1.3 Factors, prime numbers and common multiples

DIFFERENT WAYS TO PRODUCE THE SAME NUMBER

The number 80 can be produced by multiplying 4 and 20: $4 \times 20 = 80$.
The number 80 can also be produced by multiplying 5 and 16.

1. In what other ways can 80 be produced by multiplying two numbers?

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The number 80 can also be produced by multiplying 2, 10 and 4:
$2 \times 10 = 20$ and $20 \times 4 = 80$ or $10 \times 4 = 40$ and $40 \times 2 = 80$.

We can use brackets to describe what calculation is done first. So instead of writing "$2 \times 10 = 20$ and $20 \times 4 = 80$" we may write $(2 \times 10) \times 4$. Instead of writing "$10 \times 4 = 40$ and 40×2" we may write $2 \times (10 \times 4)$.

2. Show how the number 80 can be produced by multiplying four numbers. Describe how you do it in two ways: without using brackets and by using brackets.

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3. Show three different ways in which the number 30 can be produced by multiplying two numbers.

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4. (a) Can the number 30 be produced by multiplying three whole numbers? .....
   Which three whole numbers? .....
   (b) Can the number 30 be produced by multiplying four whole numbers that do not include the number 1? .....
   If you answered "yes", which four numbers? .....

The number 105 can be produced by multiplying 3, 5 and 7, hence we can write $105 = 3 \times 5 \times 7$. Mathematicians often describe this by saying "105 is the **product** of 3, 5 and 7" or "105 can be **expressed as the product** $3 \times 5 \times 7$".

5. Express each of the following numbers as a product of three numbers.
   (a) 248 ..... (b) 375 .....

The whole numbers that are multiplied to form a number are called **factors** of the number. For example, 6 and 8 are factors of 48 because $6 \times 8 = 48$.

But 6 and 8 are not the only numbers that are factors of 48. 2 is also a factor of 48 because $48 = 2 \times 24$. And 24 is a factor of 48. The numbers 3 and 16 are also factors of 48 because $48 = 3 \times 16$.

6. Describe all the different ways in which 48 can be expressed as a product of two factors.

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The number 36 can be formed by $2 \times 2 \times 3 \times 3$.
Because 2 is used twice, it is called a **repeated factor** of 36. The number 3 is also a repeated factor of 36.

7. (a) Express 48 as a product of three factors. .....
   (b) Express 75 as a product of three factors. .....

8. (a) Can 36 be expressed as a product of three factors? How? .....
   (b) Can 36 be expressed as a product of five factors? How? .....

9. Express each of the following numbers as a product of as many factors as possible, including repeated factors. Do not use 1 as a factor.

   (a) 300 .....
   (b) 310 .....
   (c) 320 .....
   (d) 330 .....
   (e) 340 .....
   (f) 350 .....

PRIME NUMBERS

1. Express each of the following numbers as a product of as many factors as possible, including repeated factors. Do not use 1 as a factor.

   (a) 36 .....
   (b) 37 .....
   (c) 38 .....
   (d) 39 .....
   (e) 40 .....
   (f) 41 .....
   (g) 42 .....
   (h) 43 .....
   (i) 44 .....
   (j) 45 .....
   (k) 46 .....
   (l) 47 .....
   (m) 48 .....
   (n) 49 .....

2. Which of the numbers in question 1 cannot be expressed as a product of two whole numbers, except as the product *1 × the number itself*?

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A number that cannot be expressed as a product of two whole numbers, except as the product *1 × the number itself*, is called a **prime number**.

3. (a) Which of the numbers in question 1 are prime? .....

   (b) Which numbers between 20 and 30 are prime? .....

   (c) Are 11 and 17 prime numbers? .....

Eratosthenes, a Greek mathematician who lived a long time ago, designed a method to find the prime numbers. The process is called "the sieve of Eratosthenes".

4. Work on the table on the right.
   Follow the steps to find all the prime numbers up to 100.

   Step 1: Cross out 1.

   Step 2: Circle 2, and then cross out all the multiples of 2.

   Step 3: Circle 3, then cross out all the multiples of 3.

   Step 4: Find the next number that has not been crossed out and cross out all its multiples.

   Continue like this.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

5. (a) What is the smallest number that can be formed as a product of three prime numbers, if the same factor may be repeated? .....

   (b) What is the smallest number that can be formed as a product of three prime numbers, if no repeated factors are allowed? .....

6. Manare did a lot of work, and found out that 840 can be formed as the product of 2, 2, 2, 3, 5 and 7. Check whether Manare is correct.

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We can say that Manare **found the prime factors** of 840, or Manare **factorised 840 completely**.

We can write:

$2 \times 2 \rightarrow 4 \times 2 \rightarrow 8 \times 3 \rightarrow 24 \times 5 \rightarrow 120 \times 7 = 840$.

7. The prime factors of some numbers are given below. What are the numbers?

   (a) 3, 5, 5 and 11 ..... (b) 3, 3, 5 and 7 ..... (c) 2, 7, 11 and 13 .....

8. Investigate which of the following statements you agree with. Give reasons for your agreement or disagreement in each case.

   (a) If a number is even, 2 is one of its prime factors. .....

   .....

   (b) If half an even number is also even, 2 is a repeated prime factor. .....

   .....

   .....

   (c) If a number is odd, 3 is one of its prime factors. .....

   .....

   (d) If a number ends in 0 or 5, then 5 is one of its prime factors. .....

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Here is a method to find the prime factors of a number:

If the number is even, divide it by 2. If the answer is even, divide by 2 again. Continue like this as long as it is possible. If the answer is odd, divide by 3, if it is possible. Continue to divide by 3 as long as it is possible. Then switch to 5. Continue like this by each time trying to divide by the next prime number.

9. Find all the prime factors of each of the following numbers. Work in your exercise book or on loose paper, and write only your answers below.

   (a) 588 ..... (b) 825 .....

   (c) 729 ..... (d) 999 .....

   (e) 538 ..... (f) 113 .....

10. Find at least three prime numbers between 800 and 850.

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HIGHEST COMMON FACTOR AND LOWEST COMMON MULTIPLE

1. (a) Factorise 195 and 385 completely.

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   (b) Is 7 a factor of both 195 and 385? .....

   (c) Is 5 a factor of both 195 and 385? .....

When a number is a factor of two or more other numbers, it is called a **common factor** of the other numbers. For example, the number 5 is a common factor of 195 and 385.

The factors of a certain number are 2; 2; 5; 7; 7; 11 and 17. The factors of another number are 2; 3; 3; 7; 7; 11; 13 and 23. The common prime factors of these two numbers are 2; 7; 7 and 11.

The biggest number that is a factor of two or more numbers is called the **highest common factor (HCF)** of the numbers.

2. Find the HCF of the two numbers in each of the following cases.

   (a) $2 \times 2 \times 5 \times 7 \times 7 \times 11 \times 17$ and $2 \times 3 \times 3 \times 7 \times 7 \times 11 \times 13 \times 23$

   .....

   (b) 24 and 40 (c) 8 and 12

   ....

   (d) 12 and 20 (e) 210 and 56

   ....

3. Write five different numbers, all different from 35, that have 35 as a highest common factor.

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4. Write the next seven numbers in each pattern:

   A: 12 24 36 48 .....

   B: 15 30 45 60 .....

The numbers in pattern A are called the **multiples** of 12. The numbers in pattern B are called the multiples of 15. The numbers, for example 60 and 120, that occur in both

patterns, are called the **common multiples** of 12 and 15. The smallest of these numbers, namely 60, is called the **lowest common multiple** (LCM) of 12 and 15.

5. Continue writing multiples of 18 and 24 below, until you find the LCM:

   18 36 .....

   24 48 .....

6. Find the HCF and LCM of the given numbers in each case below:

   (a) 5 and 7 .....

   (b) 15 and 14 .....

   (c) 20 and 30 .....

   (d) 10 and 100 .....

   (e) 8 and 9 .....

   (f) 25 and 24 .....

   (g) 8 and 12 .....

   (h) 10 and 18 .....

1.4 Properties of operations

ORDER OF OPERATIONS AND THE ASSOCIATIVE PROPERTY

Suppose you want to tell another person to do some calculations. You may do this by writing instructions. For example, you may write the instruction 200 – 130 – 30. This may be called a **numerical expression**.

Suppose you have given the instruction 200 – 130 – 30 to two people, whom we will call Ben and Sara.

This is what Ben does: 200 – 130 = 70 and 70 – 30 = 40.

This is what Sara does: 130 – 30 = 100 and 200 – 100 = 100.

To prevent such different interpretations or understandings of the same numerical expression, mathematicians have made the following agreement, and this is followed all over the world:

> In a numerical expression that involves **addition and subtraction only**, the operations should be performed **from left to right**, **unless otherwise indicated** in some way.

An agreement like this is called a **mathematical convention**.

1. Who followed this convention in the above story, Ben or Sara? .....