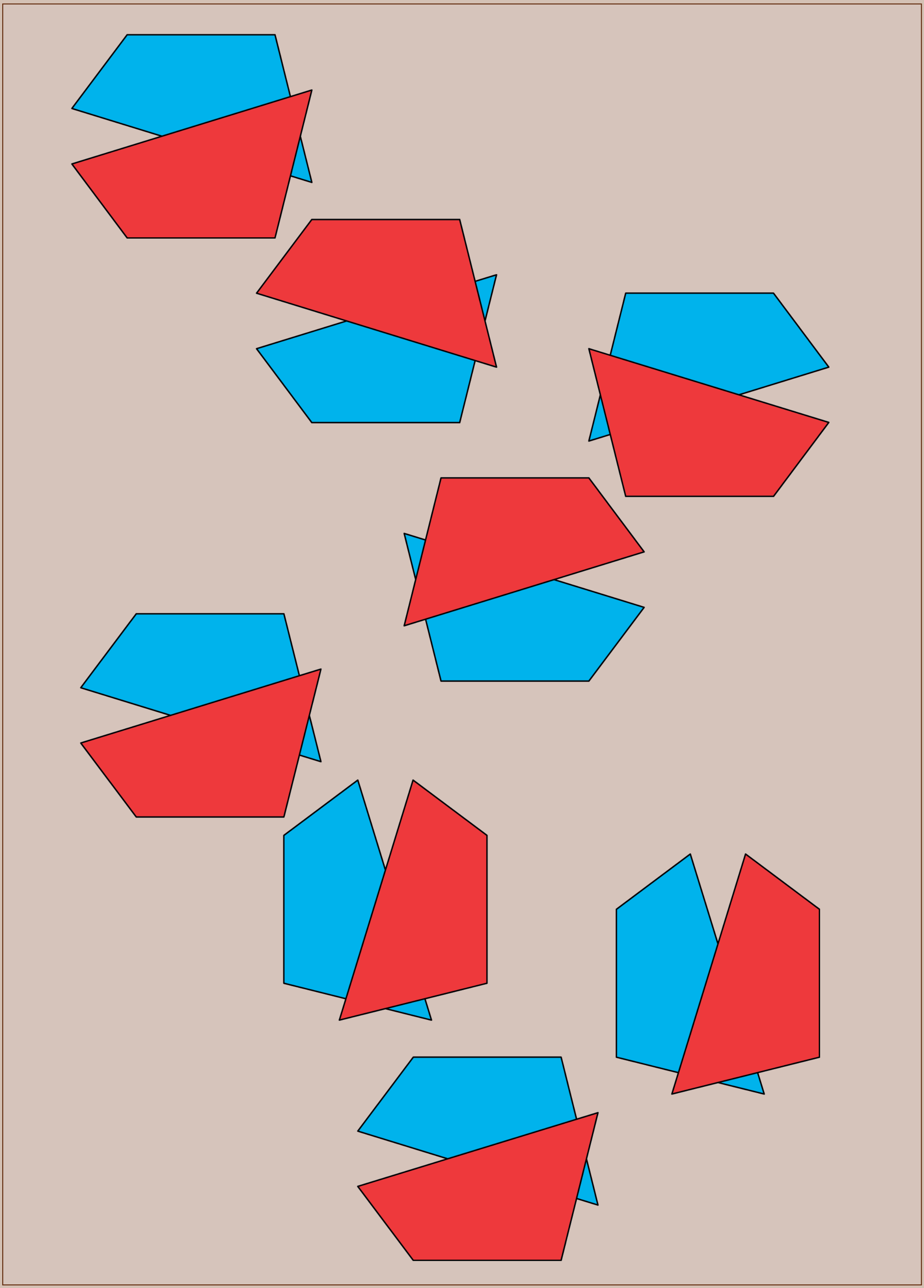
Chapter 12
Transformation geometry

In previous grades, you learnt about translating, reflecting and rotating geometric figures. These changes in the positions of figures are types of transformations. You will now learn how to plot transformations on a coordinate system. Here, you will focus on the change in the coordinates of points and geometric figures on the coordinate system. You will also revise how to enlarge and reduce figures, and investigate in more detail how the sides of enlarged and reduced figures must be in proportion. Then you will explore how enlarging or reducing a figure affects the sizes of its perimeter and area.



12 Transformation geometry

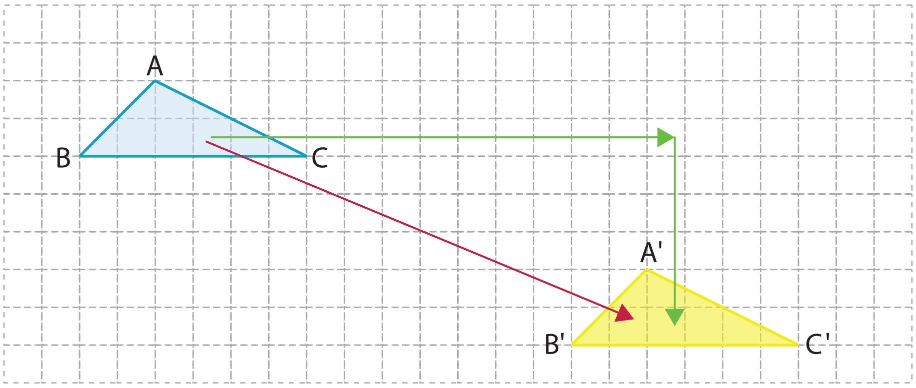
12.1 Transformations and coordinate systems

WHAT ARE TRANSFORMATIONS?

A figure can be moved from one position to another on a flat surface by **sliding** (translating), **turning** (rotating) or **flipping** it over (reflecting), or by a combination of such movements. These and other kinds of movements are also called **transformations**.

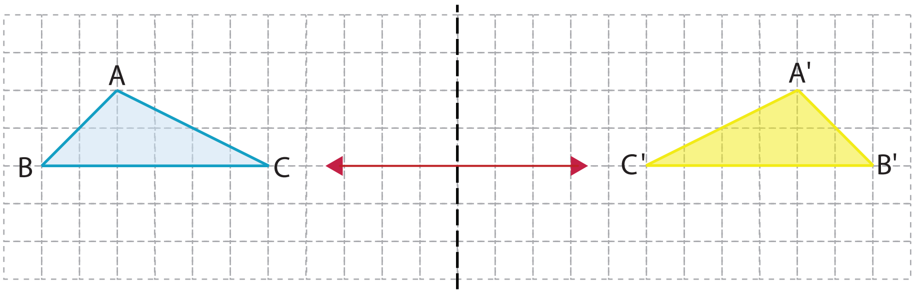
A slide, also called a ***translation***

A slide can also be performed in steps, as indicated by the green arrows.



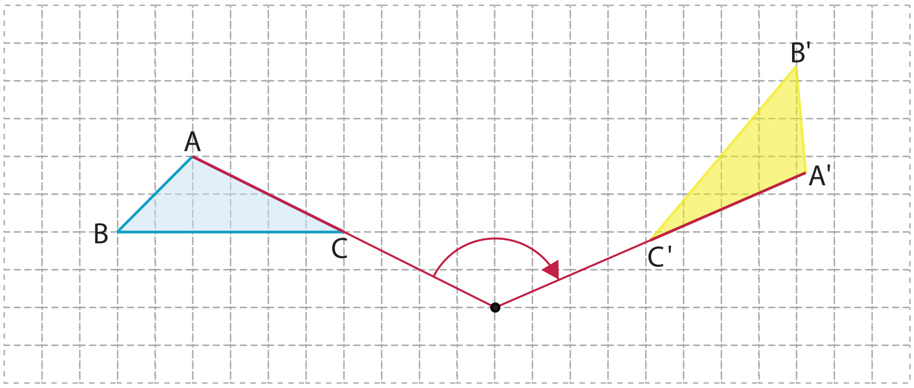
A flip-over, also called a ***reflection***

You may also think of folding the paper over on the dotted line.



A swing or turn, also called a ***rotation***

The object is swung (rotated) clockwise or anticlockwise around a point called the **centre of rotation**. It is as if you hold the object on a string.



In its new position, the figure is called the **image** of the original figure. In the diagrams above, the original figures are blue and the images are yellow. Slides, turns and flips do not change the shape or size of a figure. Hence, in these transformations, the original figure and its image are always congruent.

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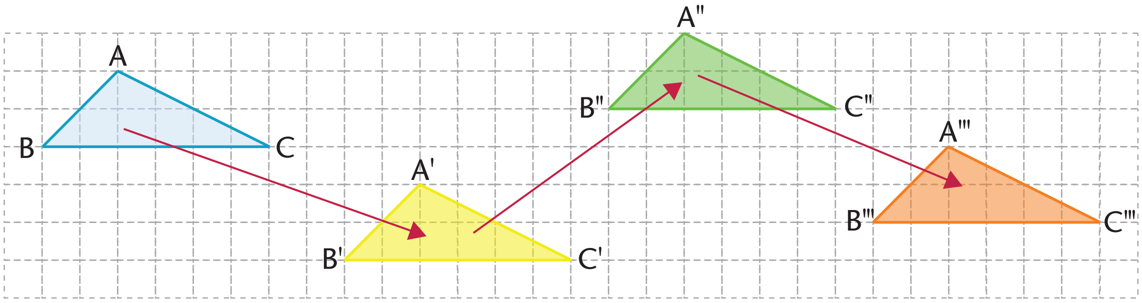
To name the image, we use the same letters as in the original figure, but we add the prime symbol (') after each letter. For example, the image of ΔABC is ΔA'B'C'. If there is a second image, we add two prime symbols, for example ΔA"B"C". If there is a third image, we use three prime symbols, for example ΔA'''B'''C''', and so on.

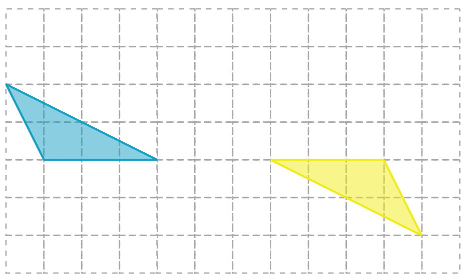


The grid in the background makes it possible to describe the different positions of the figure clearly. To do that, a **system of axes** can be drawn on the grid to form a **coordinate system**, as you will see on the next page. But first, answer the question below.

A **coordinate system** consists of numbered horizontal and vertical lines that are used to describe position.

In each case, state whether the triangle was translated, reflected or rotated.

1.



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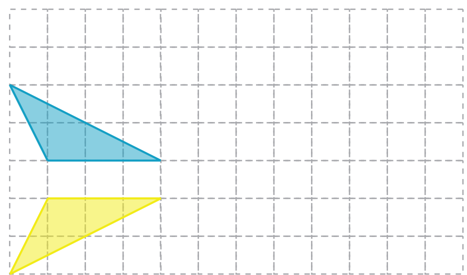

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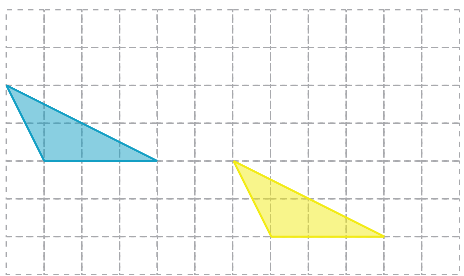

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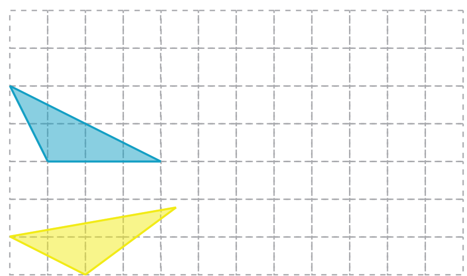

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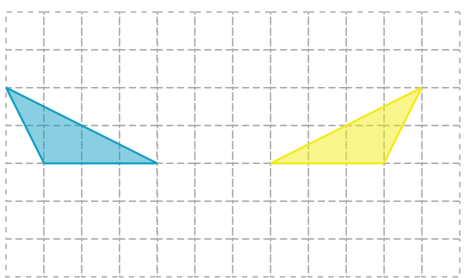

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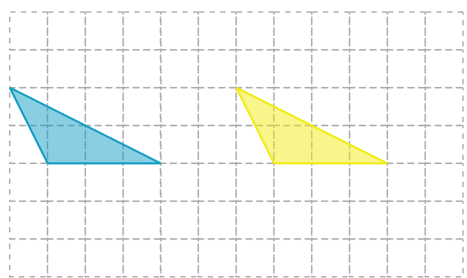
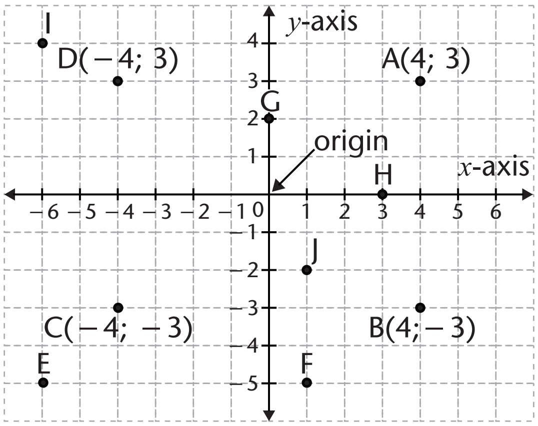


COORDINATE SYSTEMS

The position of any point on a system of coordinates can be described by two numbers, as demonstrated below for the points A, B, C and D.



In honour of the French mathematician Descartes who invented it, a coordinate system is also called *a system of Cartesian coordinates*.

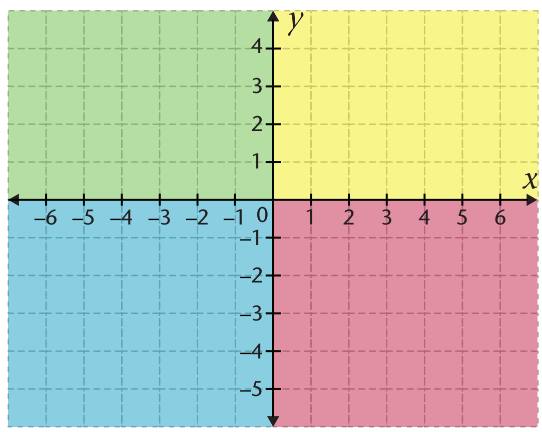
The horizontal axis on the coordinate system is called the *x*-axis and the vertical axis is called the *y*-axis. The ordered pair (4; 3) indicates that the value of the *x*-coordinate is 4 and the value of the *y*-coordinate is 3. A coordinate system is divided into four sections called **quadrants**.

1. What are the coordinates of each of the following points on the above grid?

   E .... F .... G ....

   H .... I .... J ....

The first quadrant is coloured yellow on the system on the right, the second quadrant green, the third quadrant blue and the fourth quadrant pink.



2. Mark the following points on the coloured coordinate system.

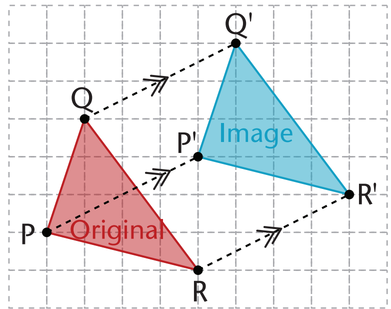
   A(5; 2) B(–4; 3)
   C(–5; 1) D(–3; –3)
   E(–6; –2) F(2; –3)
   G(5; –2) H(4; –6)

3. (a) In which quadrant are both coordinates positive? ....
   (b) In which quadrant are both coordinates negative? ....
   (c) In which quadrant is only the *x*-coordinate negative? ....
   (d) In which quadrant is only the *y*-coordinate negative? ....

12.2 Translation on the coordinate system

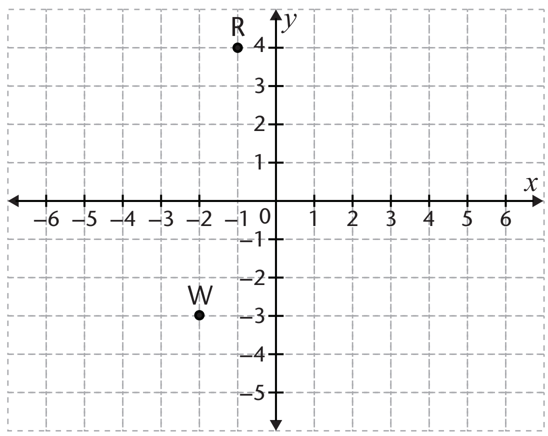
Revise the **properties of translation** from Grade 7:

- The line segments that connect any point in the original figure to its image are all equal in length. In the diagram: PP' = RR' = QQ'
- The line segments that connect any original point in the figure to its image are all parallel. In the diagram: PP'||RR'||QQ'
- When a figure is translated, its shape and size do not change. The original and its image are congruent.



TRANSLATING POINTS ON THE COORDINATE SYSTEM

1. Plot the image of each of the following translations.
   (a) R is translated 3 units down to R'.
   (b) R' is translated 4 units to the left, to R''.
   (c) W is translated 5 units to the right, to W'.
   (d) W' is translated 6 units up, to W''.



2. (a) Write down the coordinates of points A, B and C.

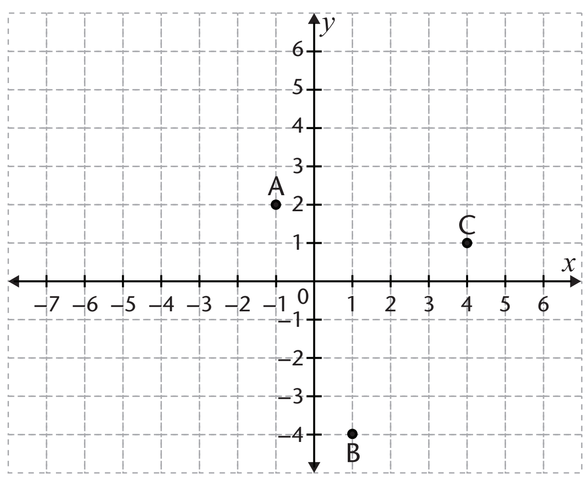
   ....

   (b) Translate A, B and C 6 units to the left and 4 units up.
   (c) Write down the coordinates of points A', B' and C'.

   ....

   (d) Join points A, B and C to form a triangle. Do the same with points A', B' and C'.
   (e) Are ΔABC and ΔA'B'C' congruent?

   ....



TRANSLATING TRIANGLES ON THE COORDINATE SYSTEM

When you plot the transformation of a shape, first plot the images of the vertices of the shape and then join the image points to create the shape.

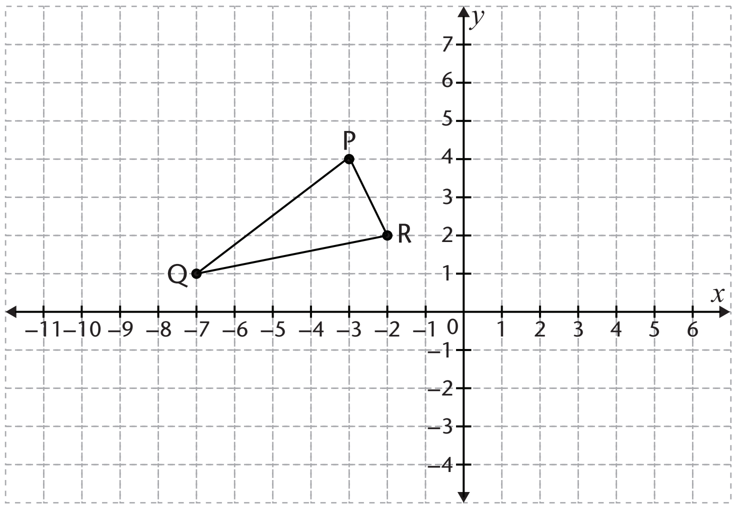
1. (a) Translate ΔPQR 6 units to the right and 2 units down. What are the coordinates of the vertices of ΔP'Q'R'?

   ....

   (b) Translate ΔPQR 4 units to the left and 3 units up. What are the coordinates of the vertices of ΔP''Q''R''?

   ....

   ....



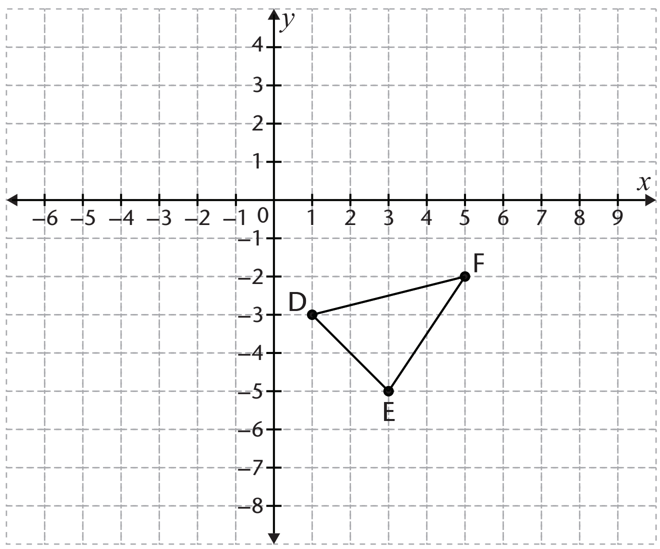
2. (a) Translate ΔDEF 4 units to the left and 2 units down. What are the coordinates of the vertices of ΔD'E'F'?

   ....

   ....

   (b) Translate ΔDEF 3 units to the right and 4 units up. What are the coordinates of the vertices of ΔD''E''F''?

   ....



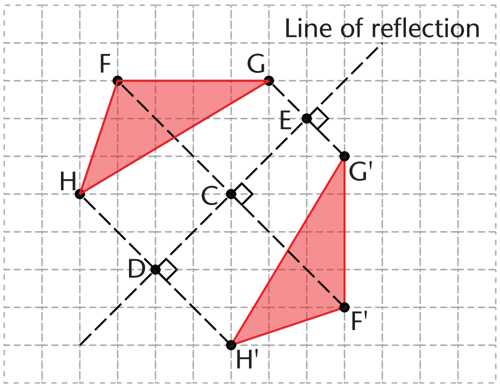
3. Write down the coordinates of the vertices of ΔKLM after each translation described in the table.

Vertices of triangle	**Translated 5 units to the right and 2 units down**	**Translated 4 units to the left and 3 units down**	**Translated 2 units to the right and 3 units up**
K(–1; 3)			
L(–2; –3)			
M(4; 0)			

12.3 Reflection on the coordinate system

Revise the **properties of reflection** from Grade 7:

- The image of ΔFGH lies on the opposite side of the **line of reflection** (mirror line).
- The distance from the original point to the line of reflection is the same as the distance from the image point to the line of reflection. In the diagram: GE = G'E; FC = F'C and HD = H'D.
- The line that connects the original point to its image point is always perpendicular (⊥) to the line of reflection. In the diagram: HH' ⊥ line of reflection, FF' ⊥ line of reflection and GG' ⊥ line of reflection.
- When a figure is reflected, the figure and its image are congruent.

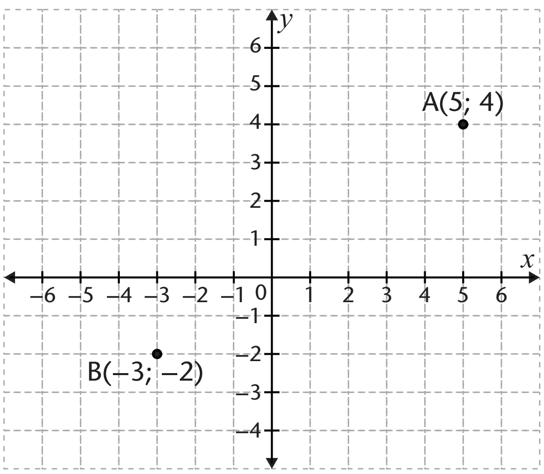


A line of reflection can run in any direction. This year, you will learn about reflections in the *x*-axis or in the *y*-axis only.

REFLECTING POINTS IN THE *x*-AXIS OR IN THE *y*-AXIS

Reflecting a point in the *x*-axis means that the *x*-axis is the line of reflection.

Reflecting a point in the *y*-axis means that the *y*-axis is the line of reflection.



1. The points A(5; 4) and B(–3; –2) are plotted on a coordinate system.

   (a) Reflect points A and B in the *x*-axis (horizontal mirror) and then in the *y*-axis (vertical mirror).

   (b) What are the coordinates of the images of point A and B when reflected in the *x*-axis?

   ....

   (c) What are the coordinates of the images of point A and B when reflected in the *y*-axis?

   ....

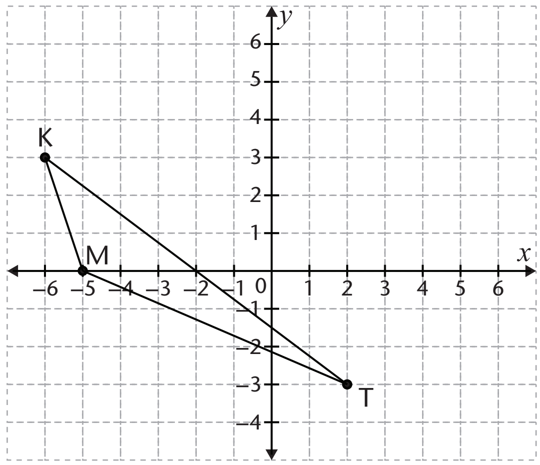
(d) Compare the coordinates of points A and B with the coordinates of their images. What do you notice?

....

....

....

2. The points K, M and T are plotted on the coordinate system.



(a) Write down the coordinates of points K, M and T.

....

(b) Reflect each point in the *x*-axis and write down the coordinates of K', M' and T'.

....

(c) Reflect points K, M and T in the *y*-axis and write down the coordinates of K'', M'' and T''.

....

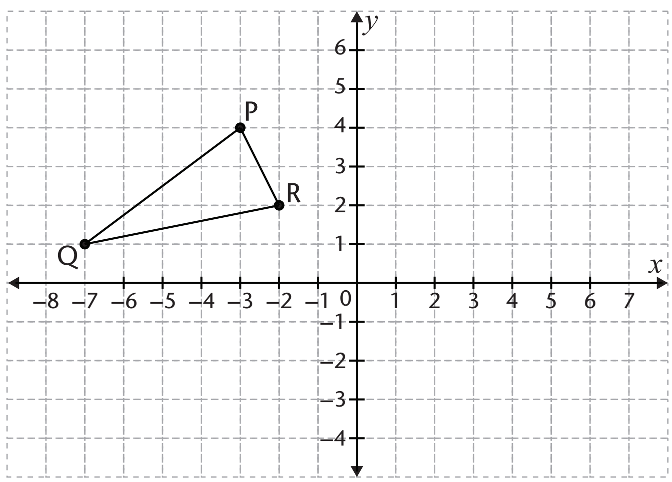
(d) Join points K, M and T to form a triangle. Do the same with points K', M' and T', and with points K'', M'' and T''.

(e) Are all three triangles congruent? ....

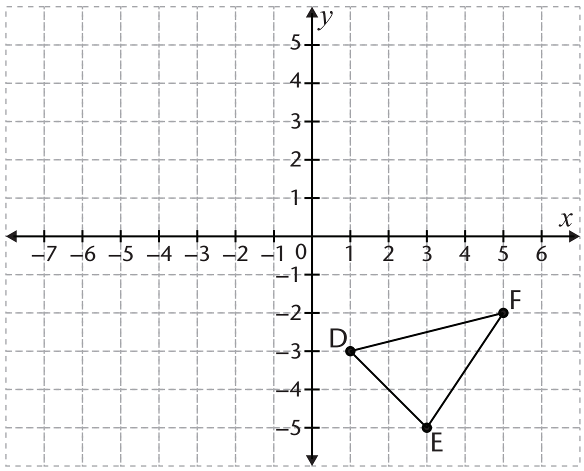
REFLECTING TRIANGLES IN THE *x*-AXIS OR IN THE *y*-AXIS

When you reflect a triangle, first reflect the vertices of the triangle and then join the reflected points.

1. (a) Reflect ΔPQR in the *x*-axis.
   (b) Reflect ΔPQR in the *y*-axis.



2. (a) Reflect ΔDEF in the x-axis.
   (b) Reflect ΔDEF in the y-axis.



3. The coordinates of the vertices of three triangles are given in the tables below. For each vertex, write down the coordinates of its reflection in the x-axis or in the y-axis as required.

(a)

Vertices of triangle	**Reflection in the x-axis**
K(–4; 5)	
L(2; –5)	
M(–5; –3)	

(b)

Vertices of triangle	**Reflection in the y-axis**
X(–1; 3)	
Y(–2; –3)	
Z(4; 1)	

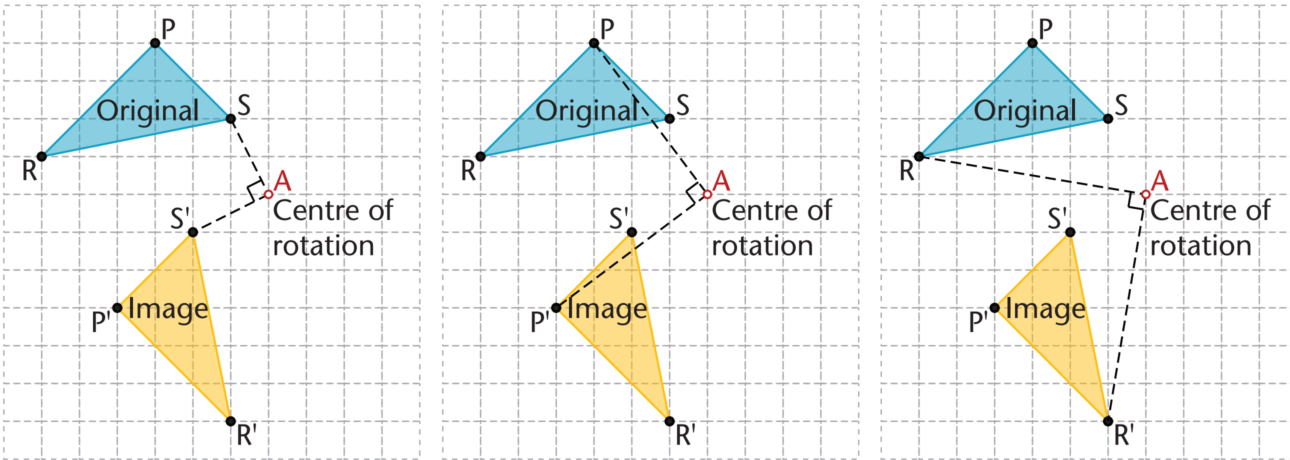
(c)

Vertices of triangle	**Reflection in the y-axis**	**Reflection in the x-axis**
D(–2; 5)		
E(0; –3)		
G(2; 0)		

12.4 Rotation on the coordinate system

The distance from the centre of rotation to any point on the original image is equal to the distance from the centre of rotation to its corresponding point on the image. In the diagrams below: SA = S'A, PA = P'A and RA = R'A.

The angle that is formed between the line connecting an original point (S or P or R) to the centre of rotation A and the line connecting the image point (S', P', R') to the centre of rotation is equal to the angle of rotation. In the diagrams: the triangle was rotated through 90°, so $S\hat{A}S' = 90°$, $P\hat{A}P' = 90°$ and $R\hat{A}R' = 90°$.



On the coordinate system, the centre of rotation can be any point. This year, you will focus on rotations about the point (0; 0), which is called the **origin**. A point, line segment or figure can be rotated clockwise or anticlockwise through any number of degrees about the centre of rotation.

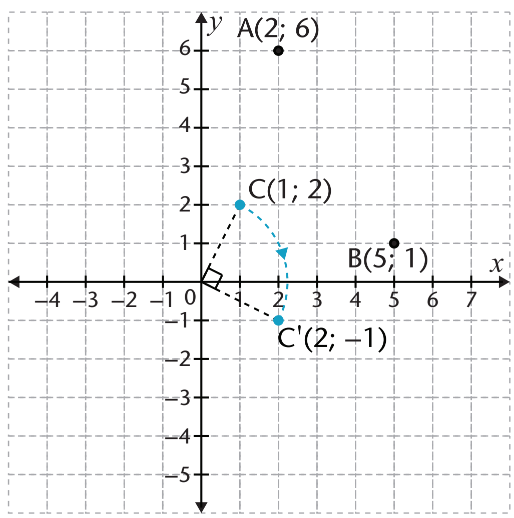
ROTATING POINTS AND FIGURES ABOUT THE ORIGIN

1. In the diagram, point C has been rotated 90° clockwise about the origin.
   (a) Rotate points A and B 90° clockwise about the origin.
   (b) Write down the coordinates of points A' and B'.

   ....

   (c) Join points A, B and C to form a triangle. Do the same with points A', B' and C'.
   (d) Are the triangle and its image congruent?

   ....



(e) Compare the coordinates of points A, B and C with the coordinates of their images. What do you notice?

....

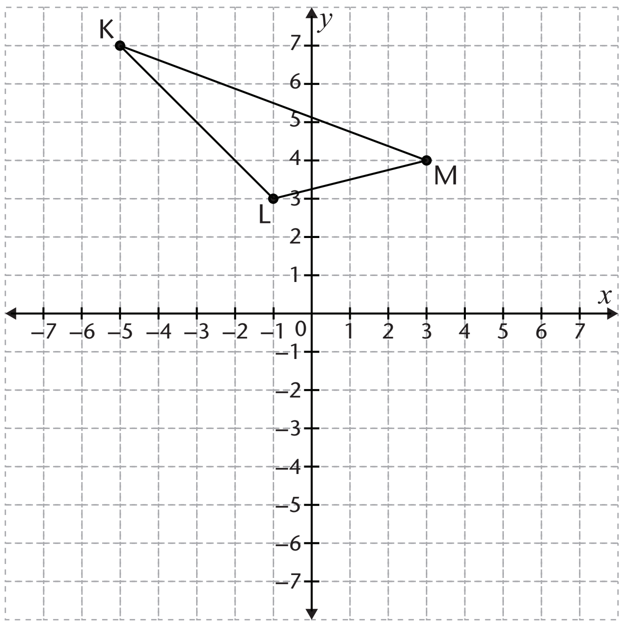
....

....

2. (a) Write down the coordinates of points K, L and M.

....

....



(b) Rotate points K, L and M 90° anticlockwise about the origin.

(c) Write down the coordinates of the image points.

....

....

(d) Rotate points K, L and M 180° about the origin.

(e) Write down the coordinates of K'', L'' and M''.

....

(f) Can you explain why there was no need to say "clockwise" or "anticlockwise" in question (d)?

....

....

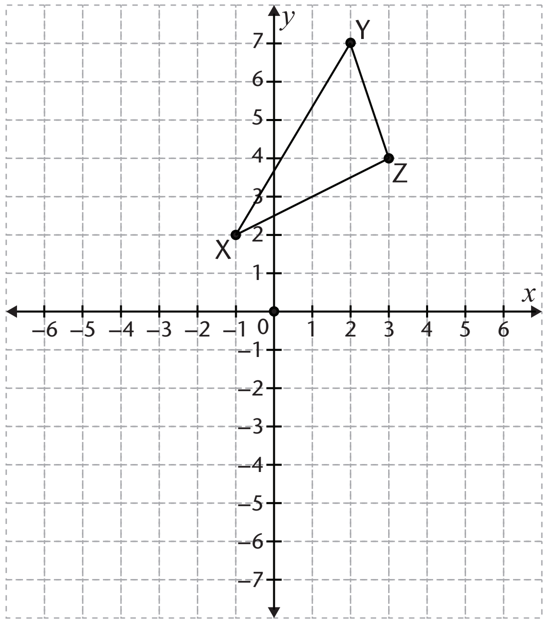
....

....

....

3. Rotate the following triangles and write down the coordinates of the vertices of each triangle after the required rotation.

(a) 180° about the origin



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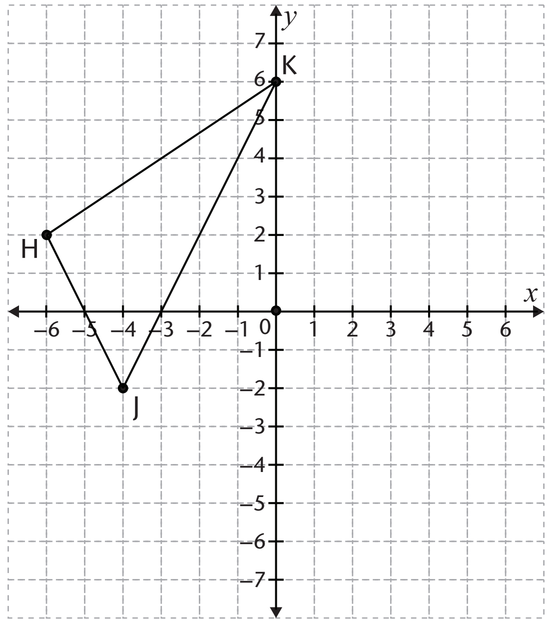
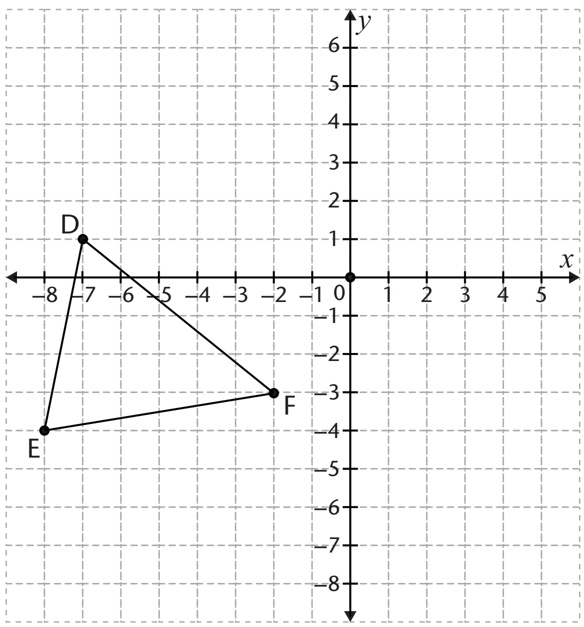
(b) 90° clockwise about the origin



....

(c) 90° anticlockwise about the origin



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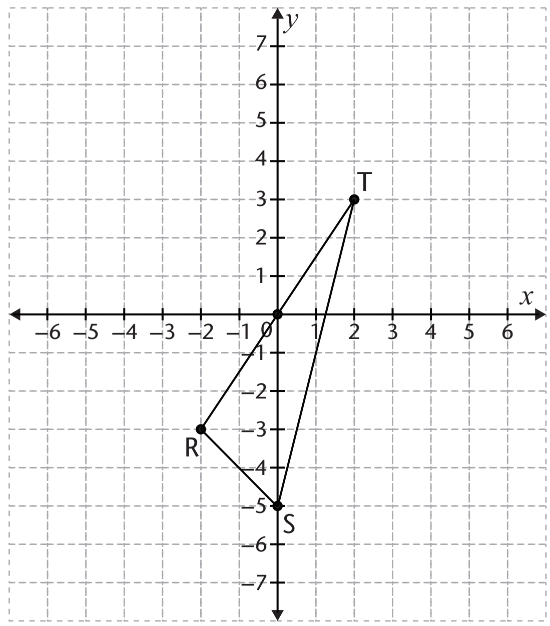
(d) 180° about the origin



....

4. Write down the coordinates of each image point after these transformations.
   (a) Rotation 180° about the origin: K(–1; 0); C(1; 1); N(3; –2)

   ....

   (b) Rotation 90° clockwise about the origin: L(1; 3); Z(5; 5); F(4; 2)

   ....

   (c) Rotation 90° anticlockwise about the origin: S(1; –4); W(1; 0); J(3; –4)

   ....

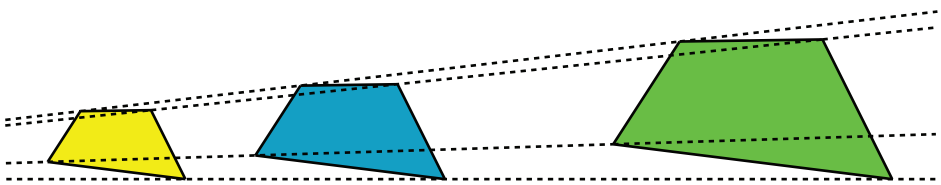
   (d) Rotation 180° about the origin: V(–5; –3); A(–3; 1); G(0; –3)

   ....

12.5 Enlargements and reductions

CALCULATE AND USE SCALE FACTORS

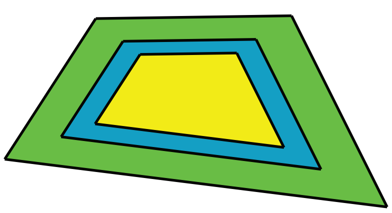
A figure may be made bigger or smaller without changing its shape.



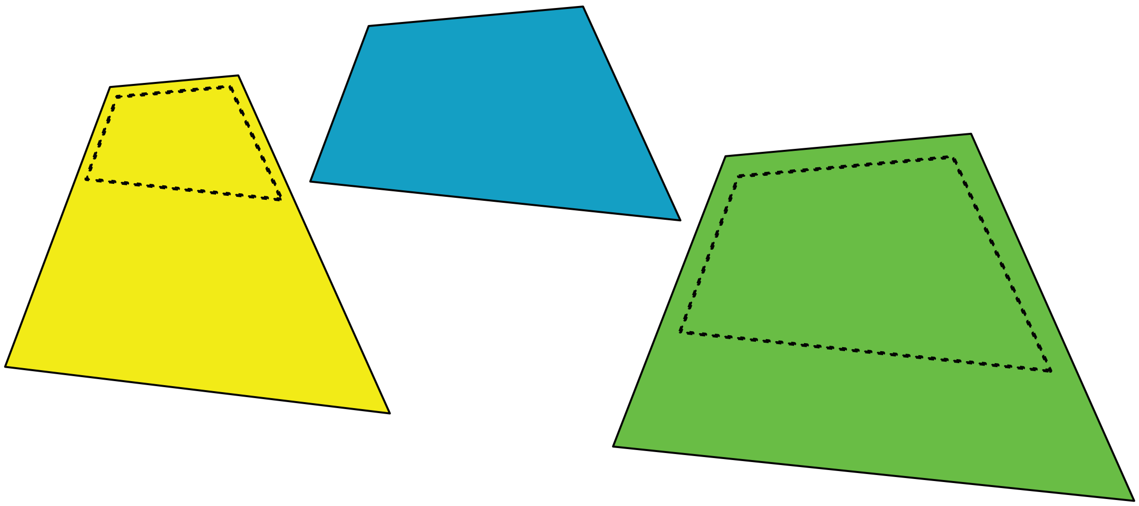
*The yellow figure is a **reduction** of the blue quadrilateral*

*The green figure is an **enlargement** of the blue quadrilateral*

A figure is only called an enlargement or reduction of another figure if the two figures have **the same shape**. The shapes can only be the same if all the corresponding angles are equal.



Even if the angles are equal, two figures may have different shapes. When the corresponding angles are equal, one figure is not necessarily an enlargement or reduction of the other.



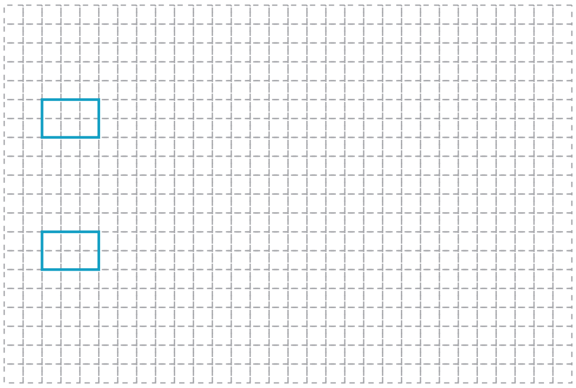
Although the angles are equal, the yellow and green figures above are *not* enlargements of the blue figure.

When a figure with straight sides is enlarged or reduced, the lengths of the sides are increased or decreased.

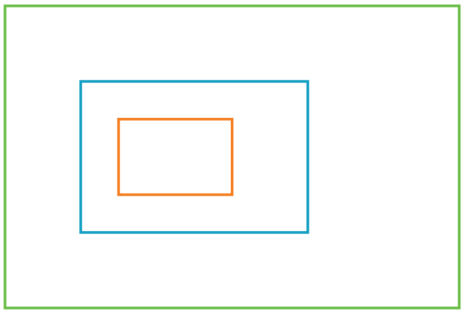
To find the lengths of the sides of the new figure, the lengths of the sides of the original figure are all multiplied by the same number. This number is called the **scale factor** of the enlargement or reduction.

The scale factor for an **enlargement** is bigger than 1. The scale factor for a **reduction** is smaller than 1.

1. Draw a bigger rectangle ABCD on the grid below, with each side 5 times as long as the blue rectangle. Also draw another bigger rectangle PQRS, with each side 5 units longer than the blue rectangle.

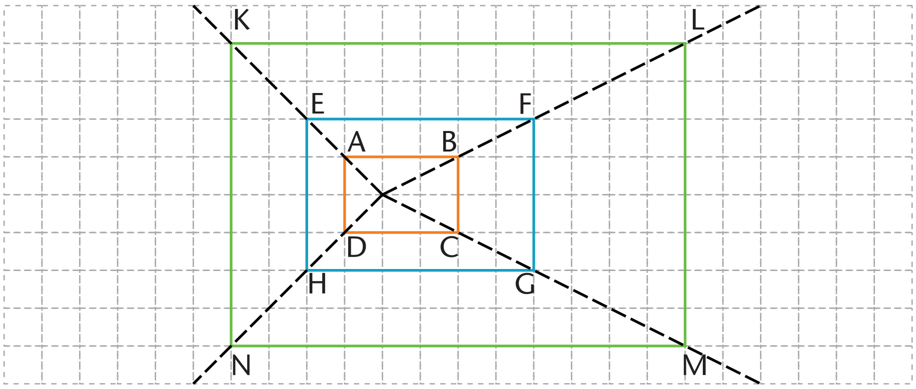


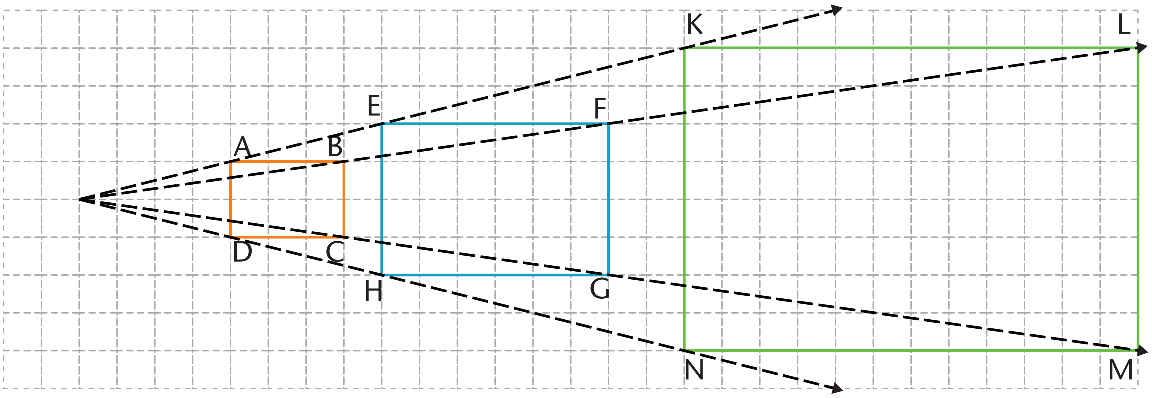
One figure is only called an enlargement or reduction of another figure if the **corresponding angles are equal** and **the ratio between the lengths of the corresponding sides is the same**, for all pairs of corresponding angles and sides in the two figures. This is demonstrated below.



The green rectangle on the right is an enlargement of the blue rectangle. The orange rectangle is a reduction of the blue rectangle.

In the two diagrams below, the same rectangles are shown on grids so that it is easy to compare the lengths of the corresponding sides and calculate the ratio between the lengths of the sides.





KLMN is an enlargement of EFGH.

Note that $\frac{LM}{FG} = 8:4 = 2$, $\frac{MN}{GH} = 12:6 = 2$, $\frac{NK}{HE} = 8:4 = 2$ and $\frac{KL}{EF} = 12:6 = 2$.

The ratio between the lengths of corresponding sides is 2, for all four pairs of corresponding sides.

We say: The **scale factor** of the enlargement from EFGH to KLMN is 2.

To avoid confusion, mathematicians normally state the dimensions of the image first when forming ratios.

ABCD is a reduction of EFGH.

Note that $\frac{BC}{FG} = 2:4 = \frac{1}{2}$, $\frac{CD}{GH} = 3:6 = \frac{1}{2}$, $\frac{DA}{HE} = 2:4 = \frac{1}{2}$ and $\frac{AB}{EF} = 3:6 = \frac{1}{2}$.

The ratio between the lengths of corresponding sides is $\frac{1}{2}$, for all four pairs of corresponding sides. The scale factor of the reduction from EFGH to ABCD is $\frac{1}{2}$.

2. (a) What is the scale factor of the enlargement from ABCD to KLMN?

....

(b) What is the scale factor of the reduction from KLMN to EFGH?

....

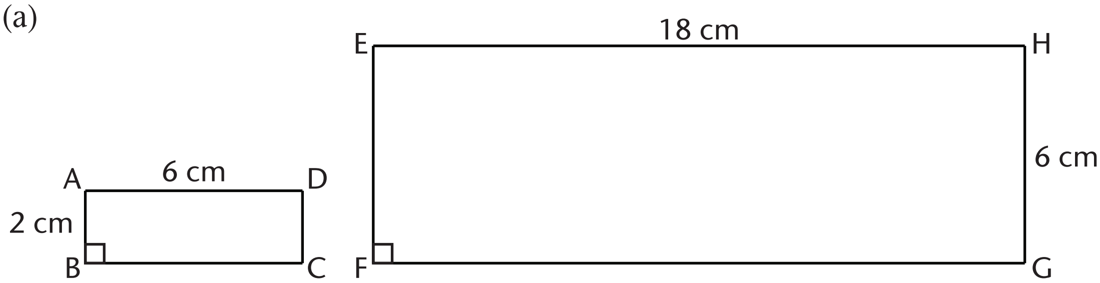
3. A rectangular shape on a photograph is 3 mm wide and 4 mm long. The photograph is enlarged with a scale factor of 5. What is the width and length of the rectangular shape on the enlarged photograph?

....

We work out the scale factor by calculating the ratios of the lengths of corresponding sides of the two figures. If the ratios are equal, we say that the corresponding sides are **in proportion**. This means that the second figure (the image) is a reduction or an enlargement of the first figure (the original).

4. Determine whether the second figure in each of the following pairs is an enlargement, a reduction, or neither of the two. Also work out the perimeters of both figures.

(a)



....

....

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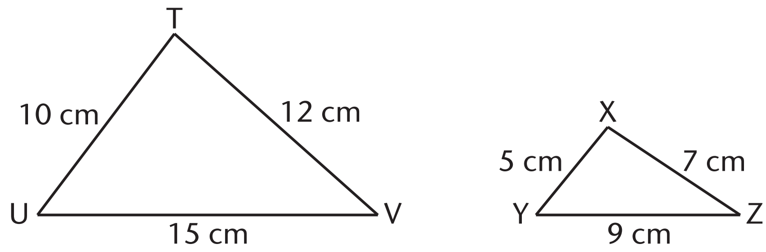
....

....

(b)



....

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....

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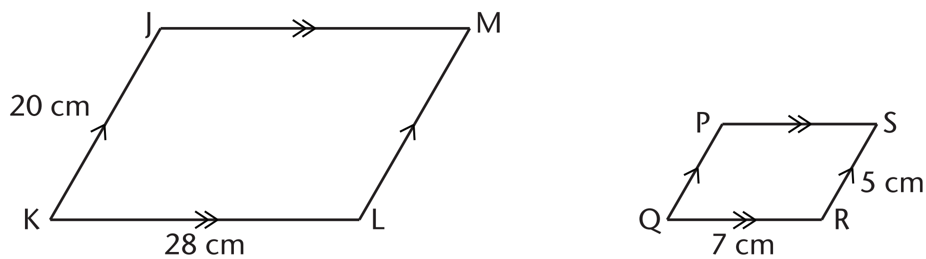
(c)



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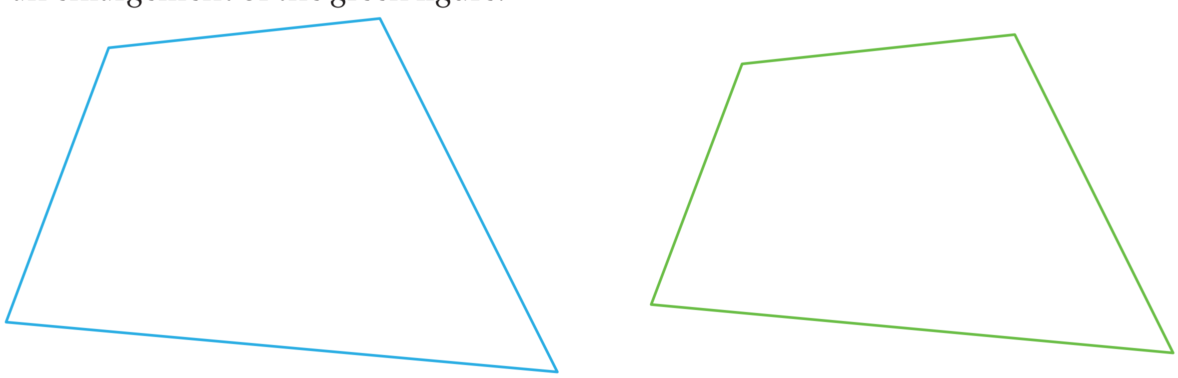
5. Take measurements and do calculations to establish whether the blue figure below is an enlargement of the green figure.



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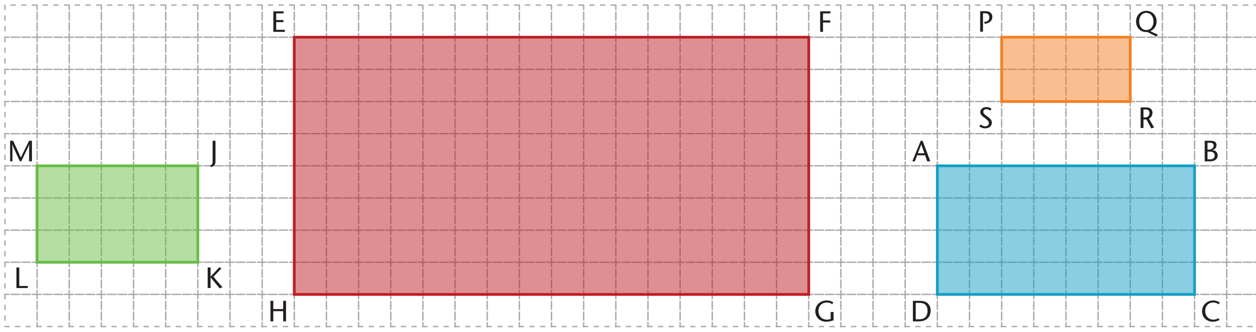
EFFECT OF ENLARGEMENTS OR REDUCTIONS ON PERIMETER AND AREA

Consider the rectangles below.



1. (a) Do you think EFGH is an enlargement of MJKL? ....
   (b) Do you think PQRS is a reduction of EFGH? ....
   (c) Do you think EFGH is an enlargement of ABCD? ....

2. (a) Calculate $\frac{EF}{MJ}$, $\frac{FG}{JK}$, $\frac{GH}{KL}$ and $\frac{HE}{LM}$.

   ....

   (b) Is rectangle EFGH an enlargement of rectangle MJKL? ....
   (c) If EFGH is an enlargement of MJKL, what is the scale factor? ....

3. (a) Calculate $\frac{PQ}{EF}$, $\frac{QR}{FG}$, $\frac{RS}{GH}$ and $\frac{SP}{HE}$.

   ....

   (b) Is rectangle PQRS a reduction of rectangle EFGH? ....
   (c) If PQRS is a reduction of EFGH, what is the scale factor?
   ....

4. (a) Calculate $\frac{EF}{AB}$, $\frac{FG}{BC}$, $\frac{GH}{CD}$ and $\frac{HE}{DA}$.

   ....

   (b) Is rectangle EFGH an enlargement of rectangle ABCD? ....
   (c) If EFGH is an enlargement of ABCD, what is the scale factor? ....

5. Do you agree or disagree with the following statements?
   (a) Perimeter of enlargement/reduction = perimeter of original × scale factor ....
   (b) Area of enlargement/reduction = area of original × (scale factor)^2 ....

CALCULATING PERIMETERS AND AREAS OF ENLARGED OR REDUCED FIGURES

1. The perimeter of rectangle DEFG = 20 cm and its area = 16 cm^2. Find the perimeter and area of the enlarged rectangle D'E'F'G' if the scale factor is 3.

   ....

   ....

2. The perimeter of ΔJKL = 120 cm and its area = 600 cm^2. Determine the perimeter and area of the reduced ΔJ'K'L' if the scale factor is 0,5.

   ....

   ....

3. The perimeter of quadrilateral PQRS = 30 mm and its area is 50 mm^2. Find the perimeter and area of quadrilateral P'Q'R'S' if the scale factor is $\frac{1}{5}$.

   ....

   ....

4. The perimeter of ΔSTU = 51 cm and its area is 12 cm^2. Calculate the perimeter and area of ΔS'T'U' if the scale factor is $\frac{1}{3}$.

   ....

   ....

5. The perimeter of a square = 48 m.

   (a) Write down the perimeter of the square if the length of each side is doubled.

   ....

   (b) Will the area of the enlarged square be twice or four times that of the original square? ....

6. The perimeter of ΔDEF = 7 cm and ΔD'E'F' = 21 cm. What is the scale factor of enlargement? How many times larger is the area of ΔD'E'F' than the area of ΔDEF?

   ....

   ....

7. The perimeter of quadrilateral ADFS = 26 cm and the perimeter of quadrilateral A'D'F'S' = 13 cm. How many times larger is the area of quadrilateral A'D'F'S' than the area of quadrilateral ADFS?

   ....

   ....