

CHAPTER 16

Probability

Probability theory deals with situations that can have many possible outcomes, only one of which actually occurs. For example, when you throw a dice, only one face shows – but any of the others could have shown. Or when you cross a street you usually get to the other side without being struck by a car – but it could have happened.

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1	1	4	4	3	2	4	1	3
2	5	3	3	2	4	1	2	4
3	4	1	5	4	5	3	3	2
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2	1	3	5	3	3	6	6	4
5	5	5	2	1	2	2	1	2

16 Probability

16.1 Possible and actual outcomes, and frequencies

WHAT CAN YOU EXPECT?

You will soon do an experiment. To do the experiment you need a bag like a plastic shopping bag or a brown paper bag. You also need three objects of the same size and shape, like three buttons, bottle tops or small square pieces of cardboard. The three objects must look different, for example they should have different colours such as yellow, red and blue. If you use cardboard squares, you can write “yellow”, “red” and “blue” on them.

1. (a) Put your three objects in your bag. You will later draw one object out of the bag, without looking inside. Can you say whether the object that you will draw will be the yellow one, the blue one or the red one?

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- (b) Discuss this with two classmates.

2. (a) Now draw an object out of the bag, write down its colour, and put it back.

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- (b) You will soon do this 12 times. Can you say how many times you will draw each of the three colours? If you think you can, write your prediction below.

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- (c) Compare your predictions with two classmates.
 - (d) Can you think of any reason why you may draw blue more often than red or yellow, when you do the experiment described in (b)?

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3. (a) Draw an object out of the bag, write down its colour, and put it back. Do this 12 times and write down the colour each time.

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- (b) Write your results in the table below.

Outcome	Yellow	Red	Blue
Number of times obtained			

What you did in question 3 is called a **probability experiment**. Each time you drew an object out of the bag, you performed a **trial**.

Each time you performed a trial, three different things could have happened. These are called the **possible outcomes**.

Each time you performed a trial, one of the possible outcomes actually occurred. This is called the **actual outcome**.

The number of times that a specific outcome occurred during an experiment is called the **actual frequency** of that outcome.

4. (a) What were the possible outcomes in the experiment that you did in question 3?

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- (b) How many trials did you perform in the experiment?

- (c) What was the actual outcome in the third trial that you performed?

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- (d) What was the actual frequency of drawing a blue object during the 12 trials in the experiment that you did?

16.2 Relative frequencies

Thomas also did the experiment in question 3 on page 221 but he performed more trials and his results were as follows:

Outcome	Yellow	Red	Blue
Number of times obtained	5	7	8

1. (a) How many trials did Thomas perform in total?

- (b) What fraction of the trials produced yellow as an outcome?

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- (c) What fraction of the trials produced red as an outcome?

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- (d) What fraction of the trials produced blue as an outcome?

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The fraction of the trials in an experiment that produce a specific outcome is called the **relative frequency** of that outcome.

$$\text{Relative frequency of an outcome} = \frac{\text{number of times the outcome occurred}}{\text{total number of trials}}$$

A relative frequency can be expressed as a common fraction, as a decimal or as a percentage. The relative frequencies in the results of the experiment Thomas did (question 1) were one quarter for yellow, 7 twentieths for red and 2 fifths for blue. Expressed as percentages, the relative frequencies were 25%, 35% and 40%. The **range** of Thomas's relative frequencies, expressed as percentages, is 15% (40% – 25%).

2. (a) Use your calculator to calculate the relative frequencies that you obtained for the three different outcomes in the experiment you did in question 3 on page 221. Express them both as fractions and percentages.

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- (b) Calculate the range of the relative frequencies of the three outcomes for the results of the experiment you did in question 3.

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- (c) You will soon repeat the experiment with 3 possible outcomes and 12 trials that you did. Do you think you will get the same results than when you first did the experiment?

3. (a) Join with three or four classmates to work as a team, and discuss question 2(c).
 (b) Assign the “names” A, B, C, D and E (if you are 5) to the team members and complete the table below for the experiment you did in question 3. Give the relative frequencies as percentages. Note that to calculate the relative frequencies for the totals as percentages, you have to use your calculators.

	Actual frequencies			Relative frequencies %			Range
	Yellow	Red	Blue	Yellow	Red	Blue	
Experiment 1 by A							
Experiment 1 by B							
Experiment 1 by C							
Experiment 1 by D							
Experiment 1 by E							
Totals for experiment 1							

- (c) Which of the ranges is the smallest?

16.3 More trials and relative frequencies

WHAT HAPPENS WHEN YOU CONDUCT MANY TRIALS?

1. Join up with your teammates of the previous session. Each of you will soon repeat the experiment you did previously. You will put a yellow object, a red object and a blue object in a bag, draw one object and note the colour. You will do this 12 times. This will be experiment 2.

- (a) Do you expect that the results will in some ways be the same as for the experiment in which you did this in the previous section? Do not talk to your teammates yet. Form your own opinion, and also consider *why* you think the results will be different or the same.

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- (b) Share your ideas with your teammates.

You will soon repeat the experiment and write the results in the rows for “experiment 2” on the table on the next page. You will repeat it once more and write the results in the rows for “experiment 3”. If you have time left, you may repeat it once more as “experiment 4”.

2. (a) Look at the table on the next page. Certain rows are for the outcomes that you and your teammates obtain. The shaded rows are for adding different sets of outcomes together. Think about what may happen and predict in what rows the ranges will be smaller than in other rows, and in what row the range will be the smallest of all.

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- (b) Share your ideas with your teammates.

3. (a) Copy the totals for “experiment 1” into the first row of the table on the next page. Do the experiment described in question 1 and enter the results in the rows for “experiment 2”. Calculate the relative frequencies and the range.
(b) Add in the results of your teammates, add up the totals and calculate the relative frequencies and the range of the totals.
4. Repeat question 3, and enter the results in the rows for “experiment 3”.

		Actual frequencies			Relative frequencies%			Range
		Yellow	Red	Blue	Yellow	Red	Blue	
1	Totals for experiment 1							
2	Experiment 2 by A							
3	Experiment 2 by B							
4	Experiment 2 by C							
5	Experiment 2 by D							
6	Experiment 2 by E							
7	Totals for experiment 2							
8	Totals for experiments 1 and 2 combined							
9	Experiment 3 by A							
10	Experiment 3 by B							
11	Experiment 3 by C							
12	Experiment 3 by D							
13	Experiment 3 by E							
14	Totals for experiment 3							
15	Totals for experiments 1, 2 and 3 combined							
16	Experiment 4 by A							
17	Experiment 4 by B							
18	Experiment 4 by C							
19	Experiment 4 by D							
20	Experiment 4 by E							
21	Totals for experiment 4							
22	Totals for experiments 1, 2, 3 and 4 combined							

