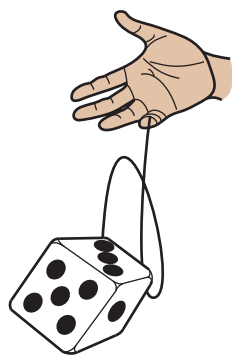


CHAPTER 14

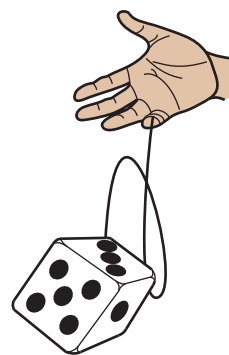
Probability

Some actions, like drawing a card from a bag with ten different coloured cards, have different possible results (outcomes). One cannot know what card will be drawn. You will learn that some predictions can be made though about what will happen if the action is repeated many, many times.

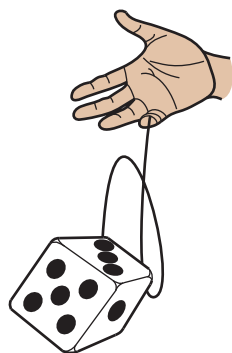
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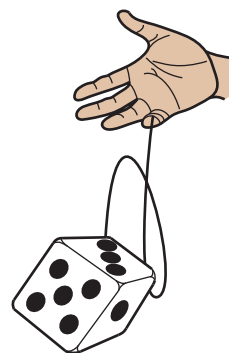
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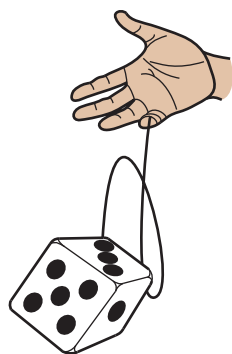
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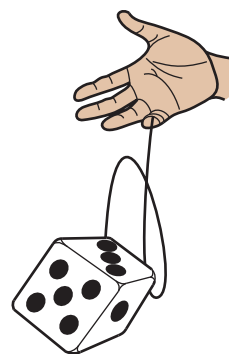
2?



6?



4?



1?

14 Probability

14.1 How often different things can happen

DIFFERENT FRACTIONS OF A WHOLE NUMBER

Jayden lives close to the sea. He goes fishing every day. Some days he catches no fish, but on some days he catches several fish. He never catches more than five fish in a day.

He has decided that he will always stop fishing when he has caught five fish in one day.

1. What are the different possible outcomes of each of Jayden's daily fishing trips?

.....

2. Jayden rolls a dice just once each day before he goes fishing.

- (a) What are the possible outcomes of rolling a dice once?

.....

- (b) Are the six outcomes of rolling a dice equally likely?

.....

- (c) Is there any reason for Jayden to believe that the outcome of his fishing trip on a day will be one less than the number that came up when he rolled the dice on that day?

.....

3. Jayden keeps a record of the outcomes of his daily dice rolls. Here is a summary of his record for 60 consecutive days.

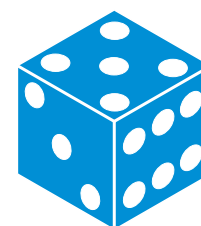
Outcome	1	2	3	4	5	6
Frequency	9	9	12	11	10	9

- (a) How many times was the outcome a 6?

- (b) What fraction is this of the total of 60 events?

- (c) On what fraction of the days was the outcome a 3?

The fraction of a number of events which have a specific outcome is called the **relative frequency** of that outcome.



4. What is the relative frequency of a:

(a) 5 in Jayden's series of 60 dice rolls?

.....

(b) 4 in Jayden's series of 60 dice rolls?

.....

The **range** of a set of numbers is the difference between the smallest and largest numbers in the set.

5. What is the **range** of the relative frequencies of the different outcomes in Jayden's series of dice rolls? Express the range as a fraction in sixtieths, and as a percentage.

.....

6. Do you think the six possible outcomes of Jayden's daily fishing trips are equally likely? Give reasons for your answer.

.....

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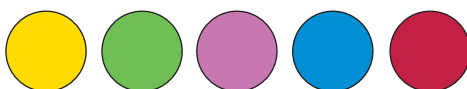
7. (a) Jayden has a book in which he keeps a record of the outcomes of his daily fishing trips. A summary of his record for a period of 200 consecutive days is given in the table below. Write the relative frequencies of the different outcomes in the table, with each expressed as two hundredths.

Outcome	0	1	2	3	4	5
Frequency	30	32	68	54	12	4
Relative frequency						

(b) What is the range of the relative frequencies in this case? Express the range as a common fraction and as a percentage.

.....

HOW OFTEN CAN WE EXPECT SOMETHING TO HAPPEN?



Imagine that you have five coloured buttons as shown above in a paper bag.

1. Imagine that you put your hand into the bag without looking inside, and grab one of the buttons.

(a) Can you say which colour that button will be?

(b) Discuss this with some classmates.

-
2. (a) What are the different possible colours of buttons that you could draw from the bag?

.....

- (b) How many different possibilities are there?

3. Read the passage below, then answer the questions that follow.

*When you draw a button from the bag, we say you perform a **trial**. The colour you draw is called the **outcome** of the trial.*

- (a) What are the different possible outcomes of the trial if you draw one button out of the bag?

.....

- (b) Imagine that you put the first button back into the bag. If you now draw one button from the bag again, what are the possible outcomes of this new trial?

.....

- (c) Imagine that you repeat the event a third time. What are the possible outcomes of this new trial?

.....

- (d) Imagine that you perform many trials. What are the possible outcomes of each repetition?

.....

4. (a) When you draw one of the five buttons many times and put it back each time, do you think you will draw one colour more often than the others?

.....

- (b) Discuss this with some classmates.

5. (a) Imagine that you draw a button out of the bag with five buttons and put it back, and repeat this 60 times. Approximately how many times do you think you will draw the red button?

.....

- (b) Approximately how many times do you think you will draw the pink button?






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- (c) Discuss this with some classmates.

When there is no reason to believe that any outcome will occur more often than any other outcome, the outcomes are said to be **equally likely**.






6. Susan decides to perform 160 trials on the bag with five buttons. In each trial she will draw one button from the bag, note its colour, and put it back. Lebogang decides she will perform 60 trials and Archie decides to perform 40 trials.

Approximately how many times do you think each of them will draw each of the buttons? Enter your expectations in the table below.

					
Susan					
Lebogang					
Archie					

7. Here are the answers that eight different people gave for Archie, with his 40 trials.

“Close to 6” means it can be 6 or another number close to 6, for example 5 or 7 or 4 or 8.

					
Answer A	close to 5	close to 5	close to 5	close to 5	close to 20
Answer B	7, 8 or 9	7, 8 or 9	7, 8 or 9	7, 8 or 9	7, 8 or 9
Answer C	6, 7 or 8	6, 7 or 8	6, 7 or 8	6, 7 or 8	6, 7 or 8
Answer D	close to 7	close to 9	close to 8	close to 10	close to 6
Answer E	close to 6	close to 6	close to 6	close to 6	close to 6
Answer F	6	9	7	8	10
Answer G	8	8	8	8	8
Answer H	close to 8	close to 8	close to 8	close to 8	close to 8

Which answers do you think are good answers, and which do you think are poor answers? For each answer explain why you think it is good or poor.

.....

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.....

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.....

.....

8. (a) How much is 1 fifth of 160, 1 fifth of 60, and 1 fifth of 40?
- (b) Look at your own answers for question 6 again. Do you still agree with your answers? If you want to give different answers now, do so and explain what made you change your position.
-

9. Willem has decided to perform as many trials as he can in an afternoon, drawing one button each time out of the bag with five coloured buttons.

(a) In close to what fraction of the trials can he expect to get yellow as the outcome?

.....

(b) In close to what fraction of the trials can he expect to get red as the outcome?

.....

10. Manare has decided to perform as many trials as he can in an afternoon, drawing one button each time out of the bag with seven different coloured buttons.



(a) In close to what fraction of the trials can he expect to get blue as the outcome?

.....

(b) In close to what fraction of the trials can he expect to get grey as the outcome?

.....

11. Miriam has decided to perform as many trials as she can in an afternoon, drawing one button each time out of a bag with twelve different coloured buttons.

In close to what fraction of the trials can Miriam expect to get each specific colour as the outcome?

The number of times that a specific outcome is obtained during a series of trials is called the **frequency** of the outcome.

12. What is the frequency for each of the following colours in answer F, in question 7 on the previous page?

(a) red (b) pink

(c) yellow (d) blue

When the different possible outcomes of an event are equally likely, it is reasonable to expect that when the event is repeated many times, the frequencies for the different outcomes will be almost equal.

AN INVESTIGATION

1. Make eight small cards or pieces of paper. On each card write a different letter. Use the letters A, B, C, D, E, F, G and H. Put the cards in a paper bag. Imagine that you draw a card out of the bag, note the letter and put it back. Imagine that you perform 40 such trials, noting the outcome each time. Then you find the frequency for each letter. To what number do you think each of the frequencies will be close?
The number you think of may be called the **expected frequency**.

2. What will be the expected frequencies for each letter if:
 - (a) 200 trials are performed?
 - (b) 1 000 trials are performed?

3. Now actually do the experiment described in question 1. Record your results with tally marks in the table below. When you have finished, count the tally marks to find the **actual frequencies**.

	A	B	C	D	E	F	G	H
Tally marks								
Actual frequency								
Expected frequency	5	5	5	5	5	5	5	5

4. Write your actual frequencies on a slip of paper, in a table like this.

	A	B	C	D	E	F	G	H
Actual frequency								

5. The next step is to collect the slips of four different classmates, and write their frequencies in rows 1, 4, 7, 10 and 13 of the table on the next page, together with your own frequencies. **Do not do it yet.** When you put the five sets of results together, and add them up, you will have the actual frequencies out of 200 trials. You will write these in row 16. In the row for expected frequencies, write the numbers to which you think the frequencies will be close.
6. Now work with your four classmates, and complete rows 1, 4, 7, 10 and 13.
7. In the first empty row after each actual frequency row, express the frequency as a fraction of the total number of outcomes in the experiment, which was 40 in each case. You need not simplify the fractions in rows 2, 5, 8, 11 and 14 of the table.
8. In rows 17 and 20, express the frequencies of rows 16 and 19 as fractions of 200.
9. In the remaining empty rows, express the fractions as percentages.

10. Calculate the ranges of the numbers in rows 3, 6, 9, 12, 15 and 18.

Row 3:
Row 6:
Row 9:
Row 12:
Row 15:
Row 18:

		A	B	C	D	E	F	G	H
1	Actual frequencies								
2									
3									
4	Actual frequencies								
5									
6									
7	Actual frequencies								
8									
9									
10	Actual frequencies								
11									
12									
13	Actual frequencies								
14									
15									
16	Total actual frequencies								
17									
18									
19	Expected frequencies								
20									
21									

11. In which row is the range the smallest? Try to explain why this is the case.

.....

.....

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.....

12. In which of the rows in question 10 are the numbers closest to the expected percentages in row 21?











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The expected relative frequency of an outcome is called the **probability** of the outcome.

13. Imagine that you have ten different coloured buttons in a bag, as shown below.



- (a) Imagine that you draw one button out of the bag.
How many different equally likely outcomes are there for this trial?
- (b) Imagine you draw one button out of the bag, look at the colour and make a tally mark in the column for that colour on the table below. Imagine you do it many times. Approximately what fraction of the total number of tally marks do you expect to be in each column?
.....
- (c) The fraction you have specified in (b) is the probability of the outcome for each of the columns. Would you expect to get precisely that fraction in each column?
.....

- (d) Hashim says he expects to have **approximately** 10 tally marks in each column, because the outcomes are equally likely. Do you agree with Hashim? Give reasons for your answer.

.....

.....

.....

14.2 Probability

In this activity you have to think about the following situation.

There are 10 coloured, numbered buttons in a bag: 6 yellow buttons, 3 blue buttons, and 1 red button.



1. (a) What fraction of the total number of buttons is yellow?
- (b) What fraction of the total number of buttons is blue?
- (c) What fraction of the total number of buttons is red?
2. Suppose you put your hand into the bag without looking inside, take one button out and note its colour, and then put it back into the bag.
If you repeat this trial many times, you will sometimes get a yellow, sometimes a blue and sometimes a red button.
 - (a) Do you think you will get blue more often than yellow? Explain your answer.
.....
.....
 - (b) Do you think you will draw yellow about twice as often as blue?
.....
 - (c) Can you be certain which colour will be drawn? Explain your answer.
.....
.....
.....
 - (d) Share your ideas with two classmates.

Here is an experiment that you will do later. **Do not do it now.**

Put 10 buttons like those on page 239, or pieces of paper or cardboard with the names of the colours written on them, in a bag. Put your hand into the bag without looking inside, and take one button out. Check what colour it is, make a tally mark in the column below for that colour, and put the button back into the bag. Do this 10 times.

Yellow	Blue	Red

Each time you perform a trial, a certain **event** takes place, and there are three possible events:

- A. The event of the colour being yellow
- B. The event of the colour being blue
- C. The event of the colour being red

3. (a) In how many different ways can event A be achieved in one trial?
- (b) In how many different ways can event B be achieved in one trial?
- (c) In how many different ways can event C be achieved in one trial?

4. (a) Suppose you do the experiment, and make 10 trials. Do you think event A will happen 3 times or maybe 4 times, event B will happen 3 times or maybe 4 times and event C will happen 3 times or maybe 4 times?

.....

.....

- (b) Share your ideas with two classmates.
- (c) Do you rather think event A will happen 6 times (or maybe 5 or 7 times), event B will happen 3 times (or maybe 2 or 4 times), and event C will happen once (or maybe twice or not at all)?

.....

.....

.....

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- (d) Share your ideas with two classmates.

5. (a) Do the experiment that is described before question 3, and write the results in the second row of the table on the next page.
- (b) Repeat the experiment, and write the results in the third row of the table.
- (c) Repeat the experiment three more times, and enter the results in the table.
- (d) Complete the last two rows of the table.

Outcome	Yellow	Blue	Red
Frequency of each colour during the first 10 trials			
Frequency of each colour during the second 10 trials			
Frequency of each colour during the third 10 trials			
Frequency of each colour during the fourth 10 trials			
Frequency of each colour during the fifth 10 trials			
Total frequencies out of 50 trials			
Total frequencies divided by 5			

When you did the experiment for the first time in question 5(a), you performed 10 **trials**: you took a button out of the bag, and inspected the colour.

Each time, there were three **possible outcomes** for the trial: the button could be **yellow**, it could be **blue** or it could be **red**.

We can also say that three different events were possible: yellow, blue and red. But if we consider the numbers on the buttons, 10 different outcomes are possible.

6. (a) How many different outcomes (numbered buttons) will produce the event yellow?
- (b) How many different outcomes will produce the event blue?
- (c) How many different outcomes will produce the event red?
7. (a) What fraction of the ten possible outcomes will produce the event yellow?
- (b) What fraction of the ten possible outcomes will produce the event blue?
- (c) What fraction of the ten possible outcomes will produce the event red?

The fractions you have given as answers for question 7 are the **probabilities** of the three different events.

8. (a) What is the probability of getting blue when one of the buttons below is drawn from a bag?



- (b) Describe in your own words what is meant by saying the probability of an event is $\frac{3}{20}$.
.....
.....

