Chapter 3
The theorem of Pythagoras

Right-angled triangles have a special feature that does not apply to other types of triangles. In this chapter, you will investigate this feature, which has come to be known as the theorem of Pythagoras. A theorem is a statement that is proved to be true through reasoning. Once you understand the theorem, you will practise applying it in various ways.

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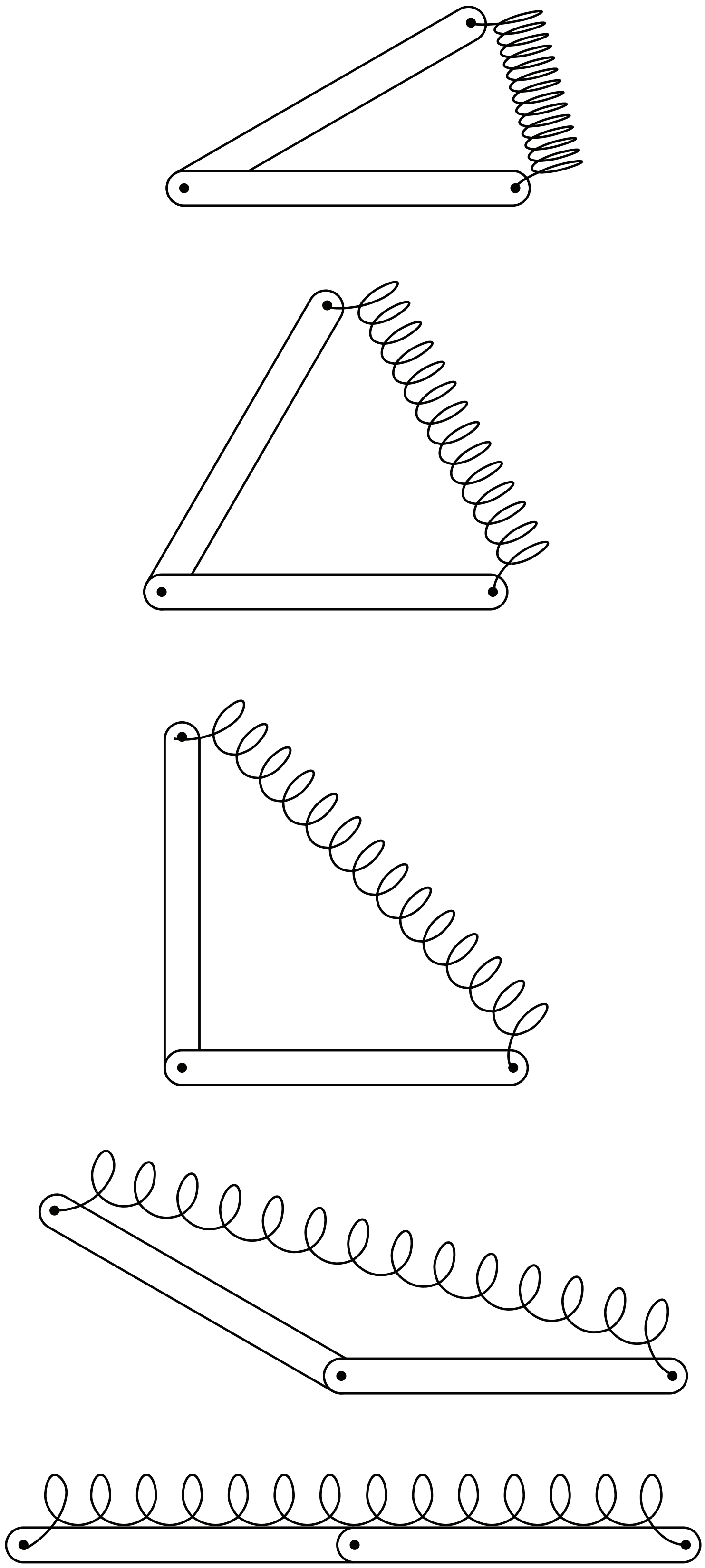

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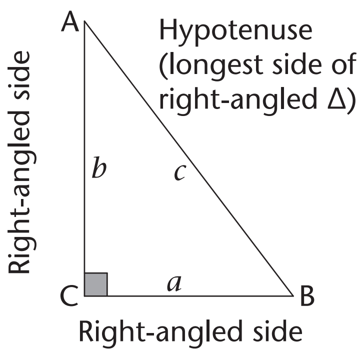
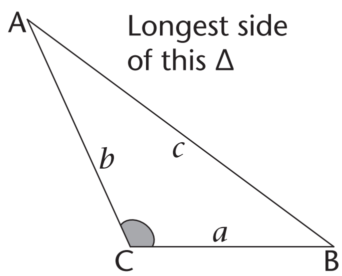
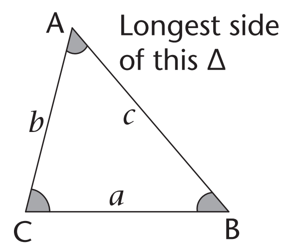
3 The theorem of Pythagoras

3.1 The lengths of sides of right-angled triangles

WHAT DO YOU REMEMBER ABOUT TRIANGLES?

Right-angled triangle (Δ) One angle is 90°.	Obtuse-angled triangle (Δ) One angle is obtuse (between 90° and 180°).	Acute-angled triangle (Δ) All angles are acute (less than 90°).
		

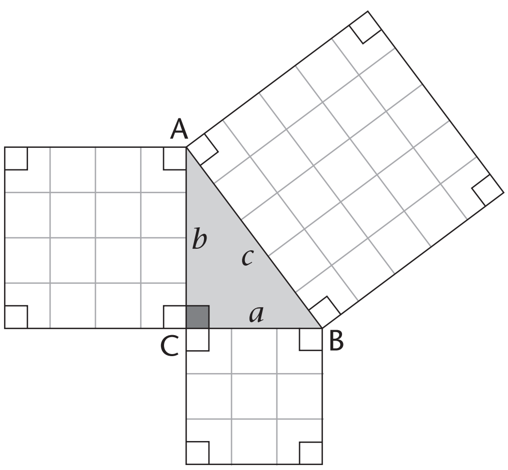
If the vertices of a triangle are labelled A, B and C, the sides opposite these vertices are often labelled as *a*, *b* and *c*, as shown in the above diagrams.

We use the word **hypotenuse** to indicate the side opposite the 90° angle of a right-angled triangle. The hypotenuse is always the longest side of a right-angled triangle. A triangle with no right angle does not have a hypotenuse.

INVESTIGATING THE RELATIONSHIP BETWEEN THE LENGTHS OF SIDES

1. Study the figures below. Each triangle in the following four figures has a square drawn on each of its sides. So, in figure (a), $a = 3$ units, $b = 4$ units and $c = 5$ units long.

(a)



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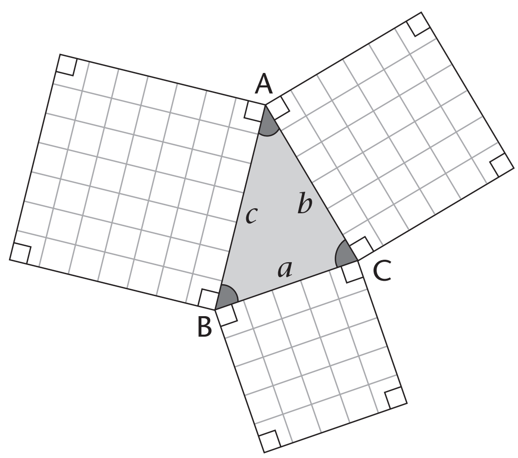

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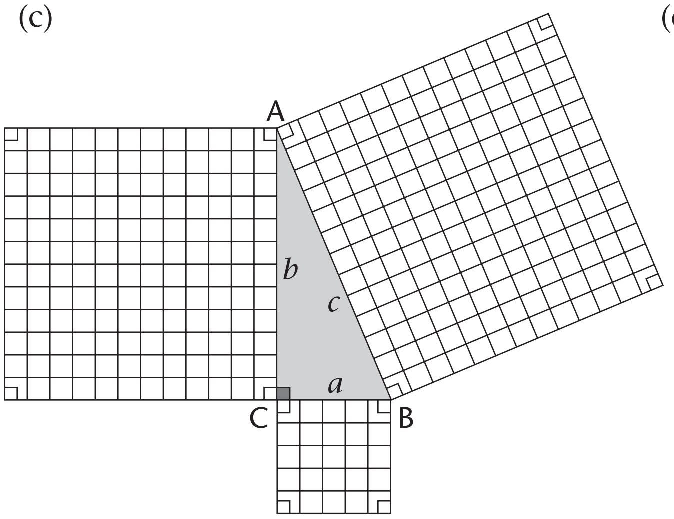
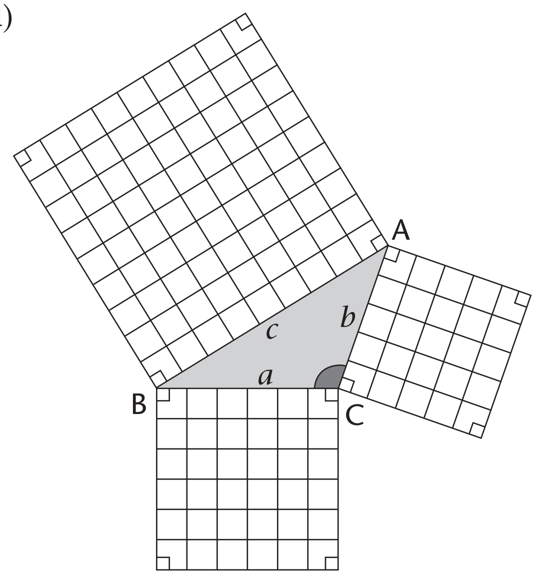
(c)



(d)



2. Refer to the four figures above to complete the table.

Figure	Type of triangle	Length of side a	Length of side b	Length of side c	a^2	b^2	c^2
(a)							
(b)							
(c)							
(d)							

3. Look at your completed table and then insert =, > or < in the following statements.

$a^2 + b^2$ $\square$ c^2 when ΔABC is an acute-angled triangle.

$a^2 + b^2$ $\square$ c^2 when ΔABC is an obtuse-angled triangle.

$a^2 + b^2$ $\square$ c^2 when ΔABC is a right-angled triangle.

4. Which of the statements below are correct? ....

   A. In any right-angled triangle, the area of the square on the hypotenuse is equal to the sum of the areas of the squares on the other two sides.
   B. If a triangle is acute-angled, then the square of the length of the longest side is equal to the sum of the squares of the lengths of the other two sides.
   C. If a triangle is right-angled, then the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other two sides.
   D. In any obtuse-angled triangle, the area of the square on the longest side is equal to the sum of the area of the squares on the other two sides.

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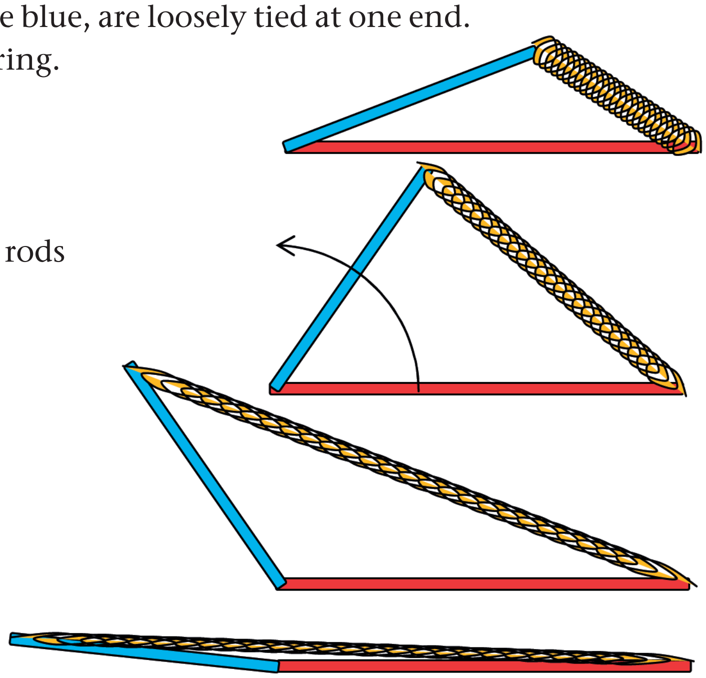
5. The following table gives the side lengths *a*, *b* and *c* of 10 triangles. Complete the table to decide what type of triangle each triangle is (acute-angled, obtuse-angled or right-angled).

a	b	c	$a^2 + b^2$	c^2	Fill in =, < or >	Type of triangle
7	8	10	$7^2 + 8^2 = 113$	$10^2 = 100$	$a^2 + b^2 > c^2$	Acute-angled
4	5	8	$4^2 + 5^2 = 41$	$8^2 = 64$	$a^2 + b^2 < c^2$	Obtuse-angled
6	8	10	$6^2 + 8^2 = 100$		$a^2 + b^2 = c^2$	Right-angled
8	13	17			$a^2 + b^2 \quad c^2$	
3	4	5			$a^2 + b^2 \quad c^2$	
5	6	7			$a^2 + b^2 \quad c^2$	
5	12	13			$a^2 + b^2 \quad c^2$	
15	8	17			$a^2 + b^2 \quad c^2$	
11	60	61			$a^2 + b^2 \quad c^2$	
12	35	37			$a^2 + b^2 \quad c^2$	

6. Two pieces of wood, one red and one blue, are loosely tied at one end. The two free ends are linked by a spring.



The angle between the two wooden rods can be changed.

Describe how this angle affects the length of the spring.

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3.2 Working with the theorem of Pythagoras

The special relationship between the lengths of the sides of a right-angled triangle is known as the **theorem of Pythagoras**. It can be stated in terms of area as follows:

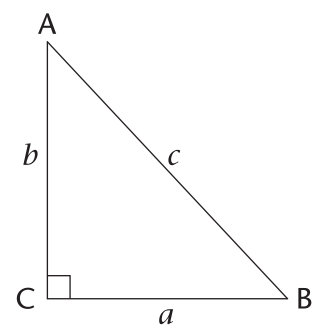
> If a triangle has a right angle, then the area of the square with a side on the hypotenuse is equal to the sum of the areas of the squares on the other two sides.

The reference to area can be left out.

> If a triangle is a right-angled triangle, then the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other two sides.

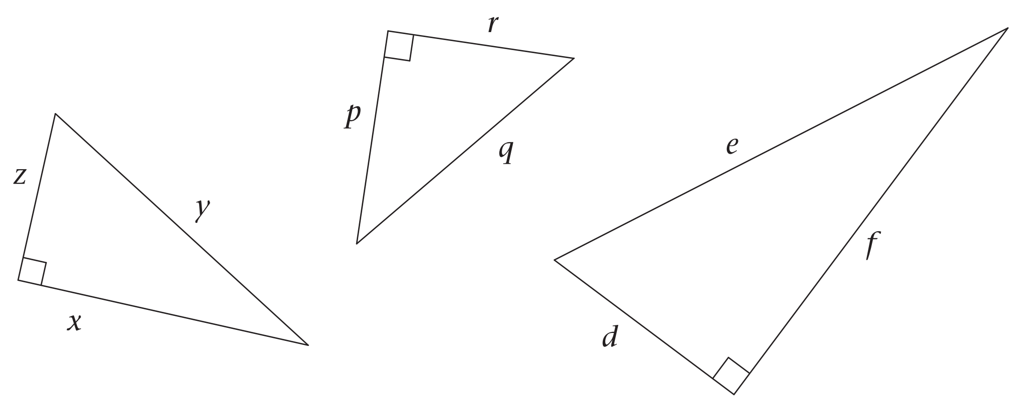
We can express the relationship between the lengths of the sides of the triangle by means of the equation $c^2 = a^2 + b^2$, where c represents the length of the hypotenuse and a and b represent the lengths of the other two sides.

A note about Pythagoras
Pythagoras lived in about 500 BCE. The theorem is named after Pythagoras because he may have been the first person to prove it. However, the theorem was known and used in other parts of the world such as Egypt 1 200 years before Pythagoras was born.



WORKING WITH THE FORMULA

1. Write a Pythagorean equation for each of the following triangles. Explain what each letter symbol represents.



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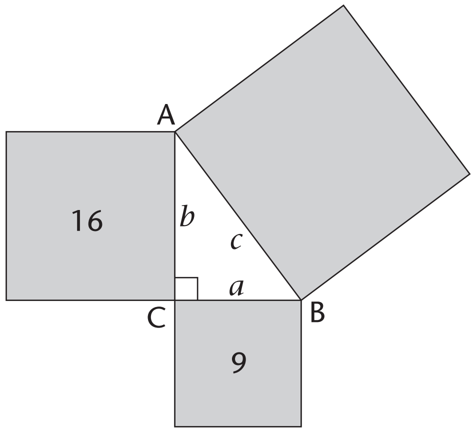
2. Study the worked example below.

Example

Consider the triangle below. Side a is 3 units long and side b is 4 units long. What is the length of side c?



If side a is 3 units long, and side b is 4 units long, then, according to Pythagoras' theorem:

$c^2 = a^2 + b^2$

$c^2 = 3^2 + 4^2$

$c^2 = 9 + 16$

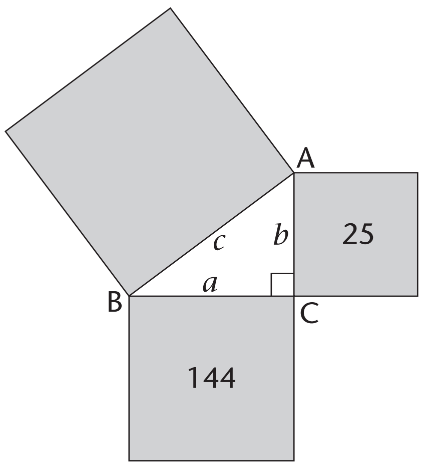
$c^2 = 25$

$\sqrt{c^2} = \sqrt{25}$

$c = 5$ units

3. The areas of some of the squares below are given. Calculate the areas of each of the squares that are not given and the lengths of all the sides.

(a)



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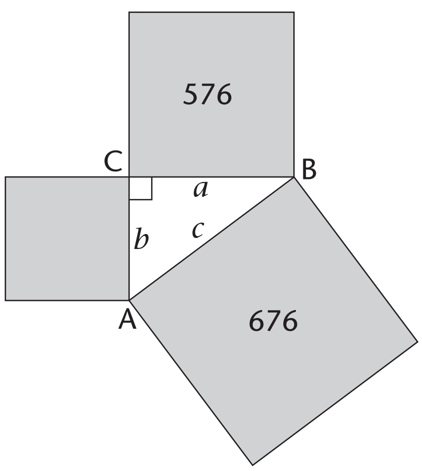
(b)



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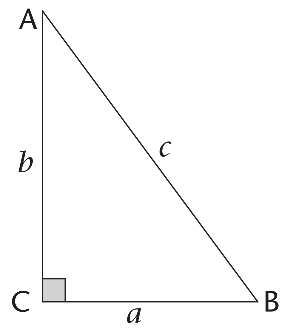
4. The following table gives information about the sides of five right-angled triangles. The letter symbol c represents the length of the hypotenuse in all cases. Use Pythagoras' theorem to complete the table, leaving answers in surd form if necessary.



a	b	c	a^2	b^2	$a^2 + b^2$	c^2
7	24					
16		34				
10				576		
			16	49		
	1		1			

3.3 Finding the missing sides in right-angled triangles

We can use the theorem of Pythagoras to calculate the length of the third side of a right-angled triangle if we know the lengths of the other two sides.

Example 1

A right-angled triangle has side $a = 6$ units and side $b = 8$ units. Calculate the length of side c.

$$
\begin{aligned}
c^2 &= a^2 + b^2 \\
&= 6^2 + 8^2 \\
&= 36 + 64 \\
&= 100 \\
\sqrt{c^2} &= \sqrt{100} \\
c &= 10 \\
\therefore c &= 10 \text{ units}
\end{aligned}
$$

Example 2

A right-angled triangle has side $a = 5$ units and side $b = 3$ units. Calculate the length of side c.

$$
\begin{aligned}
c^2 &= a^2 + b^2 \\
&= 5^2 + 3^2 \\
&= 25 + 9 \\
&= 34 \\
\sqrt{c^2} &= \sqrt{34} \\
c &= \sqrt{34} \text{ (leave in surd form)} \\
\therefore c &= \sqrt{34} \text{ units}
\end{aligned}
$$

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CALCULATING THE LENGTH OF THE HYPOTENUSE

Use the formula for the theorem of Pythagoras to calculate the length of the hypotenuse. Leave answers in surd form if necessary.

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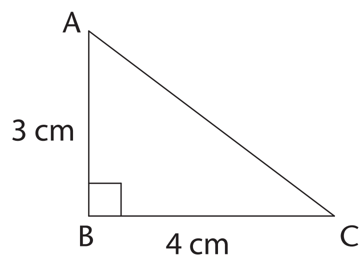

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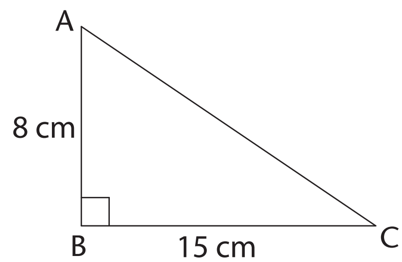
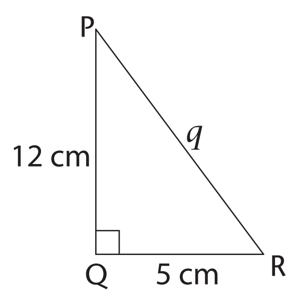


3.



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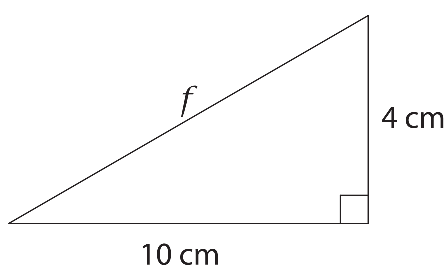

5. A right-angled triangle with hypotenuse c and sides the following lengths:
   $a = 9$ cm, $b = 40$ cm.

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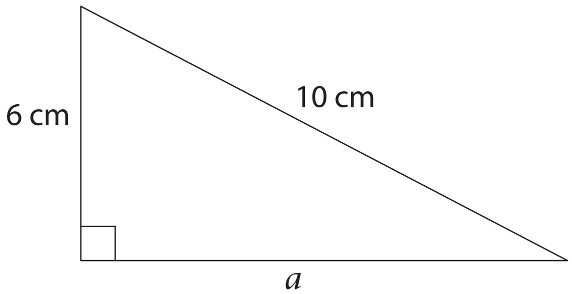
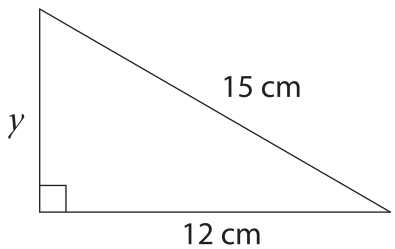
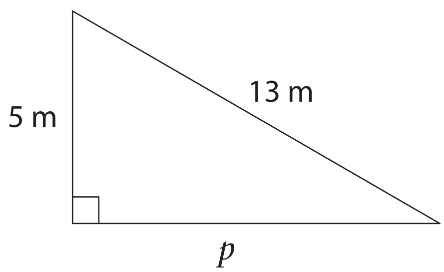
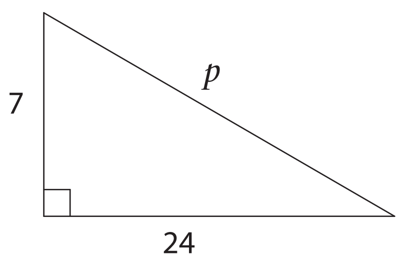
CALCULATING THE LENGTH OF ANY SIDE IN A RIGHT-ANGLED TRIANGLE

Calculate the missing sides in the following triangles. Do not use a calculator and leave the answers in the simplest surd form where necessary.

1. 

2. 

3. 

4. 

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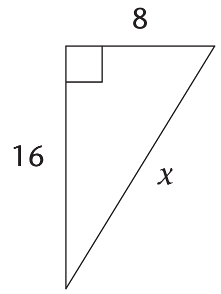
5.



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3.4 Are the triangles right-angled?

You learnt in sections 3.1 and 3.2 that in a right-angled triangle the area of the square on the hypotenuse is equal to the sum of the areas of the squares on the other two sides.

How can we tell whether a triangle is right-angled if we are given the lengths of the sides? One way is to use the "converse" of the Pythagoras theorem.

> The converse states that if the sum of the squares of the lengths of two sides equals the square of the length of the longest side, then the triangle is a right-angled triangle.

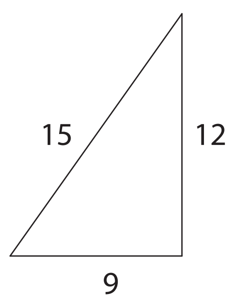
A converse is a statement that swaps around **what is given** in a theorem and **what must be determined**.

We can also state the converse as follows:

> If a triangle has side lengths a, b and c such that $c^2 = a^2 + b^2$, then the triangle is a right-angled triangle.

In the questions that follow, you have to determine whether triangles are right-angled or not. You may study the example first.

Example: Determine whether the triangle is right-angled or not.



(Length of longest side)^2 = $(15)^2$ = 225
Sum of the squares of the lengths of the other two sides
= $9^2 + 12^2$
= 81 + 144
= 225
(Longest side length)^2 = Sum of squares of other two sides lengths
And this can be written as $15^2 = 9^2 + 12^2$
$\therefore$ The triangle is right-angled.

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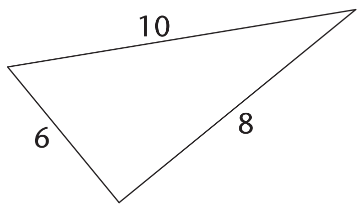
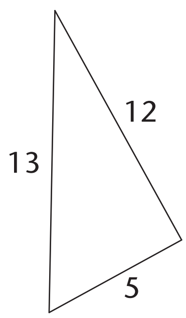
RIGHT-ANGLED OR NOT?

Determine whether the triangles are right-angled or not.

1. 

2. 

3. A triangle has sides measuring 6, 9 and 15 units.

4. Which of the following lengths of sides of a triangle will form a right-angled triangle? Answer without doing any calculations and explain your answer.

(a) 4, 2, 2 (b) 6, 8, 10 (c) 9, 12, 15
(d) 3, 4, 6 (e) $3x$, $4x$, $5x$ (f) 30, 40, 50

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