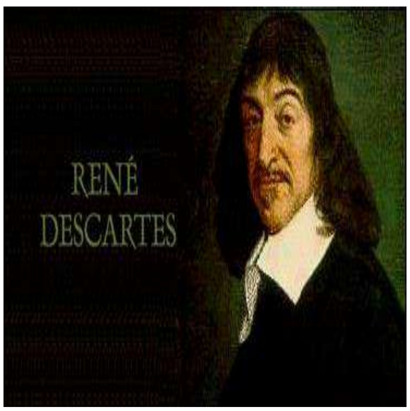
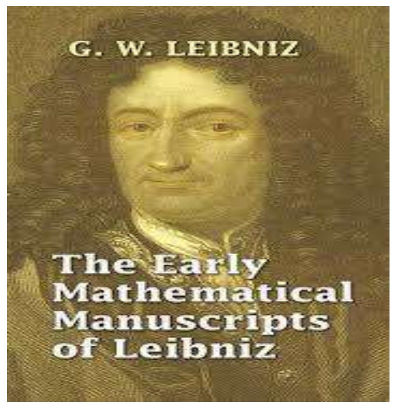
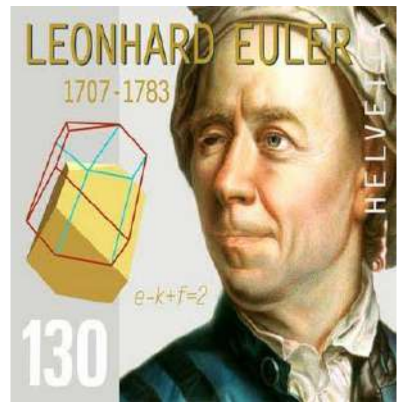
STRAND 1 → FUNCTIONS

HISTORY OF FUNCTIONS

The idea of a function was developed in the seventeenth century. During this time, Rene Descartes (1596-1650), in his book Geometry (1637), used the concept to describe many mathematical relationships. The term "function" was introduced by Gottfried Wilhelm Leibniz (1646-1716) almost fifty years after the publication of Geometry. The idea of a function was further formalized by Leonhard Euler (pronounced "oiler" 1707-1783) who introduced the notation for a function, y = f(x).







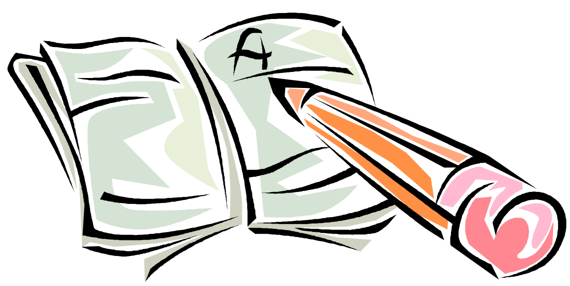
Source: http://science.irank.org/pages/2881/Function-History-functions.html

1.1 Linear and Quadratic Functions

LEARNING OUTCOMES

Students should be able to:

- Describing linear and quadratic functions
- Identifying and describing domain and range of functions
- Calculating functions using function notations
- Generating domain and range of functions as ordered



A RELATION is a set of ordered pairs

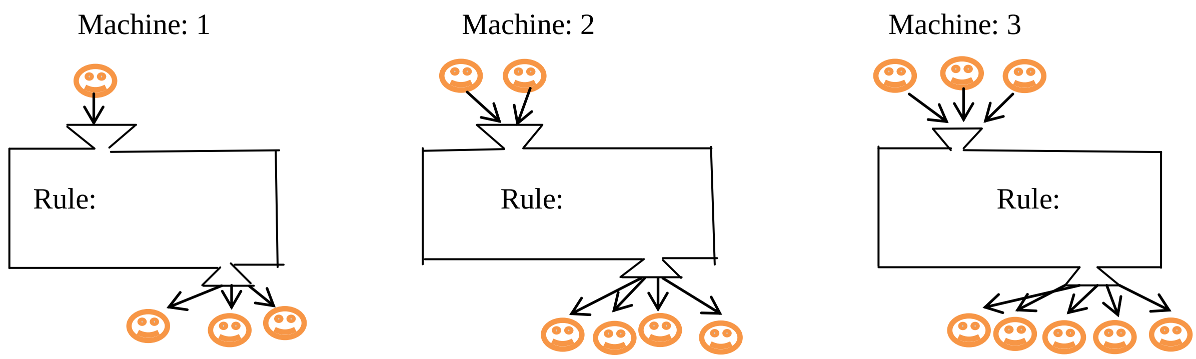
A FUNCTION is a set of information or data that has a clear output for each input. a function is a set of ordered pairs in which each x-element has only one y-element associated with it.

Types Of Function

I. Linear Function

ACTIVITY

Three number machines are given. For each number machine, state the rule being followed by the machines.



INPUT	OUTPUT
1	3
2	4
3	5

If x is to represent the input and y is to represent the output, the common rule derived from the three number machines in terms of x and y would be: $y = x + 2$

LINEAR FUNCTION: A function whereby the degree or index on the input variable is 1 e.g. $y = x + 2$ has the degree 1 i.e. the index on x the input variable is 1

Example 1.1

For the linear function, $y = x + 2$ where $x \in \{$ -2, -1, 0, 1, 2$\}$ list the function as:

(i) a set of ordered pairs

(ii) an arrow diagram

(iii) a Cartesian graph

(iv) a table of values

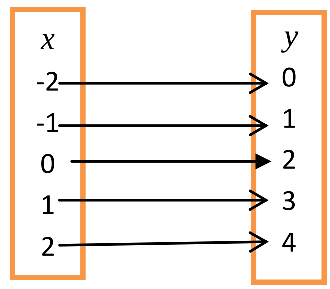
Answer

Linear function	x value	Substitute x	y value
y = x + 2	x = -2	y = (-2) + 2	y = 0
	x = -1	y = (-1) + 2	y = 1
	x = 0	y = (0) + 2	y = 2
	x = 1	y = (1) + 2	y = 3
	x = 2	y = (2) + 2	y = 4

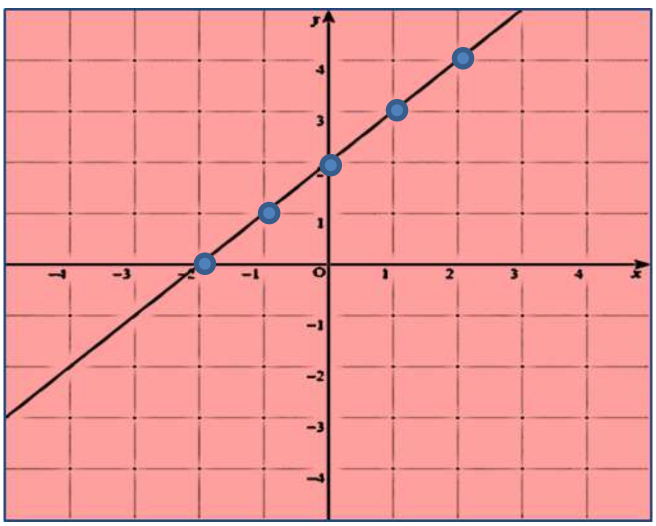
(i) Set of ordered pairs

R={(-2,0)(-1,1)(0,2)(1,3)(2,4)}

(ii) Arrow diagram



(iii) Cartesian graph



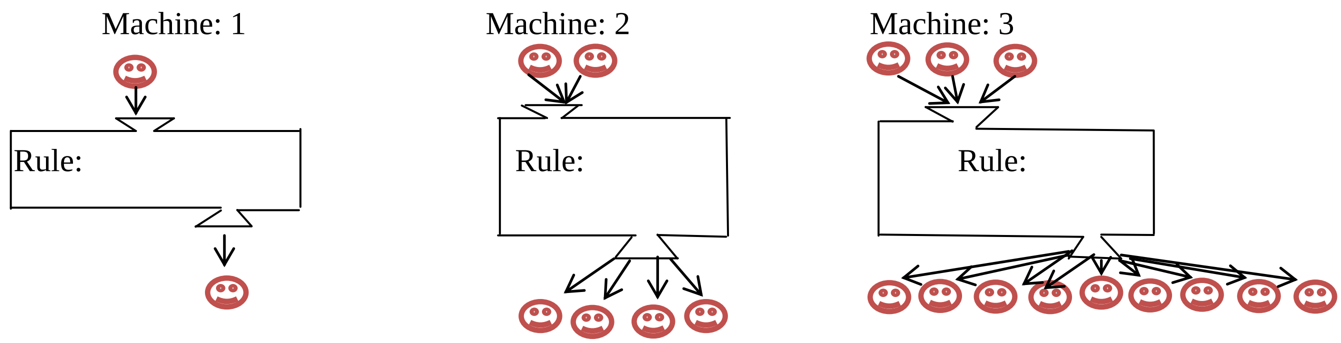
(iv) Table of values

x	-2	-1	0	1	2
y	0	1	2	3	4

II. Quadratic Function

ACTIVITY

Three number machines are given. For each number machine, state the rule being followed by the machines.



INPUT	OUTPUT
1	1
2	4
3	9

If x is to represent the input and y is to represent the output, the common rule derived from the three number machines in terms of x and y would be: $y = x^2$

QUADRATIC FUNCTION: A function whereby the degree, power or index on the input variable x is equal to 2 e.g. $y = x^2$ has the degree of 2 i.e. the index on x the input variable is 2

Example 1.2

For the quadratic function, $y = x^2$ where $x \in \{-2, -1, 0, 1, 2\}$ list the function as:

(i) a set of ordered pairs

(ii) an arrow diagram

(iii) a Cartesian graph

(iv) a table of values

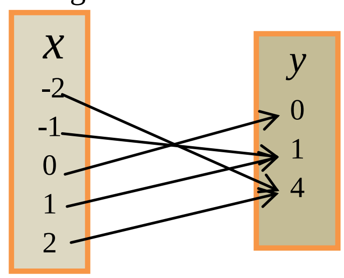
Answer

Linear function	x value	Substitute x	y value
$y = x^2$	$x = -2$	$y = (-2)^2$	$y = 4$
	$x = -1$	$y = (-1)^2$	$y = 1$
	$x = 0$	$y = (0)^2$	$y = 0$
	$x = 1$	$y = (1)^2$	$y = 1$
	$x = 2$	$y = (2)^2$	$y = 4$

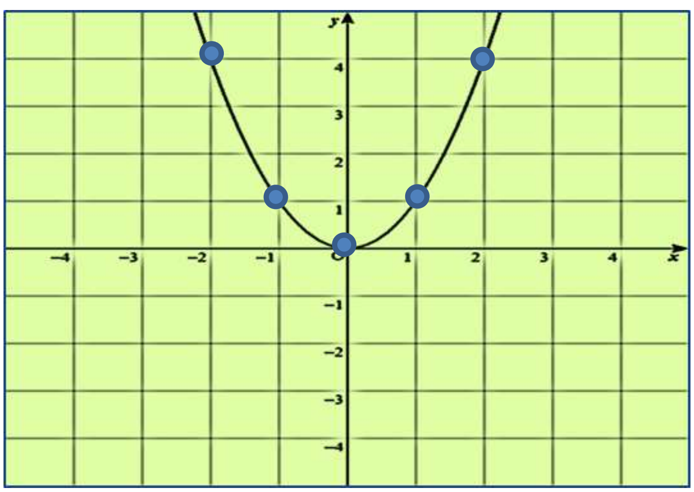
(i) Set of ordered pairs

R={(-2,4)(-1,1)(0,0)(1,1)(2,4)}

(ii) Arrow diagram



(iii) Cartesian Graph



(iv) Table of values

x	-2	-1	0	1	2
y	4	1	0	1	4

Example 1.3

For the linear function y = 3x – 1 where $x \in \{-2, -1, 0, 1, 2\}$, list the fuction as:

(i) a set of ordered pairs

(ii) an arrow diagram

(iii) a Cartesian graph

(iv) a table of values

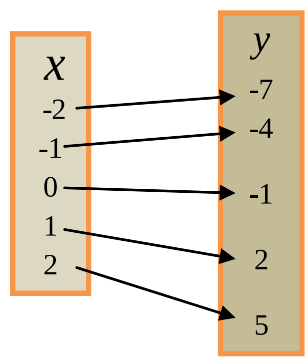
Answer

Linear function	x value	Substitute x	y value
y = 3x- 1	x = -2	y = 3(-2) - 1	y = -7
	x = -1	y = 3(-1) - 1	y = -4
	x = 0	y = 3(0) - 1	y = -1
	x = 1	y = 3(1) - 1	y = 2
	x = 2	y = 3(2) - 1	y = 5

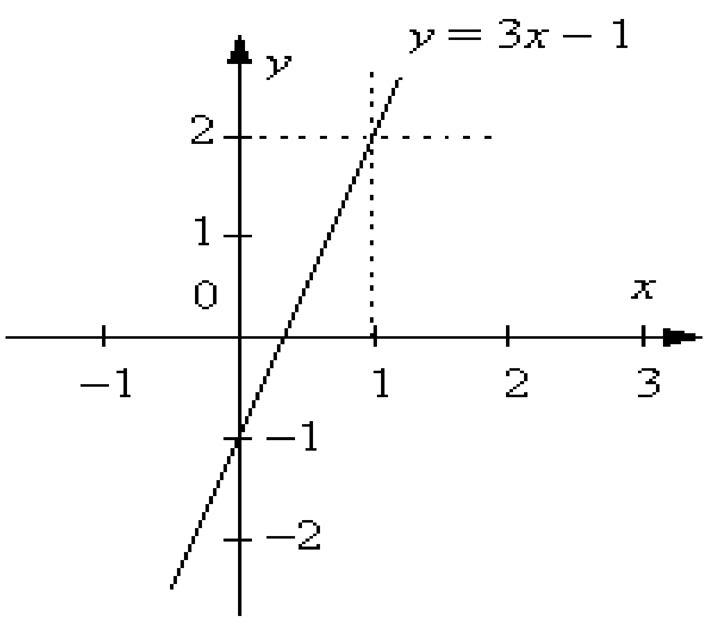
(i) Set of ordered pair

R={(-2,-7)(-1,-4)(0,-1)(1,2)(2,5)}

(ii) Arrow diagram



(iii) Cartesian graph



(iv) Table of values

x	-2	-1	0	1	2
y	-7	-4	-1	2	5

Exercise 1.1

1. For the function $y = x - 1$ where $x \in$ {-2, -1, 1, 0, 1, 2} list the function as:

   i. A set of ordered pairs

   ii. An arrow diagram

   iii. A Cartesian graph

2. For the function $y = 2x^2$ where $x \in R$ list the function as:

   i. A set of ordered pairs

   ii. An arrow diagram

   iii. A Cartesian graph

3. For the function $y = -2x + 1$ where $x \in$ {-5,-3,-1, 0, 1, 3, 5} list the function as:

   i. A set of ordered pairs

   ii. An arrow diagram

   iii. A Cartesian graph

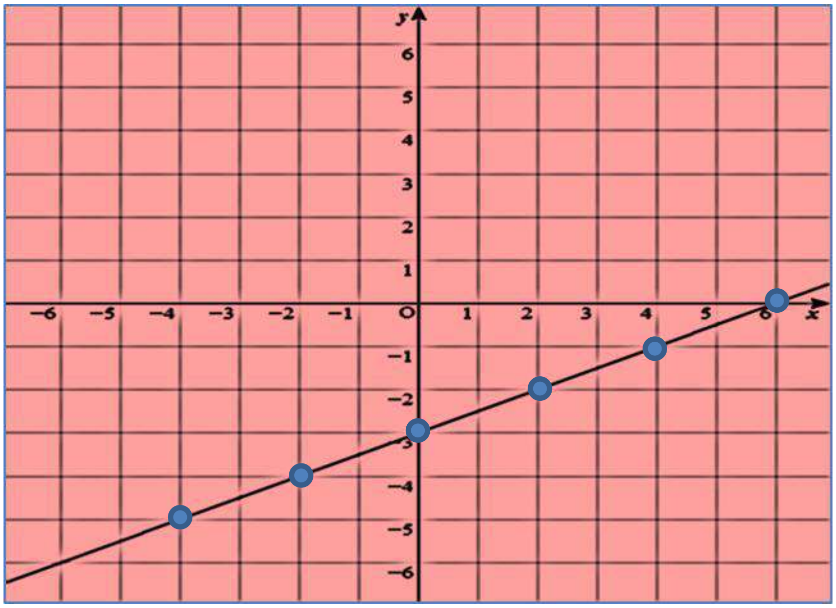
4. For the function $y = -2x^2$ where $x \in$ {-4, -2, 0, 2, 4} list the function as:

   i. A set of ordered pairs

   ii. An arrow diagram

   iii. A Cartesian graph

5. The diagram shows a function given as a Cartesian graph.



Using the points indicated on the line graph, show the function as:

i. An ordered pair

ii. An arrow diagram

iii. A rule

III. Function Notation

A way to indicate that an equation is a function.

$f(x)$ Or $g(x)$ etc. is read as f of x or g of x

$\Rightarrow$

Example 1.4

Given $f(x) = x + 2$ where $x \in$ {-2, -1, 0, 1, 2}

$f(-2) = -2 + 2 = 0$, $f(-1) = -1 + 2 = 1$,

$f(0) = 0 + 2 = 2$, $f(1) = 1 + 2 = 3$,

$f(2) = 2 + 2 = 4$

Exercise 1.2

1. If $f(x) = 3x - 4$ and $g(x) = x^2 - 2$, find:

   i. $f(2)$ ii. $f(-3)$ iii. $g(3)$ iv. $g(-4)$

2. Two functions are given as $h(x) = -\frac{1}{2}x + 3$ and $k(x) = -3x^2$. Find:

i. $k(3)$ ii. $h(4)$ iii. $k(-6)$ iv. $h(-8)$

3. $f(x) = x + 4$ and $g(x) = \dfrac{x-4}{3}$. Evaluate

i. $f(-2)$ ii. $f(\frac{2}{3})$ iii. $g(13)$ iv. $g(-14)$

v. For what value of x is $f(x) = -5$

vi. For what value of x is $g(x) = -3$

IV. Domain (D_x) And Range (R_y)

Domain (D_x): *The set of Input Values or set of* x *values that can be put in an equation.*

Range (R_y): *The set of output values or the set of possible* $y-$*values.*

Example 1.5

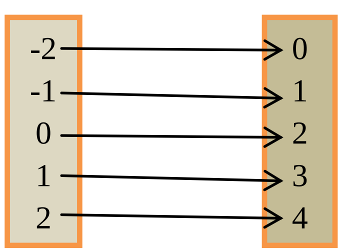
Give the domain and range for the following function.

(i) R= {(-2,0)(-1, 1)(0,2)(1,3)(2,4)}

Domain, $(D_x) =$ {-2, -1, 0, 1, 2}

Range, $(R_y) =$ {0, 1, 2, 3, 4}

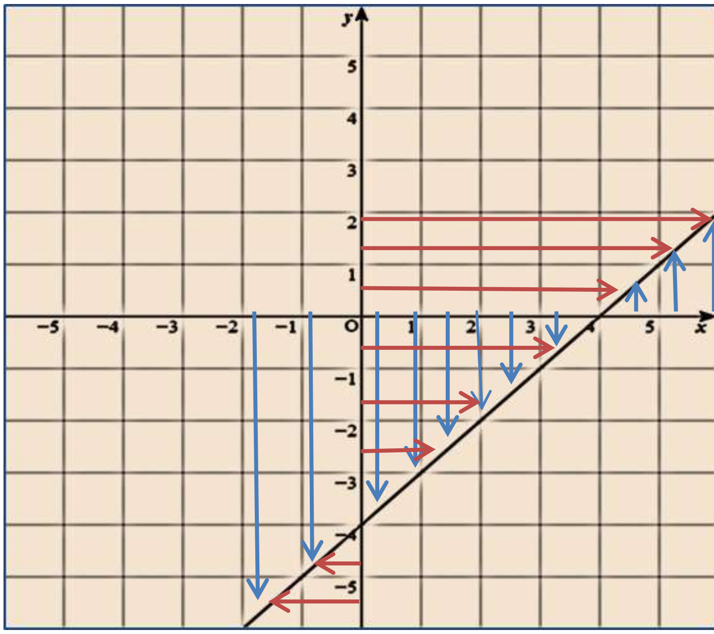
(ii)



Domain, $(D_x) =$ {-2, -1, 0, 1, 2}

Range, $(R_y) =$ {0, 1, 2, 3, 4}

(iii) $f(x) = x - 4$ where $x \in R$



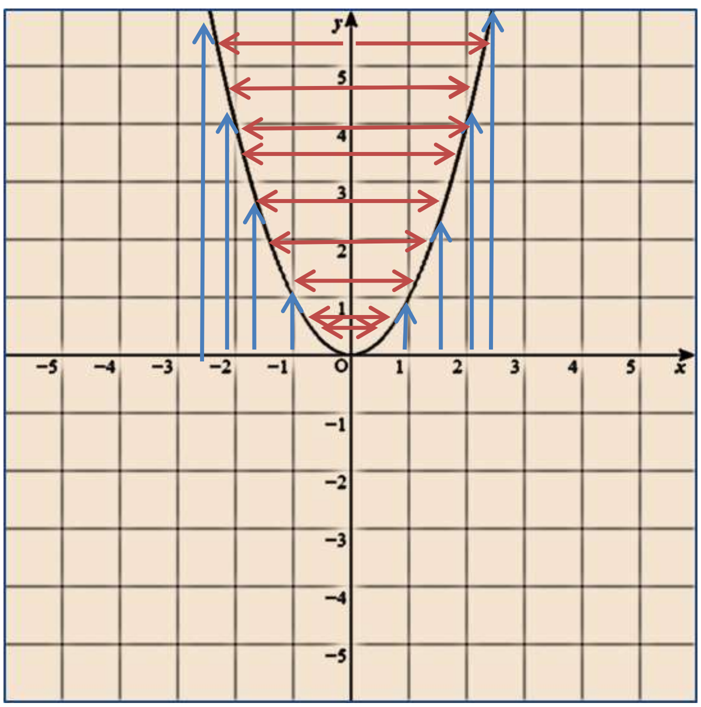
Domain $D_x = X \in R$

Why? All values of x on the $x-$axis would be mapped on the line as shown on the blue arrow ( )

Range $R_y = y \in R$

Why? All values of y on the $y-$axis would be mapped on the line as shown by the maroon arrow ( )

(iv) $y = x^2$ where $x \in R$



Domain $D_x = X \in R$

Why? All values of x on the $x-$axis would be mapped on the line as shown on the blue arrow ( )

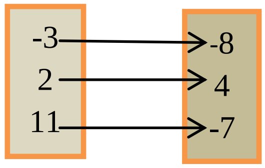
Range $y \geq 0$, $y \in R$

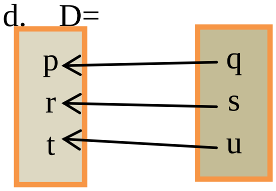
Why? Only values of y that is greater than and equal to zero on the y axis would be mapped on the line as shown on the maroon arrow ( )

Exercise 1.3

1. List the domain and range for each of the following functions

   a. A = {(1, 6), (2, 8), (3, 10), (4, 12)} b. B ={-4, 13), (-3, 6), (-2, 1), (-1, -2), (0, -3)}

c. D = 

d. D= 

2. For the functions given as a rule, list the domain and range.

   a. $y = -2x - 3,\ x \in \{0, 1, 2, 3, 4\}$

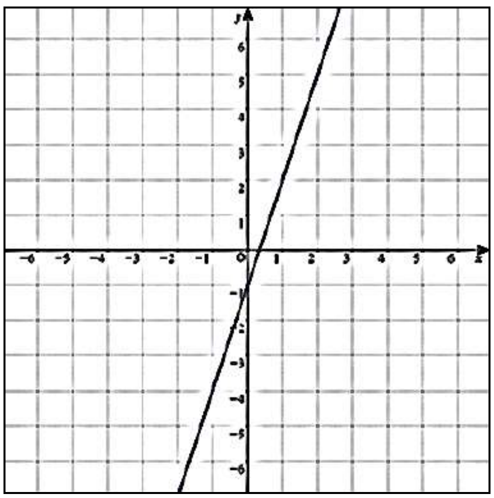
   b. $y = x^2 + 3,\ x \in \{-4, -3, -2, -1, 0\}$

   c. $f(x) = -3x^2,\ x \in \{-2, -1, 0, 1, 2\}$

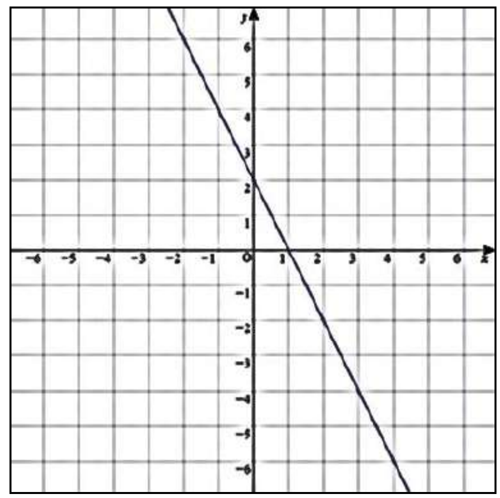
   d. $g(x) = \dfrac{2}{3}x - 3,\ x \in \{-9, -6, -3, 3, 6, 9\}$

3. List the domain and range for the functions given as a graph.

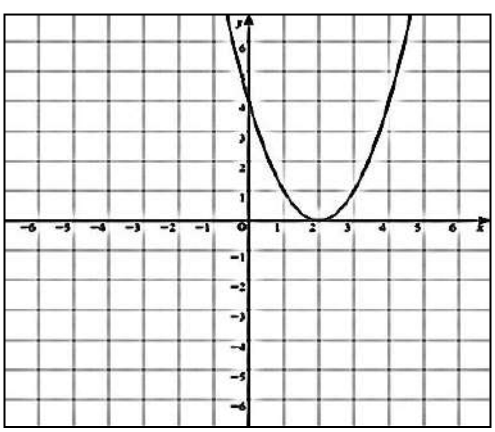
A.



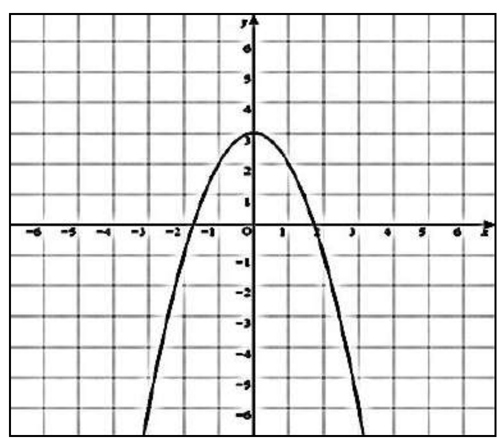
B.



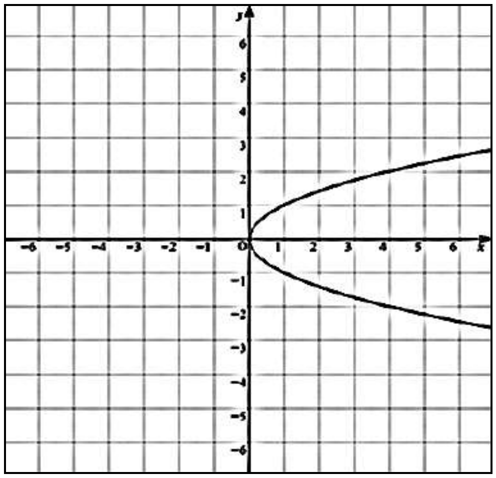
C.



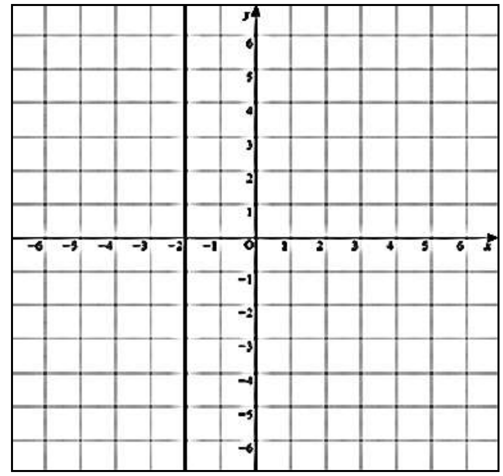
D.



E.



F.



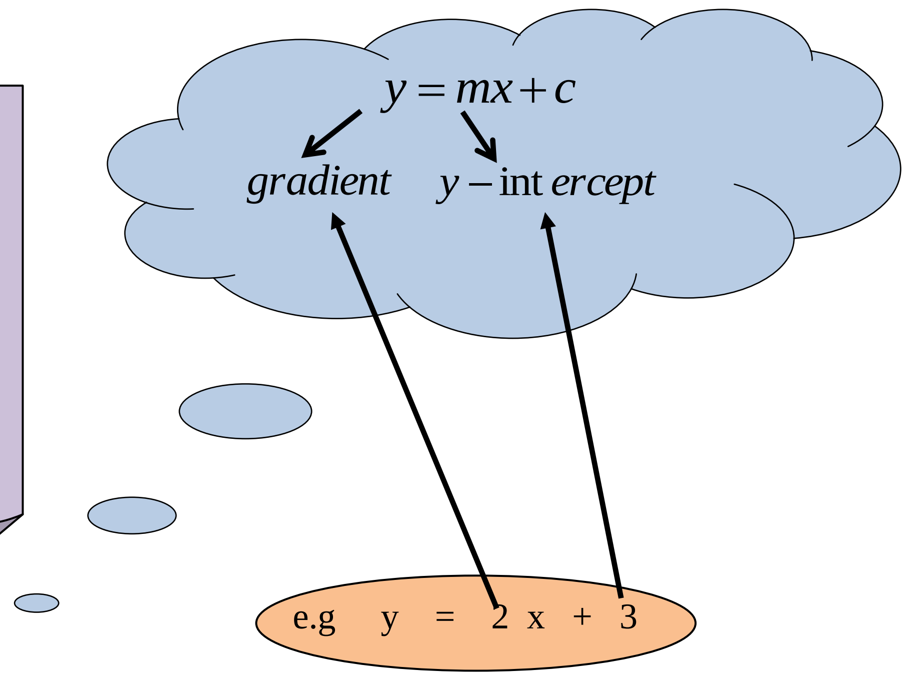
1.2 Graphing Equations and Inequations

LEARNING OUTCOMES

Students should be able to:

- Calculating intercepts and gradient of the linear equation in the form $y = mx + c$
- Draw graphs of linear equation
- Identifying intercepts from the graph of linear equation
- Determine and shade regions indicated by inequations.

Linear Equation: $y = mx + c$



I. Gradient

Definition: the slope of a line or how steep a line is.

Positive Gradient (m)

Negative Gradient (m)

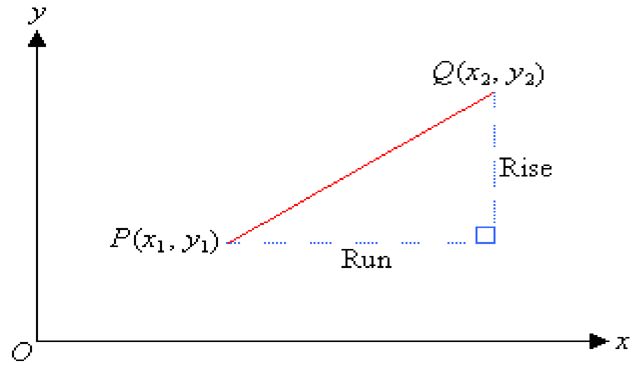
II. Calculating Gradient

Gradient (m) is equal to distance along the $y-axis$ divided by the distance along the $x-axis$.

The **gradient** of a straight line is the rate at which the line rises (or falls) vertically for every

unit across to the right. That is:

$$\text{Gradient} = \frac{\text{Rise}}{\text{Run}} = \frac{\text{Change in } y}{\text{Change in } x} = \frac{y_2 - y_1}{x_2 - x_1}$$



Note:

The gradient of a straight line is denoted by *m* where:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

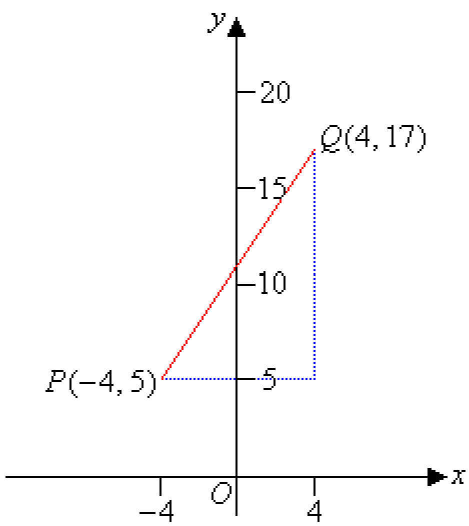
Example 1.6

Find the gradient of the straight line joining the points *P*(– 4, 5) and *Q*(4, 17).

Solution:

Let $(x_1, y_1) = (-4, 5)$ and $(x_2, y_2) = (4, 17)$.

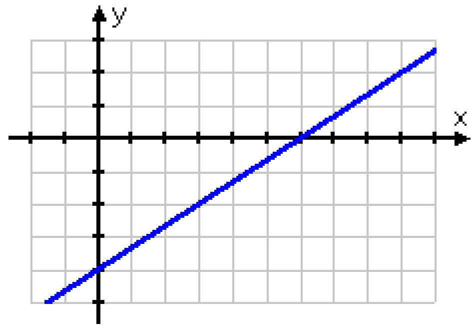
$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{17 - 5}{4 - (-4)} \\ &= \frac{12}{8} \\ &= 1.5 \end{aligned}$$



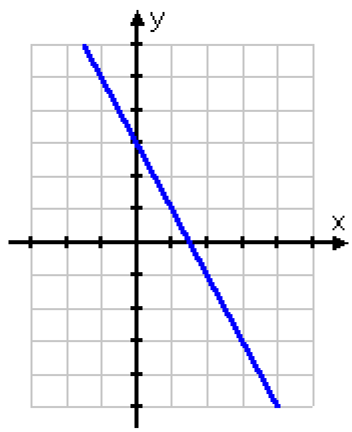
So, the gradient of the line *PQ* is 1.5.

Note:

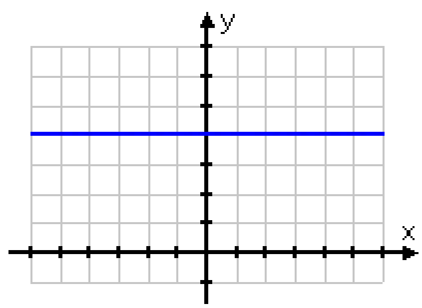
1. If the gradient of a line is **positive**, then the line **slopes upward** as the value of x increases



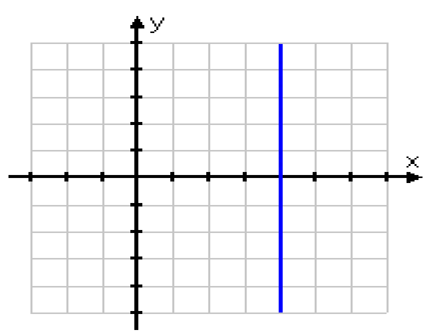
2. If the gradient of a line is **negative**, then the line **slopes downward** as the value of x increases



3. If the gradient of a line is **zero**, then the line is horizontal



4. If the gradient of a line is **undefined**, then the line is vertical



Example 1.7

Draw the graph for the linear equation: $y = 2x + 3$

Solution:

Method I

STEP: 1 Identify the gradient (m) and $y - \text{int}\,ercept$

$\therefore\ m = 2$ and $y - \text{int}\,ercept$ =3 or (0,3)

STEP: 2 Define the gradient (m) and its movement on the Cartesian plane

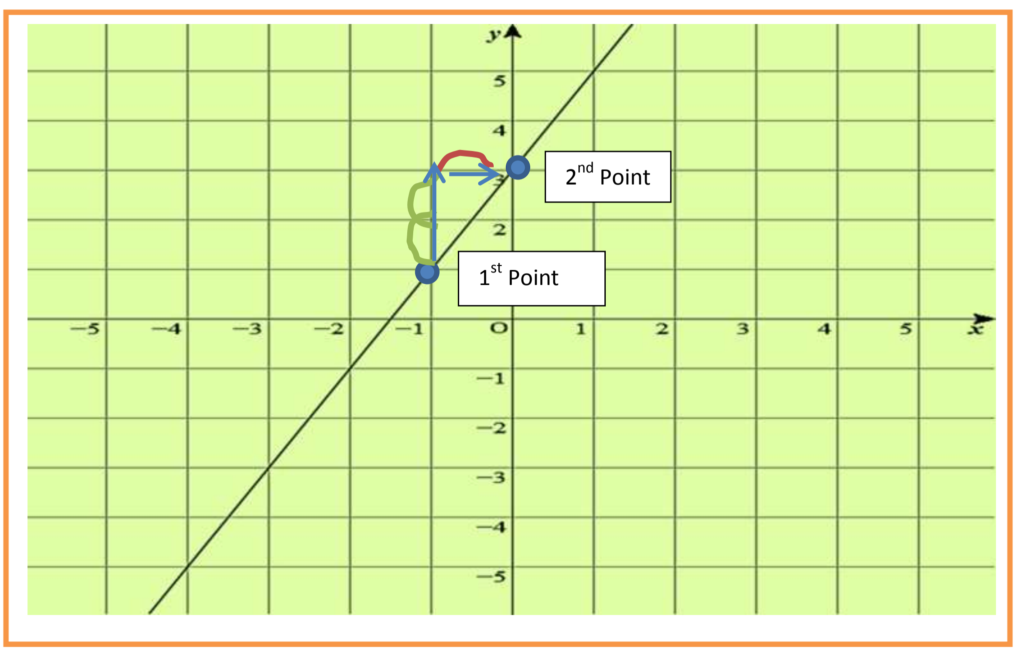
$\therefore\ m$ =2 as a fraction would read $\frac{2}{1}$ meaning 2 steps along the $y - axis$ (upward movement) and 1 movement along the $x - axis$ (movement to the right).

STEP: 3

Plot the $y - \text{int}\,ercept$ on the Cartesian plane to get the first point

STEP: 4

From the $y - \text{int}\,ercept$, using the gradient plot the second point, ie, 2 movement upwards and 1 movement to the right



Method II

Using table of values

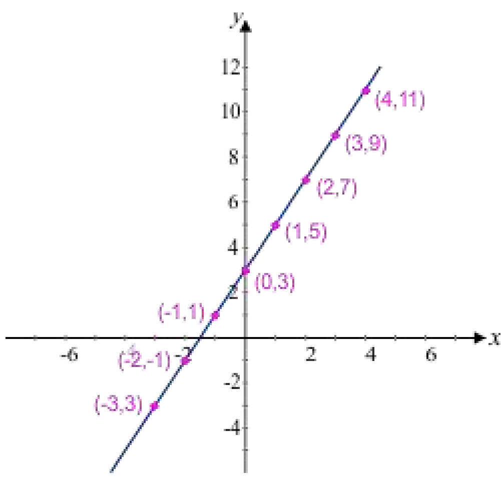
Step: 1 Choose some x values (negative, zero and positive values)

x	-2	-1	0	1	2

Step: 2 Substitute x values into the equation to get the y values

x	-2	-1	0	1	2
y	-1	1	3	5	7

Step: 3 Plot points on Cartesian plane join them with a line



Method III

Intercept method

Step: 1 Workout the x intercept by substituting y with 0, ie, y = 0

$y = 2x + 3$

$0 = 2x + 3$

$-3 = 2x$

$\frac{-3}{2} = x$ or $(\frac{-3}{2}, 0)$

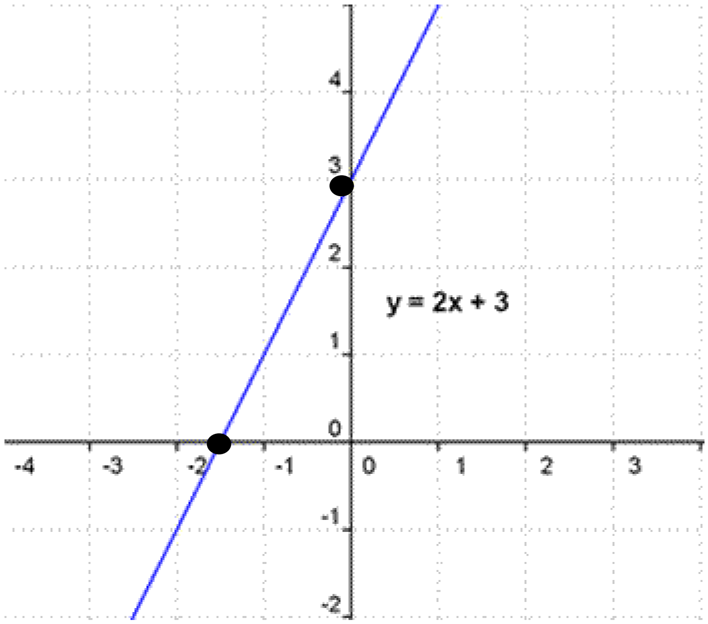
Step: 2 Workout the y intercept by substituting x with 0, ie, x = 0

y = 2x + 3

y = 2(0) + 3

y = 3 or (0, 3)

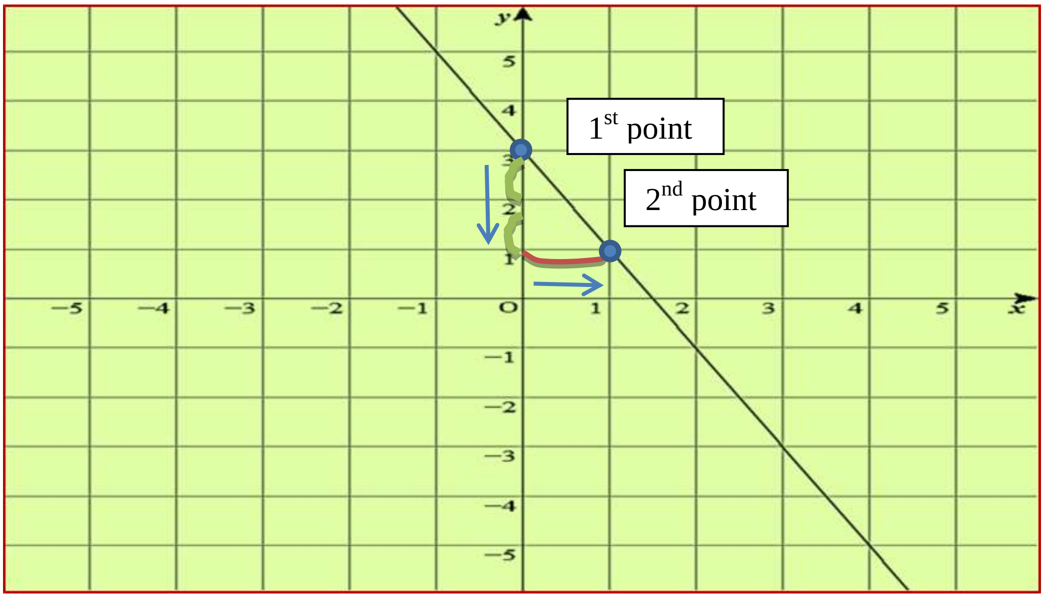
Step: 3 Plot the x and y intercepts on Cartesian plane and them with a straight line



Example 1.8

Draw the graph for the linear equation $y = -2x + 3$

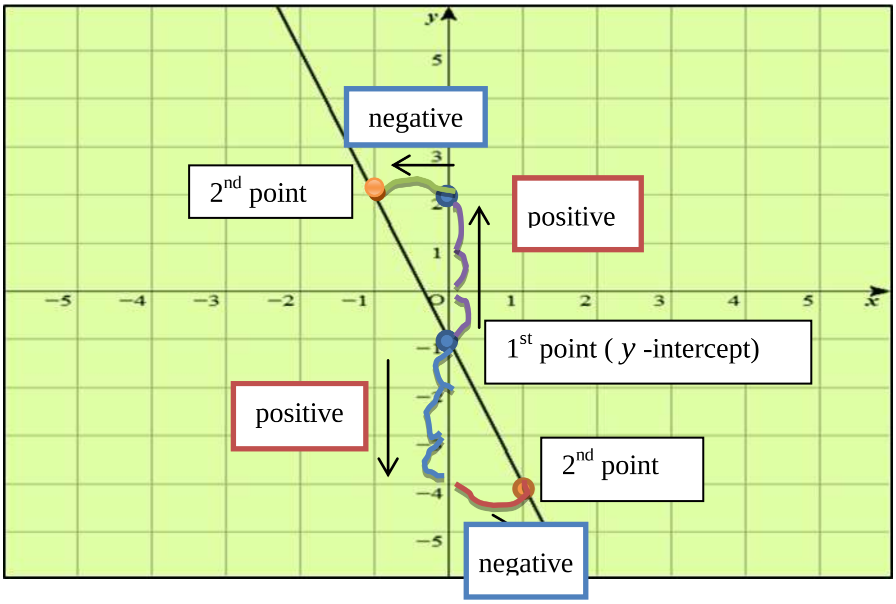
$\therefore m$ = -2 and y -intercept = 3 or (0,3)



III Working out Equation of the Line from the given Linear Graph: $y = mx + c$

Example 1.9

For the graph given below, work out the equation of the line:



STEP: 1 Identify the y-intercept i.e the point at which the linear graph crosses on the y - axis $\therefore$ y-intercept = (0, -1)

STEP:2 Choose 2 points on the linear graph with the y-intercept as the 1st point

STEP:3 Use the 1st point as the starting point, move to the 2nd point by first moving along the y-axis and then along the x-axis.

NOTE

1. ***Positive y-axis movement:-upward movement***
2. ***Negative y-axis movement:- downward movement***
3. ***Positive x-axis movement:- movement to the right***
4. ***Negative x-axis movement:- movement to the left***

STEP:4 Calculate the gradient(m) using the formula i.e gradient is equal to movement along the y-axis divided by movement along the x-axis

$\therefore m = \dfrac{3}{-1}$ or $m = \dfrac{-3}{1}$

$\therefore m = -3$ $m = -3$

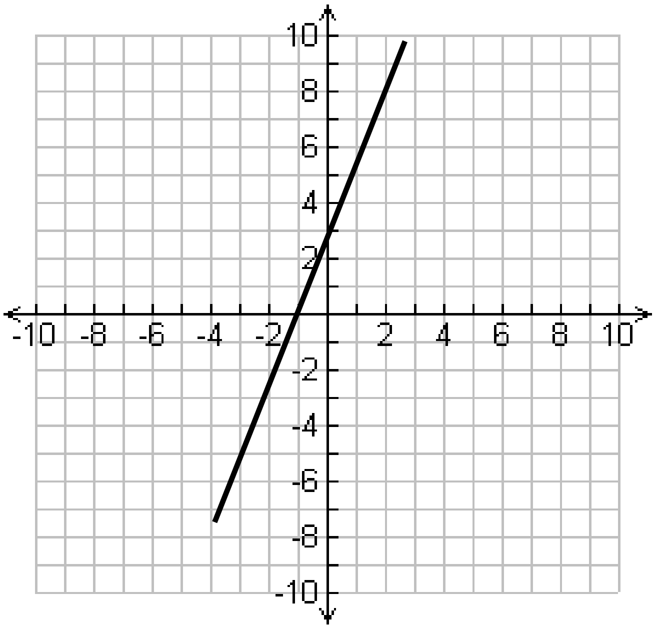
STEP:5 Substitute the value of the y-intercept and the gradient (m) in the equation

$$y = mx + c$$

$\therefore$ Equation of the line: $y = \underbrace{-3}_{\boxed{m}}x \underbrace{-1}_{\boxed{y-\text{intercept}}}$

Example 1.10

Work out the equation of the line given below



1. y intercept, y = 3 or (0, 3)

2. gradient, $m = \frac{\text{rise}}{\text{run}} = \frac{1}{1} = 1$

3. $y = mx + c$

   $y = 1x + 3$ or $y = x + 3$

"This one shows the time and money spent on making graphs."

Exercise 1.4

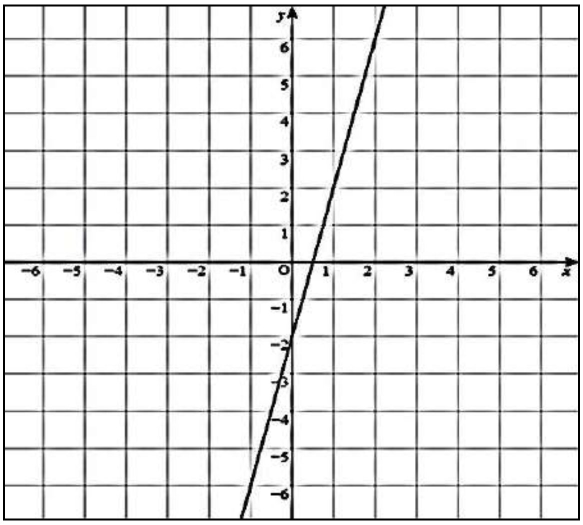
1. For the following equations, identify the gradient and the $y-intercept$.

a. $y = x + 3$ b. $y = -3x - 1$ c. $y = \frac{2}{3}x + 4$ d. $y = 2 - \frac{1}{3}x$

2. Rearrange the equations to the form $y = mx + c$ and state the gradient and $y-intercept$

a. $y - 3x = 2$ b. $y + 3 = 4x$ c. $y - 2x + 1 = 0$ d. $6x + 3y = -9$

3. A function is given by the graph below.



(a) Find the gradient of the graph

(b) Find the $y-intercept$

(c) Write the equation of the function in the form $y = mx + c$

4. Draw graphs of the following linear functions using intercept method

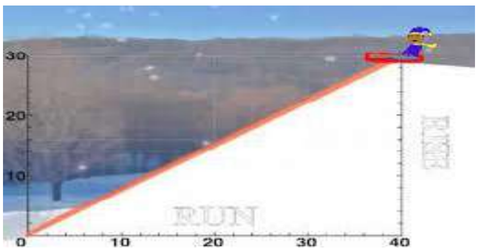
a. y = x + 3 b. y = -x – 2 c. y = -2x + 1

d. y + 3 – 3x = 0

5. Draw graphs of the following linear functions using table of values method.

a. $y = -x - 5$ b. $y = 2x + 1$ c. $y - 2 = 2x$

d. $y + 4x - 1 = 0$

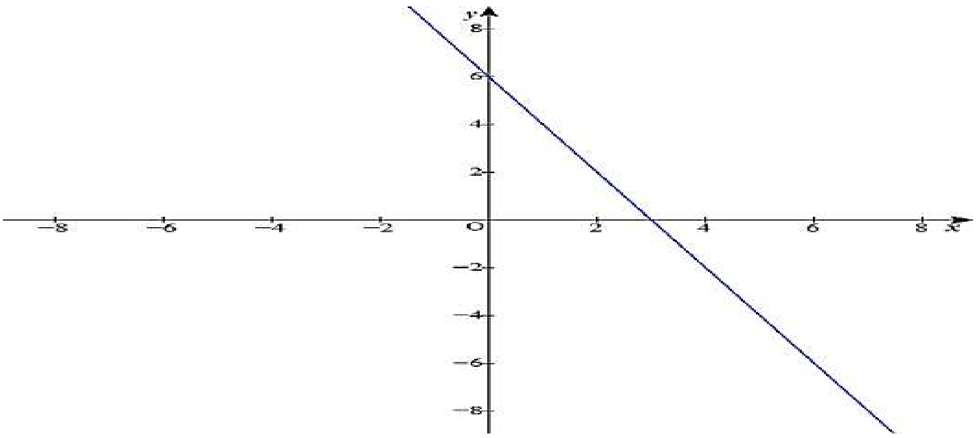


Source: https://www.google.com/search?q=google&ie=utf-8&oe=utf-8#q=images+of+cartoon+gradient+and+y+intercepts

6. Draw graphs of the following linear functions using the gradient, y intercept method.

a. $y = 2x - 1$ b. $y = \frac{2}{5}x + 1$ c. $y - 6x + 4 = -4$

7. A straight line graph is shown below.



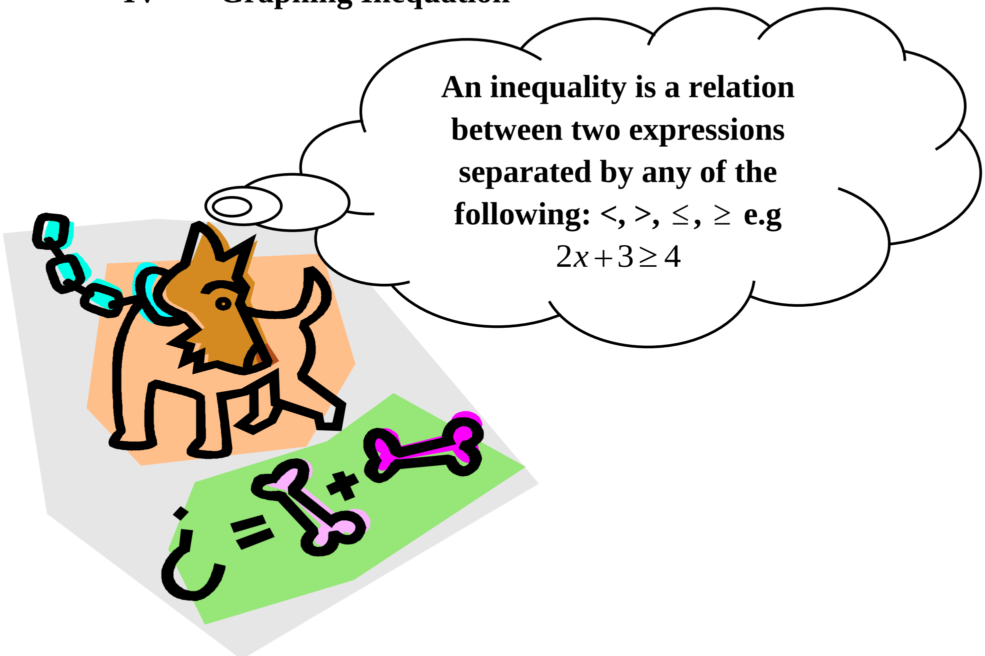
(a) Write the coordinates of the $y-\text{int}\,ercept$.

(b) Calculate the gradient, m, slope of the line.

(c) Write the equation of the straight line shown in the form $y = mx + c$.

(d) If another line $x = 2$ is drawn on the same axes, what are the coordinates of the point of intersection.

IV Graphing Inequation



Example 1.11

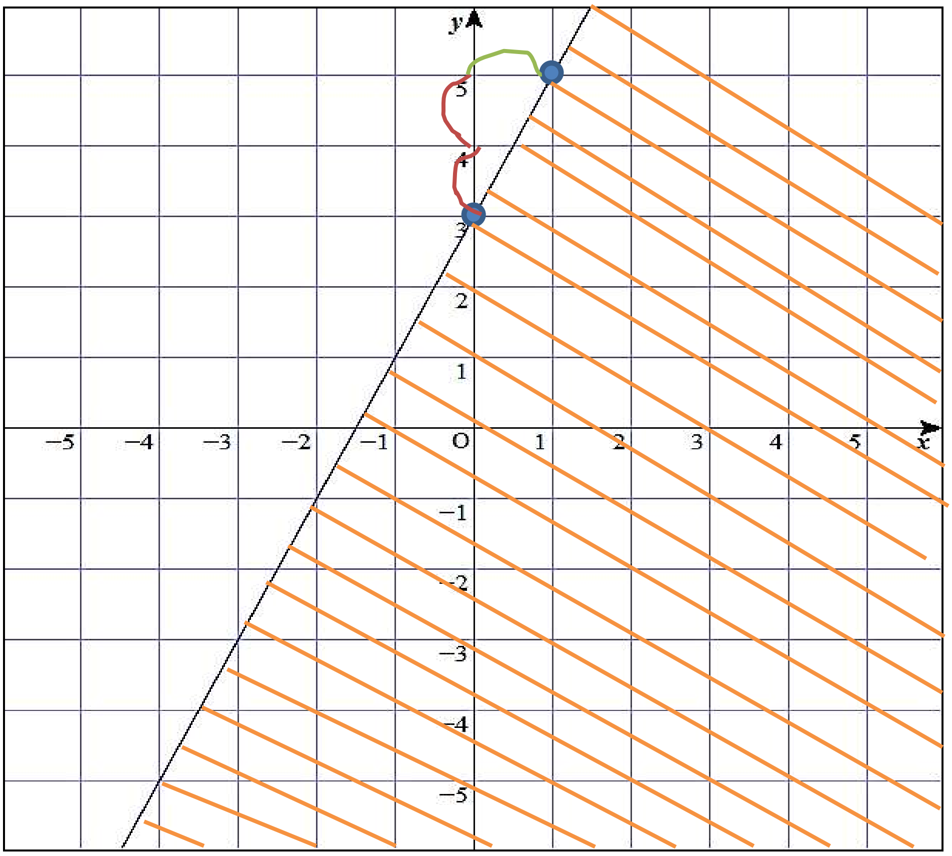
Graph the inequation $y \leq 2x + 3$

STEP: 1 Find the equal part i.e graph $y = 2x + 3$

STEP: 2 Shade the region indicated by the inequality sign

NOTE:

1. ***If the inequality sign is < or >, the line of the graph would have a broken line i.e*** - - - - - - - - - - - - - - - - -
2. ***If the inequality sign is ≤ or ≥, the line of the graph would be bold i.e*** ____________________
3. ***If the inequality sign is ≤ or <, the shading would be below the linear graph***
4. ***If the inequality sign is ≥ or >, the shading would be above the graph***

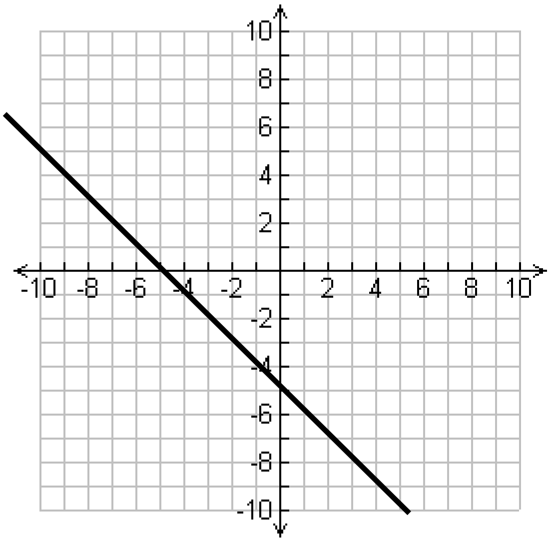


Example 1.12

Graph the inequation $y > - x - 5$

Step: 1 Graph $y = - x - 5$

y intercept = (0, - 5) x intercept = (-5, 0) gradient, m = -1

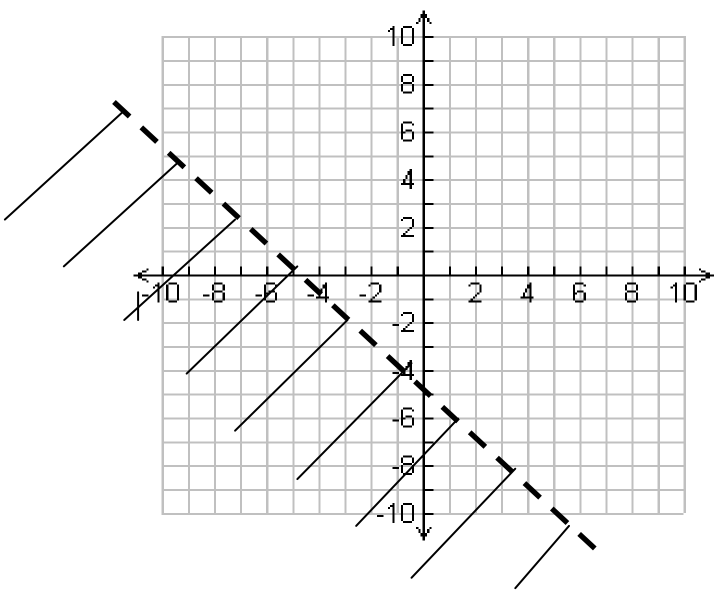


Step: 2 Look at inequality sign and draw graph accordingly

$y > - x - 5$

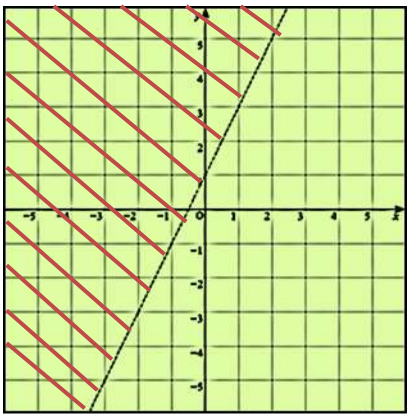
For less than - broken line is used.

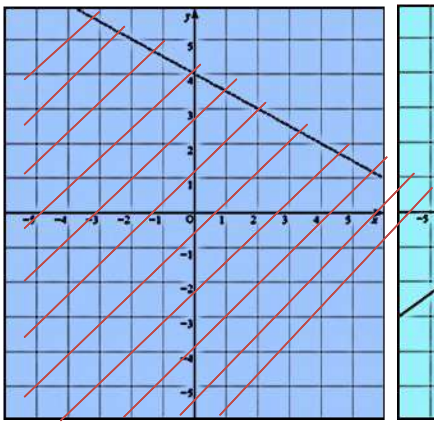
- area below the broken line is shaded

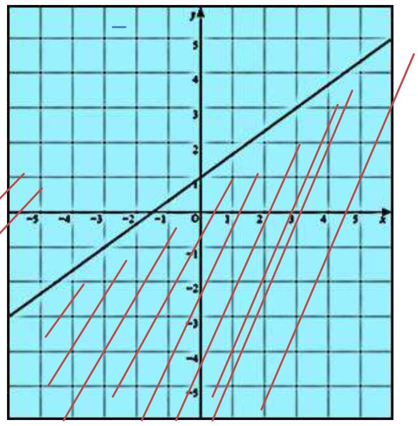


Exercise 1.5

1. For the given graphs, describe the inequalities.

a. 

b. 

c. 

2. Draw the graphs of the following inequations.

a. $y \le x - 4$ b. $y > -2x + \frac{1}{2}$ c. $y - 5x \ge -2$ d. $y + 4x - 6 < -12$

Domain and range in real life situations

- *A local youth group is planning a trip to a local amusement park. They are taking their church bus which holds 32 people. It will cost \$25 for parking and tickets to enter the park are \$22.50 per person. The equation that models this situation is: c(n) = 22.5n + 25, where c represents the cost for the group to go to the park and n represents the number of people who go on this excursion.*

Source: https://www.google.com/search?q=google&ie=utf-8&oe=utf-8#q=images+of+cartoon+youth+group

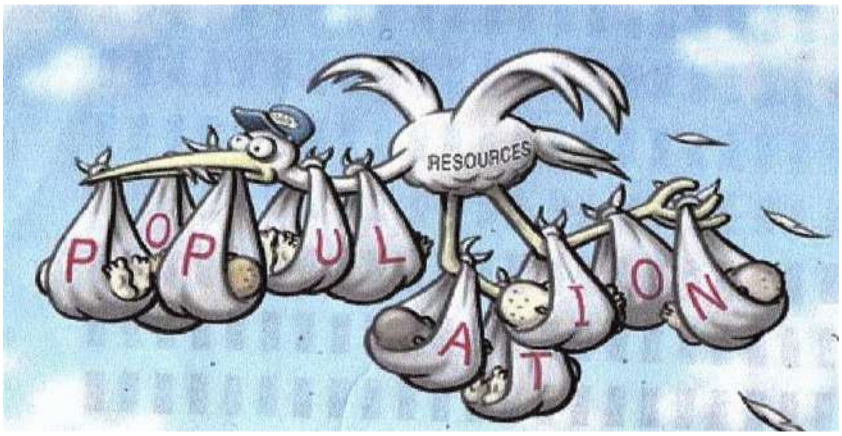
In this problem, for the domain, the problem says that the bus can only hold 32 people, so the domain has to be less than or equal to 32. However, since negative numbers are also less than 32 and impossible to have negative people (independent variable), It is a must to have a lower limit on domain of 0.To find the range values, simply use the limits set on the domain and substitute those values into the equation to find my limits on the range.

- ***A Function is given by a Table of Values below***

  The following table gives U.S. population in millions in the indicated year:

Year	*1960*	*1970*	*1980*	*1990*
U.S. Population (in millions)	*181*	*205*	*228*	*250*

Source: Statistical Abstracts of the United States,1993.



Source: https://www.google.com/search?q=google&ie=utf-8&oe=utf-8#q=images+of+cartoon+population

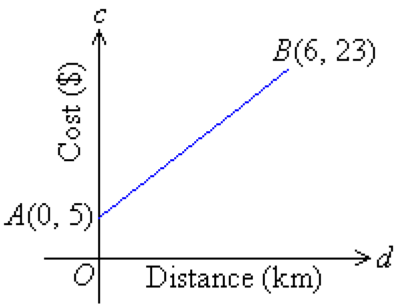
We can think of population as depending on time in years so the independent variable or input is the year and the dependent variable or output is the U. S. population. Since the table gives a unique population for each year, it represents a function. The domain is the set of years {1960, 1970, 1980, 1990} and the range is the set of populations in those years {181 million, 205 million, 228 million, 250 million}.

- *Peter needs to fill up his truck with gasoline to drive to and from school next week. If gas costs $2.79 per gallon, and his truck holds a maximum of 28 gallons, analyze the domain, range, and function values through the following questions.*

The domain is the number of gallons of gas purchased. On the graph, this is the possible x-values. The range is the costs of the gasoline. On the graph, this is the possible y-values.

Gradient in in real life situations

The cost of transporting documents by courier is given by the line segment drawn in the diagram. Find the gradient of the line segment; and describe its meaning.



Solution:

Let $(d_1, c_1) = (0, 5)$ and $(d_2, c_2) = (6, 23)$.

$$\begin{aligned} m &= \frac{c_2 - c_1}{d_2 - d_1} \\ &= \frac{23-5}{6-0} \\ &= \frac{18}{6} \\ &= 3 \end{aligned}$$

So, the gradient of the line is 3. This means that the cost of transporting documents is \$3 per km plus a fixed charge of \$5, i.e. it costs \$5 for the courier to arrive and \$3 for every kilometre travelled to deliver the documents.

GLOSSARY		
12	Cartesian plane	A plane made up of an x axis (horizontal line)and y axis (vertical line)
11	Coordinates	Ordered set of numbers that define the position of a point
14	degree	The degree of a term is the exponent of the term
5	Domain	The set of x values
8	Equation	A written statement indicating the equality of two expressions
3	Function	A set of ordered pairs in which each x value has only one y value associated with it
16	Gradient (slope)	A measure of steepness of a line
15	In equation (inequality)	A relation that holds between two values when they are different A mathematical sentence built from expressions using one or more of the symbols $<$, $>$, $\geq$, $\leq$
13	Index	A number to the right and above the base number
1	Intersection	The point where two lines intersect
4	Linear	Relating to a graph that is a linear, especially a straight line
9	Notations	Written symbols used to represent numbers
7	Quadratics	An equation where the highest exponent of the variable is a square (2)
6	Range	The set of y values
2	Relation	A set of ordered pairs
10	Variable	An element or feature that is liable to change
17	X intercept	It is where the graph cuts the x axis
18	Y intercept	It is where the graph cuts the y axis