Chapter 1
Numeric and geometric patterns

In this chapter, you will learn to create, recognise, describe, extend and make generalisations about numeric and geometric patterns. Patterns allow us to make predictions. You will also work with different representations of patterns, such as flow diagrams and tables.

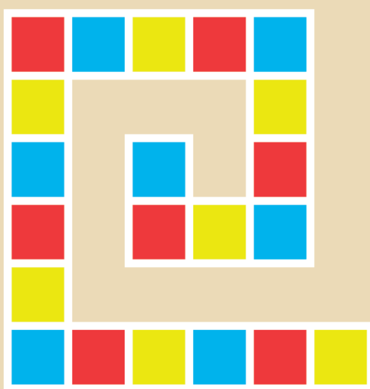
1.1 Number patterns in sequences 3
1.2 Geometric patterns 8

6

10

15

21



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1 Numeric and geometric patterns

1.1 Number patterns in sequences

WHAT COMES NEXT?

What may the next three numbers in each of these sequences be?

4; 8; 12; 16; 20; ....

4; 8; 16; 32; 64; ....

4; 8; 14; 22; 32; ....

5; 7; 4; 8; 3; 9; 2; ....

A set of numbers in a given order is called a **number sequence**. In some cases each number in a sequence can be formed from the previous number by performing the same or a similar action. In such a case, we can say there is a **pattern** in the sequence.

The numbers in a sequence are called the **terms** of the sequence. Terms that follow one another are said to be **consecutive**.

1. (a) Write down the next three numbers in each of these sequences:

   Sequence A: 4; 7; 10; 13; 16; ....

   Sequence B: 5; 10; 20; 40; 80; ....

   Sequence C: 2; 5; 10; 17; 26; ....

   (b) Write down how you decided what the next numbers would be in each of the three sequences.

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A sequence can be formed by repeatedly adding or subtracting the same number. In this case the **difference** between one term and the next is constant.

A sequence can be formed by repeatedly multiplying or dividing by the same number. In this case the **ratio** between one term and the next is constant.

A sequence can also be formed in such a way that neither the difference nor the ratio between one term and the next is constant.

In sequence A of question 1 there is a **constant difference** between consecutive terms, as shown below.

Sequence A:	4		7		10		13		16
Difference:		+ 3		+ 3		+ 3		+ 3	

In sequence B of question 1 there is a **constant ratio** between consecutive terms, as shown below.

Sequence B:	5		10		20		40		80
Ratio:		× 2		× 2		× 2		× 2	

In sequence C of question 1 there is neither a constant difference nor a constant ratio between consecutive terms. There is, however, a pattern in the differences between the terms, which makes it possible to extend the sequence. Consecutive odd numbers, starting with 3, are added to form the next term.

Sequence C:	2		5		10		17		26
Difference:		+ 3		+ 5		+ 7		+ 9	

2. Write down the next five terms in each of the sequences below. In each case, describe the relationship between consecutive terms.
   (a) 100; 95; 90; 85; ....

   ....

   (b) 0,3; 0,5; 0,7; 0,9; ....

   ....

   (c) 6; 18; 54; 162; ....

   ....

   (d) 1; 3; 6; 10; 15; ....

   ....

   ....

   ....

   (e) 20; 31; 42; 53; ....

   ....

   (f) 10; 9,7; 9,4; 9,1; ....

   ....

   (g) 18 000; 1 800; 180; 18; ....

   ....

(h) $\frac{1}{48}$; $\frac{1}{24}$; $\frac{1}{12}$; $\frac{1}{6}$; ....

....

(i) 1; 4; 9; 16; ....

....

....

(j) 625; 125; 25; 5; ....

....

In all of the above cases it was possible to extend the sequence by repeatedly adding or subtracting a number to get the next term, or by repeatedly multiplying or dividing by a number to get the next term, or by adding different numbers according to some pattern to get the next term.

The word "recur" means "to happen again". The extension of a number sequence by repeatedly performing the same or similar action is called **recursion**. The rule that describes the relationship between consecutive terms is called a **recursive rule**.

RELATIONSHIPS BETWEEN DEPENDENT AND INDEPENDENT VARIABLES

1. (a) Mr Twala pays a fee to park his car in a parking lot every day. He has to pay R3 to enter the parking lot and then a further R2 for every hour that he leaves his car there. Complete the table below to show how much his parking costs him per day for various numbers of hours.

Number of hours	1	2	3	4	5	6	7	8	9
Cost of parking in R	5	7	9						

(b) How did you complete this table? Describe your method.

....

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(c) Is there another way that you could complete the table? Describe it.

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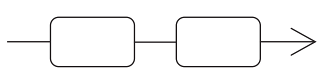
(d) Thembi multiplied the number of hours by 2 and then added 3 to calculate the cost for any specific number of hours. Complete the flow diagram to show Thembi's rule.



The rule *multiply by 2 and then add 3* describes the relationship between the two variables in this situation. The number of hours is the **independent variable**. The cost of Mr Twala's parking is the **dependent variable** because the amount he has to pay *depends on* the number of hours that he parks.

The R3 that is added is a **constant** in this situation. The number of hours and the cost are **variables**.

This rule describes how you can calculate the value of the *dependent* variable if the corresponding value of the *independent* variable is known. It differs from a recursive rule, which describes how you can calculate the value of the *dependent* variable that follows on a given value of the *dependent* variable.

In the case of a number sequence, the **position** (number) of the term can be taken as the independent variable, as shown for the sequence 15; 19; 23; 27; 31; . . . in this table:

Term number	1	2	3	4	5	6	7	8	50
Term	15	19	23	27	31				

2. (a) Complete the above table.
   (b) How did you calculate term number 50?

   ....

   ....

   ....

   (c) Lungile reasoned like this:
   I added 4 each time to complete the table. I counted backwards to see what comes before term 1. I got 11 and then I knew I had to add one 4 to 11 to get the first term.

   Lungile remembered that multiplication is done before addition, unless otherwise indicated by brackets.

   Complete the pattern below to show Lungile's thinking:

   Term 1: $11 + 1 \times 4 = 11 + 4 = 15$

   Term 2: $11 + 2 \times 4 = 11 + 8 = 19$

   Term 3: ....

   Term 4: ....

   Term 5: ....

   Term 6: ....

   Term 10: ....

   Term 50: ....

(d) Describe in your own words how term number 50 can be calculated.

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(e) Tilly reasoned like this: *The constant difference between the terms is 4. I must add four 49 times to the first term to get the 50th term. So,* $15 + 49 \times 4 = 15 + 196 = 211$.
Complete the pattern below to demonstrate Tilly's thinking:

Term 1: 15

Term 2: $15 + 1 \times 4 = 15 + 4 = 19$

Term 3: $15 + 2 \times 4 = 15 + 8 = 23$

Term 4: ....

Term 5: ....

Term 6: ....

Term 10: ....

Term 50: ....

(f) Write the rule to calculate term number 50 in your own words.

....

In the example in question 2, the term number is the independent variable and the term itself is the dependent variable. So, if we know the rule that links the dependent variable and the independent variable, we can use it to determine any term for which we know the term number.

3. Write a rule to calculate the term for any term number in the sequence 15; 19; 23; 27; 31; . . . by using

(a) Lungile's thinking.

....

(b) Tilly's thinking.

....

We can use n as a symbol for "any term number".
The rule to calculate the term for any term number when using Lungile's thinking will then be:

Term = $n \times 4 + 11$

(c) Write down the rule to calculate the term for any term number in terms of n by using Tilly's thinking.

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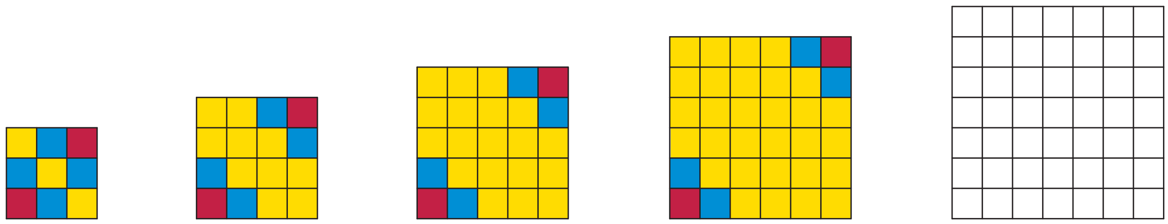
1.2 Geometric patterns

CONSTANT QUANTITIES AND VARIABLE QUANTITIES

Small yellow, blue and red tiles are combined to form larger square tiles as shown below:



Tile no. 1 *Tile no. 2* *Tile no. 3* *Tile no. 4* *Tile no. 5*

1. Draw tile no. 5 on the grid provided. (Shade the blue and red tiles in different ways. You don't have to use colours.)
2. Complete the table.

	Tile no. 1	Tile no. 2	Tile no. 3	Tile no. 4	Tile no. 5	Tile no. 10
Number of yellow tiles						
Number of red tiles						
Number of blue tiles						

3. How many red tiles are there in each bigger tile?
4. How many yellow tiles are there in each bigger tile?
5. Some of the quantities in this situation are variables and some are constants. Which are variables and which are constants?
6. Was it possible to predict the pattern on tile no. 2 by looking only at tile no. 1?

The number of red tiles is constant and the number of blue tiles is constant. It is clear that the design is such that there is always a red tile in the top right corner, and also in the bottom left corner, and that the red tiles are always "bordered" by two blue tiles each. So the number of red and blue tiles is **constant** in this situation.

The number of yellow tiles in the arrangements varies. The number of yellow tiles is a **variable** in this situation.

PATTERNS WITH MATCHES

1. A pattern with matches is shown below.

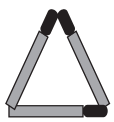


Figure 1

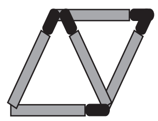


Figure 2

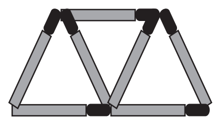


Figure 3

(a) Explain how the pattern is formed.

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(b) Complete the table.

Figure number	1	2	3	4	5	6	7	8
Number of matches	3	5	7					

(c) What rule did you use to complete the table?

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(d) How many matches are needed to form figure no. 9? ....

(e) How many matches are needed to form figure no. 17? Explain.

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....

(f) If you used the recursive rule to complete the table, it would have taken a long time to answer question (e) because you had to add the same number many times. Try to find an easier way to answer question (e). Describe your method.

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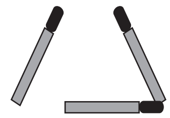
(g) Complete the pattern below.
Hint: It may help to think of figure no. 1 or term 1 like this: There is 1 match at the beginning and two more are added every time. It helps to “see” the two matches that are added each time.



Term 1: $1 + 1 \times 2 = 3$

Term 2: $1 + 2 \times 2 = 5$

Term 3: $1 + 3 \times 2 = 7$

Term 4: ....

Term 5: ....

Term 10: ....

Term 17: ....

(h) What stays the same in the pattern in (g) and what varies?

....

....

(i) Use the flow diagram below to write down the rule that you can use to calculate the number of matches needed for any figure in the pattern.

Figure number → ☐ → ☐ → *Number of matches*

(j) Can you link the number of matches added each time to the number that you multiply by in the flow diagram? Explain.

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2. Another pattern with matches is shown below.

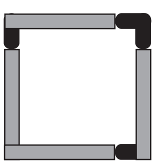

Figure 1

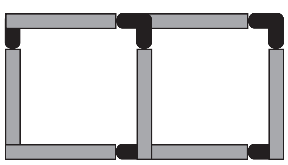

Figure 2

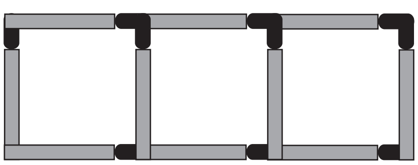

Figure 3

(a) Explain how the pattern is formed.

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(b) Complete the table.

Figure number	1	2	3	4	5	6	7	8
Number of matches	4							

(c) What rule did you use to complete the table?

....

(d) How many matches are needed for figure 9 (or term 9)? ....

(e) How many matches are needed for figure 20 (or term 20)? ....

(f) What rule did you use to calculate the number of matches in question (e)?

....

(g) Complete the pattern:

Term 1: $1 + 1 \times 3 = 4$

Term 2: $1 + 2 \times 3 = 7$

Term 3: $1 + 3 \times 3 =$ ....

Term 4: ....

Term 5: ....

Term 10: ....

Term 17: ....

(h) What stays the same in the pattern in (g) and what varies?

....

....

(i) Use the flow diagram below to write down the rule that you can use to calculate the number of matches needed for any figure in the pattern.

Figure number → ☐ → ☐ → *Number of matches*

3. Compare the way in which the number of matches increases in question 1 to the way in which it increases in question 2. What is the same and what is different?

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ALPHABETIC PATTERNS

Consider the figures below formed with red dots.

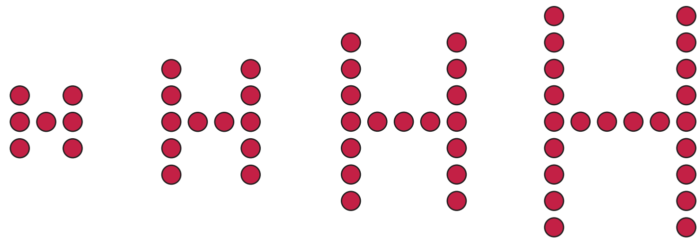


Figure 1 *Figure 2* *Figure 3* *Figure 4* *Figure 5*

1. How many dots are used to form figure 5?

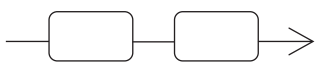
 ....

2. Draw figure 5.

3. Complete the table.

Figure number	1	2	3	4	5	6	7	8
Number of dots	7	12	17					

4. Complete the flow diagram.



5. What rule did you use to complete the table? Describe your rule.

 ....

6. Can you think of another rule to complete the table? Describe your rule.

 ....

 ....

7. Name the dependent variable and the independent variable in this situation.

 ....

 ....

SQUARES AND CUBES

1. Squares are arranged to form figures as shown below, according to a rule.

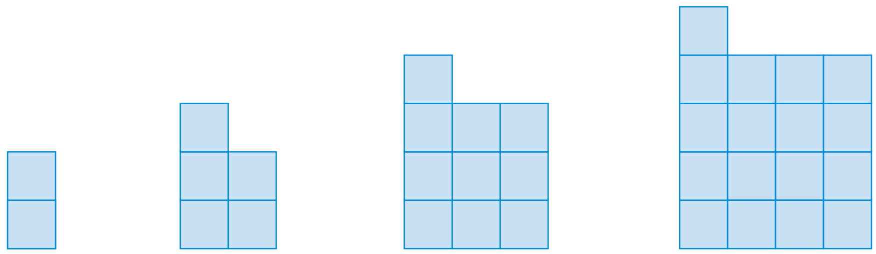


Figure 1 *Figure 2* *Figure 3* *Figure 4*

(a) Complete the table. Then determine the differences between consecutive terms.

Figure number	1	2	3	4	5	6	7	8
Number of squares	2	5						

+ 3

(b) Describe the recursive rule that you can use to extend the pattern in words.

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(c) Nombuso played around with the differences between consecutive terms. She noticed that the pattern (+ 3; + 5; + 7; ...) was similar to the one that you get when you calculate the differences between square numbers. This made her think that she should investigate square numbers to help her find a rule that could link the figure number and the number of squares.
Complete the following pattern along the lines of Nombuso's thinking:

Figure 1: $1 \times 1 + 1 = 1 + 1 = 2$

Figure 2: $2 \times 2 + 1 = 4 + 1 = 5$

Figure 3: ....

Figure 4: ....

Figure 5: ....

Figure 6: ....

Figure 7: ....

Figure 8: ....

Figure 50: ....

(d) Write a rule to calculate the number of squares for any figure number.

....

(e) Write your rule in (d) in terms of *n* where *n* is the symbol for any figure number.

....

(f) Compare the sequence in this activity to the sequence in the previous activity where dots were arranged to form the letter H. Describe the way in which the dependent variable (the output number) changed in each of the sequences.

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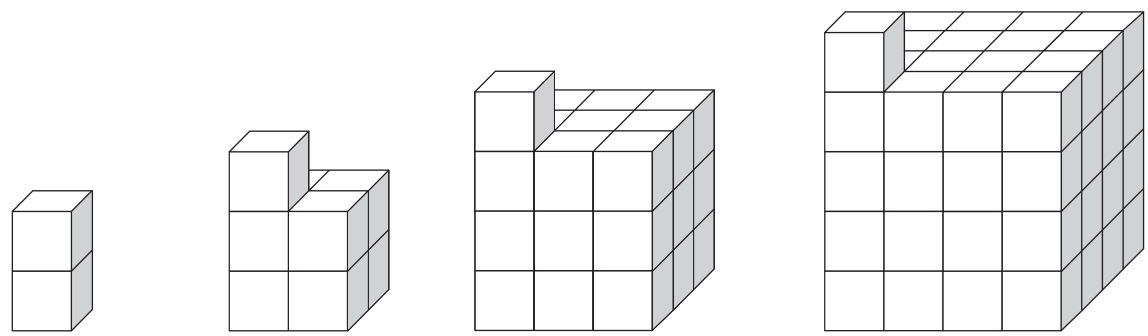
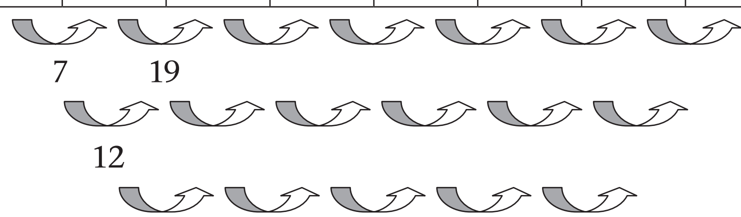
2. Identical cubes are arranged to form stacks of cubes in the following way:



Stack 1 *Stack 2* *Stack 3* *Stack 4*

(a) Complete the table. Then find the differences between consecutive terms. Do it a second and a third time. Write the differences below the arrows.

Stack number	1	2	3	4	5	6	7	8
Number of cubes	2	9	28					



(b) Describe the way in which you completed the table.

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(c) David looked carefully at the structure of the stacks and did the following to link the stack number with the number of cubes in a stack. Complete the pattern.

Stack 1: $1 \times 1 \times 1 + 1 = 1 + 1 = 2$

Stack 2: $2 \times 2 \times 2 + 1 = 8 + 1 = 9$

Stack 3: $3 \times 3 \times 3 + 1 = 27 + 1 = 28$

Stack 4: $4 \times 4 \times 4 + 1 = 64 + 1 = 65$

Stack 5: ....

Stack 6: ....

Stack 7: ....

Stack 8: ....

Stack 9: ....

Stack 10: ....

(d) How many cubes will there be in stack 50?

....

....

(e) Write the rule that you used to calculate the number of cubes in stack 50 in words.

....

(f) Write your rule in (e) in terms of *n* where *n* is the symbol for any stack number.

....

3. In questions 1(a) and 2(a) you calculated the differences between the consecutive terms.

(a) What did you find when you kept on finding the differences, as suggested in question 2(a)?

....

(b) Go back to question 1(a). What do you find when you keep on finding the differences between consecutive terms, like you did in question 2(a)?

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MY OWN PATTERNS

Use the grid, the tables and the tile template to create and describe your own geometric patterns.

Pattern A

Pattern B