Chapter 8
Integers

In this chapter you will work with numbers smaller than 0. These numbers are called negative numbers. Mathematicians have agreed that negative numbers should have certain properties that will make them useful for various purposes. You will learn about these properties and how they make it possible to do calculations with negative numbers.

Do what you can.

5 – 0 = ?	5 – 7 = ?	5 + 5 = ?
5 – 1 = ?	5 – 6 = ?	5 + 4 = ?
5 – 2 = ?	5 – 5 = ?	5 + 3 = ?
5 – 3 = ?	5 – 4 = ?	5 + 2 = ?
5 – 4 = ?	5 – 3 = ?	5 + 1 = ?
5 – 5 = ?	5 – 2 = ?	5 + 0 = ?
5 – 6 = ?	5 – 1 = ?	5 + ? = ?
5 – 7 = ?	5 – 0 = ?	5 + ? = ?
5 – 8 = ?	5 – ? = ?	5 + ? = ?
5 – 9 = ?	5 – ? = ?	5 + ? = ?
5 – 10 = ?	5 – ? = ?	5 + ? = ?
5 – 11 = ?	5 – ? = ?	5 + ? = ?
5 – 12 = ?	5 – ? = ?	5 + ? = ?
5 – 13 = ?	5 – ? = ?	5 + ? = ?

8 Integers

8.1 The need for numbers called integers

Numbers are used for many different purposes. We use numbers to say how many objects there are in a collection, for example the number of desks in a classroom. For this purpose we use the **counting numbers** 1, 2, 3, 4 . . . Numbers are also used to describe size, for example the lengths of objects. For this purpose we need more than the counting numbers, we also need **fractions**. Another purpose of numbers is to indicate position, for example the position of the right end of the red line on the pictures below.

Numbers also occur as the solutions to equations, and the natural numbers and fractions do not provide solutions for all equations. For example, there is no natural number or fraction that is the solution to the equation $10 - x = 20$. The number that provides the solution to this equation must have the property that when you subtract it, it has the same effect as when you add 10!

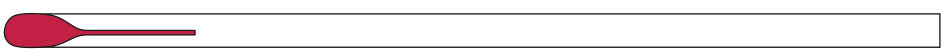
With a view to have numbers that can serve more purposes than counting and measuring, mathematicians have decided to also think of another kind of numbers which are called **integers**. The integers include the natural numbers, but for each natural number, for example 24, there is also another number called the **additive inverse**. For example, –24 is the additive inverse of 24. When you add a number to its additive inverse, the answer is 0. For example, $24 + (-24) = 0$.

SAYING HOW COLD IT IS

One of the uses of integers is for the measurement of temperature. If we say that the temperature is 0 when water freezes to become ice, we need numbers smaller than 0 to describe the temperature when it gets even colder than when water freezes. When water starts boiling, its temperature is 100 degrees on the scale called the Celsius scale.

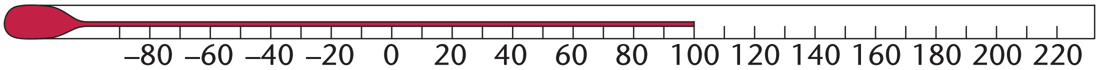
Liquids expand when heated, and shrink when cooled down. So when it is warm, the liquid in a thin tube may almost fill the tube:

When it is cold, the column of liquid will be quite short.

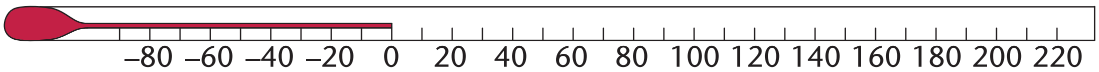


This property of liquid is used to measure temperature, and an instrument like the above is called a **thermometer**.

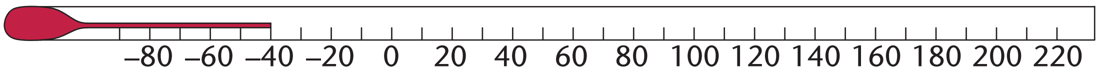
This is what a thermometer will show when it is put in water that is boiling. It shows a temperature of 100 degrees Celsius, which is written as 100 °C.



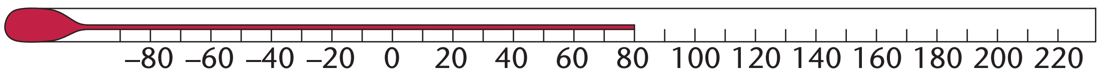
On the diagram below, you can see what a thermometer will show if it is in water that is starting to freeze. It shows a temperature of 0 °C.



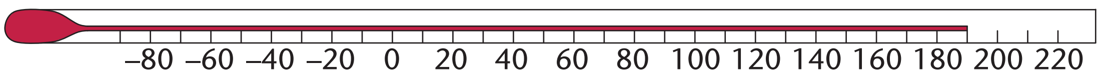
On the next diagram you can see what a thermometer will show when the temperature is –40 °C, which is colder than any winter night you may have experienced.



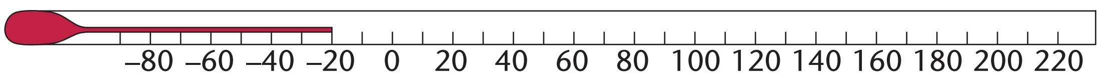
1. Write down the temperature that is shown on each of the thermometers below.



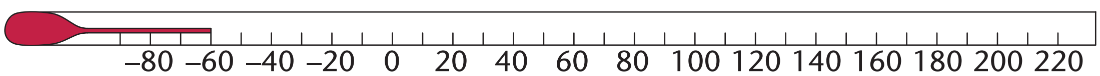
(a) ....



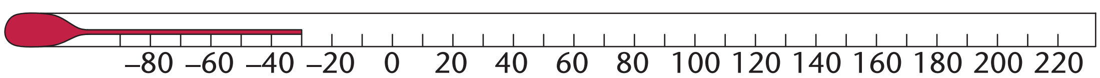
(b) ....



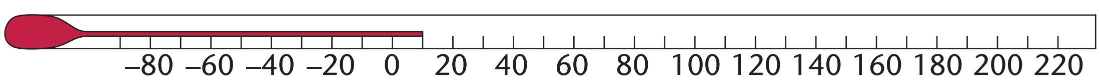
(c) ....



(d) ....



(e) ....



(f) ....

2. (a) The temperature of water in a pot is 20 °C. It is heated so that it gets 30 °C warmer. What is the temperature of the water now? ....

(b) The temperature of water in a bottle is 80 °C. During the night it cools down to 30 °C. By how much has it cooled down? ....

(c) In the middle of a very cold winter night the temperature outside is –20 °C. At nine o'clock in the morning it has become 30 degrees warmer. What is the temperature at nine o'clock? ....

3. (a) The temperature is 8 °C. What will the temperature be if it gets 10 degrees colder? ....

(b) The temperature is 8 °C. What will the temperature be if it gets 20 degrees colder? ....

(c) The temperature is –8 °C. What will the temperature be if it gets 10 degrees warmer? ....

(d) The temperature is –24 °C. What will the temperature be if it gets 10 degrees warmer? ....

4. Some numbers are shown on the number lines below. Fill in the missing numbers.

–9				–5	–4		–2	–1	0	1	2	3	4					9

														0	1	2	3	

							–10	–5	0	5	10	15						

SAYING HOW MUCH MONEY IT IS

Simon is in Grade 5. He saved money in a tin. When he turned 10, his grandmother gave him R100. He also opened his savings tin on his tenth birthday and there was R260 in the tin. Simon was very happy. He said to himself: "I am very rich!"

Simon decides to buy some things that he has always wanted. This is what he decides to buy:

- a soccer ball at R160
- a pair of sunglasses at R180
- a book about animals at R90

1. How much money did Simon have in total on the day that he thought he was rich? ....

2. What is the total cost of the three items he wants to buy?

....

3. Simon decides to first buy the soccer ball only. How much money will he have after paying for the soccer ball?

....

4. How much money will Simon have if he buys the soccer ball and the sunglasses?

....

....

5. How much money will Simon have if he buys the soccer ball and the sunglasses and the book about animals?

....

....

Simon did these calculations while he was thinking about buying the various items:

R360 – R160 = R200
R200 – R180 = R20
R20 – R90 = (–) R70 ?

6. Fatima owns a small shop. One afternoon when she closed the shop, she had R120 cash, clients owed her R90, and she owed her suppliers R310. In Fatima's view her financial position was as follows: R120 + R90 – R310 = –R100.
   (a) On another day, Fatima ended the business day with R210 cash, clients owed her R180 and she owed her suppliers R160. What was her financial position?

   ....

   ....

   (b) On another day, Fatima ended the business day with R150 cash, clients owed her R130 and she owed her suppliers R460. What was her financial position?

   ....

   ....

About 500 years ago, some mathematicians proposed that a "negative number" may be used to describe the result in a situation like the above, where a number is subtracted from a number smaller than it.

For example, we may say 10 – 20 = (–10)

This proposal was soon accepted by other mathematicians, and it is now used all over the world.

Mathematicians are people who do mathematics for a living. Mathematics is their profession, like health care is the profession of nurses and medical doctors.

7. Continue the lists of numbers below to complete the table.

(a)	(b)	(c)	(d)	(e)	(f)	(g)
10	100	3	–3	–20	150	0
9	90	6	–6	–18	125	–5
8	80	9	–9	–16	100	–10
7	70	12	–12	–14	75	–15
6	60	15	–15		50	–20
5	50					–25
4	40					
3	30					
2	20					
1	10					
0	0					
–1						

8. Calculate each of the following:

(a) 16 – 20 = ...
(b) 16 – 30 = ...
(c) 16 – 40 = ...
(d) 16 – 60 = ...
(e) 16 – 200 = ...
(f) 5 – 1 000 = ...

9. Jeminah has R200 in a savings account and R40 in her purse. Her brother owes her R50. How rich is she? In other words, how much money does she have?

....

....

10. Oops! Jeminah forgot that she borrowed R60 from her mother, and that she still has to pay R150 for a dress she bought last month. So how rich (or poor) is she really? In other words, how much money does she actually have?

....

....

11. In fact, Jeminah's financial situation is even worse. She has received an outstanding bill from her doctor, for R250. So how much money does she really have?

....

....

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ORDERING AND COMPARING INTEGERS

1. On a certain day the following minimum temperatures were provided by the weather bureau:

Bethlehem	–4 °C	Bloemfontein	–6 °C
Cape Town	7 °C	Dordrecht	–9 °C
Durban	12 °C	Johannesburg	0 °C
Pretoria	4 °C	Queenstown	–1 °C

   Arrange the temperatures from the coldest to the warmest.

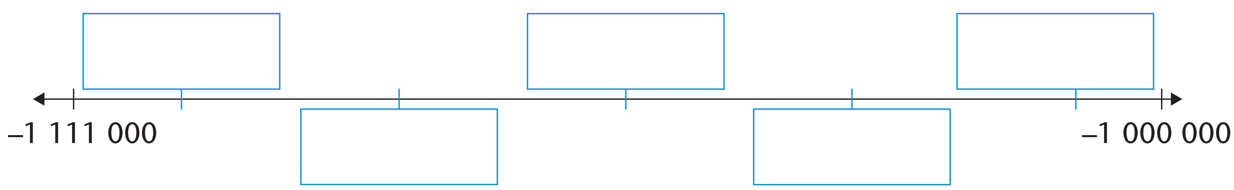
   ....

2. Place the following numbers on the number line as accurately as you can:
   50; –2; –23; 5; –36

3. In each case, place the numbers in the boxes provided:

   (a) 125 000; –178 000; –100 900; 180 500

   (b) –1 055 500; –1 010 100; –1 100 100; –1 032 800; –1 077 500



4. Insert one of the symbols > or < to indicate which number is the smaller of the two.

   (a) 978 543 ☐ 978 534 (b) –1 043 724 ☐ –1 034 724

   (c) –864 026 ☐ –864 169 (d) –103 232 ☐ –104 326

   (e) –710 742 ☐ 710 741 (f) –904 700 ☐ –904 704

8.2 Finding numbers that make statements true

The numbers 1, 2, 3, 4 and so on that we use for counting are called the **natural numbers**. Natural numbers are **whole numbers** – they do not contain fraction parts.

1. Is there a natural number that can be put in the brackets below to make the statement true?

   12 + (.....) = 17

2. In each case below, insert a natural number in the space between the brackets that will make the statement true.

   (a) 15 + (.....) = 21

   (b) 15 – (.....) = 10

   (c) (.....) + 10 = 34

   (d) (.....) – 10 = 34

   (e) 3 × (.....) = 18

Here is a different way to ask the same questions:
(a) What is x if $15 + x = 21$?
(b) What is x if $15 - x = 10$?
(c) What is x if $x + 10 = 34$?
(d) What is x if $x - 10 = 34$?
(e) What is x if $3 \times x = 18$?

3. (a) Can you think of a natural number that will make this statement true?

   2 × (....) = 5 ....

   (b) Can you think of any other number that will make the statement true?

   ....

4. (a) Can you think of a natural number that will make this statement true?

   8 + (....) = 5 ....

   (b) Can you think of any other number that will make the statement true?

   ....

We normally think of adding as making something bigger. Question 4(a) requires us to change our mind about this. We have to consider the possibility that adding a number may make something smaller.

We are looking for a number that will make the following statement true:

8 + (....) = 5

Consider this plan:
Let us agree that we will call this number *negative 3* and write it as (–3).
If we agree to this, we can say 8 + (–3) = 5.

This may seem a bit strange to you. You do not have to agree now. But even if you do not agree, let us explore how this plan may work for other numbers. What answers will a person who agrees to the plan give to the following question?

5. Calculate each of the following:

(a) $10 + (-3)$ .... (b) $12 + (-3)$ ....
(c) $12 + (-5)$ .... (d) $10 + (-9)$ ....
(e) $8 + (-8)$ .... (f) $1 + (-1)$ ....

What may each of the following be equal to?
$5 + (-8)$
$(-5) + (-8)$

You possibly agree that

$5 + (-5) = 0$ $\quad 10 + (-10) = 0$ $\quad$ and $\quad 20 + (-20) = 0$

We may say that for each "positive" number there is a **corresponding** or **opposite** negative number. Two positive and negative numbers that correspond, for example 3 and (–3), are called **additive inverses**. They wipe each other out when you add them.

When you add any number to its additive inverse, the answer is 0. For example, $120 + (-120) = 0$
So, the **set of integers** consists of all the natural numbers and their additive inverses and zero.

The number zero is regarded as an integer.

6. Write the additive inverse of each of the following numbers:

(a) 24 .... (b) –24 ....
(c) –103 .... (d) 2 348 ....

The idea of additive inverses may be used to explain why $8 + (-5)$ is equal to 3:

$$8 + (-5) = 3 + \boxed{5 + (-5)} = 3 + 0 = 3$$

7. Use the idea of additive inverses to explain why each of these statements is true:

(a) $43 + (-30) = 13$ (b) $150 + (-80) = 70$

....

....

8. Calculate each of the following:

(a) $10 + 4 + (-4)$ .... (b) $10 + (-4) + 4$ ....
(c) $3 + 8 + (-8)$ .... (d) $3 + (-8) + 8$ ....

Natural numbers can be arranged in any order to add and subtract them. It would make things easy if we agree that this should also be the case for negative numbers.

9. Calculate each of the following:

(a) $18 + 12 =$ .... (b) $12 + 18 =$ ....
(c) $2 + 4 + 6 =$ .... (d) $6 + 4 + 2 =$ .... (e) $2 + 6 + 4 =$ ....
(f) $4 + 2 + 6 =$ .... (g) $4 + 6 + 2 =$ .... (h) $6 + 2 + 4 =$ ....
(i) $6 + (-2) + 4 =$ .... (j) $4 + 6 + (-2) =$ .... (k) $4 + (-2) + 6 =$ ....
(l) $(-2) + 4 + 6 =$ .... (m) $6 + 4 + (-2) =$ .... (n) $(-2) + 6 + 4 =$ ....

10. Calculate each of the following:

(a) $(-5) + 10$ ....
(b) $10 + (-5)$ ....
(c) $(-8) + 20$ ....
(d) $20 - 8$ ....
(e) $30 + (-10)$ ....
(f) $30 + (-20)$ ....
(g) $30 + (-30)$ ....
(h) $30 + (-40)$ ....
(i) $10 + (-5) + (-3)$ ....
(j) $(-5) + 7 + (-3) + 5$ ....
(k) $(-5) + 2 + (-7) + 4$ ....

11. In each case find the number that makes the statement true. Give your answer by writing a closed number sentence.

(a) 20 + (an unknown number) = 50

....

(b) 50 + (an unknown number) = 20

....

(c) 20 + (an unknown number) = 10

....

(d) (an unknown number) + (–25) = 50

....

(e) (an unknown number) + (–25) = (–50)

....

Statements like these are also called number sentences.

An incomplete number sentence, where some numbers are not known at first, is sometimes called an **open number sentence**:

8 – (a number) = 10

A **closed number sentence** is where all the numbers are known:

8 + 2 = 10

12. Use the idea of additive inverses to explain why each of the following statements is true:

(a) $43 + (-50) = -7$

....

....

(b) $60 + (-85) = -25$

....

....

STATEMENTS THAT ARE TRUE FOR MANY DIFFERENT NUMBERS

For how many different pairs of numbers can the following statement be true, if only natural (positive) numbers are allowed?

a number + another number = 10

For how many different pairs of numbers can the statement be true if negative numbers are also allowed?

8.3 Adding and subtracting integers

PROPERTIES OF INTEGERS

1. Calculate:
   (a) 80 + (–60) = ...
   (b) 500 + (–200) + (–200) = ...

2. (a) Do you agree that 20 + (–5) = 15? ....
   (b) What do you think 20 – (–5) should be?

   ....

We normally think of addition and subtraction as actions that have opposite effects: what the one does is the opposite or **inverse** of what the other does.

3. (a) Is 100 + (–20) + (–20) = 60, or does it equal something else? ....
   (b) What do you think (–20) + (–20) should be equal to? ....

4. Complete the following as far as you can:

(a)	(b)	(c)
5 – 9 =	5 + 9 =	9 – 3 =
5 – 8 =	5 + 8 =	8 – 3 =
5 – 7 =	5 + 7 =	7 – 3 =
5 – 6 =	5 + 6 =	6 – 3 =
5 – 5 =	5 + 5 =	5 – 3 =
5 – 4 =	5 + 4 =	4 – 3 =
5 – 3 =	5 + 3 =	3 – 3 =
5 – 2 =	5 + 2 =	2 – 3 =
5 – 1 =	5 + 1 =	1 – 3 =
5 – 0 =	5 + 0 =	0 – 3 =
5 – (–1) =	5 + (–1) =	(–1) – 3 =
5 – (–2) =	5 + (–2) =	(–2) – 3 =
5 – (–3) =	5 + (–3) =	(–3) – 3 =
5 – (–4) =	5 + (–4) =	(–4) – 3 =
5 – (–5) =	5 + (–5) =	(–5) – 3 =

5. Calculate each of the following:
   (a) $20 - 20 =$ ....
   (b) $50 - 20 =$ ....
   (c) $(-20) - (-20) =$ ....
   (d) $(-50) - (-20) =$ ....

6. In each case suggest a number that may make the statement true. Also give an argument to support your proposal.
   (a) 20 + (a number) = 8

   ....

   ....

   (b) 20 + (a number) = 28

   ....

   ....

   (c) 20 – (a number) = 28

   ....

   ....

   (d) 20 – (a number) = 12

   ....

   ....

SOME HISTORY

The following statement is true if the number is 5:

15 – (a certain number) = 10

A few centuries ago, some mathematicians decided they wanted to have numbers that will also make sentences like the following true:

15 + (a certain number) = 10

But to go from 15 to 10 you have to **subtract 5**.

The number we need to make the sentence *15 + (a certain number) = 10* true must have the following strange property:

If you **add** this number, it should have the **same effect** as to **subtract 5**.

Now the mathematicians of a few centuries ago really wanted to have numbers for which such strange sentences would be true. So they thought:

"Let us just decide, and agree amongst ourselves, that the number we call *negative 5* will have the property that if you add it to another number, the effect will be the same as when you subtract the natural number 5."

This means that the mathematicians agreed that $15 + (-5)$ is equal to $15 - 5$.

Stated differently, instead of adding *negative 5* to a number, you may subtract 5.

We may agree that subtracting a negative number has the same effect as adding the additive inverse of the negative number. If we stick to this agreement, the following two calculations should have the same answer:

$10 - (-7)$ and $10 + 7$

7. Calculate.
   (a) $20 - (-10) =$ ....
   (b) $100 - (-100) =$ ....
   (c) $20 + (-10) =$ ....
   (d) $100 + (-100) =$ ....
   (e) $(-20) - (-10) =$ ....
   (f) $(-100) - (-100) =$ ....
   (g) $(-20) + (-10) =$ ....
   (h) $(-100) + (-100) =$ ....

8. Complete the following as far as you can:

(a)	(b)	(c)
$5 - (-9) =$	$(-5) + 9 =$	$9 - (-3) =$
$5 - (-8) =$	$(-5) + 8 =$	$8 - (-3) =$
$5 - (-7) =$	$(-5) + 7 =$	$7 - (-3) =$
$5 - (-6) =$	$(-5) + 6 =$	$6 - (-3) =$
$5 - (-5) =$	$(-5) + 5 =$	$5 - (-3) =$
$5 - (-4) =$	$(-5) + 4 =$	$4 - (-3) =$
$5 - (-3) =$	$(-5) + 3 =$	$3 - (-3) =$
$5 - (-2) =$	$(-5) + 2 =$	$2 - (-3) =$
$5 - (-1) =$	$(-5) + 1 =$	$1 - (-3) =$
$5 - 0 =$	$(-5) + 0 =$	$0 - (-3) =$
$5 - 1 =$	$(-5) + (-1) =$	$(-1) - (-3) =$
$5 - 2 =$	$(-5) + (-2) =$	$(-2) - (-3) =$
$5 - 3 =$	$(-5) + (-3) =$	$(-3) - (-3) =$
$5 - 4 =$	$(-5) + (-4) =$	$(-4) - (-3) =$
$5 - 5 =$	$(-5) + (-5) =$	$(-5) - (-3) =$

9. In each case, state whether the statement is true or false and give a numerical example to demonstrate your answer.
   (a) Subtracting a positive number from a negative number has the same effect as adding the additive inverse of the positive number.

   ....

   (b) Adding a negative number to a positive number has the same effect as adding the additive inverse of the negative number.

   ....

   (c) Subtracting a negative number from a positive number has the same effect as subtracting the additive inverse of the negative number.

   ....

   (d) Adding a negative number to a positive number has the same effect as subtracting the additive inverse of the negative number.

   ....

   (e) Adding a positive number to a negative number has the same effect as adding the additive inverse of the positive number.

   ....

   (f) Adding a positive number to a negative number has the same effect as subtracting the additive inverse of the positive number.

   ....

   (g) Subtracting a positive number from a negative number has the same effect as subtracting the additive inverse of the positive number.

   ....

   (h) Subtracting a negative number from a positive number has the same effect as adding the additive inverse of the negative number.

   ....

PROPERTIES OF OPERATIONS

1. Calculate the following:
   (a) $(-3) + (-5) =$ ....
   (b) $(-5) + (-3) =$ ....
   (c) $5 + (-7) =$ ....
   (d) $(-7) + 5 =$ ....
   (e) $(-13) + 17 =$ ....
   (f) $17 + (-13) =$ ....
   (g) $15 + 19 =$ ....
   (h) $19 + 15 =$ ....
   (i) $(-21) + (-15) =$ ....
   (j) $(-15) + (-21) =$ ....

In chapter 1 of Book 1 (which was about whole numbers) we said:

Addition is commutative: the numbers can be swopped around.

Or, in symbols: $a + b = b + a$, where a and b are whole numbers.

2. (a) Would you say addition is also commutative when the numbers are integers?

   ....

   (b) Explain your answer.

   ....

   ....

3. Calculate the following:

   (a) $9 - 5 =$ ....
   (b) $5 - 9 =$ ....
   (c) $(-7) - 3 =$ ....
   (d) $3 - (-7) =$ ....
   (e) $15 - (-12) =$ ....
   (f) $(-12) - 15 =$ ....
   (g) $(-40) - (-23) =$ ....
   (h) $(-23) - (-40) =$ ....

4. (a) Do you think subtraction is commutative?

   ....

   (b) Explain your answer.

   ....

   ....

In Book 1, chapter 1 we also said:

When three or more whole numbers are added, the order in which you perform the calculations makes no difference. We say: **Addition is associative**.

5. Do you think addition is also associative when we work with integers? Investigate.

   ....

   ....

   ....

   ....