

CHAPTER 12

Algebraic equations 2

You have already done some work on equations in Term 3. In this term, we extend the work you have already done to include negative numbers.

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$$5 \times 21 - 3 = 102$$

$$5 \times ? - 3 = 97$$

$$5 \times ? - 3 = 92$$

$$5 \times ? - 3 = 87$$

$$5 \times ? - 3 = 82$$

$$5 \times x - 3 = 77$$

$$5 \times ? - 3 = 72$$

$$5 \times ? - 3 = 67$$

$$5 \times ? - 3 = 62$$

$$5 \times ? - 3 = 57$$

$$5 \times ? - 3 = 52$$

$$5 \times ? - 3 = 47$$

12 Algebraic equations 2

12.1 Describing problem situations

A **closed number sentence** is a true statement about numbers, for example $21 + 5 = 26$. All the numbers are given.

In an **open number sentence**, for example $15 + x = 21$, one or more of the numbers are unknown.

An open number sentence is also called an **equation**.

1. Jan is 3 years older than his sister Amanda. Amanda is 14 years old. Write a closed number sentence to show Jan's age.

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2. Numbers are said to be consecutive if they follow one another. The numbers $-1, 0, 1$ are consecutive. The sum of $-1, 0$ and 1 is 0 .

- (a) Write a closed number sentence that shows two consecutive numbers that add up to -33 .

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- (b) Write a closed number sentence that shows two consecutive numbers whose product is 6 .

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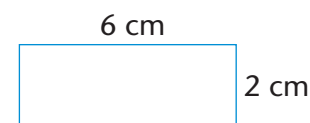
3. A cell phone costs R500 after a discount of R150 is given. Write a closed number sentence to show the original price of the cell phone.

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4. When the bus leaves the terminal, it is carrying 55 people. At the first bus stop 12 people get off the bus and 9 people get in. At the second bus stop, 12 people get in and 9 people get off the bus. Write a closed number sentence to show the number of people that are now in the bus.

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5. A rectangle is shown on the right.
Write a closed number sentence to calculate the following:



- (a) the area of the rectangle
- (b) the perimeter of the rectangle

12.2 Analysing and interpreting equations

1. The cost of a school uniform in rands is represented by x . An alteration fee of R20 is also charged. Mr Malan paid R520 for both the school uniform and the alterations done on it.

(a) Which equation describes this situation?

- A. $20 \times x = 520$ B. $x - 20 = 520$ C. $x + 20 = 520$ D. $20 + 20 = x$

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(b) What is the price of the uniform?

2. Five learners should each receive the same number of sweets. There are 60 sweets in total that they have to share.

(a) Which equation describes this situation?

- A. $5 + s = 60$ B. $5s = 60$ C. $s - 5 = 60$ D. $\frac{s}{5} = 60$

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(b) How many sweets does each learner get?

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(c) What does the letter s represent in the equation you have chosen?

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3. A taxi picks up n passengers at the airport and drives to the nearest hotel. When it leaves the hotel, the number of passengers in the taxi has decreased by 6. There are now 7 passengers in the taxi.

(a) Which equation describes this situation?

- A. $n - 6 = 7$ B. $7 - n = 6$ C. $n + 6 = 7$ D. $n - 7 = 6$

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(b) How many passengers were in the taxi when it left the airport?

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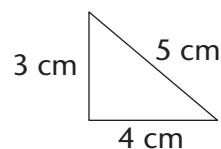
4. Write a closed number sentence for calculating the perimeter of an equilateral triangle whose sides are 5 cm long.

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Remember: An equilateral triangle is a triangle in which all three sides are equal.

5. Write a closed number sentence to calculate the perimeter of the triangle shown on the right.

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12.3 Solving and completing equations

SOLVE BY INSPECTION

1. The number sentences given below are not true. Make the number sentences true by changing the numbers in blue.

(a) $13 + 7 = 22$

(b) $50 + (-50) = -100$

(c) $7 \times 8 = 54$

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(d) $9 - (-3) = 6$

(e) $-5 + 12 = -7$

(f) $4 \times 6 = 28$

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(g) $6 - 9 = 3$

(h) $9 - 6 = -3$

(i) $5 + (-12) = 7$

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(j) $10 + (-2) = 12$

(k) $(-1) - (-1) = -2$

(l) $0 + (-2) = 0$

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2. Consider the equations given below. Check whether the value given in brackets is the solution. Simply write *yes* or *no* with an explanation.

(a) $x + 3 = 0$ ($x = -3$)

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(b) $3 - x = 4$ ($x = 1$)

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(c) $-5 + x + x = -11$ ($x = -2$)

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(d) $3 - x = 4$ ($x = -1$)

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To check whether a given value is the solution or not we have to answer the following question in our minds: **Does the given value make the equation true?** If it does, we say such a value is the **solution**.

3. Find the value of the unknown that makes the equation true in each case:

(a) $x + 6 = 8$

(b) $x + 6 = 4$

(c) $x + 6 = 0$

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(d) $6 - x = 8$

(e) $6 - x = 4$

(f) $6 - x = 0$

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(g) $\frac{x}{4} = 2$

(h) $x = 4 \times 2$

(i) $\frac{x}{2} = \frac{1}{4}$

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4. Three possible solutions are given in brackets below each equation, but only one is correct. Find the correct solution in each case.

(a) $x + 27 = 27$
 $\{-27; 0; 1\}$

(b) $12 = 4 - x$
 $\{8; 16; -8\}$

(c) $x + 3 = 0$
 $\{-3; 0; 3\}$

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(d) $5 - x = 10$
 $\{-5; 0; 5\}$

(e) $5 + x = 10$
 $\{-5; 0; 5\}$

(f) $-5 + x = 10$
 $\{-5; -15; 15\}$

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(g) $-5 - x = 10$
 $\{-5; -15; 15\}$

(h) $-5 - x = 0$
 $\{-5; -15; 15\}$

(i) $5 - x = -10$
 $\{-5; -15; 15\}$

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(j) $x = \frac{10}{10}$
 $\{0; 1; 100\}$

(k) $10x = 0$
 $\{0; 1; \frac{1}{10}\}$

(l) $\frac{x}{10} = 0$
 $\{0; 1; 10\}$

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5. What value for x would make each equation below true?

(a) Let $x = \dots$ then $x + 3 = 10$

(b) Let $x = \dots$ then $x + 3 = -4$

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(c) $x + x + x = -6$ is true for $x = \dots$

(d) $x + x + x + x = -8$ is true for $x = \dots$

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6. In each case, fill in the table until you can see for what value of x the equation given above the table is true. You may add more x values of your own choice. To save time and work, you may skip columns that you think will not help you to find the solution.

(a) $37 - 4x = 5$

x	1	10	5	6	7				
$37 - 4x$									

(b) $50 - 7x = 22$

x	1	10	5	6					
$50 - 7x$									

(c) $100 - 3x = 49$

x	10	20	25	15	16				
$100 - 3x$									

SOLVE BY TRIAL AND IMPROVEMENT

We can think of an equation as a question asking for a value that we can assign to the **unknown** to make the equation true.

Consider the equation $82 + m = 23$. We need to assign values to m until we find a value that makes the equation true, as shown in the table below.

	Equation	True/False
Let $m = -50$	$82 + (-50) = 82 - 50 = 32$	False
Let $m = -30$	$82 + (-30) = 82 - 30 = 52$	False
Let $m = -60$	$82 + (-60) = 82 - 60 = 22$	False
Let $m = -59$	$82 + (-59) = 82 - 59 = 23$	True

So $m = -59$ because $82 + (-59) = 82 - 59 = 23$

- Determine the value of t that makes the equation $28 - t = 82$ true by making use of the trial and improvement method.

	Equation	True/False

Solution:

- Consider the equation $w + 32 = -68$. Use the trial and improvement method to find the solution of the equation.

	Equation	True/False

Solution:

3. The equation $200 - 5t = 110$ is given. What value of t makes the equation true?
Use the table below to determine the solution.

	Equation	True/False

Solution:

4. What value of p makes the equation $18p = 90$ true?

	Equation	True/False

Solution:

5. What value of x makes the equation $88 - 6x = 46$ true?

	Equation	True/False

Solution:

12.4 Identifying variables and constants

1. The mass of an empty truck is 2 680 kg. The truck is used to transport cement. Each pocket of cement has a mass of 90 kg.

The combined mass of the truck and the cement can be calculated by means of the formula: $y = 90 \times x + 2\,680$.

Use the terms **variable** or **constant** to describe the meaning of each symbol used in the formula. Explain your answer.

- (a) y (b) 90 (c) x (d) 2 680

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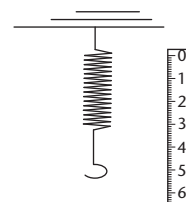
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2. A steel spring is suspended from a stand. Mass pieces of equal mass are hooked onto the bottom end of the spring. The length of the spring is measured with 1 mass piece hooked, 2 mass pieces hooked, 3 mass pieces hooked and so on. The results are shown in the table below.



Number of mass pieces	1	2	3	4	5	7	10
Length of spring in cm	48	56	64	72	80	96	120

The formula $y = 8x + 40$ is used to predict the length of the spring for the various number of mass pieces hooked.

Use the terms **variable** or **constant** to describe each symbol used in the formula. Explain your answer.

- (a) y (b) 8 (c) x (d) 40

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12.5 Numerical values of expressions

SUBSTITUTING NUMBERS INTO EXPRESSIONS

1. (a) Calculate the values of each expression for the given values of x , and write your answers in the table.

x	0	2	5	10	20	50	100
$100 - 9x$							
$100 - 8x$							
$100 - 7x$							
$100 - 6x$							
$100 - 5x$							
$100 - 4x$							
$100 - 3x$							

- (b) Which sequence in the above table decreases fastest, and which sequence decreases slowest?

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2. (a) Complete the table.

x	1	2	3	4	5	6	7
$2x + 3$							
$3x - 3$							
$3x - 2$							
$3x - 1$							

- (b) For which value of x is $2x + 3$ equal to $3x - 1$?

- (c) For which values of x is $2x + 3$ smaller than $3x - 1$?

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- (d) Do you think $2x + 3$ is smaller than $3x - 1$ for all values of x greater than 4?
You may try a few numbers to help you think about this.

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- (e) Which sequence increases fastest, the sequence generated by $2x + 3$ or the sequence generated by $3x - 3$?

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