl$

Displace means to move something out of its place.

1. Calculate the capacities of the following containers. The *inside* measurements are given. Write your answers in ml or kl.

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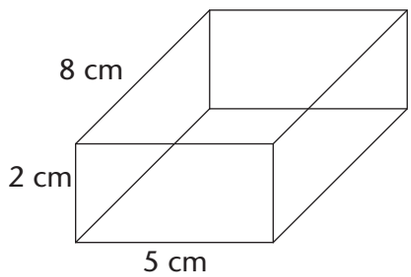
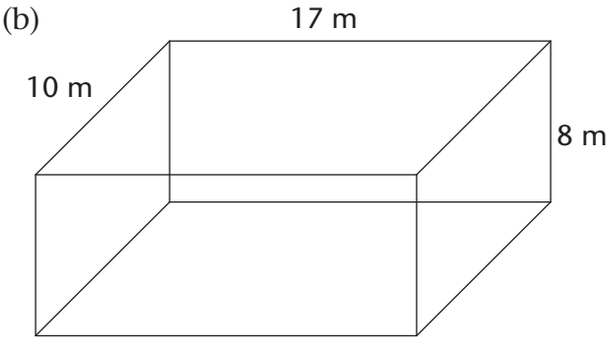


(b)



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2. Work out a possible set of inside measurements for a container with a capacity of 12 kl. Draw a sketch and write the measurements on it.