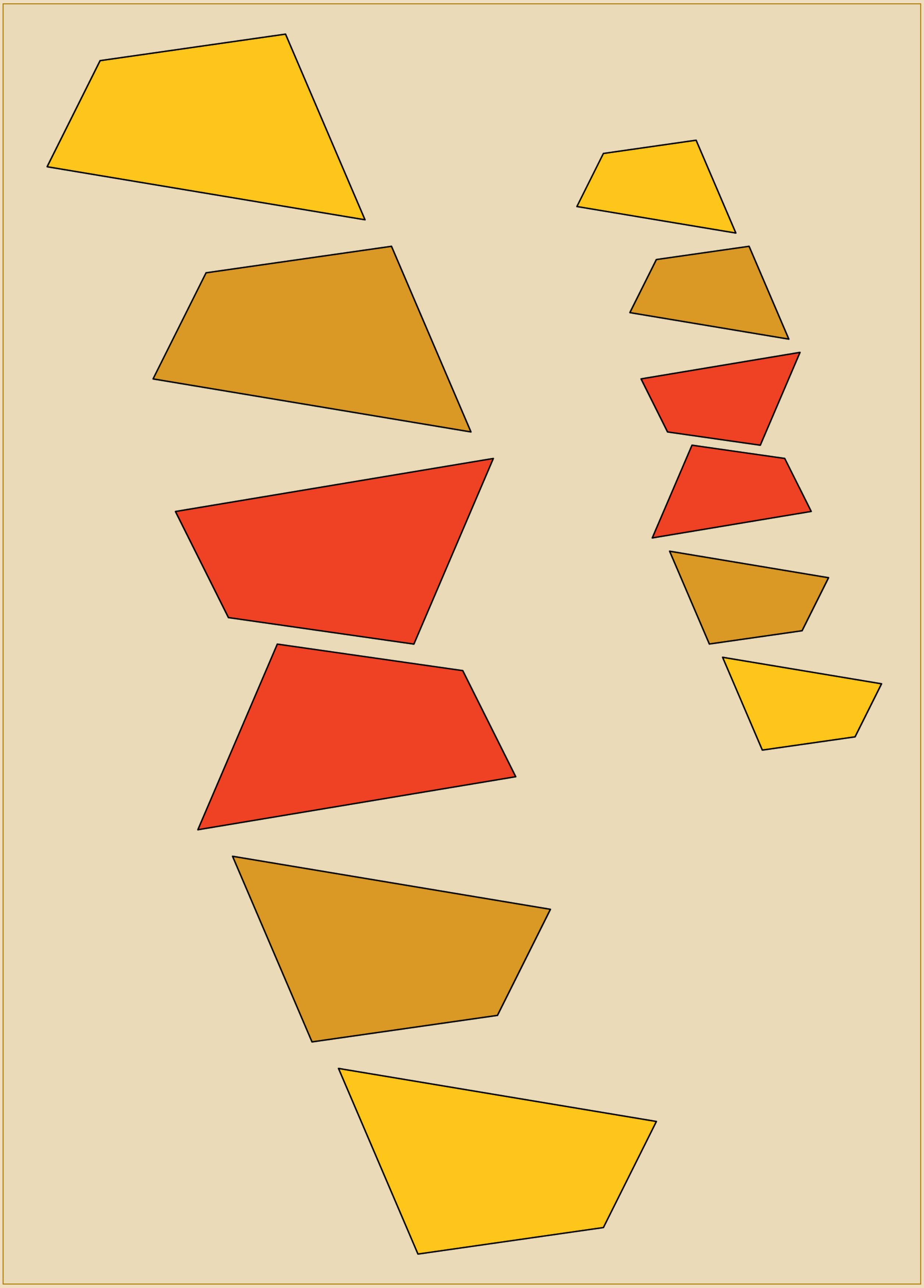
Chapter 6
Transformation geometry

In this chapter, you will revise the property of symmetry and practise identifying lines of symmetry in geometric figures. You will then investigate how figures can be reflected, rotated or translated, while the size and shape of the original figure remains the same. You will also investigate how we can change the size of a figure, but still keep the angles of the figure the same, to produce enlarged or reduced similar figures. In such figures, you will work out the factor by which the original figure was resized.



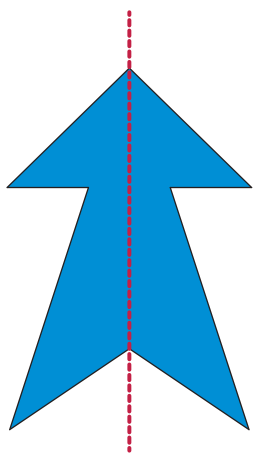
6 Transformation geometry

6.1 Lines of symmetry

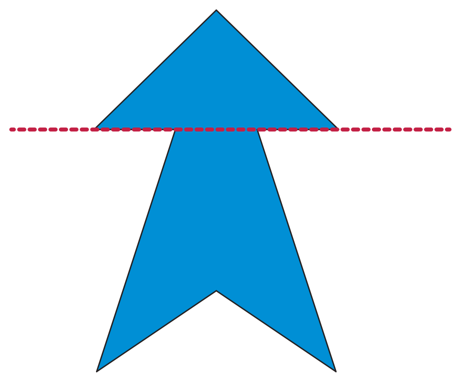
WHAT IS THE LINE OF SYMMETRY?

In the diagrams below, the red dotted lines divide the arrows into two parts. In which diagram does the red dotted line divide the arrow into two parts that are exactly the same?

....



Arrow A



Arrow B

If you were to cut out arrow A and fold it along the red dotted line, the two parts would fit perfectly on top of one another (all edges would match). The fold line is called a **line of symmetry** or an **axis of symmetry**.

A line or axis of symmetry is a line that divides a figure into two parts that have an equal number of sides, and all the corresponding sides and angles are equal. The two parts on either side of the line of symmetry are mirror images of each other. We also say the parts are **congruent**.

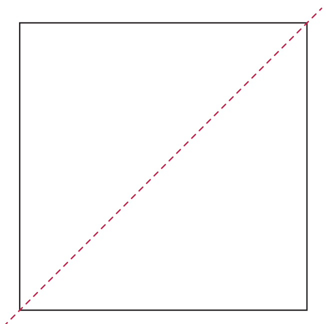
A geometric figure can have no line of symmetry, one line of symmetry, or more than one line of symmetry.

Congruent figures are figures that are the same size and shape. All the sides and angles of the figures match.

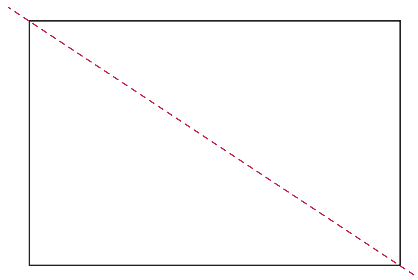
IDENTIFYING LINES OF SYMMETRY

1. (a) Make a tick next to each figure in which the red line is a line of symmetry.
   (b) In the figures where the red line is not a line of symmetry, draw in a line of symmetry if this is possible. If there is more than one line of symmetry, draw it in too. If a figure doesn't have any lines of symmetry, write this above the figure.

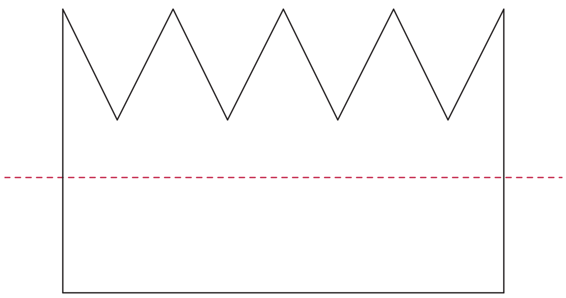
A



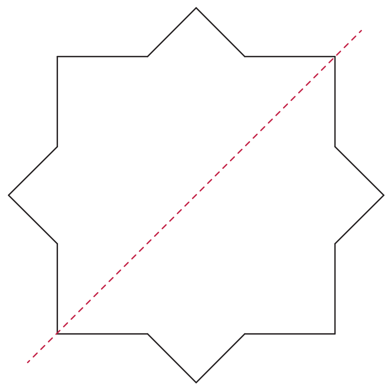
B



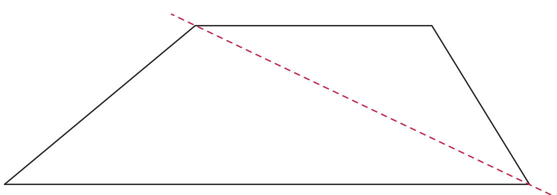
C



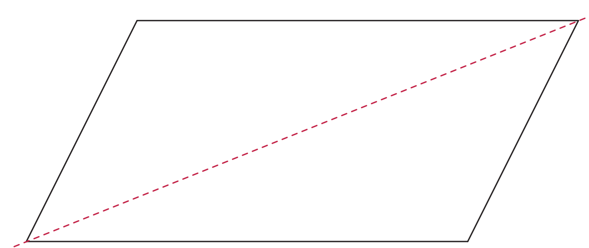
D



E

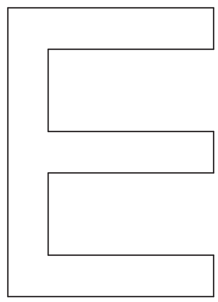


F

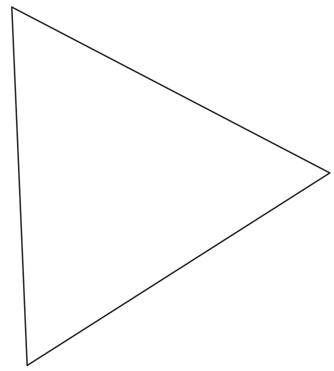


2. Draw lines of symmetry in the following geometric figures. Also write down how many lines of symmetry there are in each figure.

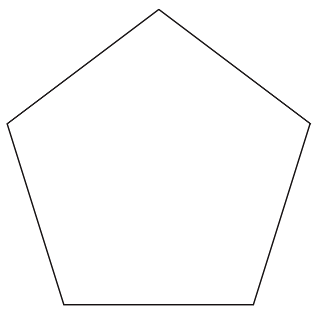
A



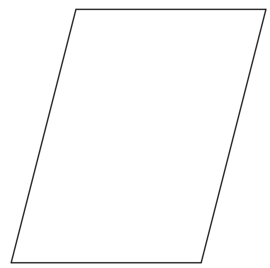
B



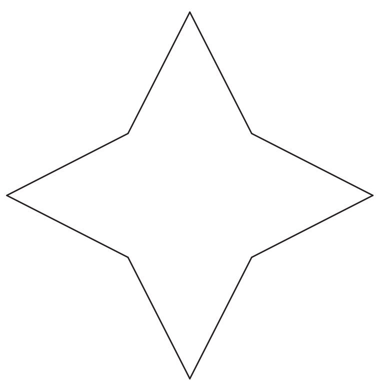
C



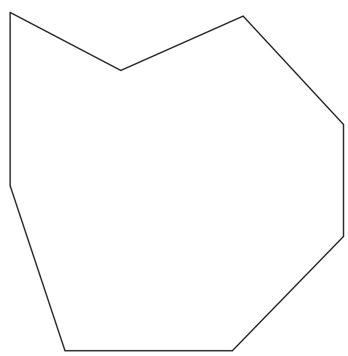
D



E

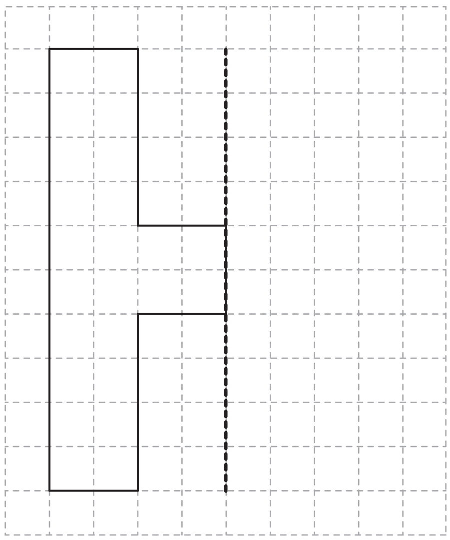


F

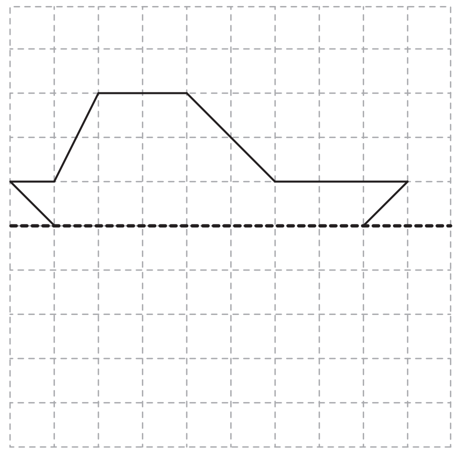


3. In each diagram, the dotted line is the axis of symmetry. Complete each figure.

A



B



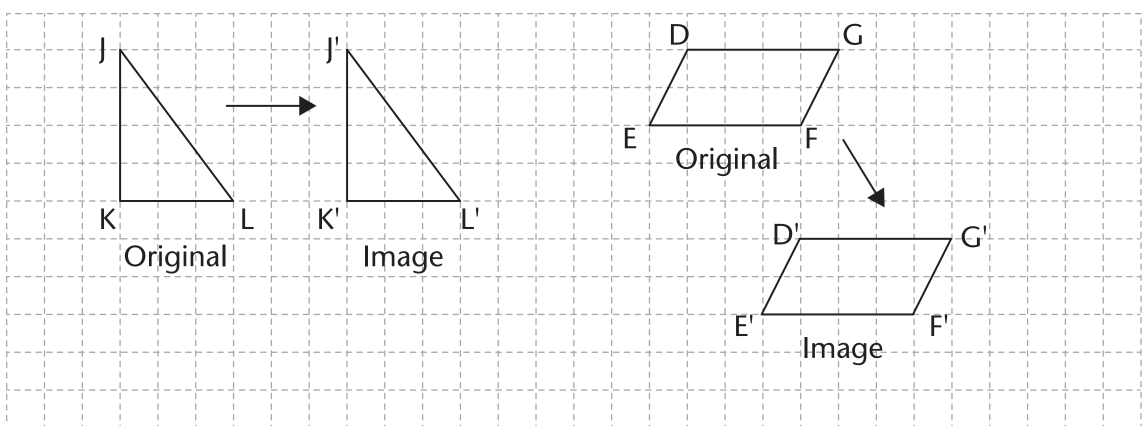
6.2 Original figures and their images

Figures can be moved around in different ways – they can be shifted, swung around and turned over. When the movement is done, the figure in its new position is called the **image** of the original figure.

> Figures can be moved in three ways: through **translation**, **reflection** and **rotation**. These transformations are often referred to as "sliding" (shifting), "flipping" (turning over) and "turning" (swinging) respectively.

6.3 Translating figures

Here are two original figures and their images after the figures were **translated**:

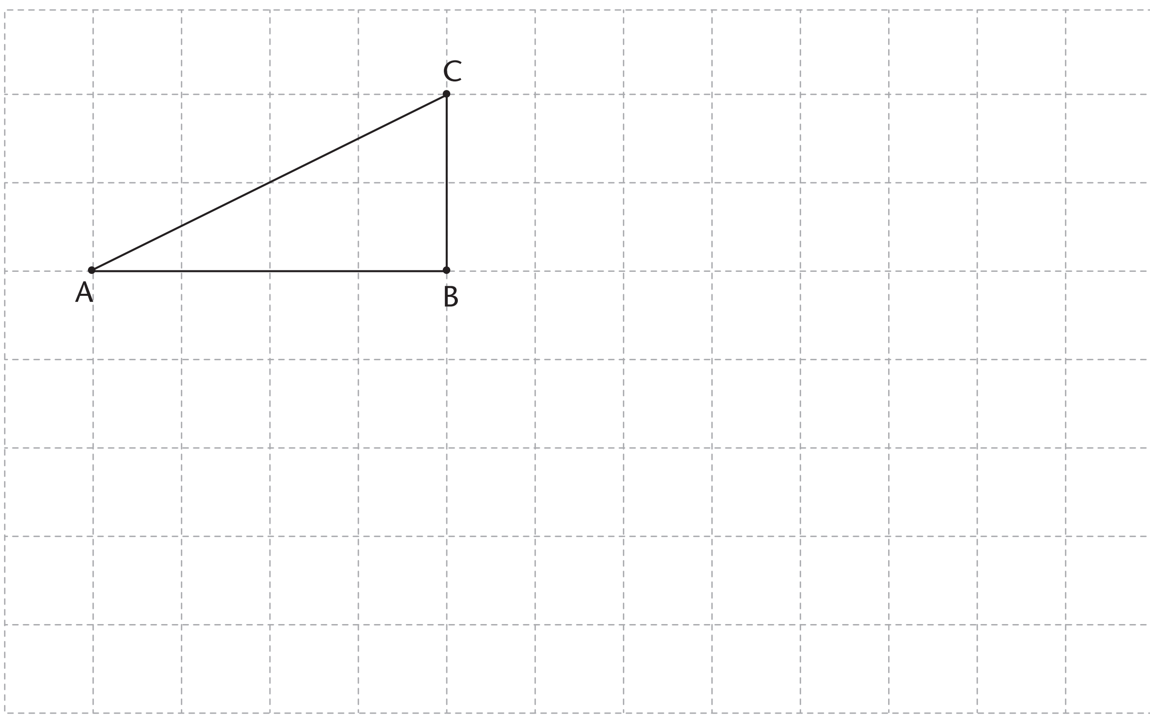


When we name the image, we use the same letters for the points that correspond to those of the original figure, but we add the prime symbol (') after each letter. The image of ΔJKL is ΔJ'K'L'. The image of parallelogram DEFG is parallelogram D'E'F'G'.

INVESTIGATING THE PROPERTIES OF TRANSLATION

In a **translation**, all the points on the figure move in the same direction by the same distance. For example, look at ΔJKL on the previous page. All of its points have moved 6 units to the right. Also look at parallelogram DEFG on the previous page. All of its points have moved 3 units to the right and 5 units down.

1. Look at ΔABC below.
   (a) Translate each of the points A, B and C 5 units to the right and 2 units down. Then join the translated points to form the image ΔA'B'C'.



Look at the completed translation.

(b) Are the side lengths of the original triangle and those of its image the same?

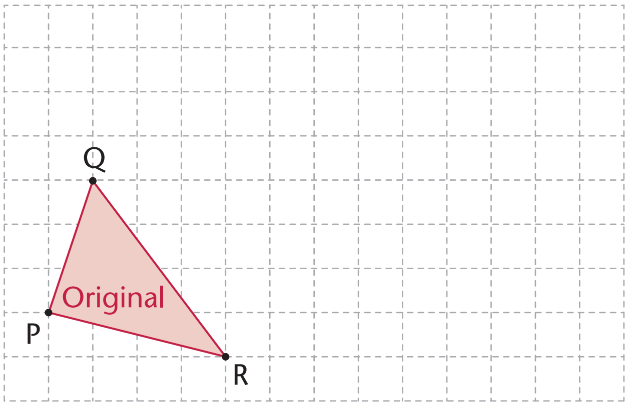
....

(c) Is the area of the original triangle the same as the area of its image?

....

2. Look at ΔPQR below.

(a) Translate each of the points P, Q and R 4 units to the right and 2 units up. Then join the translated points to form the image ΔP'Q'R'.



(b) Join point P and its image, point Q and its image, and point R and its image.

(c) Are the line segments that join the original points to their image points equal in length?

....

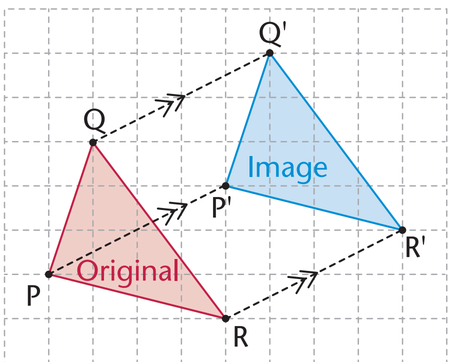
(d) Are the line segments that join the original points to their image points parallel?

....

Properties of translation

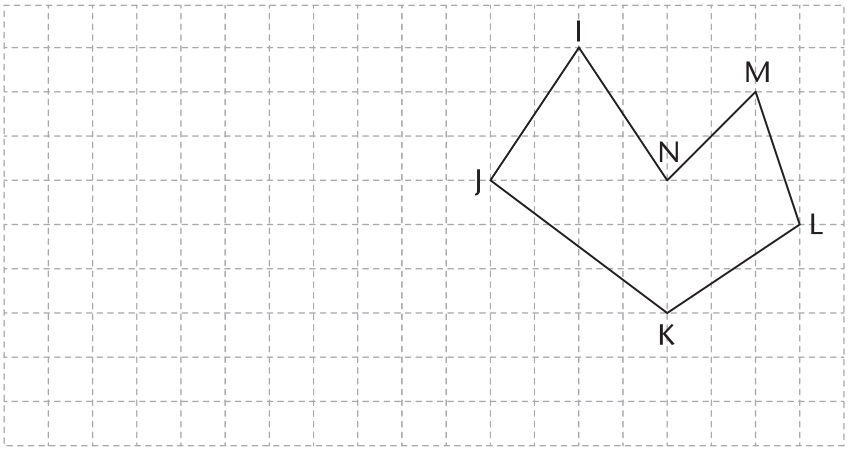
Use the diagram on the right to check if the following is true:

- The line segments that connect the vertices of the original figure to those of the image are all equal in length:
  PP' = RR' = QQ'
- The line segments that connect the vertices of the original figure to those of the image are all parallel to one another:
  PP' || RR' || QQ'
- When a figure is translated, its shape and size do not change. The original and its image are therefore congruent.

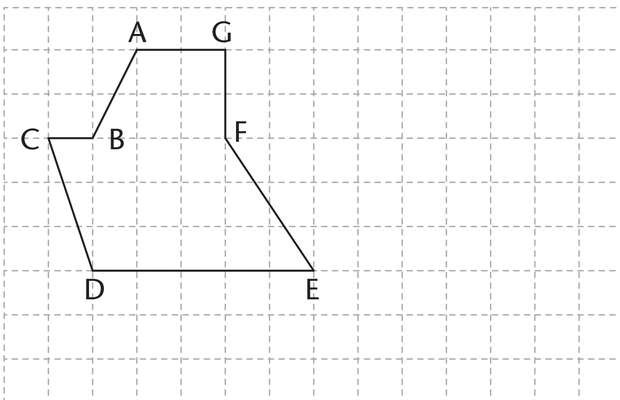


PRACTISE TRANSLATING FIGURES

1. Translate the following figure 8 units to the left and 2 units down.

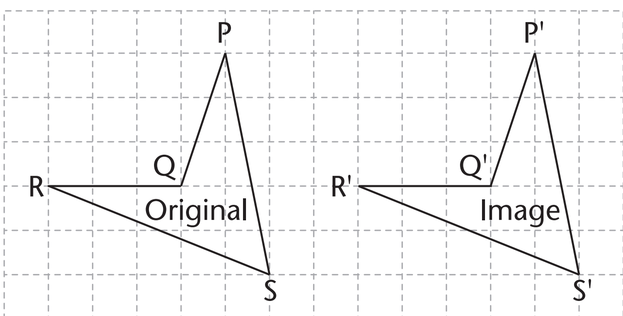


2. Translate the following figure 6 units to the right and 1 unit down.

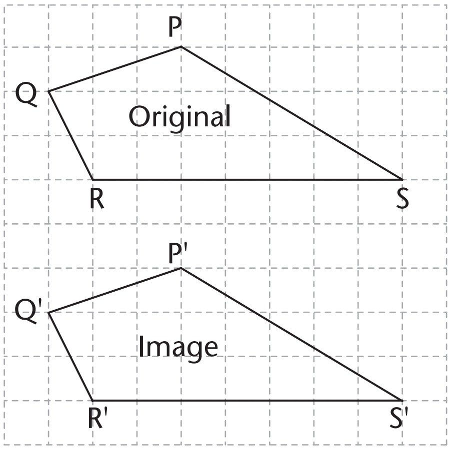


3. Describe the translation in each of the following diagrams:

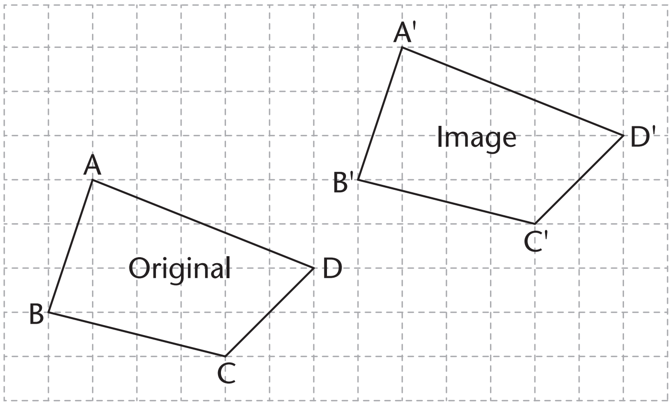
   (a) ....



(b) ....



(c) ....



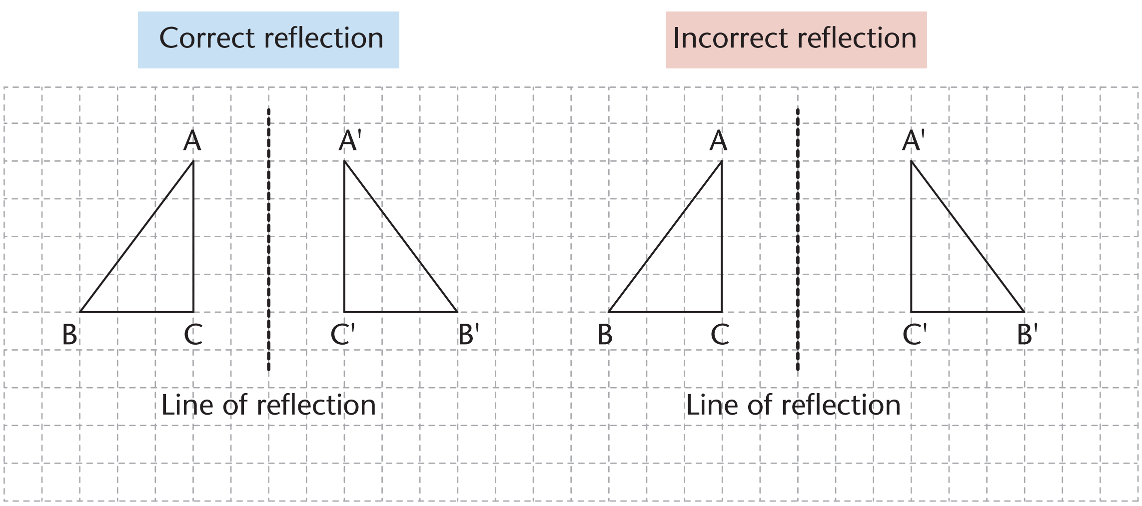
6.4 Reflecting figures

When a figure is **reflected**, it is flipped or turned over. The image that is produced is the mirror image of the original figure. The **line of reflection** is like a mirror in which the original figure is reflected.

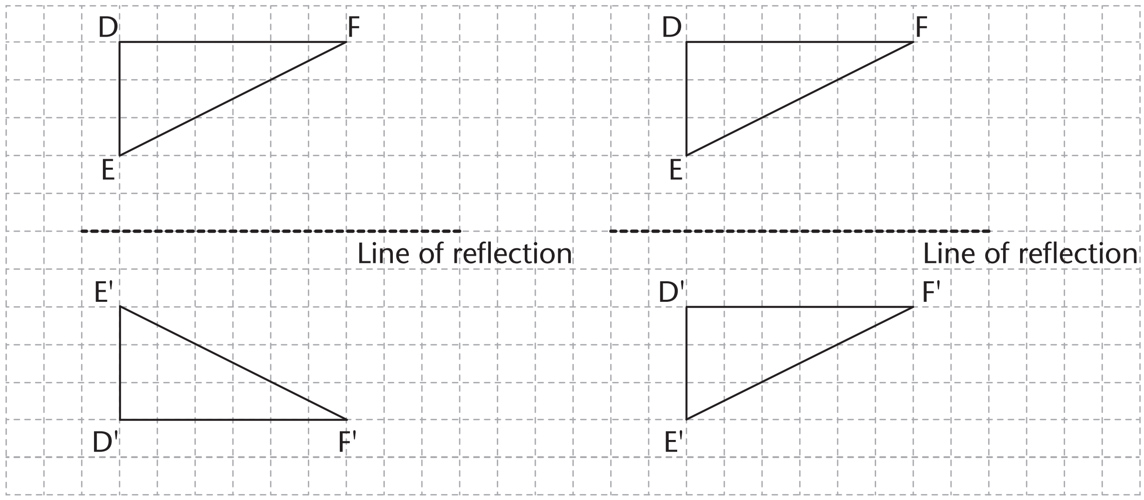
The image is produced on the opposite side of the line of reflection. Each point on the original figure and its corresponding point on the image are the same distance away from the line of reflection.

INVESTIGATING THE PROPERTIES OF REFLECTION

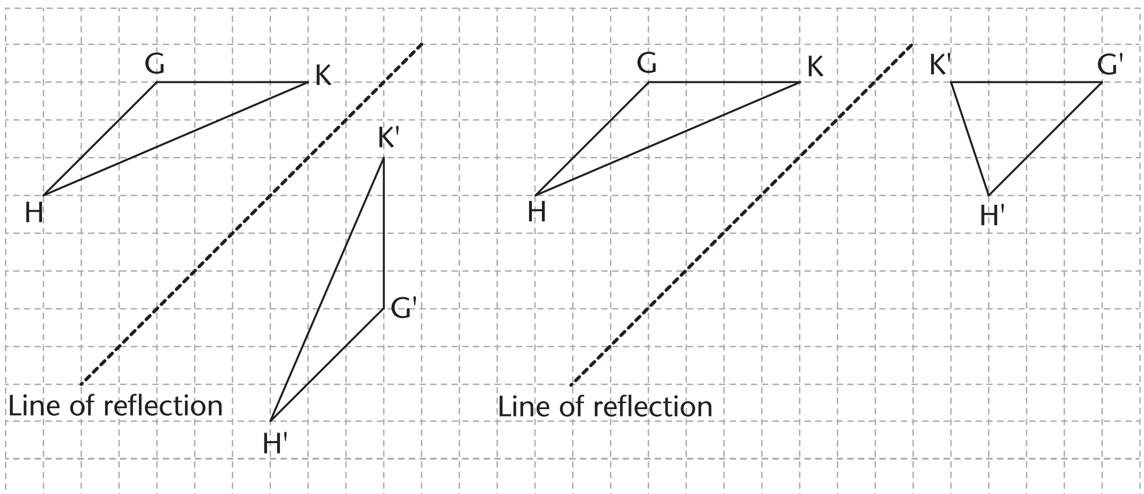
The diagrams below and on the next page show examples of figures that have been correctly and incorrectly reflected in the lines of reflection.



Correct reflection | Incorrect reflection



Correct reflection | Incorrect reflection



1. Write down the distance from each of the following points to the line of reflection.

Original figure	**Correct reflection**	**Incorrect reflection**
A: 2 units	A':	A':
B:	B':	B':
C:	C':	C':
D:	D':	D':
E:	E':	E':
F:	F':	F':
G:	G':	G':
H:	H':	H':
K:	K':	K':

2. Look at each set of *correct* reflections.
   (a) Are the side lengths of the image the same as those of the original figure?

   ....

   (b) Are the size and shape of the image the same as the size and shape of the original figure?

   ....

3. (a) In each diagram showing the *correct* reflection, draw a dotted line to join each point on the original figure to its corresponding reflected point (A to A', B to B', C to C' and so on).
   (b) Is the line that joins the original point to its correct reflection perpendicular to the line of reflection?

   ....

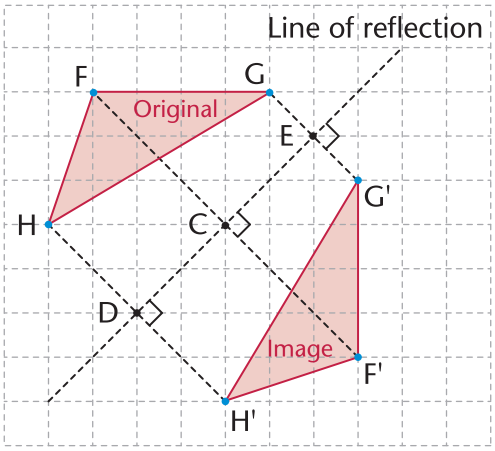
4. (a) In each diagram showing the *incorrect* reflection, draw a dotted line to join each point on the original figure to its corresponding reflected point.
   (b) Is the line that joins the original point to its incorrect reflection perpendicular to the line of reflection?

   ....

Properties of reflection

The diagram on the right shows ΔFHG and its reflection ΔF'H'G'. Notice the following properties of reflection:

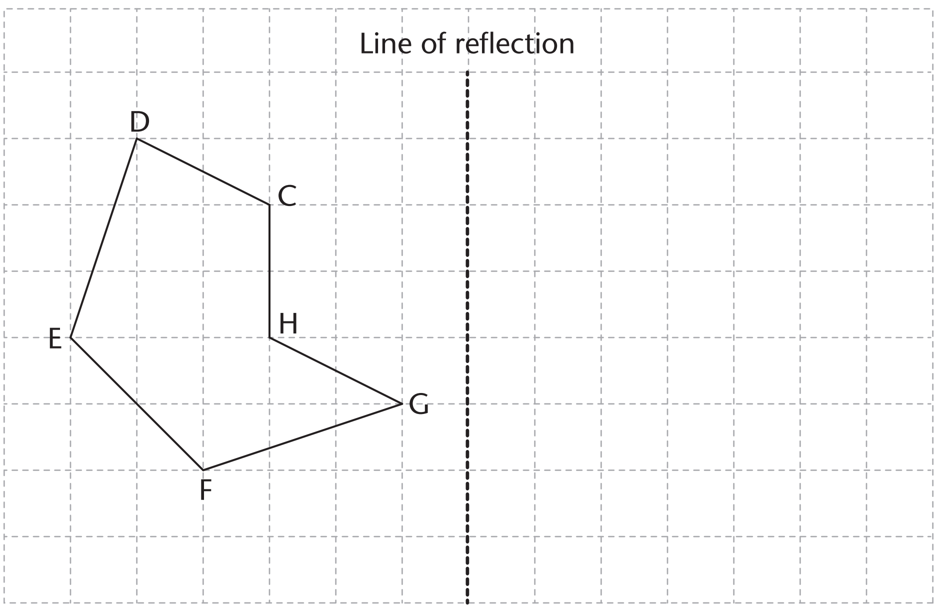
- The image of ΔFHG lies on the opposite side of the line of reflection.
- The distance from the original point to the line of reflection is the same as the distance from the reflected point to the line of reflection: GE = G'E; FC = F'C and HD = H'D
- The line that connects an original point to its image is always perpendicular (⊥) to the line of reflection: HH' ⊥ line of reflection; FF' ⊥ line of reflection, and GG' ⊥ line of reflection.
- When a figure is reflected, its shape and size do not change. The original and its image are therefore congruent.



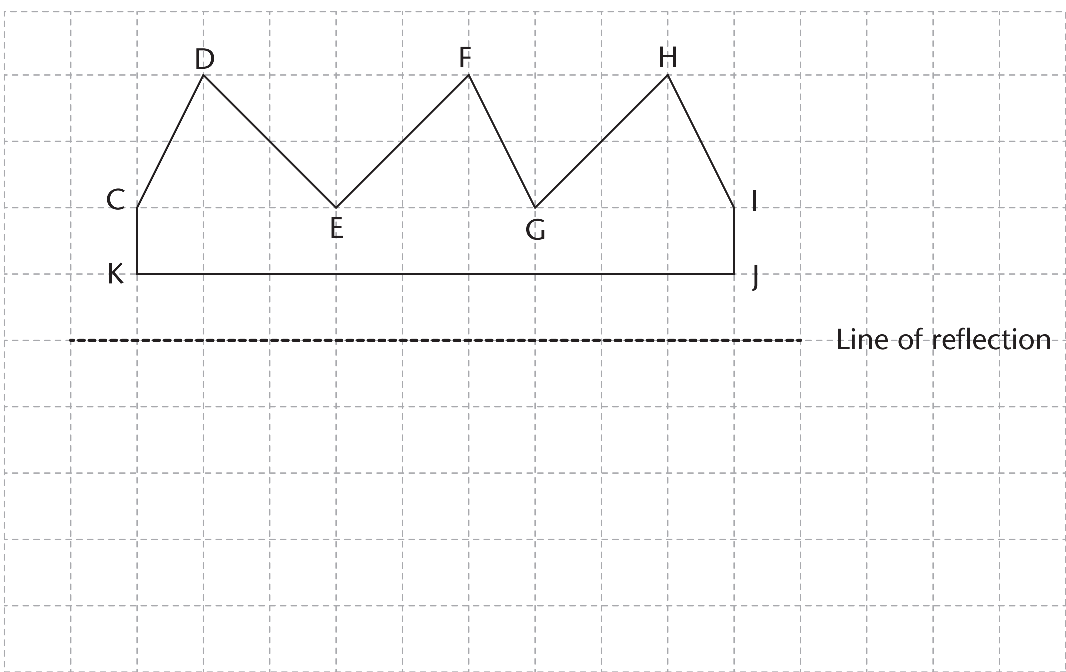
PRACTISE REFLECTING FIGURES

1. Reflect the following figures in the given line of reflection. (*Hint*: First reflect the points; then join the reflected points.)

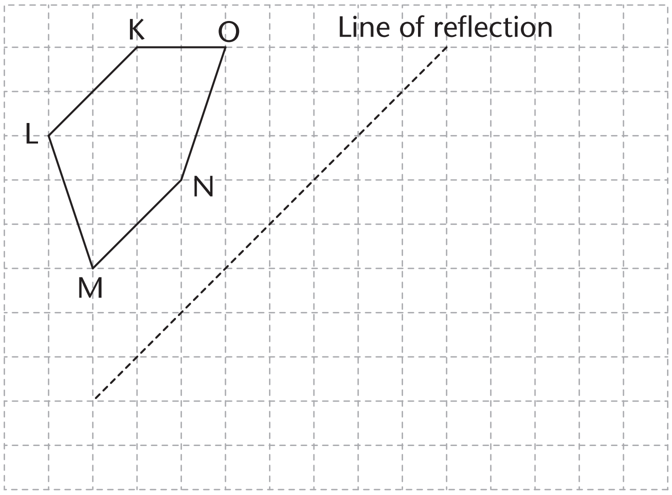
(a)



(b)

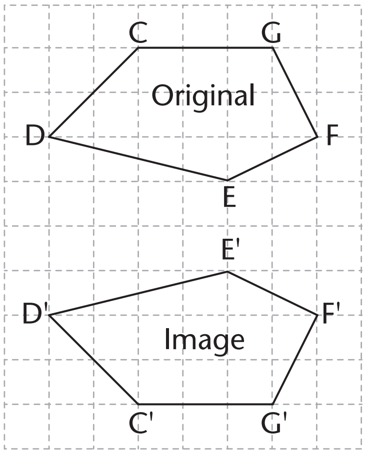


(c)

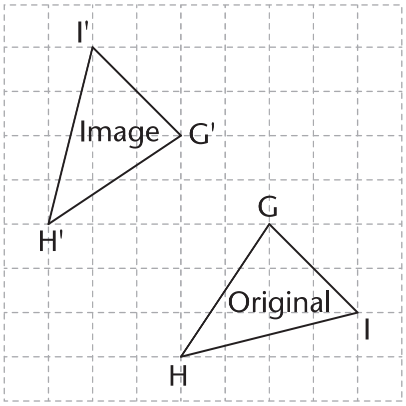


2. Draw the line of reflection.

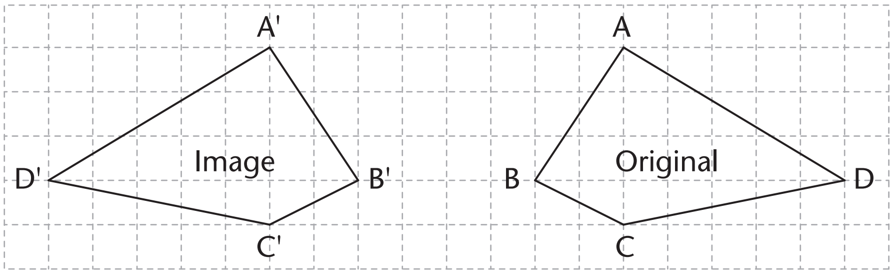
(a)



(b)



(c)



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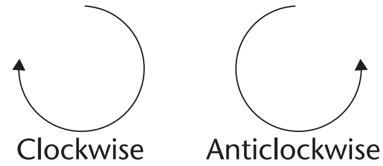
6.5 Rotating figures

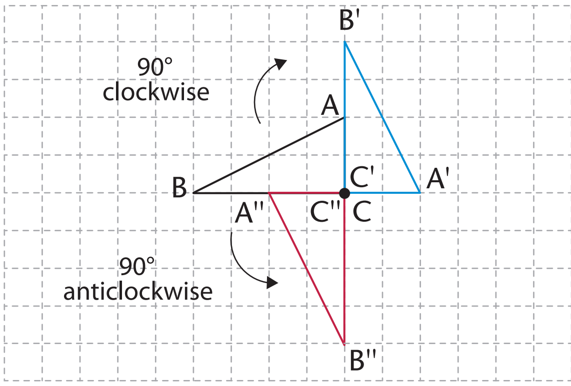
When a figure is **rotated** it is turned in a clockwise direction or in an anticlockwise direction around a particular point. This point is called the **centre of rotation** and could be inside the figure or outside of the figure.

The following diagrams show ΔABC rotated 90° clockwise and 90° anticlockwise about different centres of rotation.

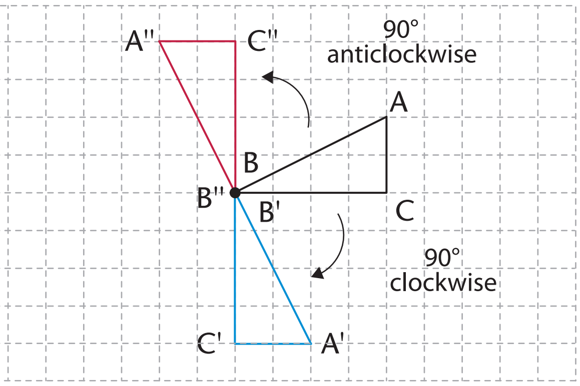


In this case, **about** means "around".

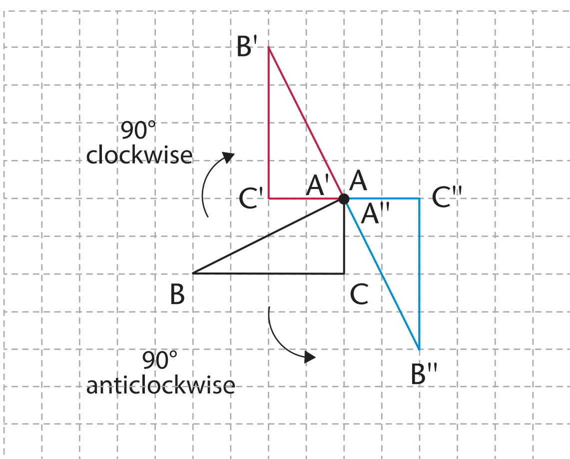
Centre of rotation is at C



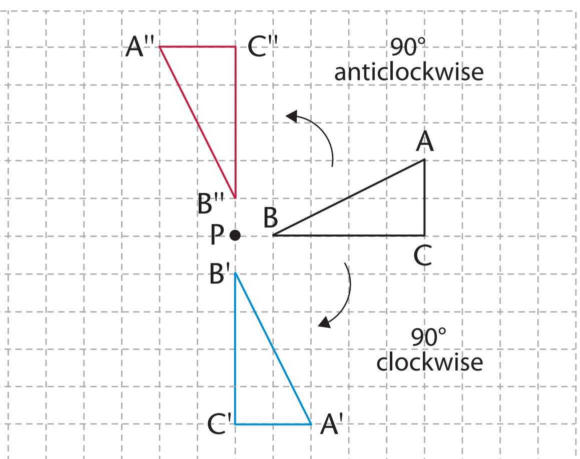
Centre of rotation is at B



Centre of rotation is at A

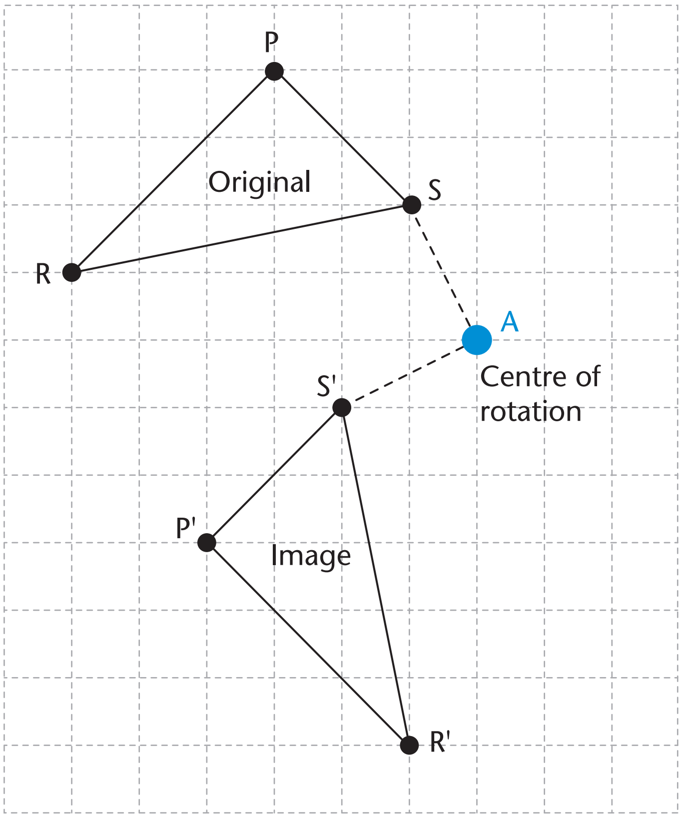


Centre of rotation is at P



INVESTIGATING THE PROPERTIES OF ROTATION

In the following diagrams, the centre of rotation is point A. ΔPRS has been rotated anticlockwise through 90° about point A.



1. Lines have been drawn to join A to point S, and A to point S'.
   (a) Measure the distance from A to S.

   ....

   (b) Measure the distance from A to S'.

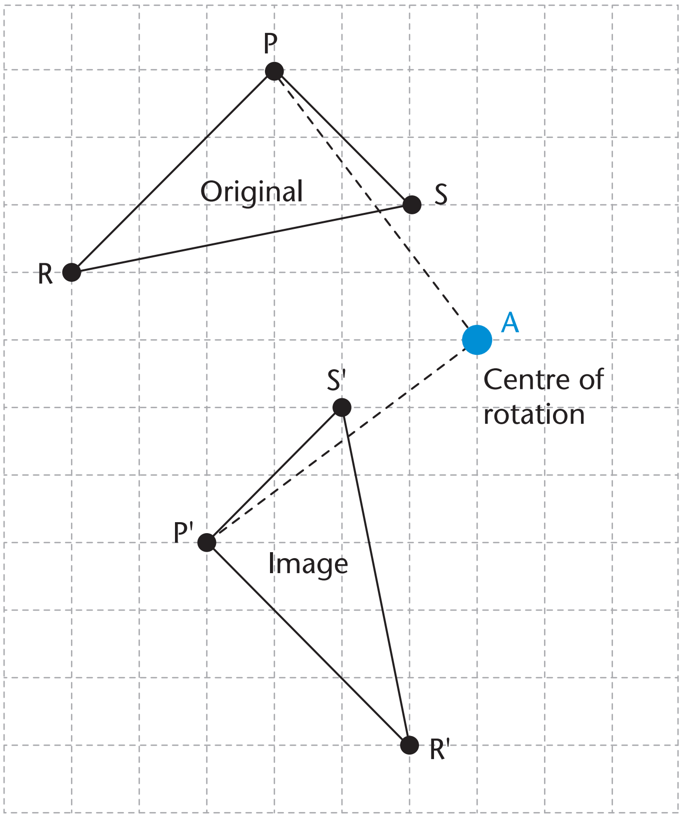
   ....

   (c) What do you notice about the distances in (a) and (b) above?

   ....

   (d) Measure the size of the angle SAS'. What do you notice?

   ....



2. Lines have been drawn to join A to P, and A to P'.
   (a) Measure the distance from A to P.

   ....

   (b) Measure the distance from A to P'.

   ....

   (c) What do you notice about the distances in (a) and (b) above?

   ....

   (d) Measure the size of the angle PAP'. What do you notice?

   ....

3. Lines have been drawn to join A to R, and A to R'.
   (a) Measure the distance from A to R.

   ....

   (b) Measure the distance from A to R'.

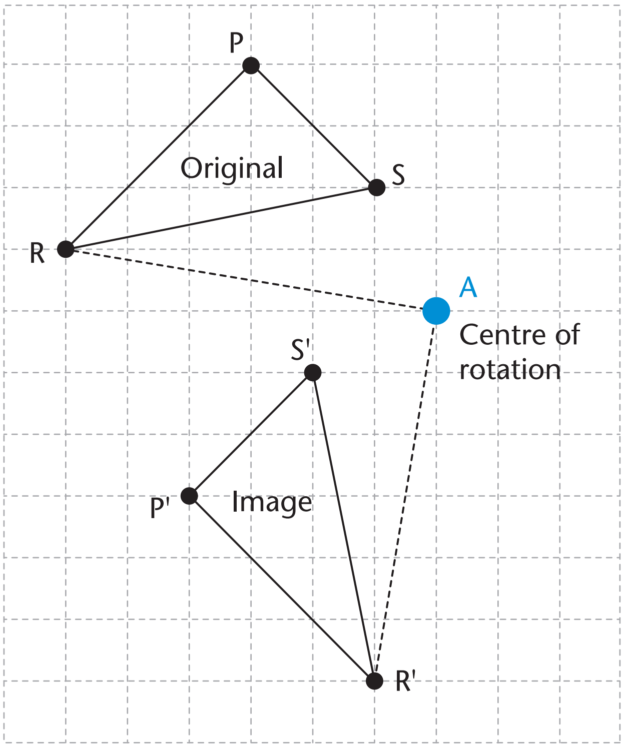
   ....

   (c) What do you notice about the distances in (a) and (b) above?

   ....

   (d) Measure the size of the angle RAR'. What do you notice?

   ....

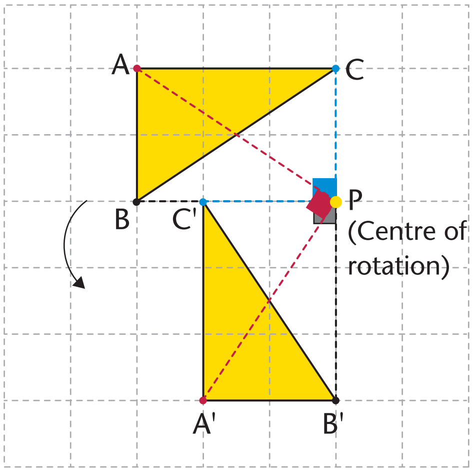


4. In any of the diagrams in questions 1 to 3 above, measure the sides of the original triangle and the corresponding sides of the image. What do you notice?

....

Properties of rotation

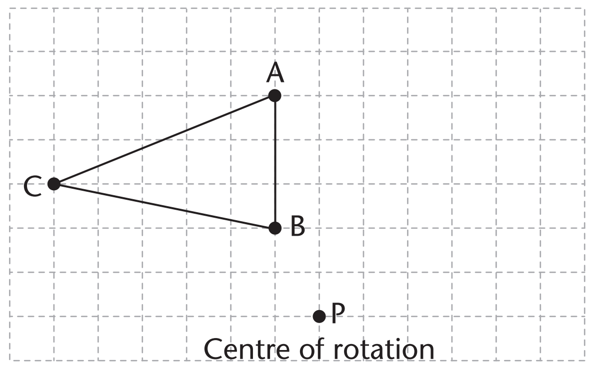
- The distance from the centre of rotation to any point on the original is equal to the distance from the centre of rotation to the corresponding point on the image. In the diagram on the right: PA = PA', PB = PB' and PC = PC'.
- The angle formed by the connecting lines between any point on the original figure, the centre of rotation and the corresponding point on the image is equal to the angle of rotation. For example, if the image is rotated through 90°, this angle will be equal to 90°. If the image is rotated through 45°, the angle will be 45°.
- When a figure is rotated, its shape and size do not change.



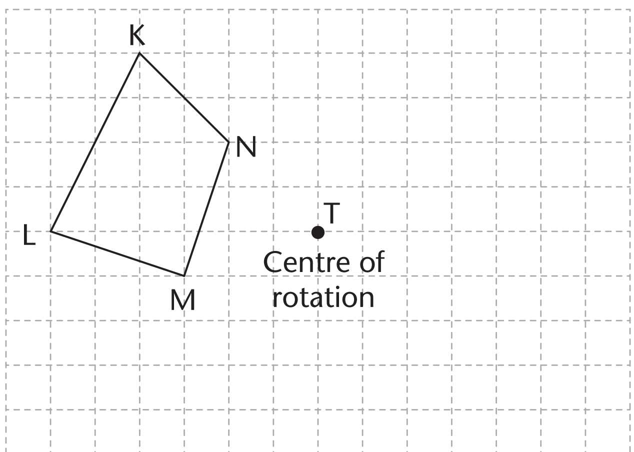
PRACTISE ROTATING FIGURES

1. Rotate triangle ΔABC 90° clockwise about point P as follows:
   (a) Plot the image of each vertex on the grid. Remember:
   - The image point must be the same distance from P as the original point.
   - The angle that is formed between the line connecting an original point to point P and the line connecting its image point to point P must be the same as the angle of rotation. In this case, it must be 90°.

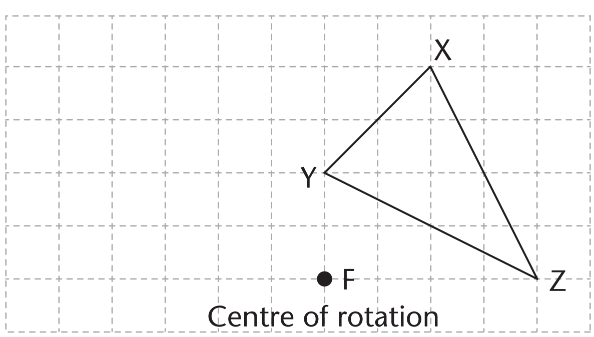
   (b) Join the image points to create ΔA'B'C'.



2. Rotate KLMN 180° about point T.



3. Rotate ΔXYZ 90° anticlockwise about point F.

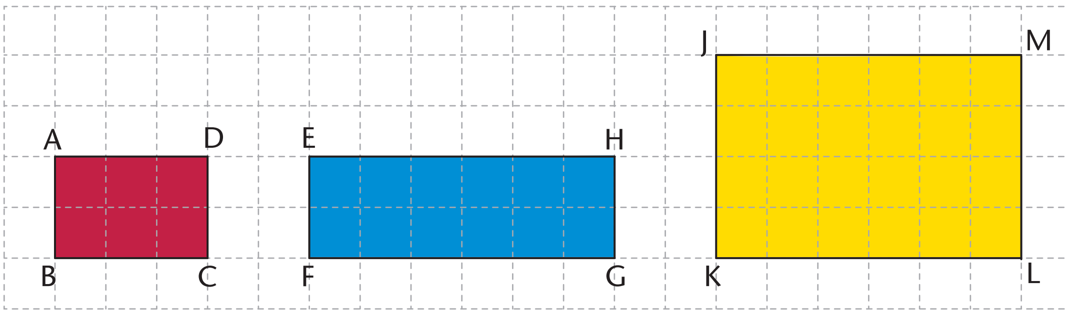


6.6 Enlarging and reducing figures

Enlarging a figure means that we make it bigger in a specific way. **Reducing** a figure means that we make it smaller in a specific way. Enlarging or reducing figures is also called **resizing**.

INVESTIGATE THE PROPERTIES OF ENLARGEMENTS AND REDUCTIONS

1. Look at the following rectangles and answer the questions below.



(a) Rectangle EFGH:

How many times is FG longer than BC? ....

How many times is EF longer than AB? ....

(b) Rectangle JKLM:

How many times is KL longer than BC? ....

How many times is JK longer than AB? ....

When the lengths of **all the sides** of a figure are **multiplied by the same number** to produce a second figure, the second figure is an **enlargement** or **reduction** of the first figure.

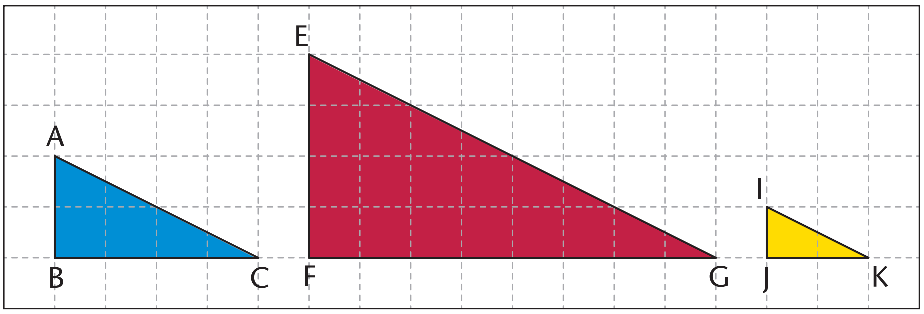
The number by which the sides are multiplied to produce an enlargement or reduction is called the **scale factor**. The scale factor in question 1(b) above is 2. We say that figure ABCD has been enlarged (or resized) by a scale factor of 2 to produce figure JKLM.

Figure EFGH is not an enlargement of figure ABCD because not *all* its sides have been increased by the *same* scale factor.

The scale factor

- When the scale factor is 1, the image is the same size as the original.
- When the scale factor is <1, the image is a reduction. For example, if the scale factor is $\frac{1}{2}$ or 0,5, each side of the image is half the length of its corresponding side in the original figure.
- When the scale factor is >1, the image is an enlargement. For example, if the scale factor is 2, each side of the image is double the length of its corresponding side in the original figure.

2. Look at the following triangles and answer the questions that follow.



(a) How many times is:

- FG longer than BC? ......
- EF longer than AB? ......
- EG longer than AC? ......
- JK shorter than BC? ......
- IJ shorter than AB? ......
- IK shorter than AC? ......

(b) Is ΔEFG an enlargement of ΔABC? Explain your answer.

....

(c) Is ΔIJK a reduction of ΔABC? Explain your answer.

....

Similar figures

When figures are enlarged or reduced, the enlarged or reduced image is **similar** to the original figure. ΔABC, ΔEFG and ΔIJK above are all similar. We also say that the lengths of their corresponding sides are **in proportion**.

If two or more figures are **similar**:
- their corresponding angles are equal, and
- their corresponding sides are longer or shorter by the same scale factor.

PRACTISE RESIZING FIGURES

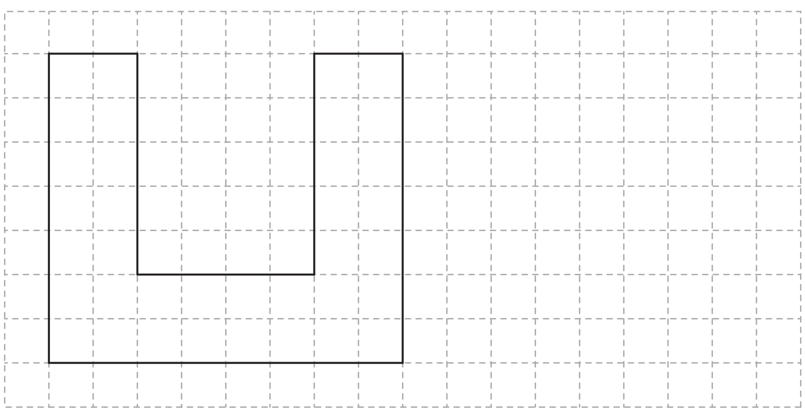
1. State whether the following scale factors will produce a larger or smaller image:

   (a) 5 .... (b) 0,25 ....

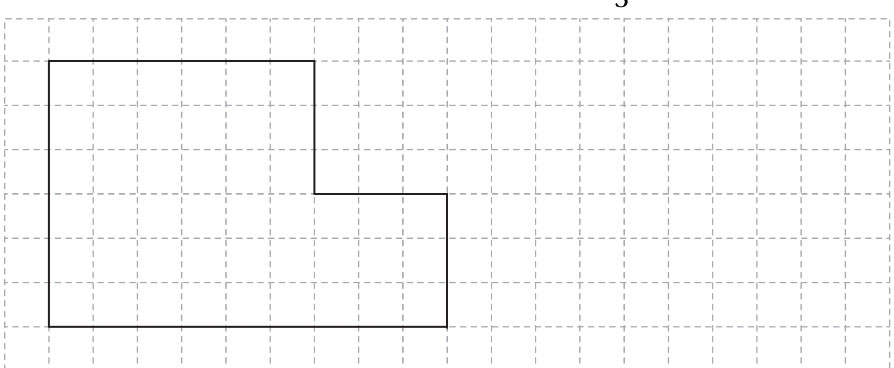
   (c) 1,2 .... (d) $\frac{3}{8}$ ....

2. Enlarge the triangle below with a scale factor of 2.

3. Resize the following figure. Use a scale factor of 0,5.

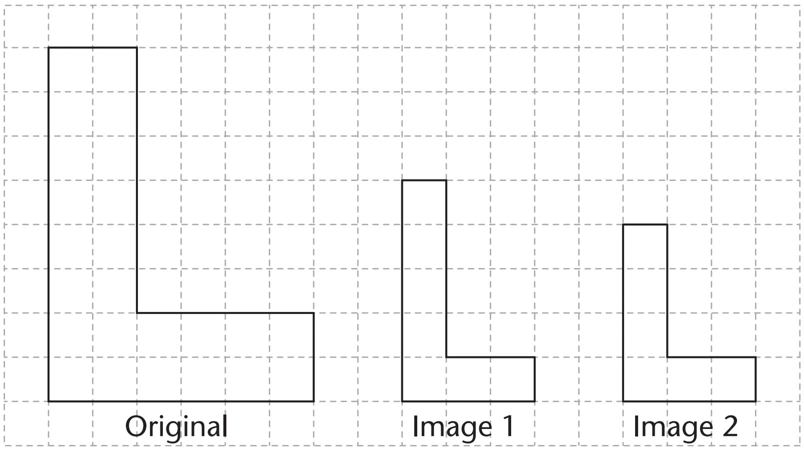


4. Resize the figure below. Use a scale factor of $\frac{1}{3}$.



5. (a) Which image below is similar to the original? ....

(b) State the scale factor by which it has been resized. ....



6. What scale factors were used to produce image 1 and image 2 from the original?

Image 1: ....

Image 2: ....