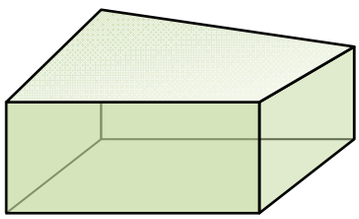
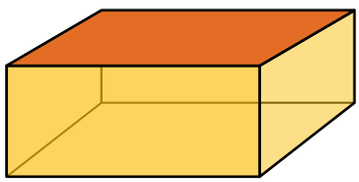
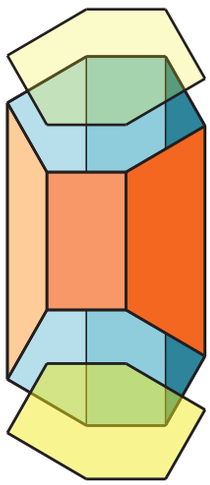
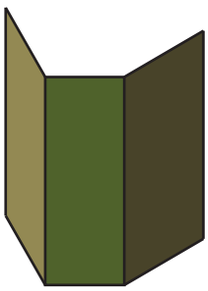
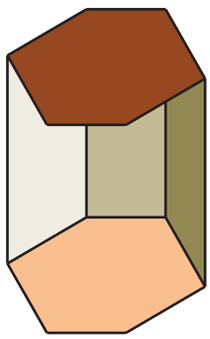
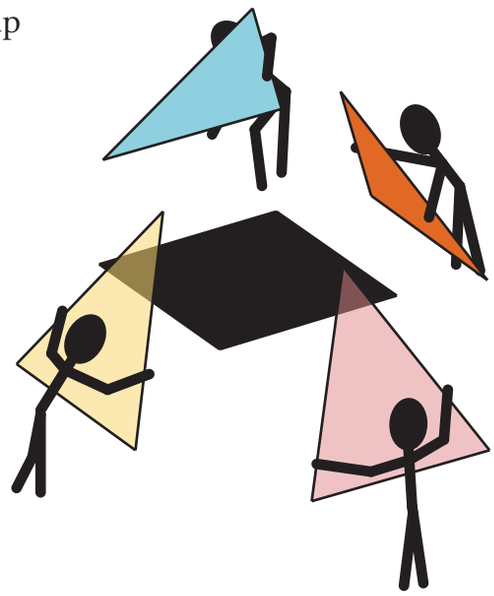
Chapter 13
Geometry of 3D objects

In this chapter, you will revise what you should already know about different types of 3D objects and how they can be described in terms of the number and shape of their faces, number of vertices and number of edges. You will draw accurate nets and construct models of prisms and pyramids. You will learn about a surprising relationship between the numbers of vertices, edges and faces of different polyhedra. You will also investigate the so-called "Platonic solids".

The four men want to put the four glass sheets up around the black base to build a pyramid.

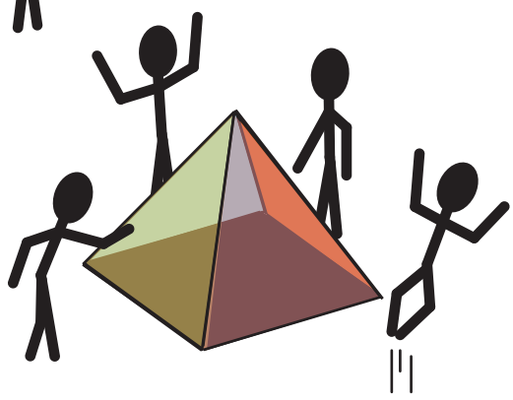


The yellow and pink glass sheets are in place now.

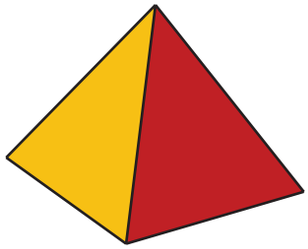
The job is done, the pyramid stands!

Do you want to be inside it?

They now decide to paint the four glass sides so that one cannot see inside.



Now you cannot see inside, and you cannot tell that this diagram represents a 3D object.

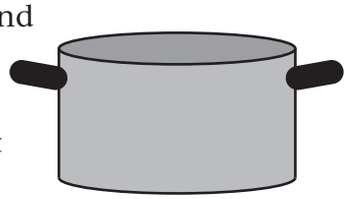


13 Geometry of 3D objects

13.1 Revision: 3D objects

THINK OF SPACE WHILE YOU LOOK AT PICTURES AND DRAWINGS

Most objects we see around us, like fruit, animals, trees, people and motor cars, have curved or round surfaces. Some objects, like a saucepan or other cooking vessel, have both round and flat surfaces. The circular bottom of a saucepan must be flat so that it makes good contact with the stove plate.

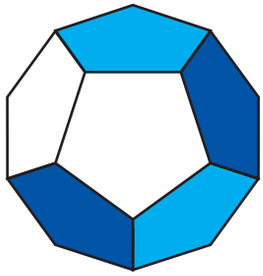


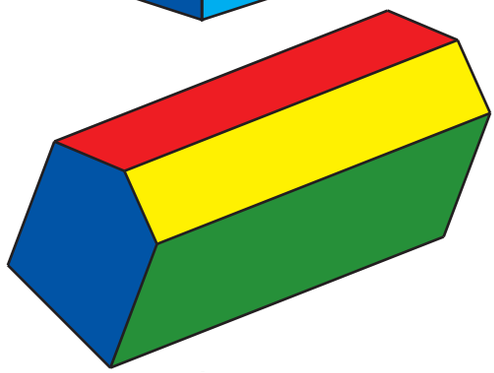
1. (a) Should the top of a table or desk be a flat or curved surface? .....

   (b) We eat with knives, forks and spoons. Which of these objects normally have curved surfaces?

   .....

This chapter is about objects that only have flat surfaces, like those shown below.





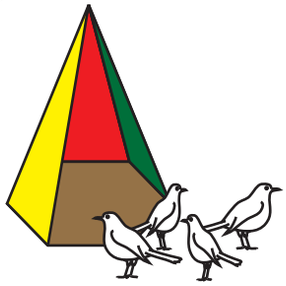
The front, right and top faces in the above drawing are made of clear plastic so that you can see the faces behind them.

Note this strange box with different colours on its different faces.

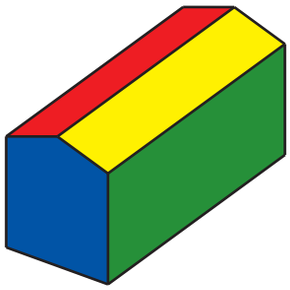
2. Do you think there is enough space for all the birds in the tent shown on the right-hand side?

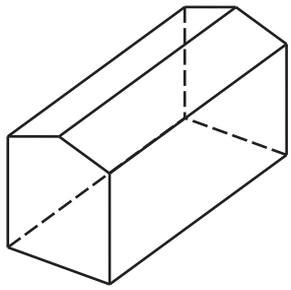
   .....

   .....



3. The unusual box, shown on the previous page, with flat faces (surfaces) only is shown again below. In the drawing of the same box on the right, dotted lines are used to indicate edges and surfaces that are hidden in the coloured drawing.





(a) How many faces (different flat surfaces) does this object have altogether? .....

(b) How many faces cannot be seen in the coloured drawing on the left? .....

(c) How many of the faces are rectangles? .....

(d) How many of the faces are pentagons? .....

A 3D object with **flat faces (surfaces) only** is called a **polyhedron** (plural: **polyhedra**).

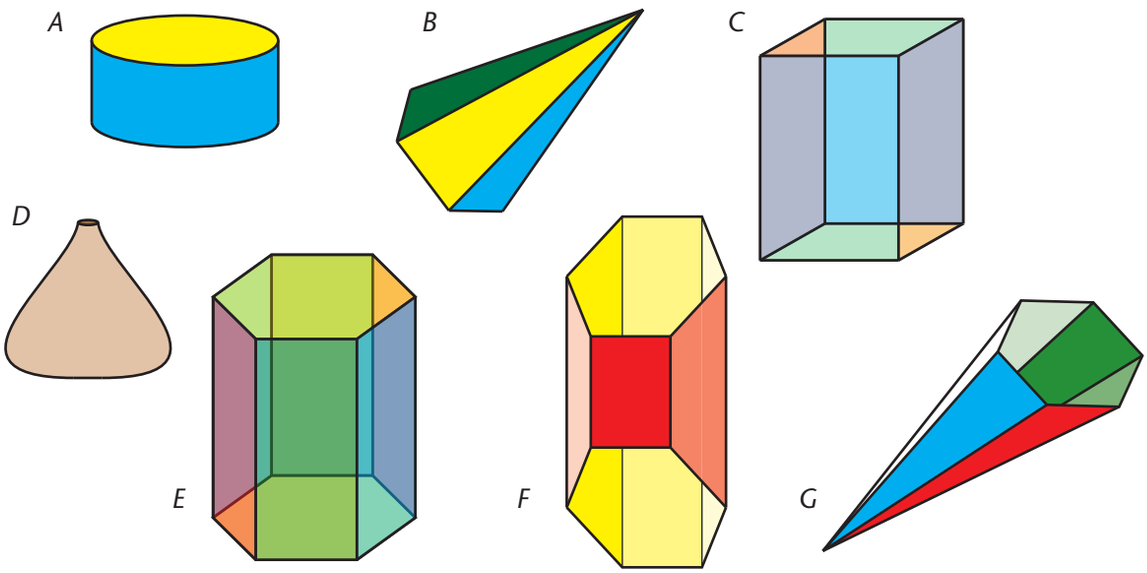
A straight **edge** is formed where two flat surfaces meet. The point where two or more edges meet is called a **vertex** (plural: **vertices**).

The word **polyhedron** means 'many-seated' and describes the shape of such an object with many flat faces.

4. (a) How many edges does the coloured polyhedron in question 3 have? .....

(b) How many vertices does it have? .....

5. Which of the objects below are polyhedra? .....



TWO SPECIAL TYPES OF POLYHEDRA

Polyhedra like C and E at the bottom of the previous page are called **prisms**.
Polyhedra like B and G are called **pyramids**.

1. Describe the differences between prisms and pyramids.

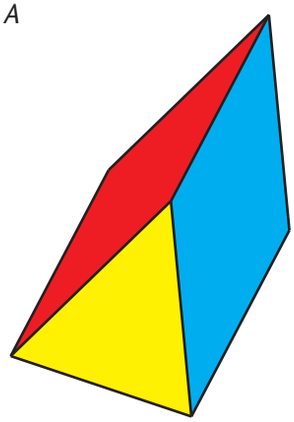
.....

.....

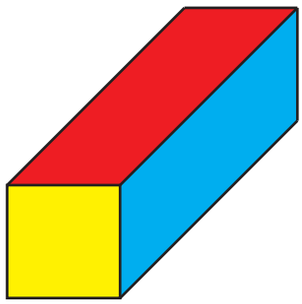
.....

Here are some more pictures of **prisms**.

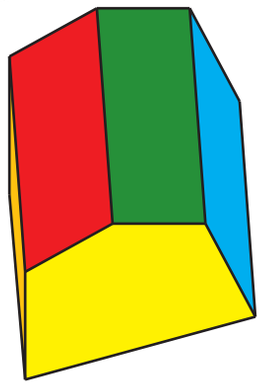
A



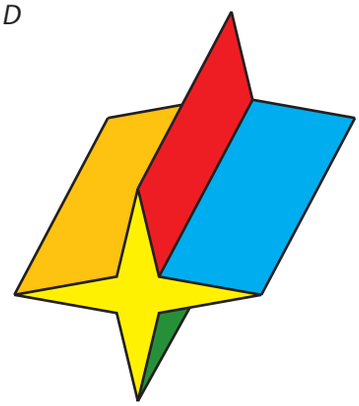
B



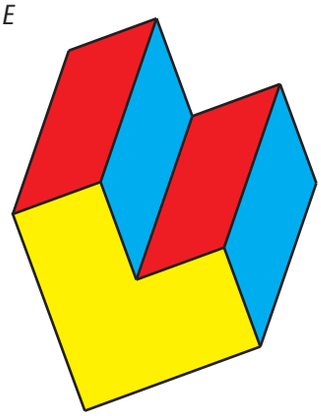
C



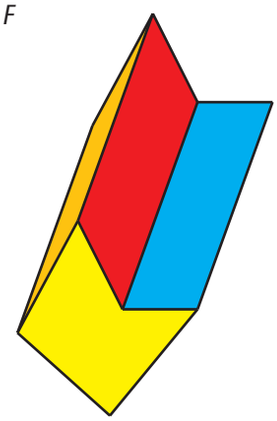
D



E



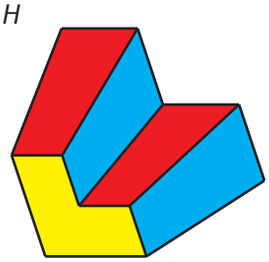
F



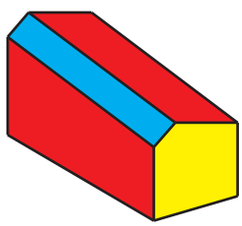
The objects shown by the four pictures below are polyhedra but they are *not* prisms or pyramids.

G

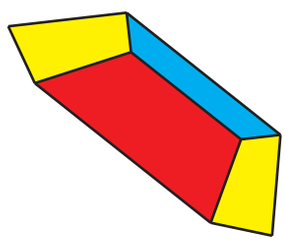
H



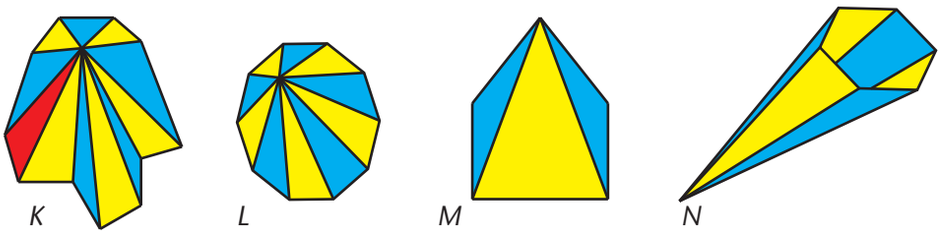
I



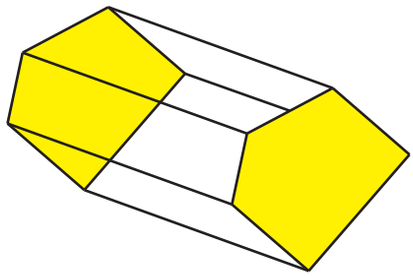
J



Here are some pictures of **pyramids**. More pictures of pyramids are shown at the bottom of this page and also on the next page.



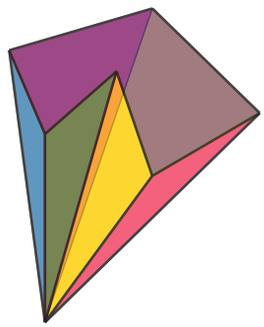
A **prism** has two identical, parallel faces (called **bases**) that are connected by parallelograms (called **lateral faces**). In the case of right prisms, the lateral faces are perpendicular to the bases and the lateral faces are rectangles.



A prism with pentagonal bases like this one is called a **pentagonal prism** because the base is a pentagon.

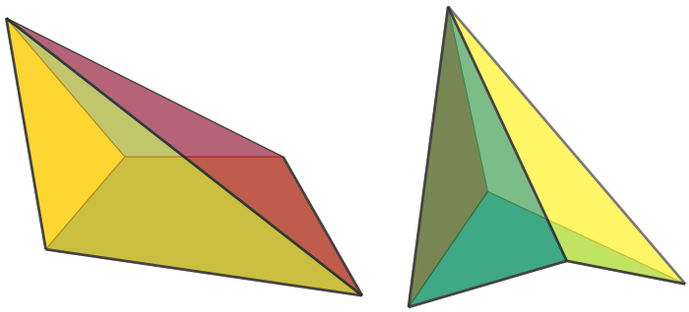
2. (a) Which pictures on the previous page also show pentagonal prisms? .....
   (b) Which picture on the previous page shows a hexagonal prism? .....
   (c) Which picture on the previous page shows an octagonal prism? .....

A **pyramid** has only one base. The lateral faces of a pyramid are triangles that meet at the **apex**.



The pyramid on the right is called a **hexagonal-based pyramid**.

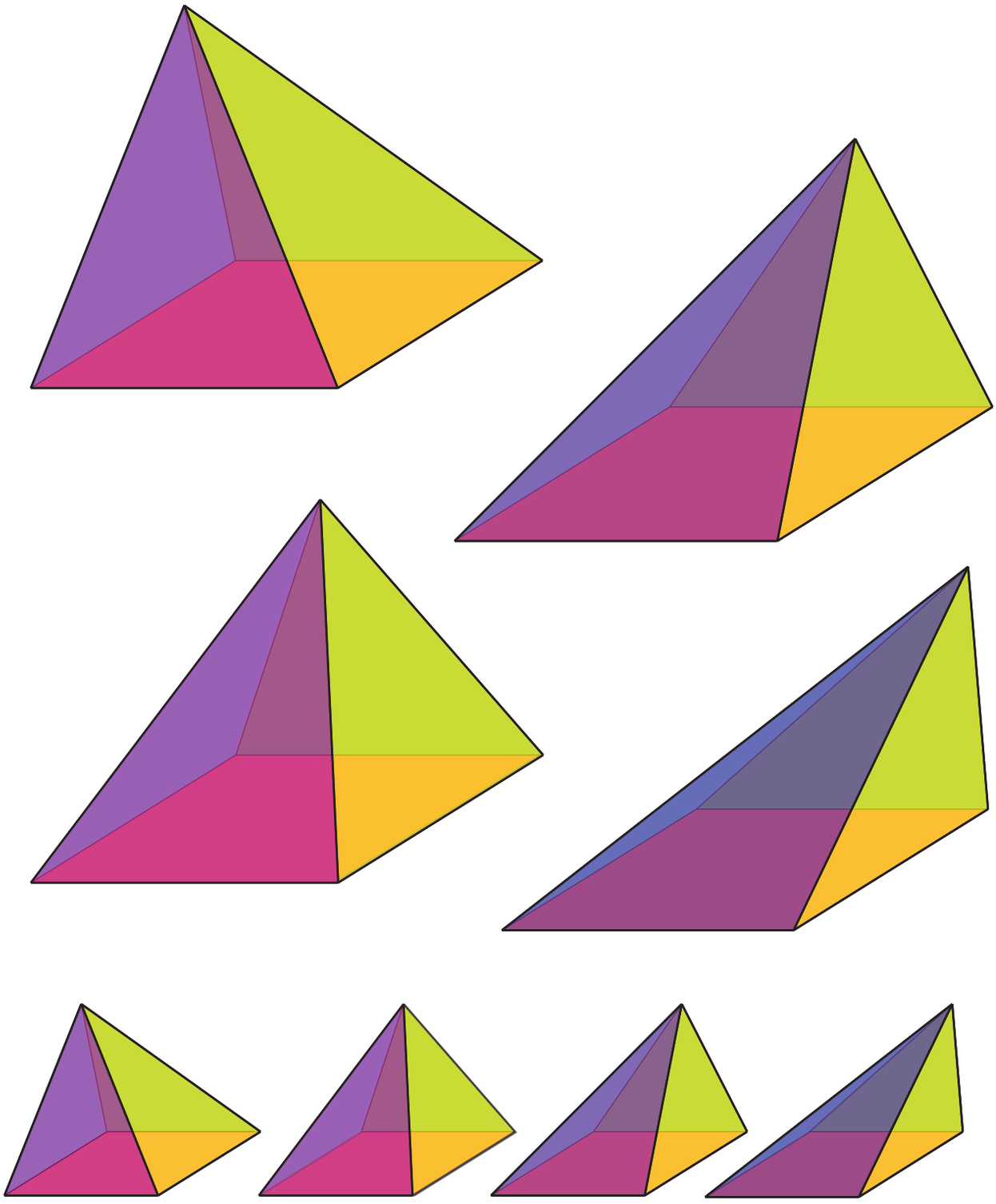
The two pyramids below have quadrilaterals as bases and are called **quadrilateral-based pyramids**.



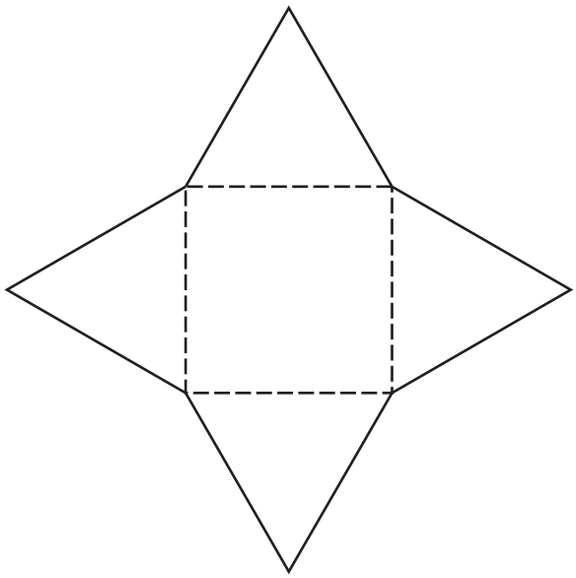
A triangular-based pyramid is also called a **triangular pyramid**; a square-based pyramid is also called a **square pyramid**; a hexagonal-based pyramid is also called a **hexagonal pyramid**, etc.

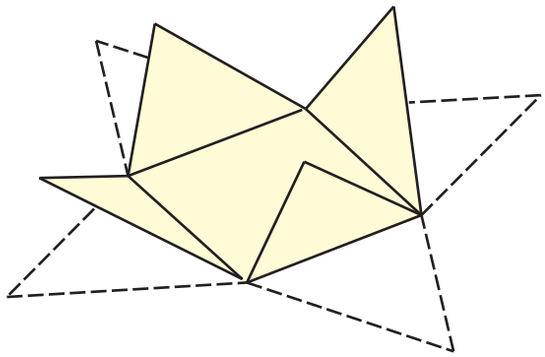
3. Which picture at the top of this page shows a hexagonal pyramid? .....

Pictures of different **square-based pyramids** are shown below.

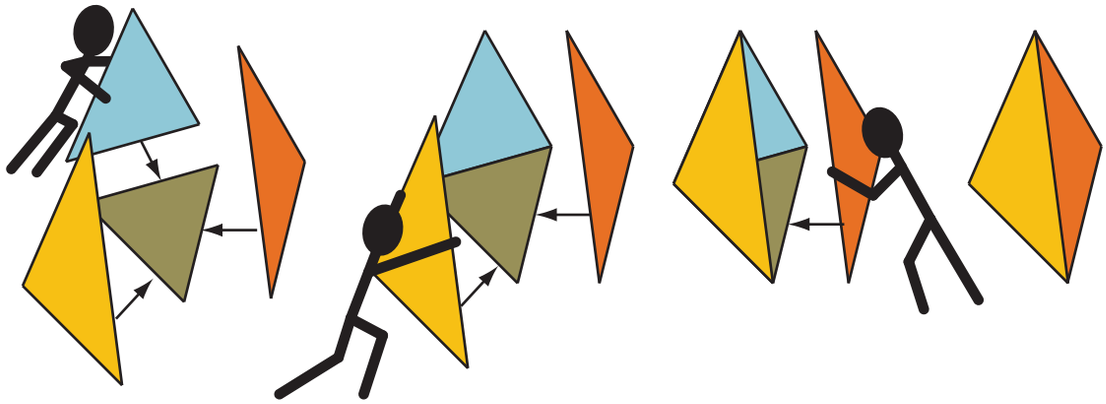


You can make a square-based pyramid by drawing and cutting out a diagram like the one on the left below, and folding the triangles up on the dotted lines as shown on the right.



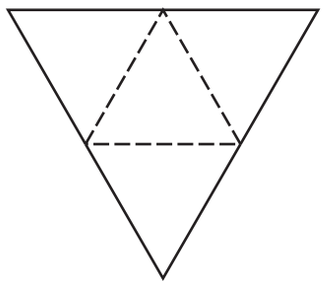


These men are building a **triangular-based** pyramid.



A triangular-based pyramid is also called a **tetrahedron**, which literally means "four-face".

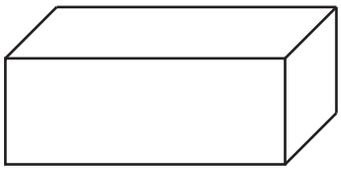
A tetrahedron with four identical faces that are equilateral triangles is called a **regular tetrahedron**.



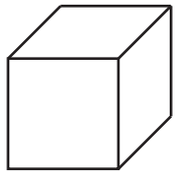
If you draw and cut out a figure like the one on the right, and fold the triangles up on the dotted lines, you can make a regular tetrahedron. A diagram like this, that can be cut out and folded to make a model of a polyhedron, is called a **net**.

A **regular polyhedron** has identical faces that are regular polygons, i.e. with all sides and angles equal.

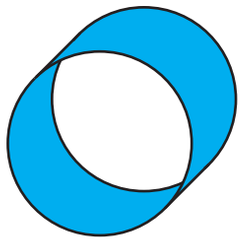
A rectangular prism is also called a **cuboid**.



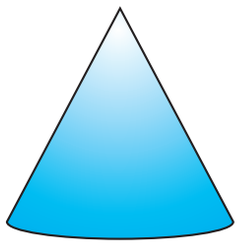
A cuboid with square faces is also called a **cube**.



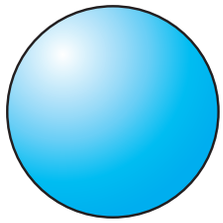
An object with two identical circular bases and one curved surface is called a **cylinder**.



A "pyramid" with a round base is called a **cone**.



An object with the shape of a ball, in other words one curved surface with every point on its surface the same distance from its centre, is called a **sphere**.



Cylinders, cones and spheres are *not* polyhedra since they have curved surfaces. Remember, a polyhedron has faces, edges and vertices. The faces are the flat surfaces. An edge is a line along which two faces of a 3D object meet; an edge connects two vertices. A vertex is the point where the edges meet.

4. Label parts (a) to (f) on the figure below.

   (a) .....

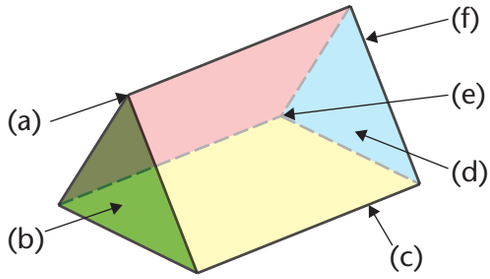
   (b) .....

   (c) .....

   (d) .....

   (e) .....

   (f) .....



Triangular prism

5. Learners in a Grade 8 class made 3D objects from cardboard. Can you say which kind of figure the following three learners made?

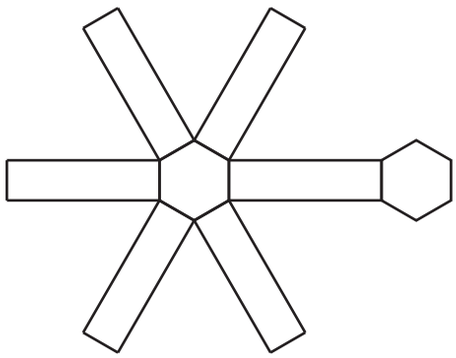
   (a) Adam's object had 8 vertices and 12 edges. .....

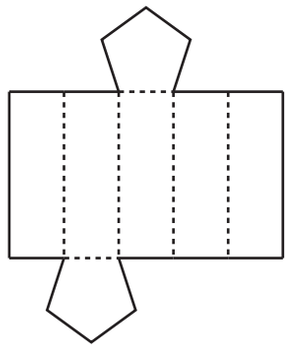
   (b) Lea's object had 4 vertices and 4 faces. .....

   (c) Mary's object had 12 edges and 6 congruent faces. .....

6. Complete the table for prisms. Count the bases as faces too. If you find this difficult, it may help you to make quick rough sketches of nets for some prisms, like the sketches given below the table.

Number of sides in each base	Number of faces	Number of vertices	Number of edges	Faces + vertices	Edges + 2
3	5	6	9		
4	6		12		
5					
6					
8					
10					





7. Complete the table for pyramids. Count the bases as faces too.

Number of sides in each base	Number of faces	Number of vertices	Number of edges	Faces + vertices	Edges + 2
3	4	4	6		
4					
5					
6					
7					
9					

8. Consider your answers for questions 6 and 7. Is the statement below true both for prisms and pyramids?

   the number of faces + the number of vertices = 2 + the number of edges

   This statement is called **Euler's formula** for polyhedra.

   .....

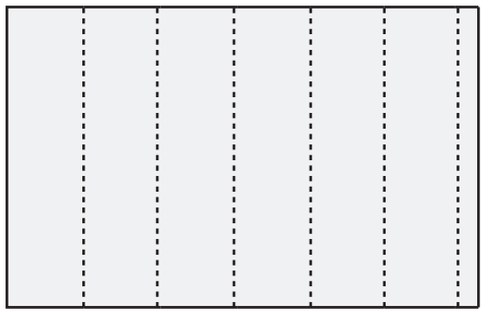
9. Is Euler's formula true for the polyhedra G, H, I and J on page 199? .....

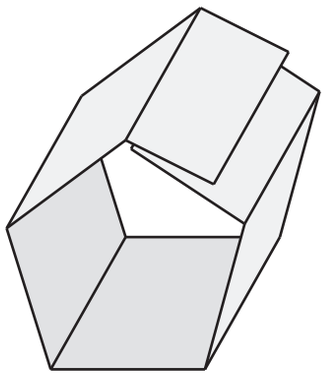
13.2 Nets and models of prisms and pyramids

A QUICK WAY TO MAKE PRISMS AND PYRAMIDS

Fold sections about two fingers wide on a sheet of A4 paper, more or less as shown by the dotted lines in the sketch on the right.

Fold the sheet into a "tube" with 5 or 6 faces along its length, as shown below.

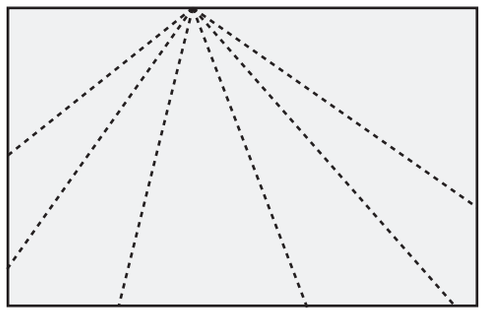


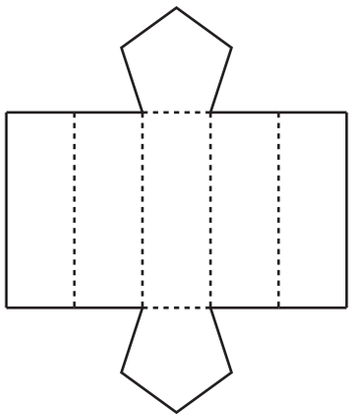


With a little extra work, you can now make a paper prism. You need to cut out two bases so that they fit well.

You can make prisms with triangular, square, rectangular, hexagonal and other shape bases in this way.

You can make a pyramid in the same way, but it is more difficult. Draw dotted lines on a sheet of A4 paper as shown in the sketch on the right. Fold the paper along the dotted lines. It is quite difficult to know where and how to cut so that the base is a flat surface.



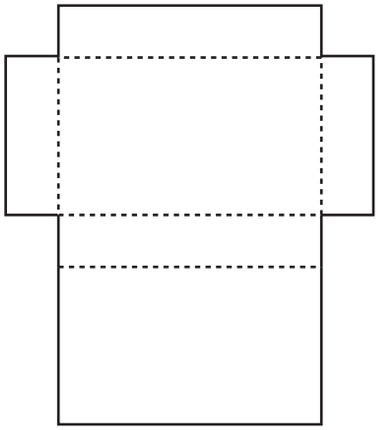


Apart from the difficulty of getting the base of the pyramid flat, the above method has the disadvantage that you have to use separate pieces of paper or other material to make one object. It would be better to make the whole object by folding one piece of paper. For example, a prism with pentagonal bases can be made by drawing, cutting out and folding a sheet of paper as shown on the left. This diagram is called a **net** of a prism with a regular pentagonal base.

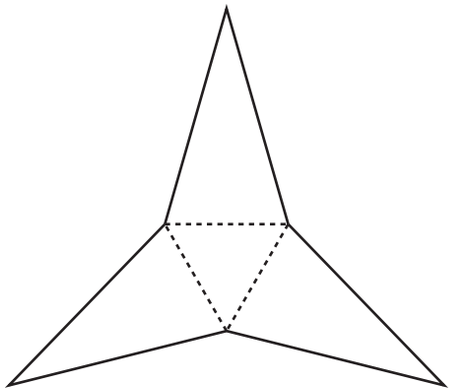
NETS FOR DIFFERENT POLYHEDRA

1. Name the polyhedron that can be made from each of the following nets.

(a)



(b)



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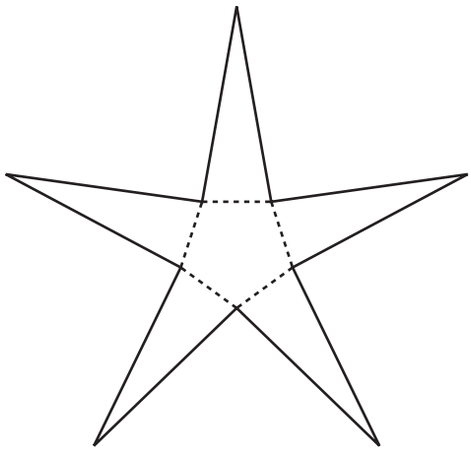
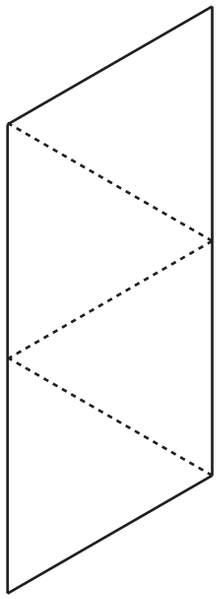
(c)



(d)



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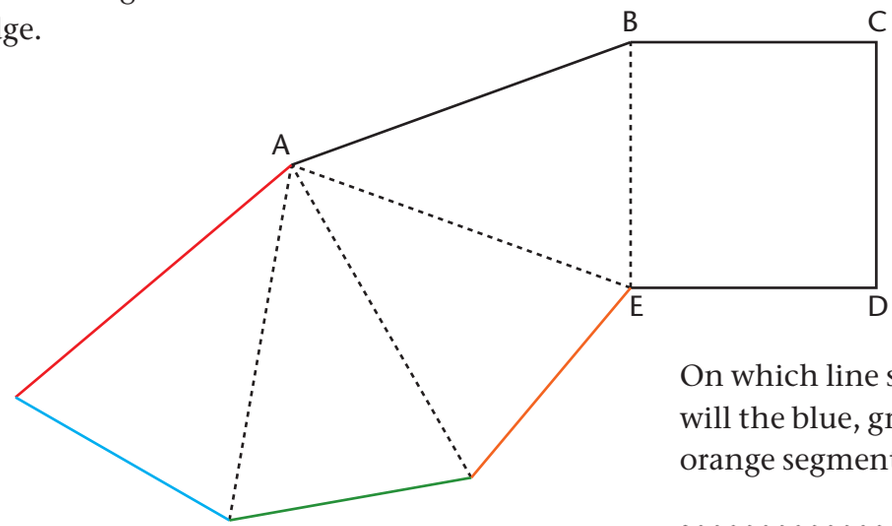
2. (a) Name the polyhedron that can be formed by cutting out the diagram below on the solid lines and folding it on the dotted lines.

. . . . . .

(b) When this net is folded to make a polyhedron, the red line segment will fall on AB to form an edge.



On which line segments will the blue, green and orange segments fall?

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3. (a) Name the polyhedron that can be formed by cutting out the diagram below on the solid lines and folding it on the dotted lines.

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(b) On which segments will the red, blue and green segments fall to form edges of this polyhedron?

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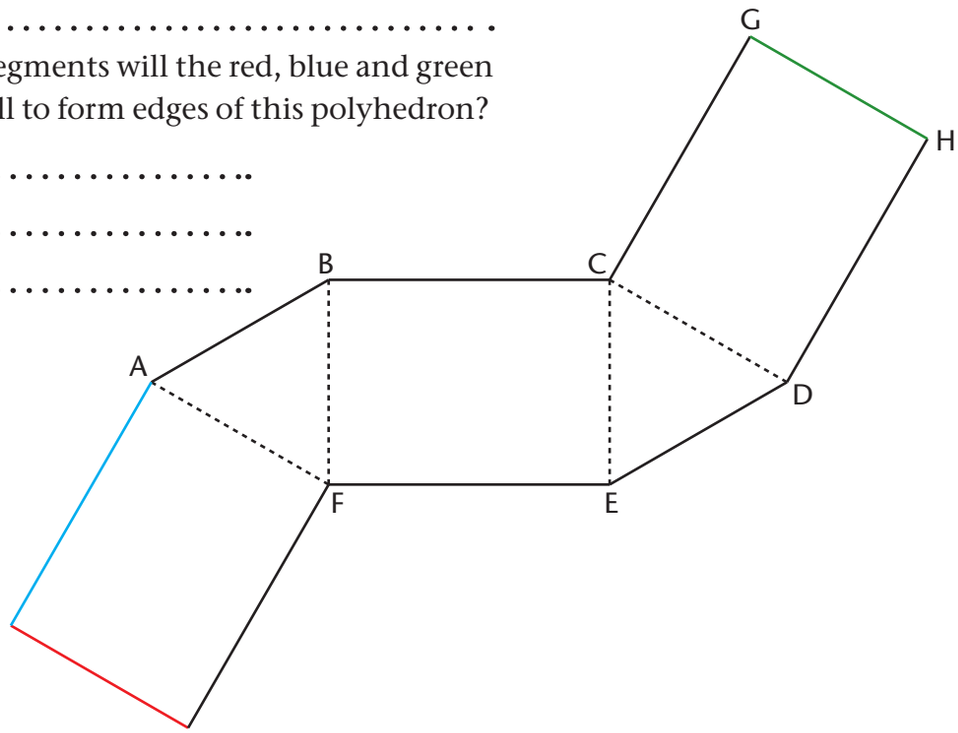


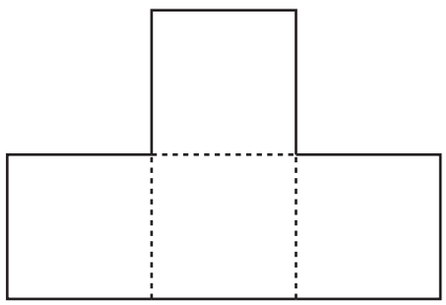
4. Some of the diagrams below and on the next page are the nets for the following objects:

a square-based pyramid	a triangular prism
a hexagonal pyramid	a hexagonal prism
a cuboid	a cube

Under each diagram, write the name of the object for which the diagram is a net. There may be more than one net for some of the objects. Write "none" if the diagram is not a net for any prism or pyramid.

A diagram is only called a **net** of an object if the cut-out diagram can be folded to form **all** the faces of the object.

(a)



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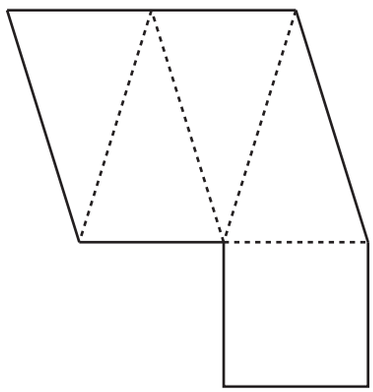
(b)



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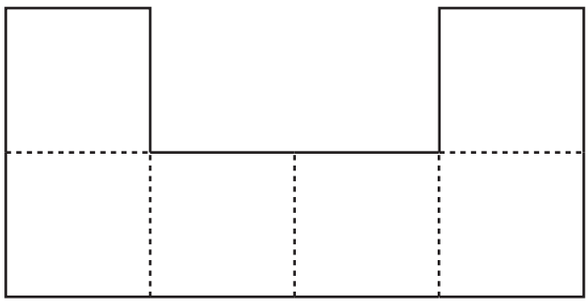
(c)



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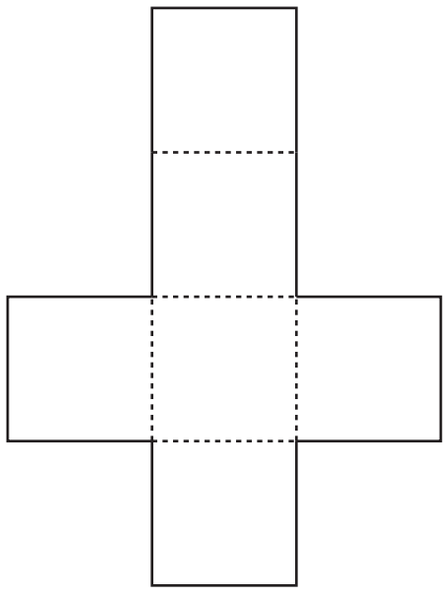
(d)



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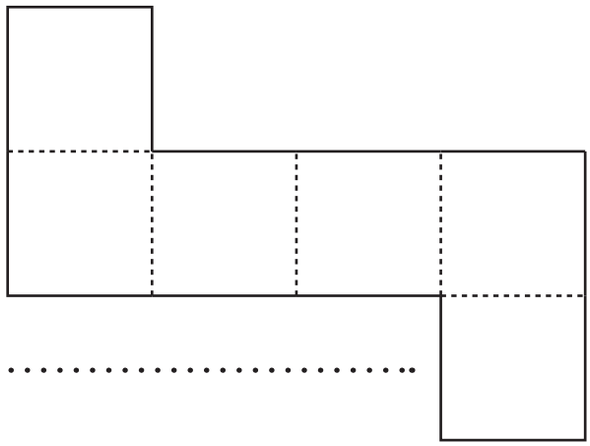
(e)



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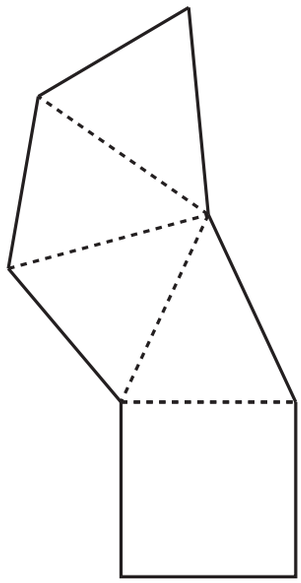
(f)



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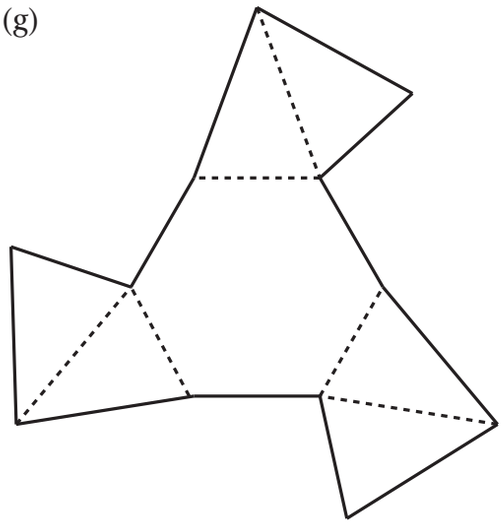
(g)



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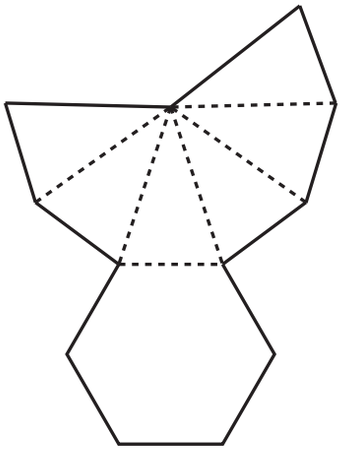
(h)



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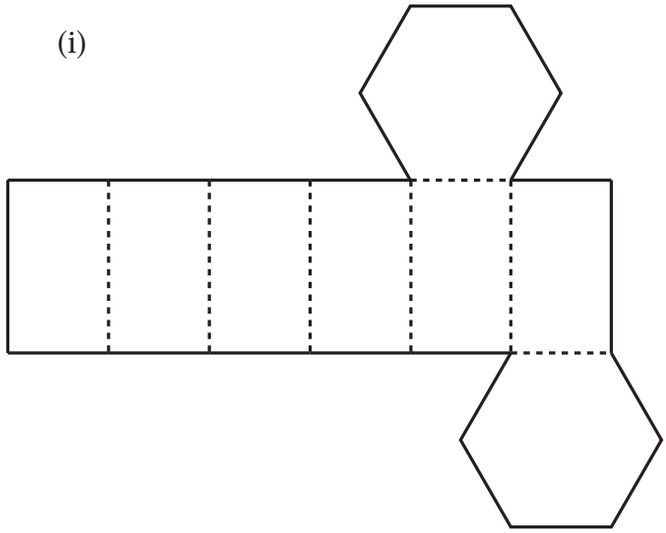
(i)



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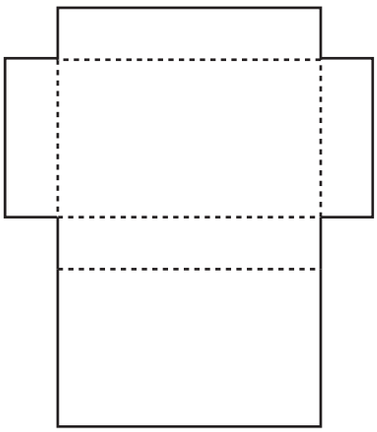
(j)



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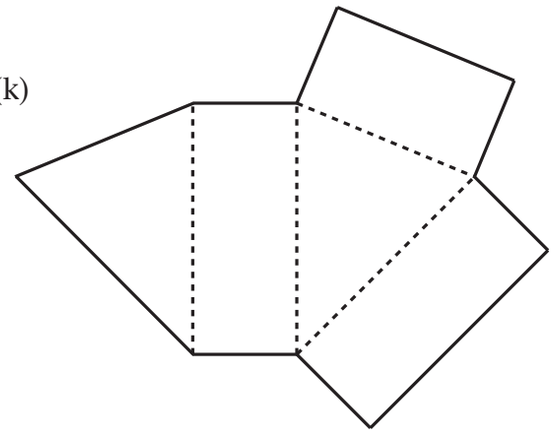
(k)



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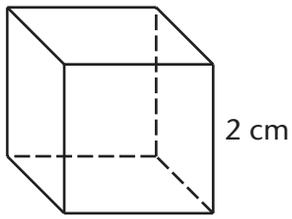
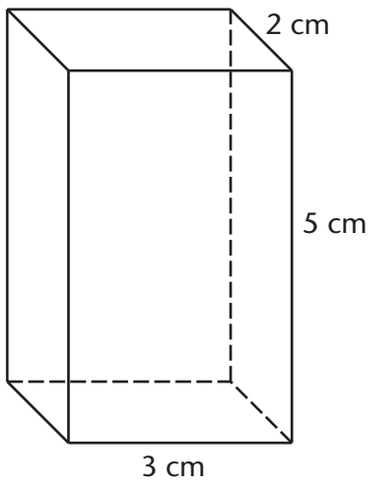
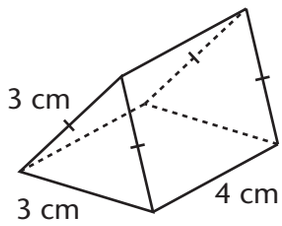
5. Draw a net for each of the following objects. Be accurate in your measurements.
   (a) Cube



   (b) Rectangular prism



   (c) Triangular prism



6. (a) Copy the nets in question 5 onto cardboard, but multiply the length of each side by 2. Be accurate in your constructions.
   (b) Cut out, fold and use sticky tape to paste the nets to build the 3D models.

7. The first diagram below is a net for a prism with quadrilateral bases. Which of the diagrams (a), (b), (c) and (d) are nets for the same prism, and which diagrams are not nets for the prism?

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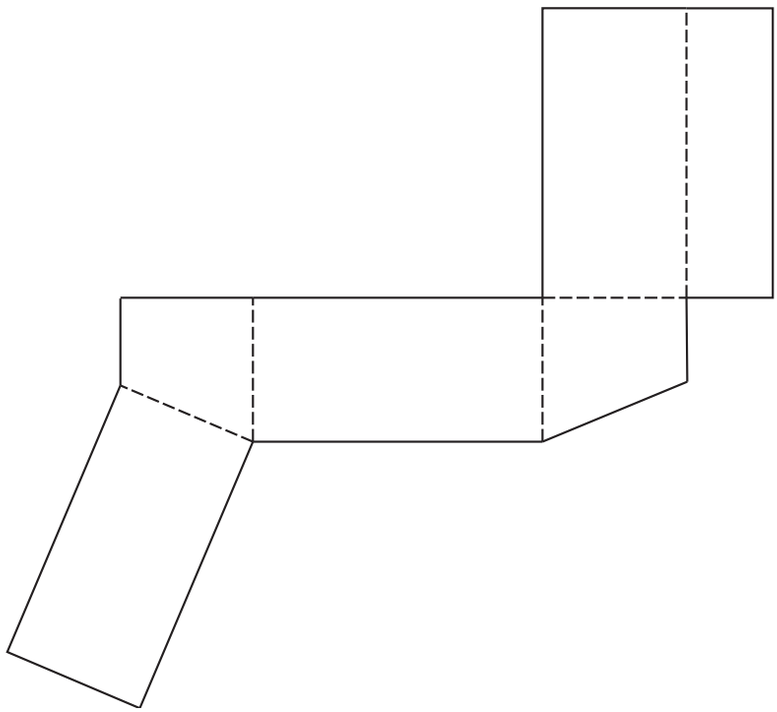
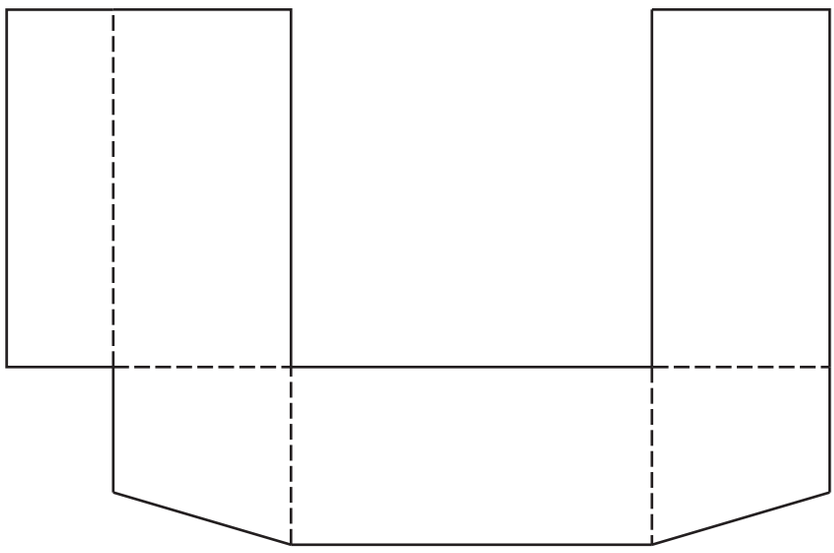
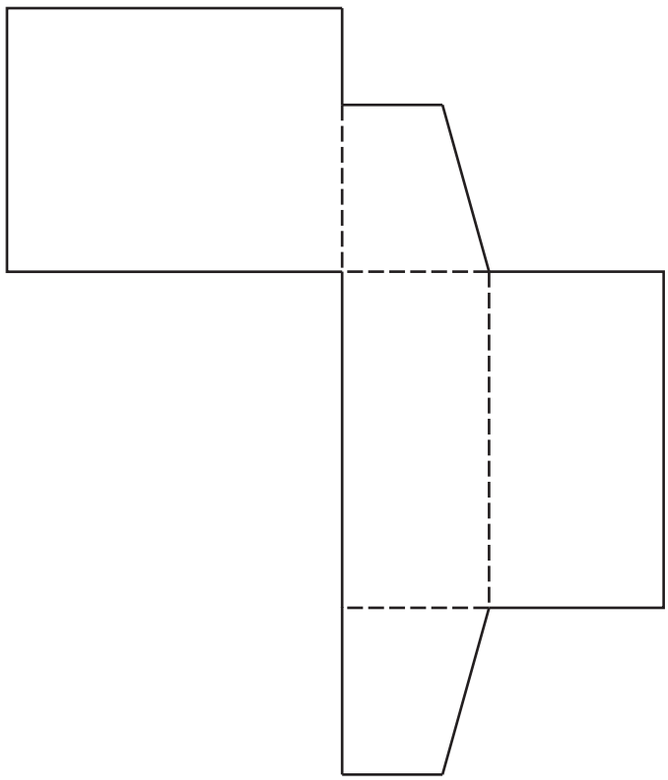


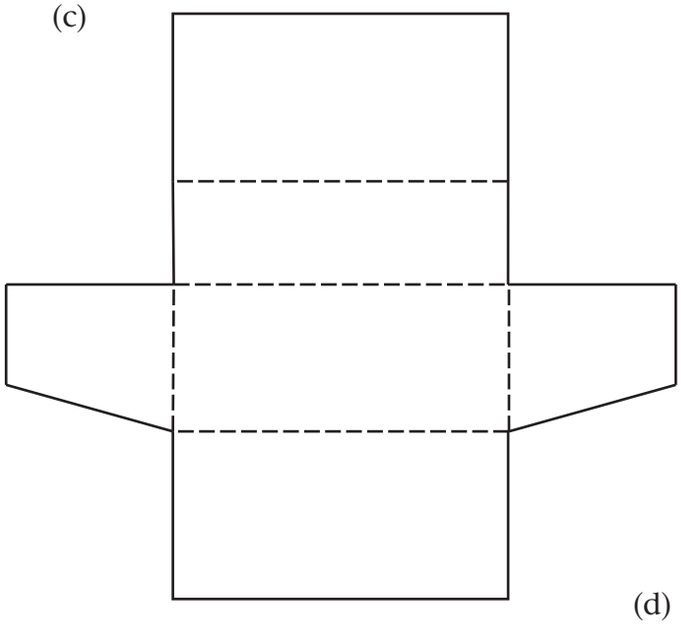
(a)



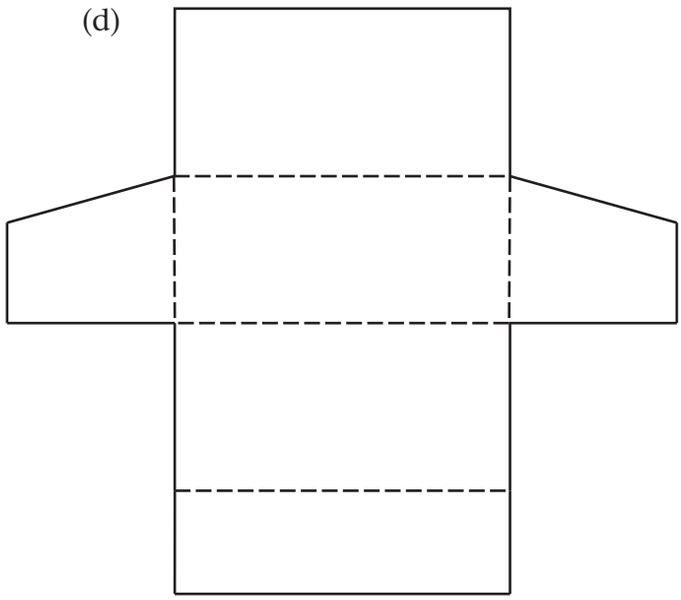
(b)



(c)

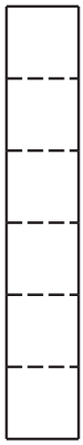


(d)

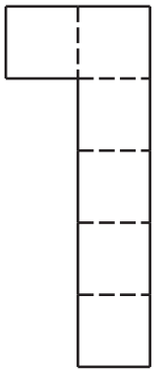


8. Which of these diagrams will work as nets for a cube?

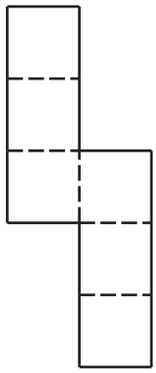
(a)



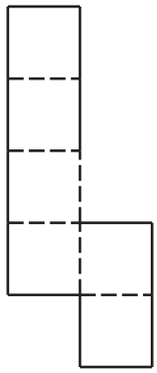
(b)



(c)



(d)



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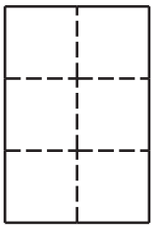
(e)



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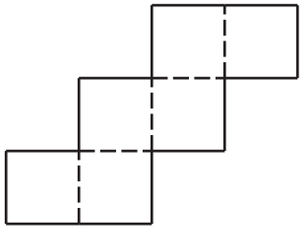
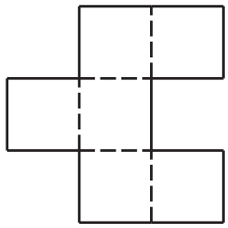
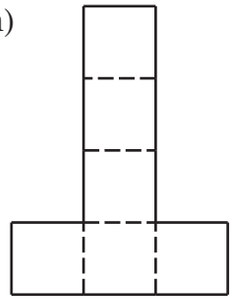


(g)



(h)



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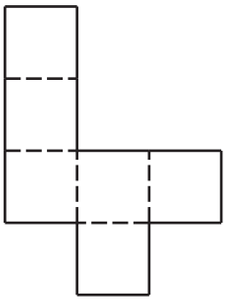
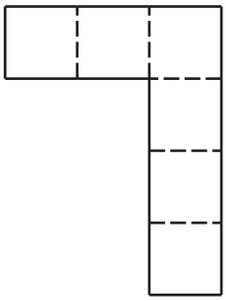
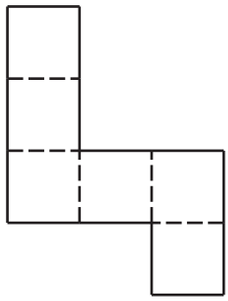
(i)



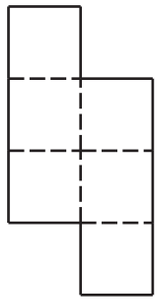
(j)



(k)



(l)



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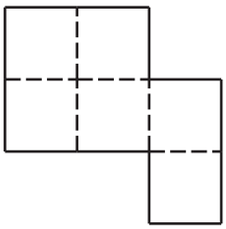
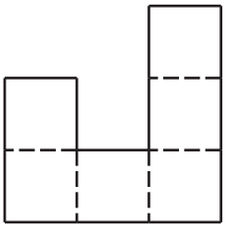
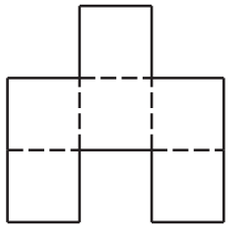
(m)



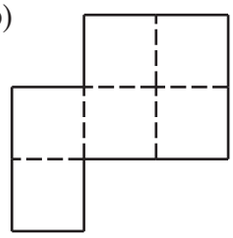
(n)



(o)



(p)



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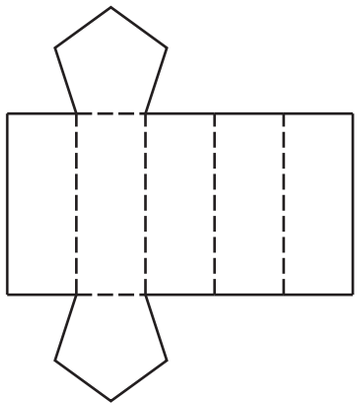
9. In each case below, state whether the diagram will work or not work as a net for making a pentagonal prism. The base need not be a regular pentagon. In the cases where the diagram will not work, explain why.

(a)



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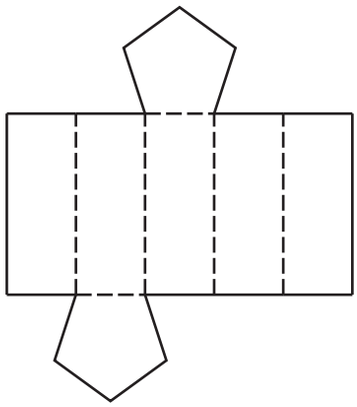
(b)



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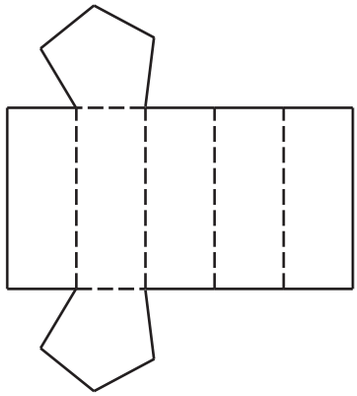
(c)



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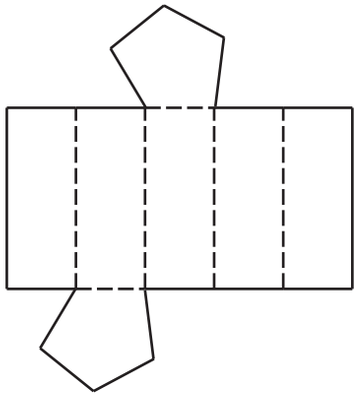
(d)



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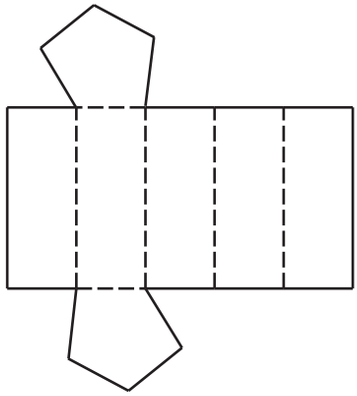
(e)



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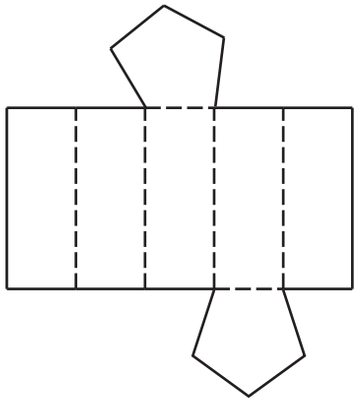
(f)



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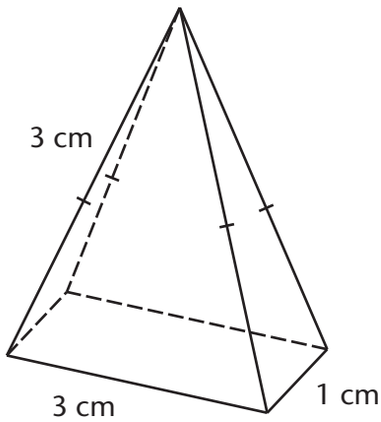
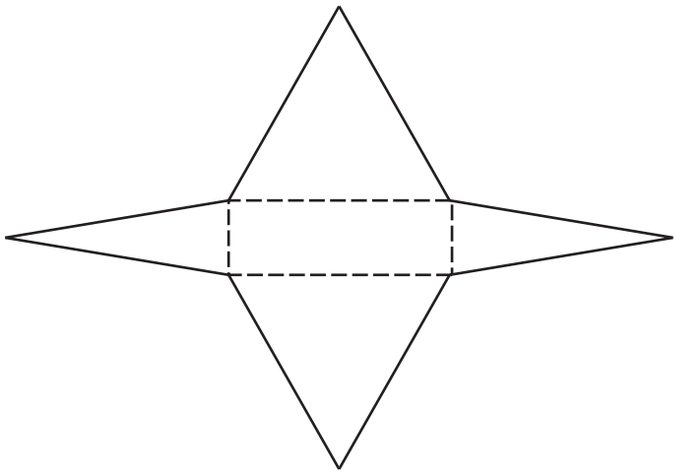
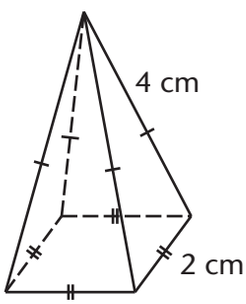
DRAWING NETS AND CONSTRUCTING 3D MODELS OF PYRAMIDS

1. Write the measurements on the sides of the net given.

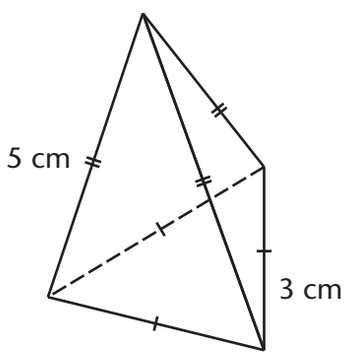




2. Draw accurate nets of the following pyramids.
   (a) Square-based pyramid



   (b) Triangular pyramid

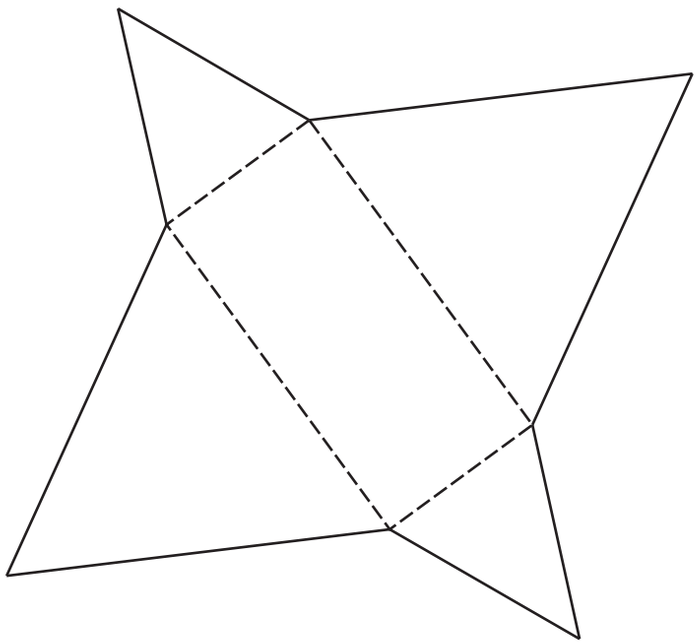


3. (a) Copy the nets you have drawn in question 2 onto cardboard or paper, but multiply the measurements by 2.
   (b) Then cut out, fold and paste the net to make a model of each 3D object.

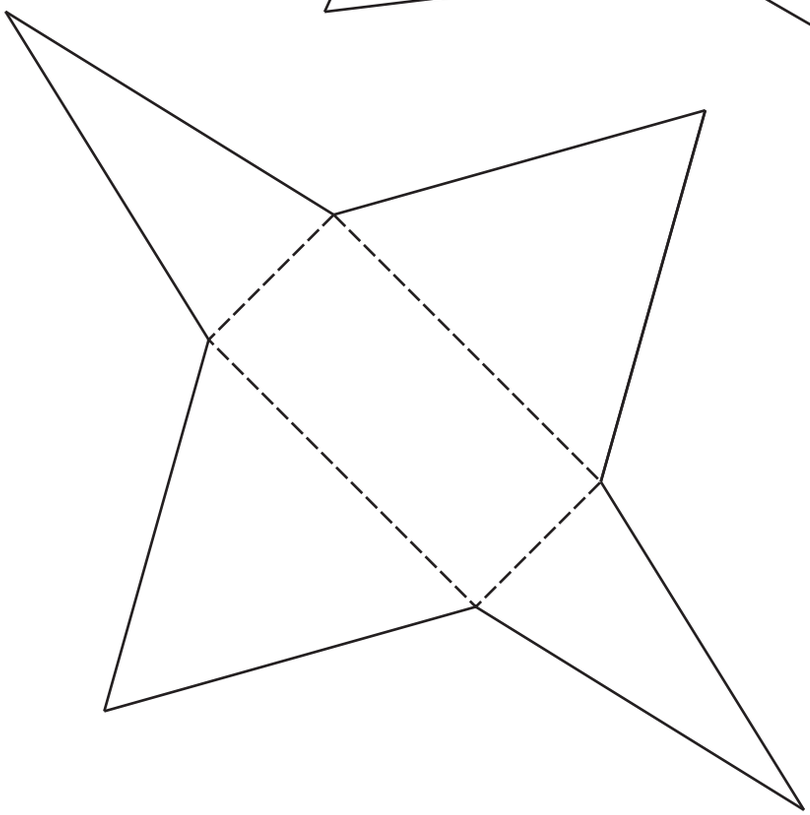
WHAT MAKES A NET WORK?

Which of the diagrams below will not work as nets to make a rectangular pyramid? You may have to take measurements to be sure in some cases, or make a copy and cut the diagram out and fold it. In each case where you say no, explain why you think it will not work.

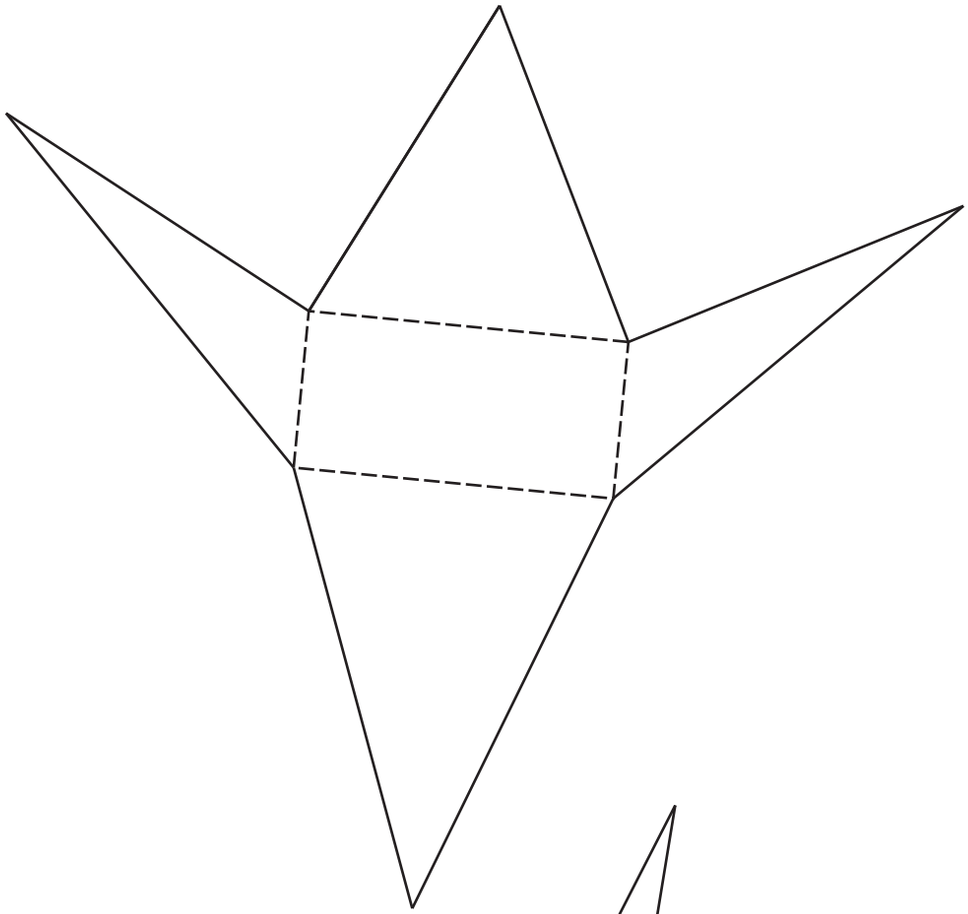
1.



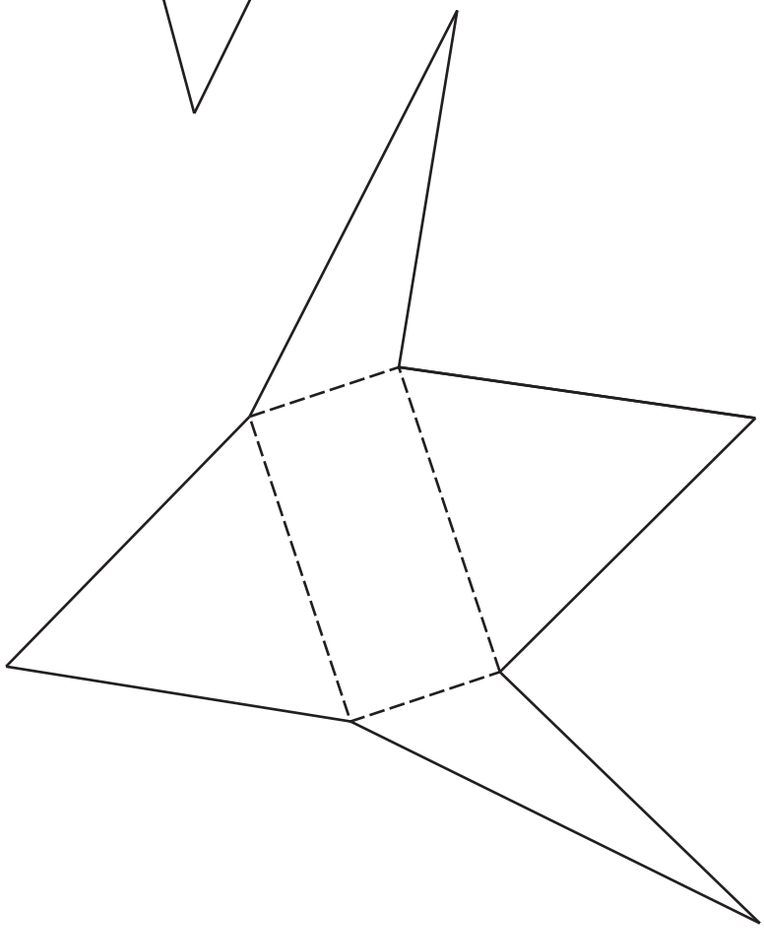
2.



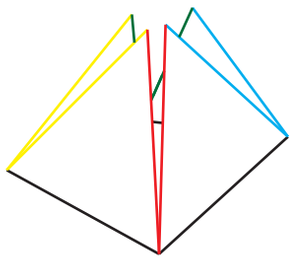
3.



4.



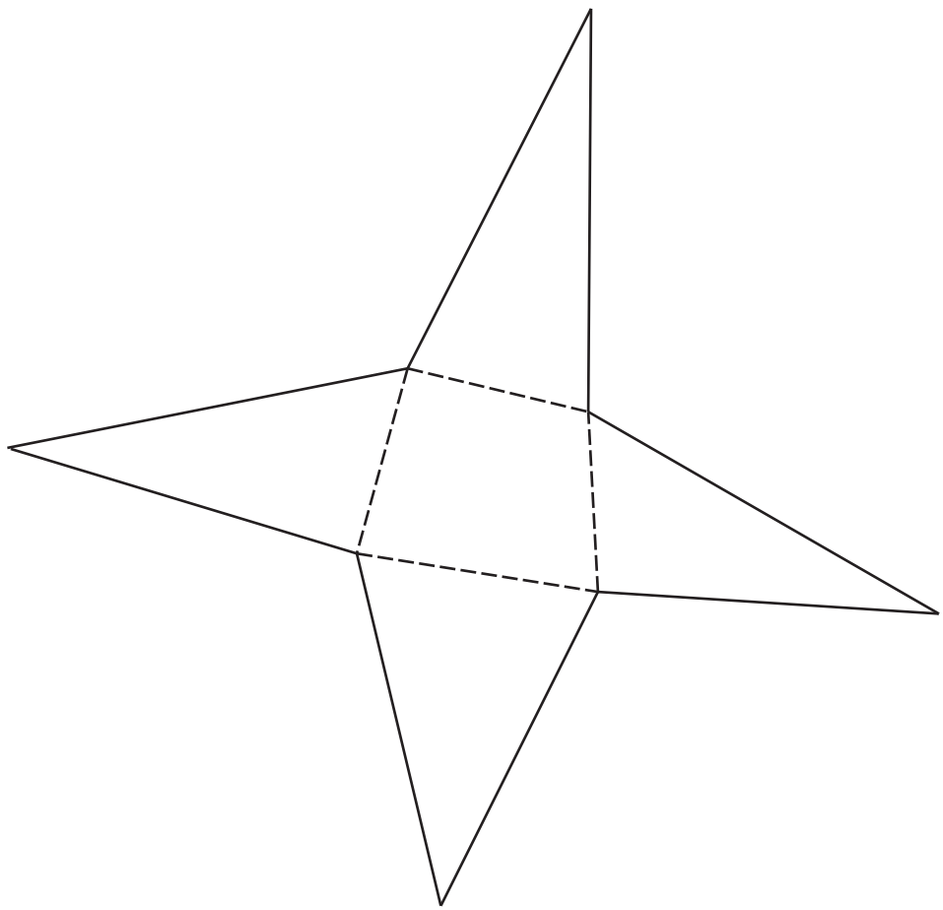
CIRCLES AND PYRAMIDS



In order to meet at the apex, one side of each triangular face of a pyramid must be the same length as the closest side of the triangle next to it.

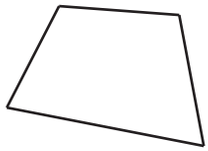
This means that certain line segments in the net of a pyramid must be equal.

1. Mark the line segments that should be equal in the diagram below, so that a pyramid can be made by folding a cut-out of the diagram on the dotted lines.



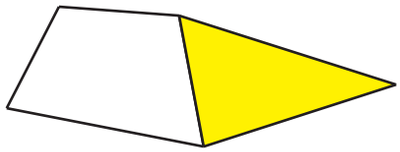
2. Make an accurate copy of the above diagram on stiff paper or cardboard. Cut it out and fold along the dotted lines. See if you can make a pyramid in this way.

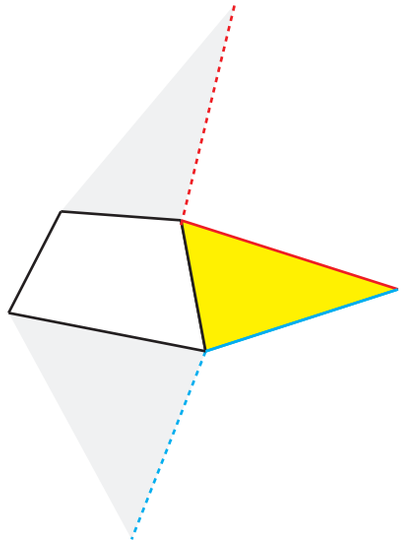
Any polygon can form the base of a pyramid.



If you want to draw a net for a pyramid, you can start by drawing a polygon.

Then draw any triangle on one side of the polygon.



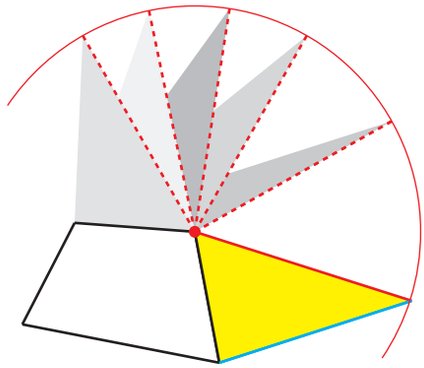


The triangles that will be adjacent to the first one must each have one side equal to the matching side of the first triangle, as indicated with the solid and dotted red and blue line segments in the sketch on the right.

The dotted line segments can be in other positions too, as long as they have the same lengths as the coloured sides of the first triangle.

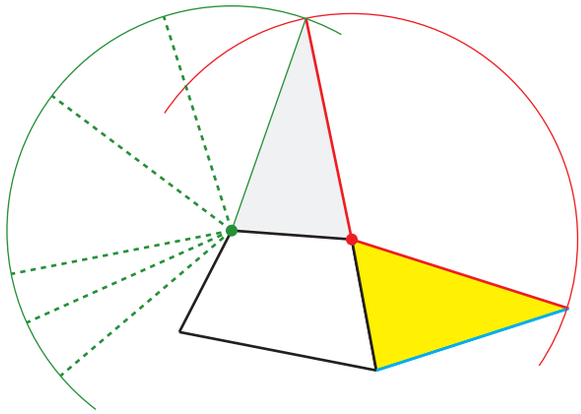
This means that once the first triangular face is drawn, there are many different possibilities for each of the two triangles that will be adjacent to it on the pyramid.

The circle with the red dot as midpoint, on the sketch on the right, shows the possibilities for one triangular face.



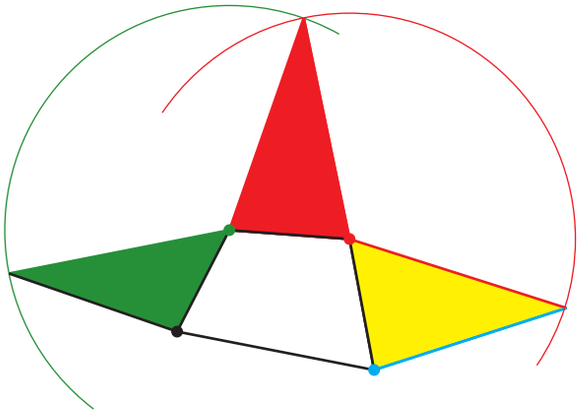
Once a triangle on the upper edge of the base is chosen, a circle can be drawn around the green vertex to indicate the possibilities for the third triangular face. The radius of this circle is the length of second leg of the second triangle, shown in solid green on the sketch.

Any line segment drawn from the green circle to the green vertex can be a side of the third triangular face.



Only one triangular face remains to be drawn now. We will refer to it as the blue triangle.

The black and blue dots on the sketch show where two vertices of the blue triangle should be.



3. Roughly draw the fourth triangular face for a pyramid on the above sketch. Think how long the sides should be so that the diagram will work precisely as a net to make a pyramid.

4. (a) How can the black dot and the green triangle help you to get some idea as to where the third vertex of the blue triangle should be?

.....

.....

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(b) How can the blue dot and the yellow triangle help you to get some idea as to where the third vertex of the blue triangle should be?

.....

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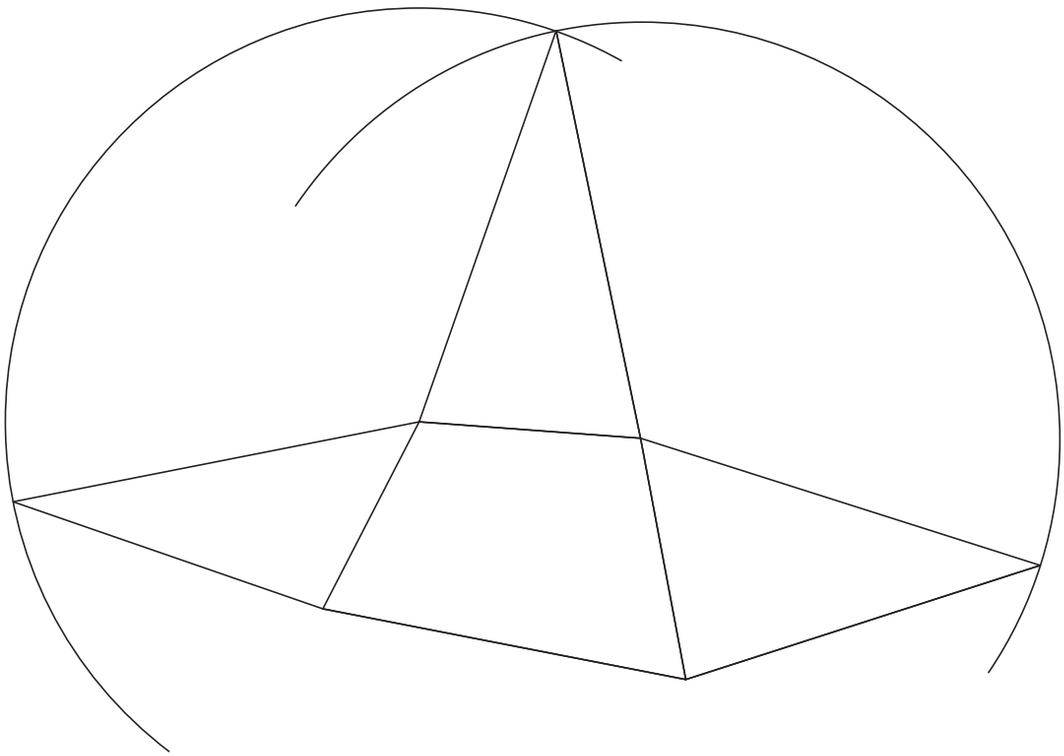
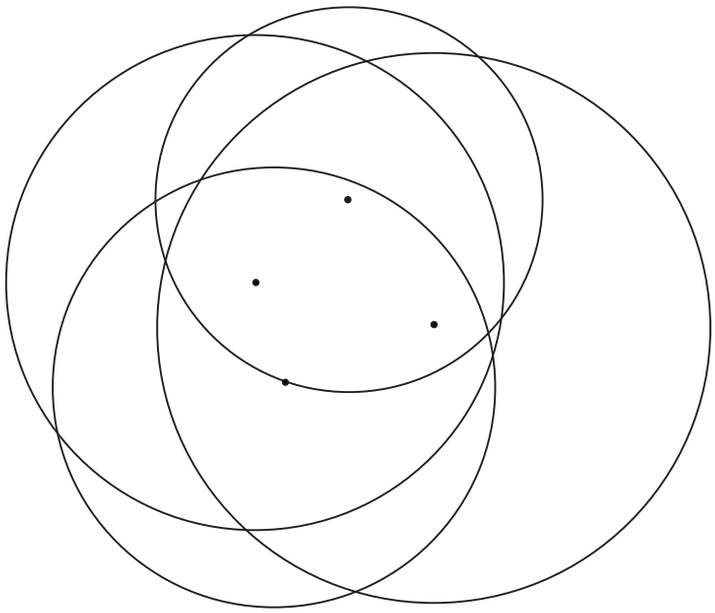
5. An enlargement of the sketch given at the top of this page is given on the next page. Use your pair of compasses to find the third vertex of the face that is not yet drawn, and complete the net for the pyramid. Then make a copy of the diagram on stiff paper or cardboard, cut it out and fold it to see whether it forms a pyramid.



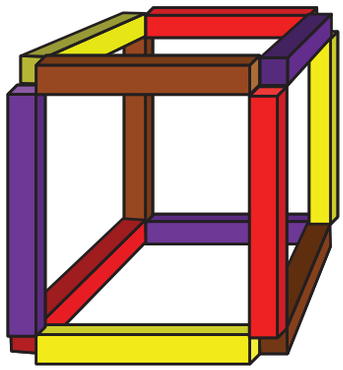
6. Join points on this sketch to draw a net for a pyramid.



13.3 Platonic solids

MAKING POLYHEDRA WITH IDENTICAL FACES AND EQUAL EDGES

A cube is a special type of polyhedron. It has 6 identical faces, and its 12 edges are all equal in length.



1. How many vertices does a cube have? .....
2. Can you think of an object which has faces that are identical triangles, and all its edges are equal? Try to draw a rough sketch of the net for such an object in your exercise book.

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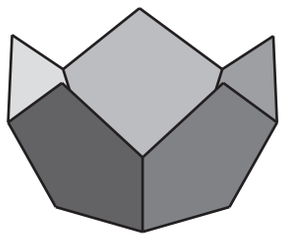
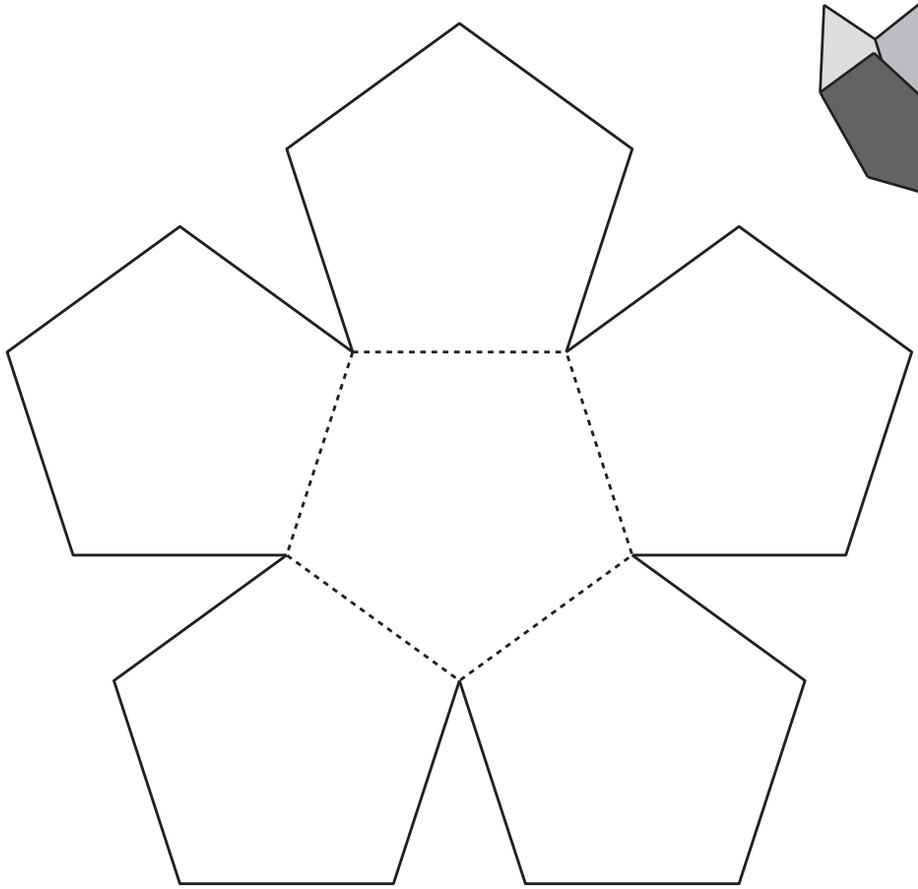
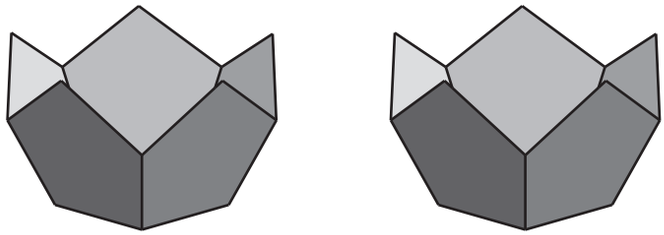
3. (a) Make a copy of the diagram below. Cut it out and fold it along the dotted lines. Attach the faces with sticky tape to make an open container with pentagonal faces.

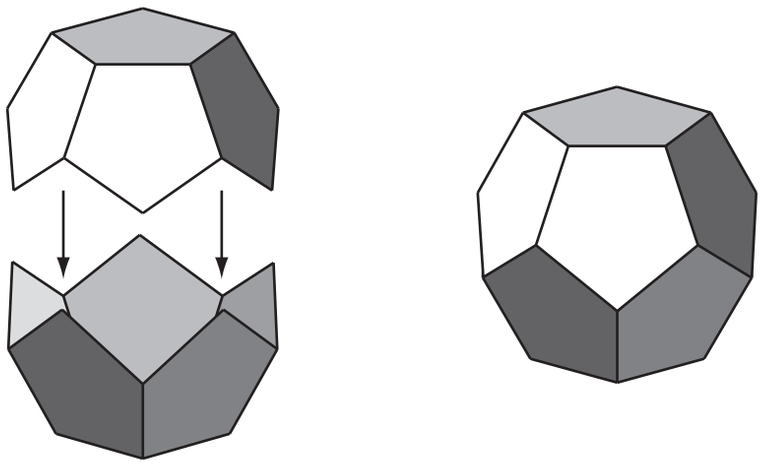




(b) Make another copy of the pentagon diagram, and make a second container.



(c) Turn one of your containers upside down and put the two together to form a polyhedron.



(d) How many faces does your polyhedron have? .....

(e) Are the faces identical, and what shape are they?

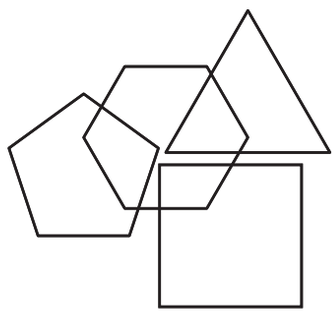
.....

(f) Are the edges equally long, and how many edges are there?

.....

A polyhedron of which all the faces are identical regular polygons is called a **Platonic solid**, because the Greek philosopher Plato was fascinated by such objects.

A **regular polygon** is a polygon with equal sides and equal angles.



4. Do you think the diagram below can be used as a net to make a Platonic solid? If it is possible, how many faces, how many edges and how many vertices will the polyhedron have?

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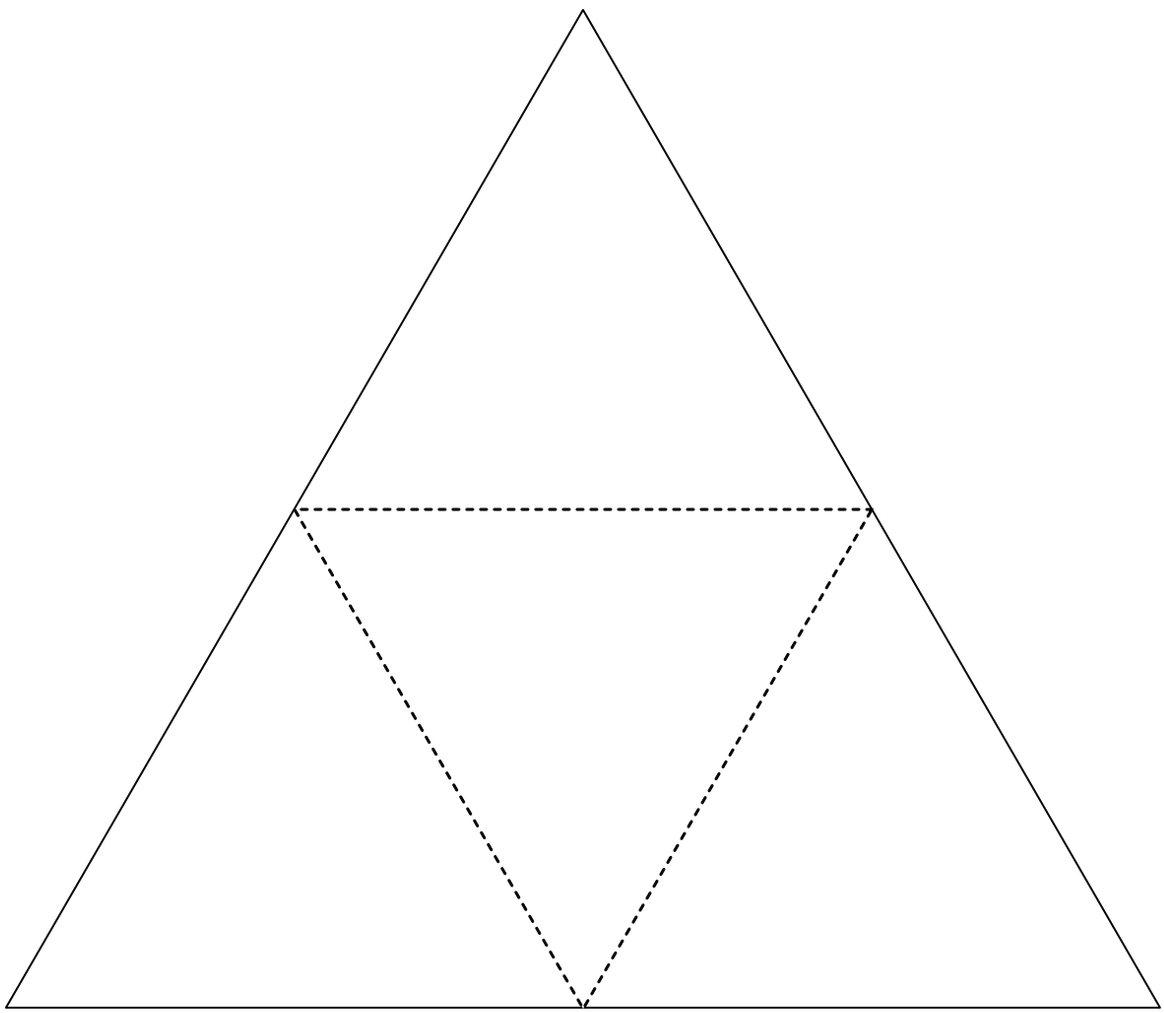
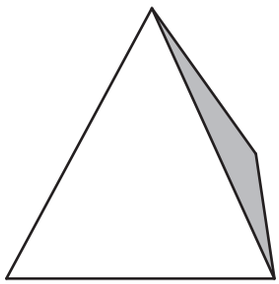


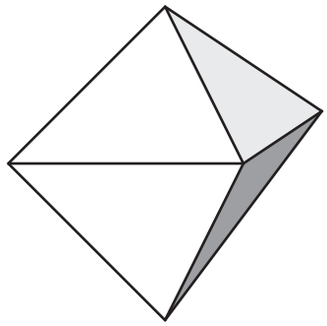
5. (a) Make two accurate copies of the above diagram. Cut out on the solid lines and fold on the dotted lines to form two identical polyhedra. Polyhedra like this are called **regular tetrahedra**.
   (b) Try to combine your two regular tetrahedra to make another solid, with 6 faces, 9 edges and 5 vertices.

THE PLATONIC SOLIDS

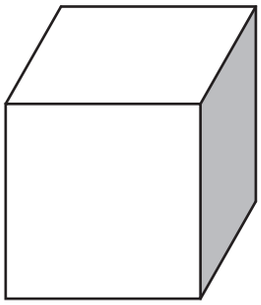
The Platonic solids have special names, and these are given below.
There are only five Platonic solids.



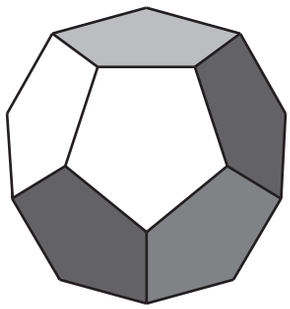
A **tetrahedron** consists of 4 equilateral triangles. It has 6 edges and 4 vertices.



An **octahedron** consists of 8 equilateral triangles. It has 12 edges and 6 vertices.

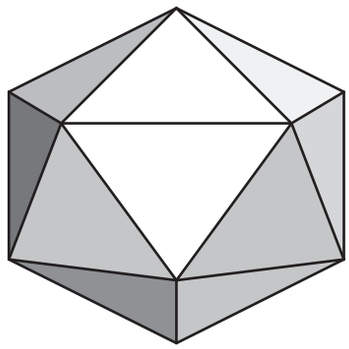


A **hexahedron** (also known as a **cube**) consists of 6 squares. It has 12 edges and 8 vertices.



A **dodecahedron** consists of 12 regular pentagons. It has 30 edges and 20 vertices.

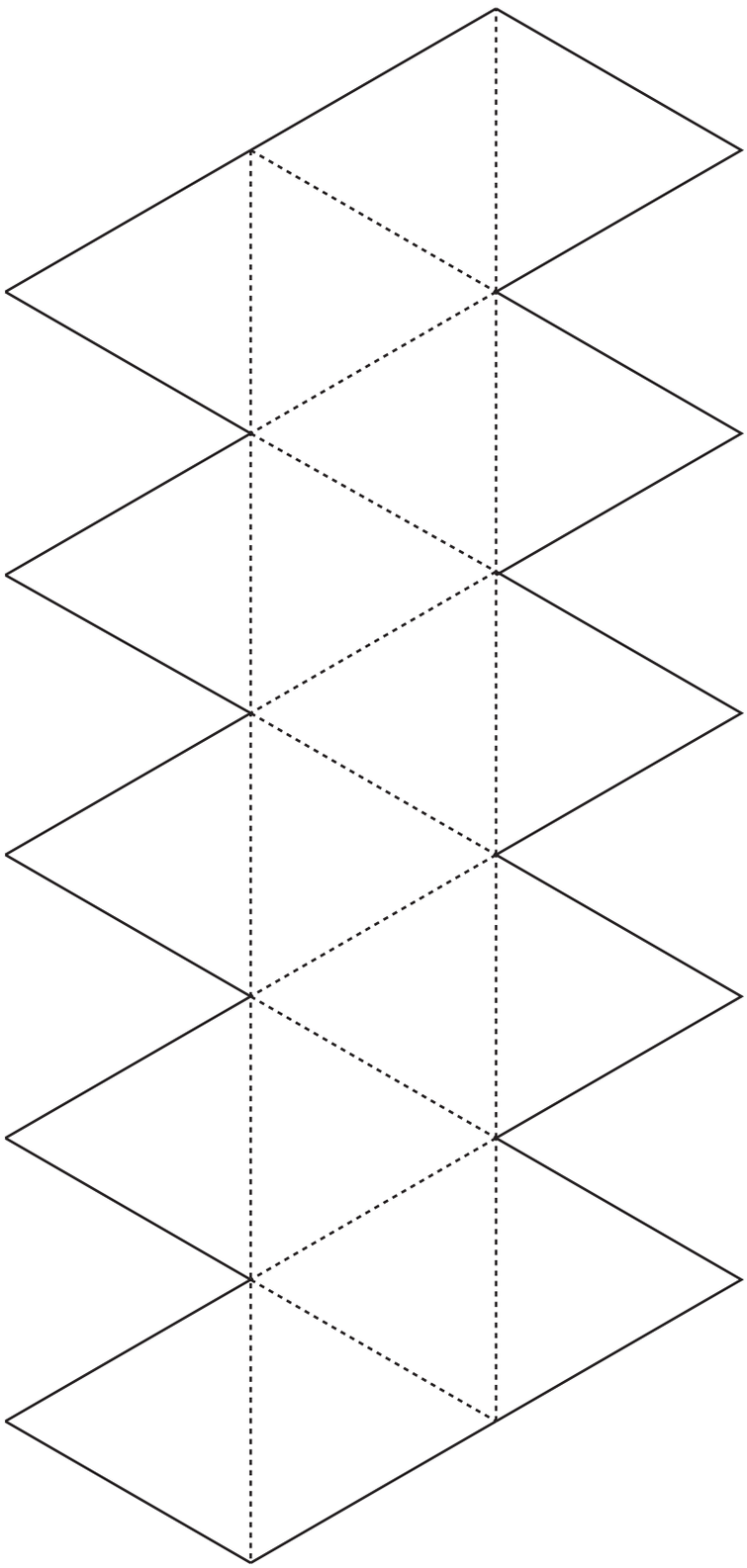
An **icosahedron** consists of 20 equilateral triangles. It has 30 edges and 12 vertices.



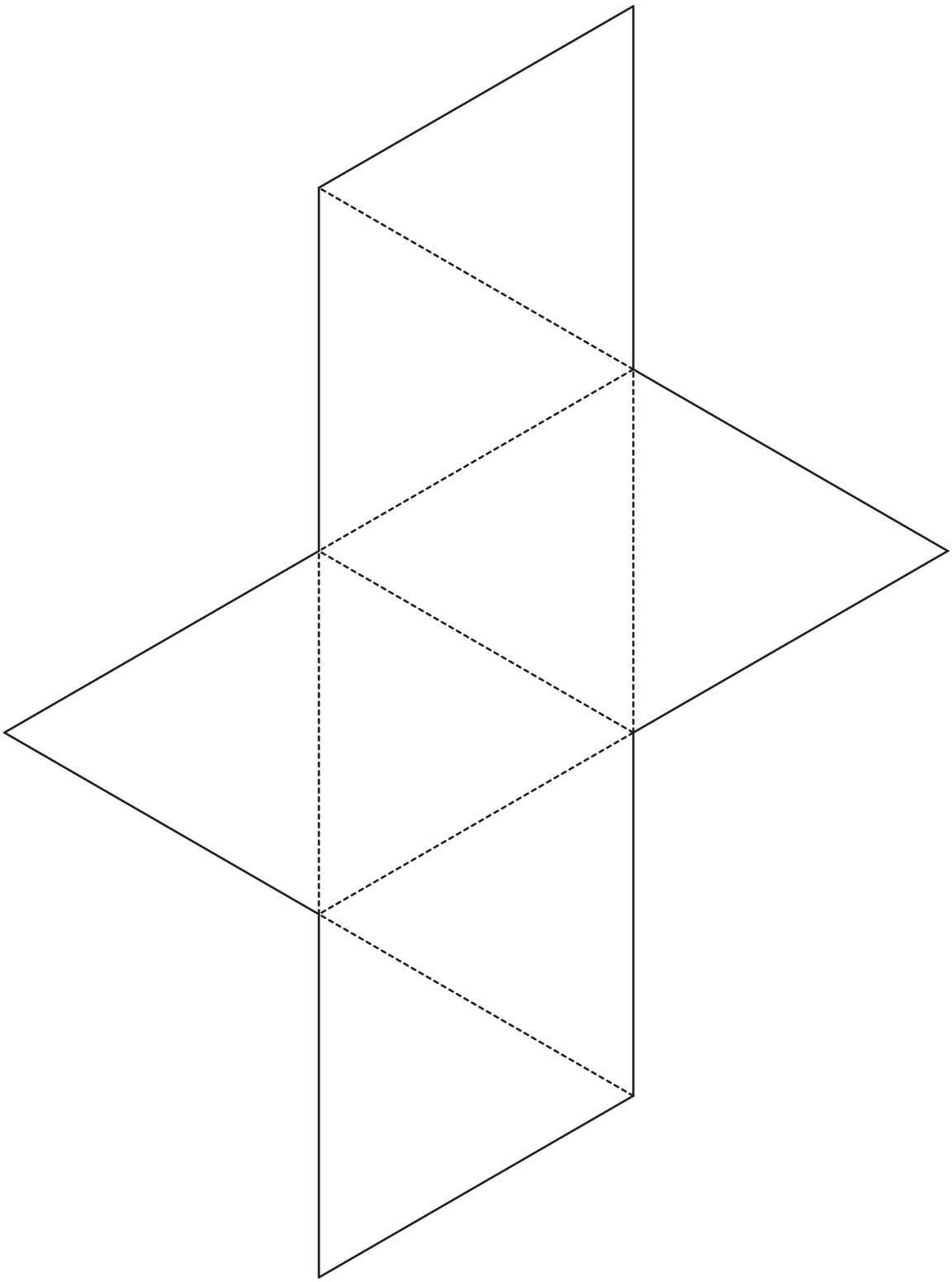
1. Nets for some of the Platonic solids are given on the following pages. Write the names of the objects next to the nets that can be used to make them.

2. Investigate whether Euler's formula is true for the Platonic solids.

(a) ……..



(b) ……..



(c) .....