

# CHAPTER 9

## Numeric patterns

In this chapter you will analyse, extend and form number patterns with integers, including negative numbers.

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-100	-91	-82	-73	-64	-55	-46	-37
-100	-92	-84	-76	-68	-60	-52	-44
-100	-93	-86	-79	-72	-65	-58	-51
-100	-94	-88	-82	-76	-70	-64	-58
-100	-95	-90	-85	-80	-75	-70	-65
-100	-96	-92	-88	-84	-80	-76	-72
-100	-97	-94	-91	-88	-85	-82	-79
-100	-98	-96	-94	-92	-90	-88	-86
-100	-99	-98	-97	-96	-95	-94	-93
-100	-100	-100	-100	-100	-100	-100	-100
-100	-101	-102	-103	-104	-105	-106	-107
-100	-102	-104	-106	-108	-110	-112	-114
-100	-103	-106	-109	-112	-115	-118	-121
-100	-104	-108	-112	-116	-120	-124	-128
-100	-105	-110	-115	-120	-125	-130	-135
-100	-106	-112	-118	-124	-130	-136	-142
-100	-107	-114	-121	-128	-135	-142	-149
-100	-108	-116	-124	-132	-140	-148	-156

# 9 Numeric patterns

## 9.1 Investigating and extending numeric patterns

### PATTERNS IN TWO DIRECTIONS

- The numbers in each row of the table form a sequence, but not all the numbers are given.

A						4	6	8	10					
B						10	8	6	4					
C						5	8	11	14					
D						20	17	13	8					

- Fill in the missing numbers.
  - What is the constant difference in sequence A? .....
  - What is the constant difference in sequence C? .....
- The first term of a certain sequence is 100 and the constant difference is 20.
    - What is the second term, and the third term, and the fourth term?  
.....
    - What is the 10th term in this sequence?  
.....

A constant-difference sequence is formed by adding the constant difference each time to form the next term.

- The first term of a certain sequence is 100 and the constant difference is  $-20$ .
  - What is the second term, and the third term, and the fourth term?  
.....
  - What is the 10th term in this sequence?  
.....
- What is the constant difference in sequence B in question 1?  
.....
  - What is the constant difference in sequence D in question 1?  
.....

5. The sixth terms of sequences E, F and G are given in the table. Fill in the other terms.

Term number	1	2	3	4	5	6	7	8	9
E with constant difference 10						30			
F with constant difference -5						30			
G with constant difference -10						30			

6. Investigate each of the patterns below. Find the pattern and write the next four terms in the sequence.

(a) 1 4 9 16 25 .....

(b) 3 6 11 18 27 .....

(c) 20 19 17 14 10 .....

(d) 20 25 29 32 34 .....

7. Make some numeric patterns of your own.

(a) .....

(b) .....

(c) .....

(d) .....

(e) .....

(f) .....

(g) .....

(h) .....

## 9.2 Making patterns from rules

1. (a) Start at 30. Add  $-5$  and write the answer. Add  $-5$  again and write the answer. Continue until you have a number sequence with 10 terms.

.....

- (b) Start at  $-30$ . Add  $-5$  and write the answer. Add  $-5$  again and write the answer. Continue until you have a number sequence with 10 terms.

.....

- (c) Start at  $-30$ . Add 5 and write the answer. Add 5 again and write the answer. Continue until you have a number sequence with 10 terms.

.....

2. (a) The first term of a sequence is  $-10$  and there is a constant difference of 5 between the terms. Write down the first ten terms of the sequence.

.....

- (b) The first term of a sequence is  $-10$  and there is a constant difference of  $-5$  between the terms. Write down the first ten terms of the sequence.

.....

3. Choose a number to be your first term and another number to be a constant difference. Write the first ten terms of your sequence.

.....

4. Choose a number smaller than  $-10$  to be your first term and another number to be a constant difference. Write the first ten terms of your sequence.

.....

5. Choose a number to be your first term and a negative number to be a constant difference. Write the first ten terms of your sequence.

.....

6. Choose a negative number to be your first term and another negative number to be a constant difference. Write the first ten terms of your sequence.

.....

7. Choose a number to be your tenth term and another number to be a constant difference. Write the first ten terms of your sequence.

.....

8. Choose a negative number to be your tenth term and another negative number to be a constant difference. Write the first ten terms of your sequence.

.....

## 9.3 Making patterns from expressions

1. (a) Complete the table.

$x$	0	1	2	3	4	5	6	7	8
$2 \times x - 10$									

- (b) Do the output values of  $2 \times x - 10$  in the above table form a pattern with a constant difference? If they do, what is the constant difference?

.....

- (c) Complete the table.

$x$	0	1	2	3	4	5	6	7	8
$3 \times x - 20$									

- (d) What is the constant difference in (c)?

.....

- (e) Complete the table.

$x$	0	1	2	3	4	5	6	7	8
$2 - 3 \times x$									

- (f) What is the constant difference in (e)?

.....

.....

- (g) Complete the table.

$x$	0	1	2	3	4	5	6	7	8
$1 - 2 \times x$									

- (h) What is the constant difference in (g)?

.....

2. Look at the pattern:  $-15; -19; -23; -27; -31; \dots$

In this pattern,  $-19$  is followed by  $-23$  and  $-23$  is followed by  $-27$ .

- (a) What number in the pattern is followed by  $-19$ ? .....
- (b) What number in the pattern is followed by  $-31$ ? .....
- (c) In the pattern,  $-19$  follows on  $-15$  and  $-23$  follows on  $-19$ .  
What number follows on  $-31$ ? .....

3. A certain pattern is formed by a common difference of 6.
- (a) What number follows on 23 in this pattern? .....
  - (b) What number is followed by 23 in this pattern? .....
  - (c) What number follows on 47 in this pattern? .....
  - (d) What number is followed by 47 in this pattern? .....

Consider the sequence: 10    6    2    -2    -6    ...    ...    ...  
 In this sequence, 2 follows on 6. They are called consecutive terms.

When one number follows another in a sequence they are called **consecutive terms**.

4. Write down any two consecutive terms in the pattern formed by  $2 \times x + 3$ , when the input numbers are consecutive whole numbers.
- .....

5. Each of the patterns below was formed by using one of the following expressions. Establish which pattern belongs to each expression.

- (a)  $2 \times x + 5$  .....
- (b)  $3 \times x + 2$  .....
- (c)  $4 \times x + 1$  .....
- (d)  $5 \times x + 6$  .....
- (e)  $6 \times x - 5$  .....
- (f)  $7 \times x - 2$  .....
- (g)  $1 - 4 \times x$  .....
- (h)  $5 - 5 \times x$  .....
- (i)  $-5 - 6 \times x$  .....

A.	6	11	16	21	26	Expression .....
B.	13	17	21	25	29	Expression .....
C.	20	23	26	29	32	Expression .....
D.	1	-3	-7	-11	-15	Expression .....
E.	31	33	35	37	39	Expression .....
F.	-20	-25	-30	-35	-40	Expression .....
G.	25	31	37	43	49	Expression .....
H.	26	33	40	47	54	Expression .....
I.	-11	-17	-23	-29	-35	Expression .....

Sequence I in question 5 is a **decreasing** sequence; the numbers become smaller as the sequence progresses:

-11   -17   -23   -29   -35

Sequence H is an **increasing** sequence; each term is bigger than the previous term:

26   33   40   47   54

6. (a) Which sequences in question 5 are increasing sequences? .....
- (b) Which sequences in question 5 are decreasing sequences? .....
7. (a) By how much does sequence A increase from one term to the next? .....
- (b) By how much does sequence B increase from one term to the next? .....
- (c) Which of the sequences in question 5 increases by the biggest amount from one term to the next, and by how much does it increase?

.....

Sequence G increases by 6 from term to term, and sequence E increases only by 2.

We may say that sequence G **increases faster** than sequence E.

8. (a) Which of the sequences in question 5 decreases fastest? .....
- (b) Which of the sequences in question 5 decreases slowest? .....
9. (a) Write 5 consecutive terms of a sequence which decreases faster than sequence D in question 5.

.....

- (b) Write 5 consecutive terms of a sequence which increases slower than sequence B in question 5.

.....

10. (a) Each of the expressions below can be used to produce a sequence. Which of the expressions will produce the sequence that increases fastest?

$3 \times x + 5$        $2 \times x + 10$        $6 \times x - 1$        $20 + 3 \times x$        $4 \times x - 9$

.....

- (b) Think of a way in which you can test your answer, and do it.

.....

.....

.....

11. In each case state whether the sequence will be decreasing or increasing.

$10 + 3 \times x$        $10 - 3 \times x$        $10 \times x + 3$        $3 \times x - 10$

.....      .....      .....      .....