STRAND 2 → ALGEBRA

HISTORY OF INEQUALITY SYMBOLS

Equal Sign

- *Before the equal sign came into popular use, equality was expressed in words. According to Lankham, Nachtergaele, and Schilling at University of California-Davis, the first use of the equal sign (=) came in 1557. Robert Recorde, 1510 to 1558, was the first to use the symbol in his work, "The Whetstone of Witte." Recorde, a Welsh physician and mathematician, used two parallel lines to represent equality because he believed they were the most equal things in existence.*

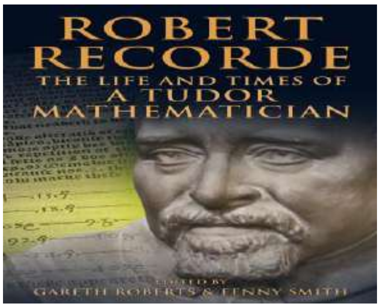
Inequalities

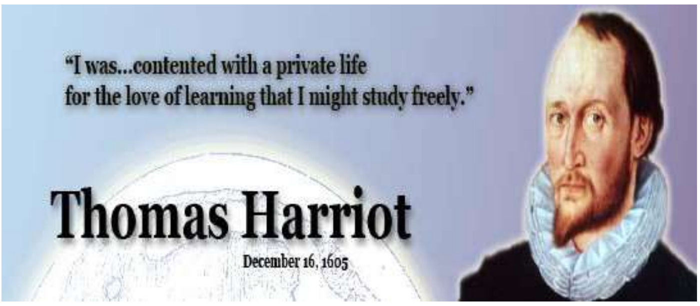
- *The signs for greater than (>) and less than (<) were introduced in 1631 in "Artis Analyticae Praxis ad Aequationes Algebraicas Resolvendas." The book was the work of British mathematician, Thomas Harriot, and was published 10 years after his death in 1621. The symbols actually were invented by the book's editor. Harriot initially used triangular symbols which the editor altered to resemble the modern less/greater than symbols. Interestingly, Harriot also used parallel lines to denote equality. However, Harriot's equal sign was vertical (II) rather than horizontal (=).*

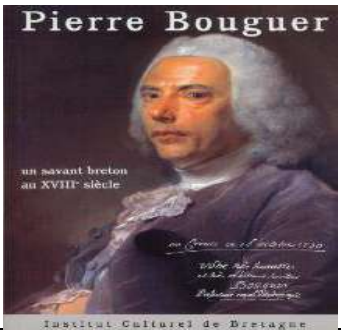
Less/Greater Than or Equal To

- *The symbols for less/greater than or equal to (< and >) with one line of an equal sign below them (≤ and ≥), were first used in 1734 by French mathematician, Pierre Bouguer.*

  John. Wallis, a British logician and mathematician, used similar symbols in 1670. Wallis used the greater than/less than symbols with a single horizontal line above them







Source: http://www.ehow.com/info_8143072_history-equality-symbols-math.html

2.1 Factorisation and Simplification of Algebraic Expressions

Factorisation

LEARNING OUTCOME

Students should be able to:

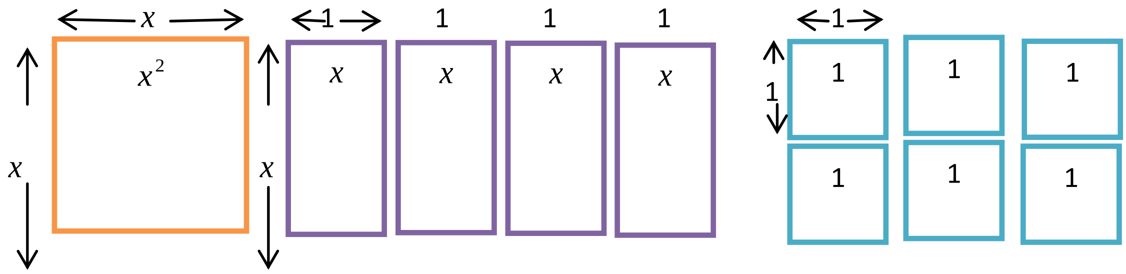
- Factorizing algebraic expressions by applying:
  - ❖ Common Factor method.
  - ❖ Grouping method
  - ❖ Difference of squares method.
  - ❖ Perfect square method.

Activity:

Let us make various rectangles out of squares and rectangles.

HOW?

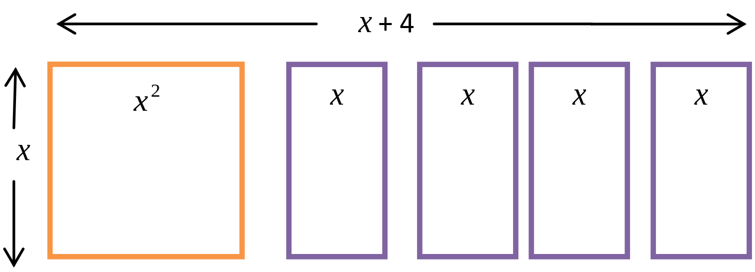
Step: 1 Cut out the following squares and rectangles.



Step: 2 Make rectangles from the cut out squares and rectangles in step 1 above.

Let's say "I use one piece of x^2 and 4 pieces of x"

What is the area of the new rectangle?



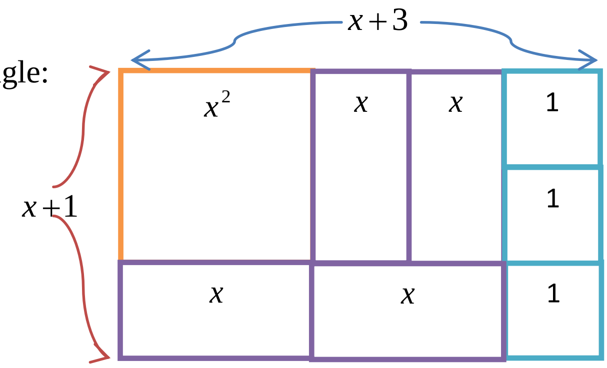
Step: 3 Now make some more rectangles using the square and rectangular pieces indicated below.

a) 1 piece of $\boxed{x^2}$, 2 pieces of $\boxed{x}$ and 2 pieces of $\boxed{1}$.

b) 1 piece of $\boxed{x^2}$, 3 pieces of $\boxed{x}$ and 2 pieces of $\boxed{1}$

Note: If we are to make a rectangle out of 1 piece of $\boxed{x^2}$, 4 pieces of $\boxed{x}$ and 3 pieces of $\boxed{1}$

Then we would get this rectangle:



Therefore:

1. The sum of the areas of all the pieces would be x^2+4x+3
2. The area of the new and bigger rectangle would be $(x+1)(x+3)$

Finally: x^2+4x+3 is the **product** of $(x+1)(x+3)$ which means that $(x+1)$ and $(x+3)$ are **factors** of x^2+4x+3.

Factorization is rewriting an expression or polynomial as a product of its factors.

Factorization

$$x^2+4x+3=(x+1)(x+3)$$

Expansion

Using Formula for Factorization:

1. Common Factor Method:

When all terms in a polynomial share a common factor, the expression can be factorized by taking the common factor outside the brackets as in the distributive law.

Example 2.1

Factorize: $2x^2+3xy$

$$\therefore 2x^2 = 2\times x\times x$$

And $$3xy = 3\times x\times y$$

The two terms have x as a common factor which is put outside the brackets to be distributed to the left over terms as shown: $2x^2+3xy = x(2x+3y)$

Example 2.2

Factorize: $3x^2 + 6xy - 9x$

$$\therefore 3x^2 = 3 \times x \times x$$
$$6xy = 2 \times 3 \times x \times y$$
$$-9x = 3 \times -3 \times x$$

$$3x^2 + 6xy - 9x = 3x(x + 2y - 3)$$

2. Grouping Method

This is done by grouping a pair of terms. Then, factor each pair of two terms

Example 2. 3

Factorize: $p^2 + 6p + 2p + 12$

Note: A. Make 2 groups having the same common factor then factorize:

$$(p^2 + 6p) + (2p + 12)$$

Group 1		Group 2
$p^2 + 6p$ $= p(p+6)$	+	$2p + 12$ $= 2(p+6)$

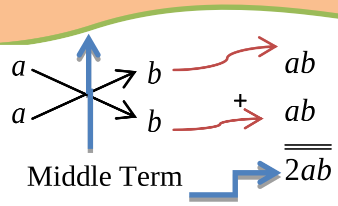
B. What is common to both groups will make one factor and the second factor would be made up of the left over factors.

$$p^2 + 6p + 2p + 12 = (p+6)(p+2)$$

Generalization:

Find factors of the first term and the second term separately, and then cross multiply. The middle term should come from adding the diagonals.

i.e. $a^2 + 2ab + b^2$



3. Quadratics

3a Difference of Squares: $a^2 - b^2 = (a+b)(a-b)$

When the sum of two numbers multiplies their difference $(a + b)(a - b)$ then the product is the difference of their squares, $a^2 - b^2$.

Proof

1. $a^2 - b^2 = (a + b)(a - b)$

Use the distributive property to expand the right - hand side.

$$= a^2 - ab + ba - b^2.$$

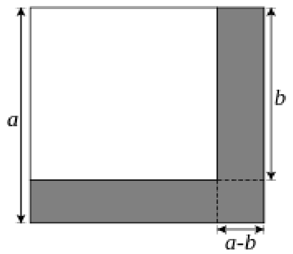
Apply the commutative Law to ba so that $ba = ab$ $(b \bullet a = a \bullet b)$ or $ba - ab = 0$

$$= a^2 - b^2$$

Thus *LHS* $= RHS$

2. The difference of two squares can also be shown geometrically.

In the diagram given below, the shaded part represents the difference between the bigger square (a^2) and the smaller square (b^2) which is equal to $a^2 - b^2$.



The area of the shaded part can be worked out by adding the following:

Rectangle	1	$= a(a - b)$
Rectangle	2	$= b(a - b)$
		$= a(a - b) + b(a - b)$

This can be factorized to $(a + b)(a - b)$

Thus $a^2 - b^2 = (a + b)(a - b)$

Example 2.4

Factorize: $x^2 - 16$

$$\begin{array}{ccc} x & & 4x \\ x & & x \end{array} \qquad \begin{array}{c} -4x \\ \hline\hline 0x \end{array}$$

$= (x-4)(x+4)$

<u>Note:</u> $0x$ means that there is no middle term

Example 2.5

Factorize: $4x^2 - 25$

$$\begin{array}{ccc} 2x & & -5 \\ 2x & & 5 \end{array} \qquad \begin{array}{c} -10x \\ +10x \\ \hline\hline 0x \end{array}$$

$= (2x-5)(2x+5)$

3b Perfect Squares: $a^2 + 2ab + b^2 = (a+b)^2$ **or** $a^2 - 2ab + b^2 = (a-b)^2$

Example 2.6

Factorize: $x^2 + 4x + 4$

$$\begin{array}{ccc} x & & 2 \\ x & & 2 \end{array} \quad + \quad \begin{array}{c} 2x \\ 2x \\ \hline\hline 4x \end{array}$$

$= (x+2)(x+2)$

$= (x+2)^2$

Example 2.7

Factorize: $x^2 - 4x + 4$

$$\begin{array}{ccc} x & & -2 \\ x & & -2 \end{array} \quad + \quad \begin{array}{c} -2x \\ -2x \\ \hline\hline -4x \end{array}$$

$= (x-2)(x-2)$

$= (x-2)^2$

Exercise 2.1

Factorize the following expressions

1. a) $2x+2$ b) $x^2 - 3x$ c) $-3xz + 6yz$

   d) $9x - 6x^2$ e) $4x - 8y + 6$ f) $4y^2 - 2y^3$

   g) $a^2 + a + 2a + 2$ h) $b^2 - 4b + 2b - 8$ i) $y^2 - 3y - 4y + 12$

2. a) $x^2 + 12x + 36$ b) $x^2 - 22x + 121$ c) $x^2 + 16x + 64$

   d) $4x^2 + 12x + 9$ e) $9x^2 - 12x + 4$ f) $\frac{x^2}{4} - 2x + 4$

3. a) $x^2 - 36$ b) $x^2 - 9^2$ c) $9x^2 - 81$

   d) $\frac{1}{4}x^2 - 9$ e) $\frac{x^2}{36} - \frac{y^2}{49}$ f) $4a^2 - \frac{y^2}{16}$

4. Use the pattern $a^2 - b^2 = (a+b)(a-b)$ to evaluate $32^2 - 31^2$

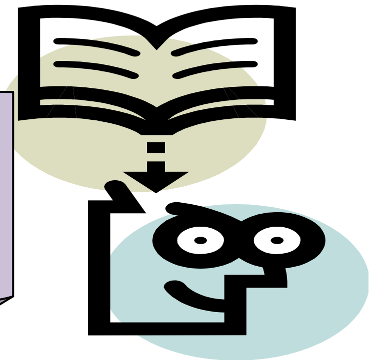
5. Marika bought a square plot of land whose length is $(4y+1)$ cm and then made a footpath on which he could walk around while tending to his vegetable garden. If the vegetable plot which is also square in shape but inside the square plot of land has length $(3y)$ cm, calculate the area of the footpath.

Square Roots

LEARNING OUTCOME

Students should be able:

- Introduce the formal treatment of square roots



WHAT CAN I MULTIPLY BY ITSELF TO GIVE THIS????

SYMBOL: $\sqrt{}$ - makes mathematics looks important and is called the **radical.**

The **square root** of a number is a value that can be multiplied by itself to give the original number.

Example 2.8

$\sqrt{4} = \pm\sqrt{4}$

$= \pm 2$

i.e. $2 \times 2 = 4$
$-2 \times -2 = 4$

Example 2. 9

$\sqrt{25} = \pm\sqrt{25}$

$= \pm 5$

i.e. $5 \times 5 = 25$
$-5 \times -5 = 25$

Square Root Generalizations:

1. $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$

   e.g. $\sqrt{12} = \sqrt{3} \times \sqrt{4}$

2. $\sqrt{a^2} = \sqrt{a \times a}$ e.g. $\sqrt{2^2} = \sqrt{2 \times 2}$

   $= \sqrt{a} \times \sqrt{a}$ $\quad \sqrt{2} \times \sqrt{2}$
   $= a$ $\quad = 2$

3. A positive number has 2 square roots i.e absolute values are equal but signs are different. e.g square root of 4 are $\sqrt{4}$ and $-\sqrt{4}$ or $\pm\sqrt{4}$

4. $\dfrac{\sqrt{a}}{\sqrt{b}} = \sqrt{\dfrac{a}{b}}$ e.g $\dfrac{\sqrt{25}}{\sqrt{16}} = \sqrt{\dfrac{25}{16}} = \dfrac{5}{4}$

5. $\sqrt{0} = 0$

6. $\sqrt{a+b} \neq \sqrt{a}+\sqrt{b}$ e.g $\sqrt{4+9} \neq \sqrt{4}+\sqrt{9}$

$\sqrt{13} \neq 2+3$

Exercise 2.2

Calculate the following:

a. $\sqrt{5}\times\sqrt{10}$

b. $\sqrt{9}\times\sqrt{16}$

c. $\sqrt{6}\times\sqrt{42}$

d. $2\sqrt{3}\times\sqrt{3}$

e. $3\sqrt{5}\times 2\sqrt{5}$

f. $\sqrt{x^2}\times\sqrt{x^2}$

g. $\sqrt{(x+1)^2}$

h. $\sqrt{(2x-1)^2}+\sqrt{x^2}$

i. $\dfrac{\sqrt{4x^2}}{\sqrt{16}}$

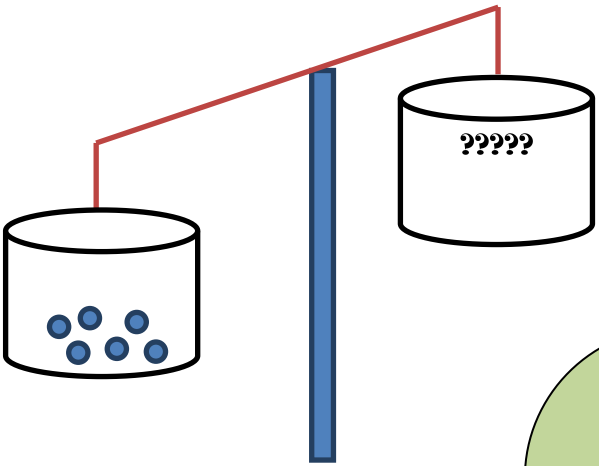
j. $\dfrac{\sqrt{9}}{\sqrt{4}}(\sqrt{(x-1)^2})$

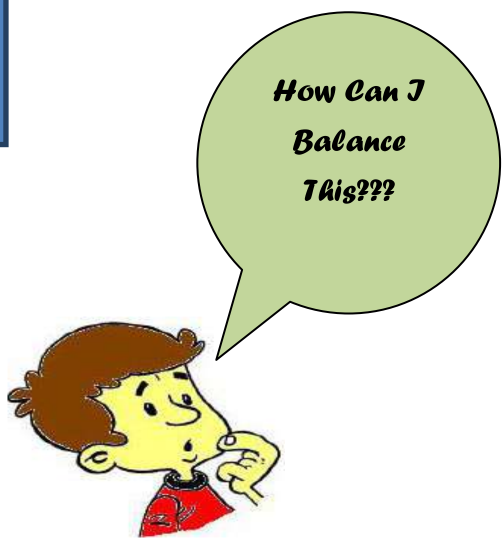
2.2 Solving Equations & Inequations

LEARNING OUTCOMES

Students should be able to:

- Solve linear equations involving variables on both sides of the equation.
- Solve linear inequations involving variables on both sides of the inequality sign.
- Solve equations by applying





What is an equation?

A mathematical statement that says two things are equal.

2.2.1 Linear Equations Having Variables on Both Sides

Example: 2.10

Solve for the missing variable in the given equation

a. $2x+3=x-2$

Step: 1 Collect the like terms

$2x-x=-2-3$

When collecting the like terms; to move one term from the right of the equal sign to the left or vice-versa, the sign changes.

Step: 2 Simplify both sides

$x=-5$

b. $x-2=4x+4$

$\therefore -2-4=4x-x$

$\therefore -6=3x$

$\therefore \frac{-6}{3}=\frac{3x}{3}$

$\therefore -2=x$

- Remove brackets
- Collect like terms
- Simplify like terms
- Find the unknown

c. $\therefore \frac{2x}{3}=x+4+2$

$\therefore 2x=3(x+6)$

$\therefore 2x-3x=18$

$\therefore -x=18$

$\therefore \frac{-x}{-1}=\frac{18}{-1}$

$\therefore x=-18$

Variable should be alone on one side by removing other terms to the other side using opposite operations

Exercise 2.3

Solve for the missing variable in the following equations.

a. $6y=4y+12$ b. $5b+7=3b+15$ c. $4a-10=2a+14$

d. $15-5n=6-2n$ e. $12+5x=x-2$ f. $11w-9=7w-30$

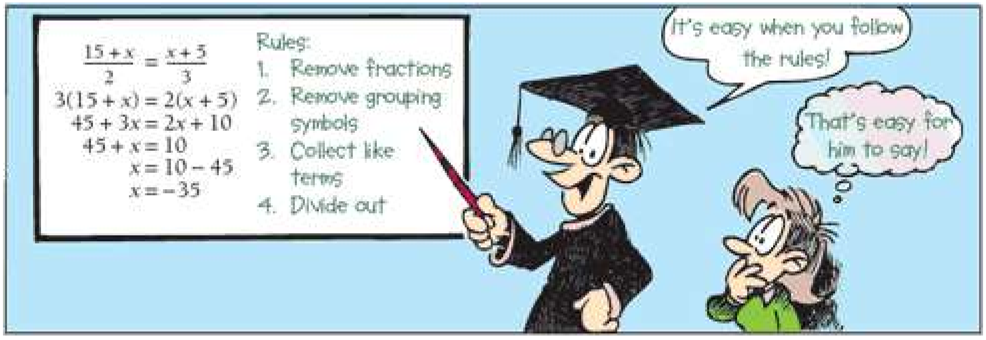
g. $2(3x-1)=5(2x-6)$ h. $h-7=2(h-4)$ i. $2(2q-5)=q+11$

j. $3x-2=2(x+4)-1$ k. $z+3(z+2)=14+2z$ l. $2(x-5)+3(x+3)=24$

Linear equations in real life situations

Equations may look scary, but you use and solve linear equations every day of your life, whether you know it or not.

One of the realities of life is how so much of the world runs by mathematical rules. As one of the tools of mathematics, linear systems have multiple uses in the real world.



Source: https://www.google.com/search?q=google&ie=utf-8&oe=utf-8#q=images+of+cartton+linear+equations

Life is full of situations when the output of a system doubles if the input doubles, and the output cuts in half if the input does the same. That's what a linear system is, and any linear system can be described with a linear equation.

In the Kitchen

Source: https://www.google.com/search?q=google&ie=utf-8&oe=utf-8#q=images+of+kitchens

If you've ever doubled a favorite recipe, you've applied a linear equation. If one cake equals 1/2 cup of butter, 2 cups of flour, 3/4 tsp. of baking powder, three eggs and 1 cup of sugar and milk, then two cakes equal 1 cup of butter, 4 cups of flour, 1 1/2 tsp. of baking powder, six eggs and 2 cups of sugar and milk. To get twice the output, you put in twice the input. You might not have known you were using a linear equation, but that's exactly what you did.

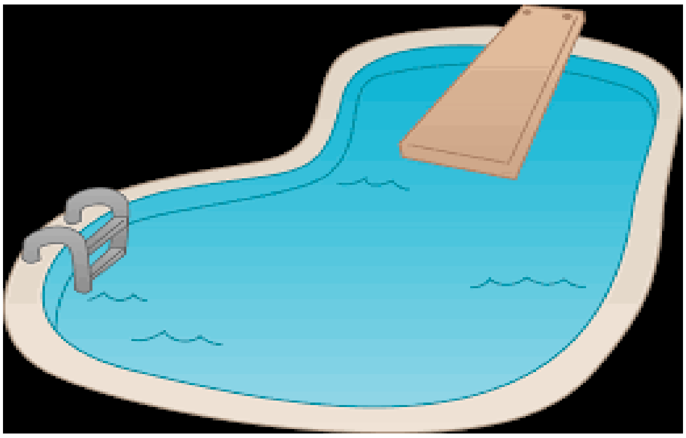
Melting Snow

Source: https://www.google.com/search?q=google&ie=utf-8&oe=utf-8#q=image+of+melting+snow+

*Suppose a water district wants to know how much snowmelt runoff it can expect this year. The melt comes from a big valley, and every year the district measures the snowpack and the water supply. It gets 60 acre-feet from every 6 inches of snow pack. This year surveyors measure 6 feet and 4 inches of snow. The district put that in the linear expression (60 acre-feet/6 inches) * 76 inches. Water officials can expect 760 acre-feet of snowmelt from the water.*

Question

It's springtime and Mrs Bula wants to fill her swimming pool. She sees that it takes 25 minutes to raise the pool level by 4cm. She needs to fill the pool to a depth of 1metre; she has 44 more cm to go.



Source: https://www.google.com/search?q=google&ie=utf-8&oe=utf-8#q=pictures+of+cartoon+swimming+pools

She figures out her linear equation: 44 cm (25 minutes/4 cm is 275 minutes, so she knows she has four hours and 35 minutes more to wait.*

Question
Brian has noticed that it's springtime. The grass has been growing. It grew 2 cm in two weeks. He doesn't like the grass to be taller than 2 1/2 cm, but he doesn't like to cut it shorter than 1 3/4 cm. How often does he need to cut the lawn?

Source: http://landscaping.about.com/od/lawns/a/spring_lawns.htm

*He needs to put that calculation in his linear expression, where (14 days/2 cm) * 3/4 cm tells him he needs to cut his lawn every 5 1/4 days. He just ignores the 1/4 and figures he'll cut the lawn every five days.*

Question
A 45 feet of wood to use for making a bookcase. If the height and width are to be 10 feet and 5 feet, respectively, how many shelves can be made between the top and bottom of the frame?

Source: https://www.google.com/search?q=google&ie=utf-8&oe=utf-8#q=images+of+bookcases

To solve this equation, we can use a linear relationship:

$Nv+Mh=45$

where v and h respectively represent the length in feet of vertical and horizontal sections of wood. N and M represent the number of vertical and horizontal pieces, respectively. Knowing that there will be only two vertical pieces, this formula can be simplified to:

$2 \cdot 10+M \cdot 5=45$

Question

Consider a shirt that costs $24 when on a 40% discount. If the original price is x, find x

Answer

$x - 0.4 \cdot x = 24$

Solving for x, we find that the original price was $40.

Using similar models we can solve equations pertaining to distance, speed, and time (Distance = Speed * Time); density (Density = Mass/ Volume); and any other relationship in which all variables are first order.

Source: Boundless. "Linear Equations and Their Applications." *Boundless Algebra*. Boundless, 03 Jul. 2014. Retrieved 03 Feb. 2015 from https://www.boundless.com/algebra/textbooks/boundless-algebra-textbook/functions-equations-and-inequalities-3/linear-equations-and-functions-22/linear-equations-and-their-applications-121-5519/

Everywhere

It's not hard to see other similar situations. If you want to buy drinks for the big party and you've got $60 in your pocket, a linear equation tells you how much you can afford. Whether you need to bring in enough wood for the fire to burn overnight, calculate your paycheck, figure out how much paint you need to redo the upstairs bedrooms or buy enough gas to make it to and from your Aunt Sylvia's, linear equations provide the answers. Linear systems are, literally, everywhere

2.2.2 Linear Inequations Having Variables on Both Sides

What is an inequation?

It is an algebraic sentence which has the inequality signs ($<, >, \leq, \geq$) instead of the equal sign (=).

Very Important Note!!!

- *Inequations are solved in the same way as equations.*
- *Inequations usually have many solutions as compared to equations.*

  e.g. $\begin{array}{ll} x+5=7 & x+5>7 \\ \therefore x=2 & \therefore x>2 \end{array}$
- *When an inequation is divided by -1, the inequality sign is reversed to make the statement true.*

One solution

Infinite solution

i.e. any number greater than 2

Symbol	*Meaning*
$\leq$	*Less than or equal to*
$\geq$	*Greater than or equal to*
$<$	*Less than*
$>$	*Greater than*

Exercise 2.4

Solve the following inequations.

a. $5x+4>x+10$

b. $4-y\leq 7-2y$

c. $w+2<2w-9$

d. $15-q\geq 2q+27$

e. $3(h+7)\leq 2(h+5)$

f. $\dfrac{y}{3}>7-\dfrac{y}{2}$

Linear inequations in real life situations

Inequalities are very common in daily life. For example:

You can work a total of no more then 41 hours each week at your two jobs. House cleaning pays \$5 per hour and your sales job pays \$8 per hour. You need to earn at least \$254 each week to pay your bills. Write a system of inequalities that shows the various numbers of hours you can work at each job.

Source: https://www.google.com/search?q=google&ie=utf-8&oe=utf-8#q=images+of+house+cleaning

x = housecleaning
y = sales job
Hours: $x + y \leq 41$
Money: $5x + 8y \geq 254$

Fuel x costs $2 per gallon and fuel y costs $3 per gallon. You have at most $18 to spend on fuel. Write and graph a system of linear inequalities to represent this situation.

x = fuel x
y = fuel y

Price: $2x + 3y \leq 18$
Gallons of x: $x \geq 0$
Gallons of y: $y \geq 0$

A salad contains fish and chicken. There are at most 6 pounds of fish and chicken in the salad. Write and graph a system of inequalities to represent this situation.

Source: http://photobucket.com/images/salad?page=1

x = fish
y = chicken

Total Pounds: $x + y \leq 6$
Pounds of fish: $x \geq 0$
Pounds of chicken: $y \geq 0$

Mary babysits for \$4 per hour. She also works as a tutor for \$7 per hour. She is only allowed to work 13 hours per week. She wants to make at least \$65. Write and graph a system of inequalities to represent this situation.

Source: https://www.google.com/search?q=google&ie=utf-8&oe=utf-8#q=images+of+babysitter

x = babysitting
y = tutoring

Hours: $x + y \leq 13$
Money: $4x + 7y \geq 65$

Exercise 2.5

1. Seru has \$500 at his savings account at the beginning of summer. He wants to have at least \$200 at the end of the summer. He withdraws \$25 each week for food and clothes.

Source: https://www.google.com/search?q=google&ie=utf-8&oe=utf-8#q=images+of+bank

(a) Write an equation that represents Seru's situation
(b) How many weeks can Seru withdraws money from his account? Justify your answer.

2. Marshall Taxi charges a \$1.50 rate in addition to \$0.85 for every kilometer. Mr Maklu has no more than \$10 to spend on a ride.

   (a) Write an inequation that represents Mr Maklu's situation
   (b) How many kilometers can he travel without exceeding his limit? Show your working.

2.2.3 Solving Quadratic Equations

2.2.3.1 Using the Square Root Law

What would be the difference between the solutions in a linear equation to that of a quadratic equation?

Linear equations have one solution while quadratic equations have 2 solutions.

Example 2: 11

Find all solutions to the following equations.

a)
$$x^2 = 9$$
$$\therefore \sqrt{x^2} = \pm\sqrt{9}$$
$$\therefore x = \pm 3$$
$$i.e. \begin{pmatrix} x = 3 \\ x = -3 \end{pmatrix}$$

To remove x^2 we will have to $\sqrt{\ }$ on both sides since square root ($\sqrt{\ }$) is the opposite of square.

b)
$$x^2 - 5 = 11$$
$$\therefore x^2 = 11 + 5$$
$$\therefore x^2 = 16$$
$$\therefore \sqrt{x^2} = \pm\sqrt{16}$$
$$\therefore x = \pm 4$$
$$i.e. \begin{pmatrix} x = 4 \\ x = -4 \end{pmatrix}$$

The square root of a number will therefore have 2 values i.e. the positive value and the negative value.

2.2.3.2 Using the Null Factor Law

If multiplying any two numbers is zero, then one or both of the numbers are zero, i.e. if $ab = 0$ **,then** $a = 0$ **or** $b = 0$. This is the Null Factor Law which is often used to solve quadratic functions or other functions which could have more than 2 solutions.

Example 2. 12

Find all solutions to the following equations.

a. $(x+2)(x-3) = 0$

using the null factor l

$\therefore x+2=0$ and $x-3=0$

l

$\therefore x = 0-2$ and $x = 3$

l

$\therefore x = -2$ and $x = 3$

b. $(x+3)(x-1)(x+4) = 0$

$\therefore x+3=0$, $x-1=0$ and $x+4=0$

$\therefore x = 0-3$, $x = 0+1$ and $x = 0-4$

$\therefore x = -3$, $x = 1$ and $x = -4$

2. $(x-4)(x+5)(x\cdot$

4. $(x-4)^2 = 25$

5. $2x(x-2) = 0$

6. $(2x+1)(3x+6) = 0$

7. $(x-2)^2 + 2 = 18$

8. $x^2 - 64 = 0$

9. $x^2 - 10x = -25$

2.3 Formula Manipulation

LEARNING OUTCOME

Students should be able to:

- manipulate the original formula using the inverse operations

$A = \pi r^2$ what will r equal to if $A = 25cm^2$ and $\pi = 3.14$

Definition:1

Formula: An equation which tells how variables are related to one another.

Definition:2

Subject of formula: Single variable on the left hand side of the equation with everything else going on the right hand side

Definition: 3

Changing the subject of formula: Begins with the variable to become the new subject and applying inverse operations on the other variables.

Example 2. 13

Given the formula for the area of a circle is $A = \pi r^2$, make r the subject of formula.

$$A = \pi r^2$$

$$\therefore \frac{A}{\pi} = \frac{\pi r^2}{\pi}$$

Divide by π on both sides of the equation to remove π from the right hand side and move the variable to the left hand side using the same operation.

$$\therefore \frac{A}{\pi} = r^2$$

$$\therefore \sqrt{\frac{A}{\pi}} = \sqrt{r^2}$$

$\sqrt{\ }$ on both sides to remove square (^{2})from the right hand side and move the variable to the left hand side using the same operation

$$\therefore \sqrt{\frac{A}{\pi}} = r$$

Now r is the new subject of formula

Exercise 2.6

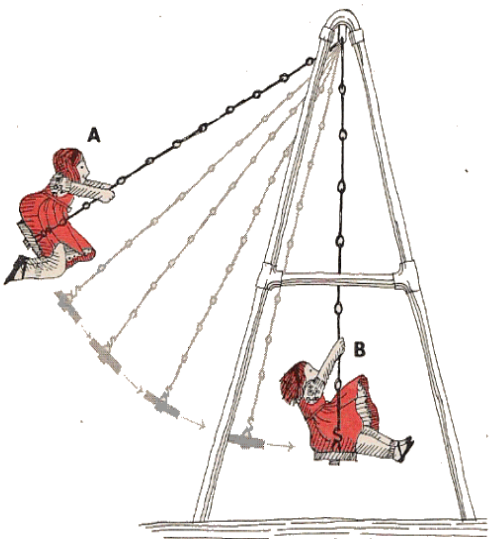
For each of the equations given, make the variables given in brackets the new subject of formula.

1. $v = u + at \ (u)$
2. $v = u + at \ (t)$
3. $y = mx + c \ (x)$
4. $2p - a = 3p + b \ (p)$
5. $P = (m + M)f \ \ (M)$
6. $V = \frac{1}{3}\pi r^2 h \ (r)$
7. $A = \frac{1}{2} b \times h \ (b)$
8. $y = 5x - \frac{z}{3} \ (x)$
9. $2s = (u + v)t \ \ (t)$

Formula manipulation is also applied in others subjects such as Physics, Chemistry, Economics

Physics

Kinetic and Potential Energy



$$E = mgh + \tfrac{1}{2}mv^2$$

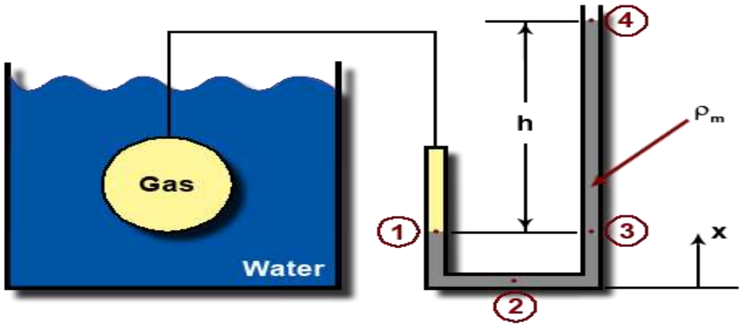
Source: https://www.google.com/search?q=google&ie=utf-8&oe=utf-8#q=kinetic+and+potential+energy

Make *m* the subject of the formula
Where m = mass of the body,

h is the height attained due to the body's displacement and
g is the acceleration due to gravity which is constant on earth
v is the velocity of the body

Chemistry

Ideal gas law



$$\boxed{PV = nRT}$$

Source: https://www.google.com/search?q=google&ie=utf-8&oe=utf-8#q=images+of+ideal+gas

Make n the subject of the formula

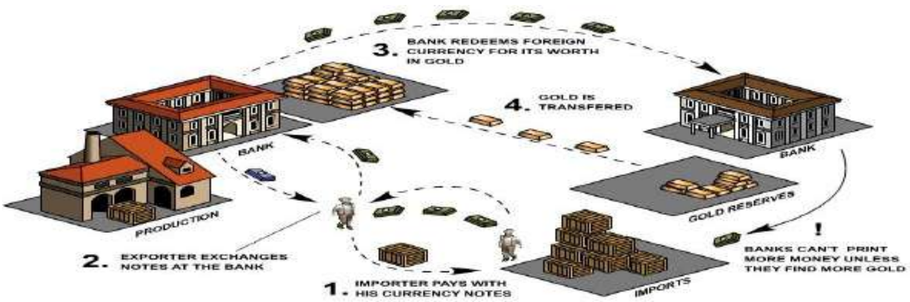
- n = number of moles
- R = universal gas constant = 8.3145 J/mol K
- P = Pressure
- V = Volume
- T = Temperature

Economics

Velocity Of Circulation

Make Q the subject of the formula

$$\boxed{M.V = P . Q}$$



Source: https://www.google.com/search?q=google&ie=utf-8&oe=utf-8#q=images+of+velocity+of+circulation+of+money

Where *M* is Money Supply, *V* is Velocity of Circulation (the average number of times money changes hand), *P* is Average Price Level (a measure of inflation), and *Q* is Quantity of Goods and Services bought or sold in the economy in a year (also known as the Gross National Product [GNP]).

GLOSSARY		
11	Absolute values	The magnitude of a quantity irrespective of sign The distance of a quantity from zero
18	Acceleration	Rate at which the speed is changing
1	Algebra	A branch of mathematics in which symbols represent numbers of a specified set and are used to represent quantities and to express general relationships that hold for all members of the given set
4	Common factor	A number or quantity that divides two or more numbers exactly
7	Commutative law	Swap numbers and still maintain the same answer
6	Distributive law	Multiplying a number by a group of numbers added together is the same as multiplying each separately
26	Economy	Consists of production distribution or trade and consumption of limited goods and services by different agents
15	Energy	Property of objects, transferable among themthrough fundamental interactions, which can be converted into different forms but not created or destroyed
9	Evaluate	To find the numerical value
2	Factorise	The resolution of an expression into factors such that when these factors are multiplied together, they give the original expression
12	Formula	Relationship between two or more variables
13	Formula manipulation	Involves rearranging variables to make an algebraic expression better suit the requirement. During this arrangement, the value of the expression does not change
20	Ideal gas	A gas whose pressure, volume and pressure are related by the ideal gas law
25	Inflation	The rate at which the general level of prices for goods and services is rising and subsequently purchasing power is decreasing
16	Kinetic energy	Energy of motion
21	Moles	A unit of measurement used in chemistry to express the amount of a chemical substance
8	Perfect square	A number that can be expressed as the product of two equal integers
5	Polynomial	An expression consisting of variables and coefficients that involves the operations of addition, subtraction and multiplication and non - negative integer exponents
17	Potential energy	Energy that an object has due to its position in a force field
22	Pressure	A measure of the force applied over a unit area
10	Radical	An expression that has a square root or cube root
3	Simplify	To rewrite an expression as simple as possible
14	Subject of a formula	The variable on its own, usually on the left hand side of a formula
23	Universal gas	A physical constant used in many thermochemical equations and relationships
19	Velocity	Rate of travel of an object along with its direction
24	Velocity of circulation	The average number of times a unit of money changes hands in an economy during a given period.