CHAPTER 1

Algebraic expressions

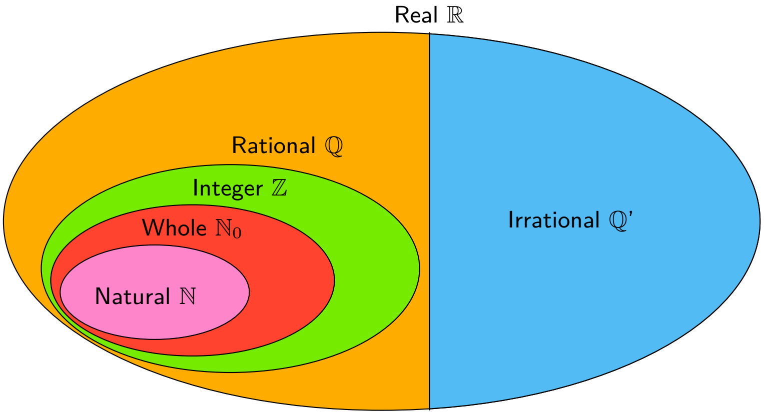
1.1 Introduction EMA2

Over human history, all peoples and cultures have contributed to the field of Mathematics. Topics like algebra may seem obvious now, but for many centuries mathematicians had to make do without it. Over the next three grades, you will explore more advanced and abstract mathematics. It may not be obvious how this applies to everyday life, but the truth is, mathematics is required for nearly everything you will do one day. Enjoy your mathematical journey. Remember, there is no such thing as a “maths person” or “not a maths person”. We can all do mathematics, it just takes practice.

Figure 1.1: Some examples of early tally sticks. These were used to help people count things such as the number of days between events or the number of livestock they had.

In this chapter, we will begin by revising the real number system and then learn about estimating surds and rounding real numbers. We will also be expanding on prior knowledge of factorisation and delve into more complex calculations involving binomial and trinomial expressions.

1.2 The real number system EMA3



We use the following definitions:

- $\mathbb{N}$: natural numbers are $\{1; 2; 3; \ldots\}$
- $\mathbb{N}_0$: whole numbers are $\{0; 1; 2; 3; \ldots\}$
- $\mathbb{Z}$: integers are $\{\ldots; -3; -2; -1; 0; 1; 2; 3; \ldots\}$

VISIT:
The following video shows an example of determining which of the above sets of numbers a particular number is in.
See video: 2DBH at www.everythingmaths.co.za

NOTE:
Not all numbers are real numbers. The square root of a negative number is called a non-real or imaginary number. For example $\sqrt{-1}$, $\sqrt{-28}$ and $\sqrt{-5}$ are all non-real numbers.

1.3 Rational and irrational numbers EMA4

DEFINITION: *Rational number*

A rational number ($\mathbb{Q}$) is any number which can be written as:

$$\frac{a}{b}$$

where a and b are integers and $b \neq 0$.

The following numbers are all rational numbers:

$$\frac{10}{1} \; ; \; \frac{21}{7} \; ; \; \frac{-1}{-3} \; ; \; \frac{10}{20} \; ; \; \frac{-3}{6}$$

We see that all numerators and all denominators are integers.

This means that all integers are rational numbers, because they can be written with a denominator of 1.

DEFINITION: *Irrational numbers*

Irrational numbers ($\mathbb{Q}'$) are numbers that cannot be written as a fraction with the numerator and denominator as integers.

Examples of irrational numbers:

$$\sqrt{2} \; ; \; \sqrt{3} \; ; \; \sqrt[3]{4} \; ; \; \pi \; ; \; \frac{1+\sqrt{5}}{2}$$

These are not rational numbers, because either the numerator or the denominator is not an integer.

Decimal numbers EMA5

All integers and fractions with integer numerators and non-zero integer denominators are rational numbers. Remember that when the denominator of a fraction is zero then the fraction is undefined.

You can write any rational number as a decimal number but not all decimal numbers are rational numbers. These types of decimal numbers are rational numbers:

- Decimal numbers that end (or terminate). For example, the fraction $\frac{4}{10}$ can be written as 0,4.
- Decimal numbers that have a repeating single digit. For example, the fraction $\frac{1}{3}$ can be written as $0{,}\dot{3}$ or $0{,}\overline{3}$. The dot and bar notations are equivalent and both represent recurring 3's, i.e. $0{,}\dot{3} = 0{,}\overline{3} = 0{,}333\ldots$.
- Decimal numbers that have a recurring pattern of multiple digits. For example, the fraction $\frac{2}{11}$ can also be written as $0{,}\overline{18}$. The bar represents a recurring pattern of 1's and 8's, i.e. $0{,}\overline{18} = 0{,}181818\ldots$.

NOTE:
You may see a full stop instead of a comma used to indicate a decimal number. So the number 0,4 can also be written as 0.4

Notation: You can use a dot or a bar over the repeated digits to indicate that the decimal is a recurring decimal. If the bar covers more than one digit, then all numbers beneath the bar are recurring.

If you are asked to identify whether a number is rational or irrational, first write the number in decimal form. If the number terminates then it is rational. If it goes on forever, then look for a repeated pattern of digits. If there is no repeated pattern, then the number is irrational.

When you write irrational numbers in decimal form, you may continue writing them for many, many decimal places. However, this is not convenient and it is often necessary to round off.

NOTE:
Rounding off an irrational number makes the number a rational number that approximates the irrational number.

Worked example 1: Rational and irrational numbers

QUESTION

Which of the following are not rational numbers?

1. $\pi = 3{,}14159265358979323846264338327950288419716939937510...$
2. 1,4
3. 1,618033989...
4. 100
5. 1,7373737373...
6. $0{,}\overline{02}$

SOLUTION

1. Irrational, decimal does not terminate and has no repeated pattern.
2. Rational, decimal terminates.
3. Irrational, decimal does not terminate and has no repeated pattern.
4. Rational, all integers are rational.
5. Rational, decimal has repeated pattern.
6. Rational, decimal has repeated pattern.

Converting terminating decimals into rational numbers EMA6

A decimal number has an integer part and a fractional part. For example, 10,589 has an integer part of 10 and a fractional part of 0,589 because $10 + 0{,}589 = 10{,}589$.

Each digit after the decimal point is a fraction with a denominator in increasing powers of 10.

For example:

- 0,1 is $\frac{1}{10}$
- 0,01 is $\frac{1}{100}$
- 0,001 is $\frac{1}{1000}$

This means that

$$\begin{aligned}
10{,}589 &= 10 + \frac{5}{10} + \frac{8}{100} + \frac{9}{1000} \\
&= \frac{10\,000}{1000} + \frac{500}{1000} + \frac{80}{1000} + \frac{9}{1000} \\
&= \frac{10\,589}{1000}
\end{aligned}$$

VISIT:
The following two videos explain how to convert decimals into rational numbers.
Part 1
See video: 2DBJ at www.everythingmaths.co.za
Part 2
See video: 2DBK at www.everythingmaths.co.za

Converting recurring decimals into rational numbers EMA7

When the decimal is a recurring decimal, a bit more work is needed to write the fractional part of the decimal number as a fraction.

Worked example 2: Converting decimal numbers to fractions

QUESTION

Write $0{,}\dot{3}$ in the form $\frac{a}{b}$ (where a and b are integers).

SOLUTION

Step 1: Define an equation

$$\text{Let } x = 0{,}33333\ldots$$

Step 2: Multiply by 10 on both sides

$$10x = 3{,}33333\ldots$$

Step 3: Subtract the first equation from the second equation

$$9x = 3$$

Step 4: Simplify

$$x = \frac{3}{9} = \frac{1}{3}$$

Worked example 3: Converting decimal numbers to fractions

QUESTION

Write $5,\dot{4}3\dot{2}$ as a rational fraction.

SOLUTION

Step 1: Define an equation

$$x = 5{,}432432432\ldots$$

Step 2: Multiply by 1000 on both sides

$$1000x = 5432{,}432432432\ldots$$

Step 3: Subtract the first equation from the second equation

$$999x = 5427$$

Step 4: Simplify

$$x = \frac{5427}{999} = \frac{201}{37} = 5\frac{16}{37}$$

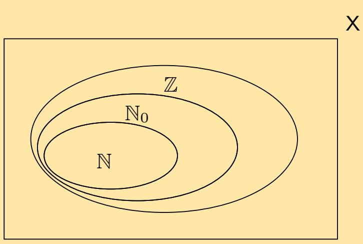
In the first example, the decimal was multiplied by 10 and in the second example, the decimal was multiplied by 1000. This is because there was only one digit recurring (i.e. 3) in the first example, while there were three digits recurring (i.e. 432) in the second example.

In general, if you have one digit recurring, then multiply by 10. If you have two digits recurring, then multiply by 100. If you have three digits recurring, then multiply by 1000 and so on.

Not all decimal numbers can be written as rational numbers. Why? Irrational decimal numbers like $\sqrt{2} = 1{,}4142135\ldots$ cannot be written with an integer numerator and denominator, because they do not have a pattern of recurring digits and they do not terminate.

Exercise 1 – 1:

1. The figure here shows the Venn diagram for the special sets $\mathbb{N}, \mathbb{N}_0$ and $\mathbb{Z}$.

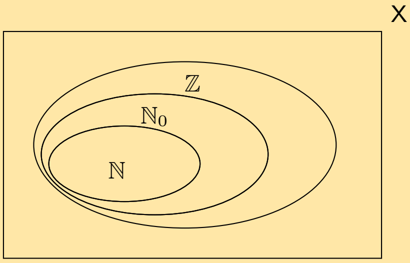


a) Where does the number $-\frac{12}{3}$ belong in the diagram?

b) In the following list, there are two false statements and one true statement. Which of the statements is **true**?

i. Every integer is a natural number.
ii. Every natural number is a whole number.
iii. There are no decimals in the whole numbers.

2. The figure here shows the Venn diagram for the special sets $\mathbb{N}, \mathbb{N}_0$ and $\mathbb{Z}$.



a) Where does the number $-\frac{1}{2}$ belong in the diagram?

b) In the following list, there are two false statements and one true statement. Which of the statements is **true**?

i. Every integer is a natural number.
ii. Every whole number is an integer.
iii. There are no decimals in the whole numbers.

3. State whether the following numbers are real, non-real or undefined.

a) $-\sqrt{3}$ b) $\dfrac{0}{\sqrt{2}}$ c) $\sqrt{-9}$ d) $\dfrac{-\sqrt{7}}{0}$ e) $-\sqrt{-16}$ f) $\sqrt{2}$

4. State whether the following numbers are rational or irrational. If the number is rational, state whether it is a natural number, whole number or an integer.

a) $-\frac{1}{3}$ b) 0,651268962154862... c) $\dfrac{\sqrt{9}}{3}$

d) π^2 e) π^4 f) $\sqrt[3]{19}$

g) $\left(\sqrt[3]{1}\right)^7$ h) $\pi + 3$ i) $\pi +$ 0,858408346

5. If a is an integer, b is an integer and c is irrational, which of the following are rational numbers?

a) $\dfrac{5}{6}$ b) $\dfrac{a}{3}$ c) $\dfrac{-2}{b}$ d) $\dfrac{1}{c}$

6. For each of the following values of a state whether $\dfrac{a}{14}$ is rational or irrational.

a) 1 b) -10 c) $\sqrt{2}$ d) 2,1

7. Consider the following list of numbers:

$$-3\ ;\ 0\ ;\ \sqrt{-1}\ ;\ -8\frac{4}{5}\ ;\ -\sqrt{8}\ ;\ \frac{22}{7}\ ;\ \frac{14}{0}\ ;\ 7\ ;\ 1{,}\overline{34}\ ;\ 3{,}3231089...\ ;\ 3+\sqrt{2}\ ;\ 9\frac{7}{10}\ ;\ \pi\ ;\ 11$$

Which of the numbers are:

a) natural numbers
b) irrational numbers
c) non-real numbers
d) rational numbers
e) integers
f) undefined

8. For each of the following numbers:
   - write the next three digits and
   - state whether the number is rational or irrational.

   a) $1{,}1\dot{5}$ b) 2,121314... c) 1,242244246...
   d) 3,324354... e) $3{,}3243\dot{5}\dot{4}$

9. Write the following as fractions:

   a) 0,1 b) 0,12 c) 0,58 d) 0,2589

10. Write the following using the recurring decimal notation:

    a) 0,1111111... b) 0,1212121212... c) 0,123123123123... d) 0,11414541454145...

11. Write the following in decimal form, using the recurring decimal notation:

    a) $\frac{25}{45}$ b) $\frac{10}{18}$ c) $\frac{7}{33}$ d) $\frac{2}{3}$
    e) $1\frac{3}{11}$ f) $4\frac{5}{6}$ g) $2\frac{1}{9}$

12. Write the following decimals in fractional form:

    a) $0{,}\dot{5}$ b) $0{,}6\dot{3}$ c) $0{,}\dot{4}$ d) $5{,}\overline{31}$ e) $4{,}\overline{93}$ f) $3{,}\overline{93}$

For more exercises, visit www.everythingmaths.co.za and click on 'Practise Maths'.

1. 2DBM	2. 2DBN	3a. 2DBP	3b. 2DBQ	3c. 2DBR	3d. 2DBS	3e. 2DBT
3f. 2DBV	4a. 2DBX	4b. 2DBY	4c. 2DC2	4d. 2DC3	4e. 2DC4	4f. 2DC5
4g. 2DC6	4h. 2DBZ	4i. 2DBW	5. 2DC7	6. 2DC8	7. 2DC9	8a. 2DCB
8b. 2DCC	8c. 2DCD	8d. 2DCF	8e. 2DCG	9a. 2DCH	9b. 2DCJ	9c. 2DCK
9d. 2DCM	10a. 2DCN	10b. 2DCP	10c. 2DCQ	10d. 2DCR	11a. 2DCS	11b. 2DCT
11c. 2DCV	11d. 2DCW	11e. 2DCX	11f. 2DCY	11g. 2DCZ	12a. 2DD2	12b. 2DD3
12c. 2DD4	12d. 2DD5	12e. 2DD6	12f. 2DD7			

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1.4 Rounding off EMA8

Rounding off a decimal number to a given number of decimal places is the quickest way to approximate a number. For example, if you wanted to round off 2,6525272 to three decimal places, you would:

- count three places after the decimal and place a | between the third and fourth numbers;
- round up the third digit if the fourth digit is greater than or equal to 5;
- leave the third digit unchanged if the fourth digit is less than 5;
- if the third digit is 9 and needs to be rounded up, then the 9 becomes a 0 and the second digit is rounded up.

So, since the first digit after the | is a 5, we must round up the digit in the third decimal place to a 3 and the final answer of 2,6525272 rounded to three decimal places is 2,653.

VISIT:
The following video explains how to round off.
See video: 2DD8 at www.everythingmaths.co.za

Worked example 4: Rounding off

QUESTION

Round off the following numbers to the indicated number of decimal places:

1. $\frac{120}{99} = 1{,}\dot{1}\dot{2}$ to 3 decimal places.
2. $\pi = 3{,}141592653...$ to 4 decimal places.
3. $\sqrt{3} = 1{,}7320508...$ to 4 decimal places.
4. 2,78974526 to 3 decimal places.

SOLUTION

Step 1: Mark off the required number of decimal places

If the number is not a decimal you first need to write the number as a decimal.

1. $\frac{120}{99} = 1{,}212|121212\ldots$
2. $\pi = 3{,}1415|92653\ldots$
3. $\sqrt{3} = 1{,}7320|508\ldots$
4. $2{,}789|74526$

Step 2: Check the next digit to see if you must round up or round down

1. The last digit of $\frac{120}{99} = 1{,}212|12121\dot{2}\dot{1}\dot{2}$ must be rounded down.
2. The last digit of $\pi = 3{,}1415|92653\ldots$ must be rounded up.
3. The last digit of $\sqrt{3} = 1{,}7320|508\ldots$ must be rounded up.
4. The last digit of $2{,}789|74526$ must be rounded up.
   Since this is a 9 we replace it with a 0 and round up the second last digit.

Step 3: Write the final answer

1. $\frac{120}{99} = 1{,}212$ rounded to 3 decimal places.
2. $\pi = 3{,}1416$ rounded to 4 decimal places.
3. $\sqrt{3} = 1{,}7321$ rounded to 4 decimal places.
4. 2,790

Exercise 1 – 2:

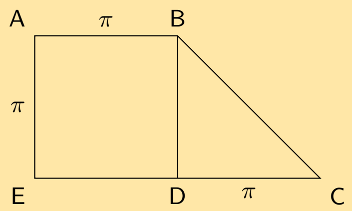
1. Round off the following to 3 decimal places:
   a) 12,56637061... b) 3,31662479... c) 0,2666666...
   d) 1,912931183... e) 6,32455532... f) 0,05555555...
2. Round off each of the following to the indicated number of decimal places:
   a) 345,04399906 to 4 decimal places.
   b) 1361,72980445 to 2 decimal places.
   c) 728,00905239 to 6 decimal places.

d) $\frac{1}{27}$ to 4 decimal places.

e) $\frac{45}{99}$ to 5 decimal places.

f) $\frac{1}{12}$ to 2 decimal places.

3. Study the diagram below



a) Calculate the area of $ABDE$ to 2 decimal places.

b) Calculate the area of BCD to 2 decimal places.

c) Using you answers in (a) and (b) calculate the area of $ABCDE$.

d) Without rounding off, what is the area of $ABCDE$?

4. Given $i = \frac{r}{600}$; $r = 7{,}4$; $n = 96$; $P = 200\ 000$.

a) Calculate i correct to 2 decimal places.

b) Using you answer from (a), calculate A in $A = P(1 + i)^n$.

c) Calculate A without rounding off your answer in (a), compare this answer with your answer in (b).

5. If it takes 1 person to carry 3 boxes, how many people are needed to carry 31 boxes?

6. If 7 tickets cost R 35,20, how much does one ticket cost?

For more exercises, visit www.everythingmaths.co.za and click on 'Practise Maths'.

1a. 2DD9 1b. 2DDB 1c. 2DDC 1d. 2DDD 1e. 2DDF 1f. 2DDG 2a. 2DDH 2b. 2DDJ
2c. 2DDK 2d. 2DDM 2e. 2DDN 2f. 2DDP 3. 2DDQ 4. 2DDR 5. 2DDS 6. 2DDT

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1.5 Estimating surds EMA9

If the n^{th} root of a number cannot be simplified to a rational number, we call it a surd. For example, $\sqrt{2}$ and $\sqrt[3]{6}$ are surds, but $\sqrt{4}$ is not a surd because it can be simplified to the rational number 2.

In this chapter we will look at surds of the form $\sqrt[n]{a}$ where a is any positive number, for example, $\sqrt{7}$ or $\sqrt[3]{5}$. It is very common for n to be 2, so we usually do not write $\sqrt[2]{a}$. Instead we write the surd as just $\sqrt{a}$

It is sometimes useful to know the approximate value of a surd without having to use a calculator. For example, we want to be able to estimate where a surd like $\sqrt{3}$ is on the number line. From a calculator we know that $\sqrt{3}$ is equal to 1,73205.... It is easy to see that $\sqrt{3}$ is above 1 and below 2. But to see this for other surds like $\sqrt{18}$, without using a calculator you must first understand the following:

If a and b are positive whole numbers, and $a < b$, then $\sqrt[n]{a} < \sqrt[n]{b}$

A perfect square is the number obtained when an integer is squared. For example, 9 is a perfect square since $3^2 = 9$.

Similarly, a perfect cube is a number which is the cube of an integer. For example, 27 is a perfect cube, because $3^3 = 27$.

Consider the surd $\sqrt[3]{52}$. It lies somewhere between 3 and 4, because $\sqrt[3]{27} = 3$ and $\sqrt[3]{64} = 4$ and 52 is between 27 and 64.

VISIT:
The following video explains how to estimate a surd.
See video: 2DDV at www.everythingmaths.co.za

Worked example 5: Estimating surds

QUESTION

Find two consecutive integers such that $\sqrt{26}$ lies between them. (Remember that consecutive integers are two integers that follow one another on the number line, for example, 5 and 6 or 8 and 9.)

SOLUTION

Step 1: Use perfect squares to estimate the lower integer
$5^2 = 25$. Therefore $5 < \sqrt{26}$.

Step 2: Use perfect squares to estimate the upper integer
$6^2 = 36$. Therefore $\sqrt{26} < 6$.

Step 3: Write the final answer
$5 < \sqrt{26} < 6$

Worked example 6: Estimating surds

QUESTION

Find two consecutive integers such that $\sqrt[3]{49}$ lies between them.

SOLUTION

Step 1: Use perfect cubes to estimate the lower integer
$3^3 = 27$, therefore $3 < \sqrt[3]{49}$.

Step 2: Use perfect cubes to estimate the upper integer
$4^3 = 64$, therefore $\sqrt[3]{49} < 4$.

Step 3: Write the answer
$3 < \sqrt[3]{49} < 4$

Step 4: Check the answer by cubing all terms in the inequality and then simplify
$27 < 49 < 64$. This is true, so $\sqrt[3]{49}$ lies between 3 and 4.

Exercise 1 – 3:

1. Determine between which two consecutive integers the following numbers lie, without using a calculator:

   a) $\sqrt{18}$ b) $\sqrt{29}$ c) $\sqrt[3]{5}$ d) $\sqrt[3]{79}$ e) $\sqrt{155}$
   f) $\sqrt{57}$ g) $\sqrt{71}$ h) $\sqrt[3]{123}$ i) $\sqrt[3]{90}$ j) $\sqrt[3]{81}$

2. Estimate the following surds to the nearest 1 decimal place, without using a calculator.

   a) $\sqrt{10}$ b) $\sqrt{82}$ c) $\sqrt{15}$ d) $\sqrt{90}$

3. Consider the following list of numbers:

   $\frac{27}{7}$; $\sqrt{19}$; 2π ; 0,45 ; $0{,}\overline{45}$; $-\sqrt{\frac{9}{4}}$; 6 ; $-\sqrt{8}$; $\sqrt{51}$

   Without using a calculator, rank all the numbers in ascending order.

For more exercises, visit www.everythingmaths.co.za and click on 'Practise Maths'.

1a. 2DDW 1b. 2DDX 1c. 2DDY 1d. 2DDZ 1e. 2DF2 1f. 2DF3 1g. 2DF4 1h. 2DF5
1i. 2DF6 1j. 2DF7 2a. 2DF8 2b. 2DF9 2c. 2DFB 2d. 2DFC 3. 2DFD

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1.6 Products EMAB

Mathematical expressions are just like sentences and their parts have special names. You should be familiar with the following words used to describe the parts of mathematical expressions.

$$3x^2 + 7xy - 5^3$$

Name	Examples
term	$3x^2$; $7xy$; -5^3
expression	$3x^2 + 7xy - 5^3$
coefficient	3 ; 7
exponent	2 ; 1 ; 3
base	x ; y ; 5
constant	3 ; 7 ; 5
variable	x ; y
equation	$3x^2 + 7xy - 5^3 = 0$

Multiplying a monomial and a binomial EMAC

A monomial is an expression with one term, for example, $3x$ or y^2. A binomial is an expression with two terms, for example, $ax + b$ or $cx + d$.

Worked example 7: Simplifying brackets

QUESTION

Simplify:

$$2a\,(a-1) - 3\left(a^2 - 1\right)$$

SOLUTION

$$\begin{aligned} 2a(a-1) - 3\left(a^2-1\right) &= 2a(a) + 2a(-1) + (-3)\left(a^2\right) + (-3)(-1) \\ &= 2a^2 - 2a - 3a^2 + 3 \\ &= -a^2 - 2a + 3 \end{aligned}$$

Multiplying two binomials EMAD

Here we multiply (or expand) two linear binomials:

$$(ax+b)(cx+d)$$

$$\begin{aligned} (ax+b)(cx+d) &= (ax)(cx) + (ax)d + b(cx) + bd \\ &= acx^2 + adx + bcx + bd \\ &= acx^2 + x(ad+bc) + bd \end{aligned}$$

Worked example 8: Multiplying two binomials

QUESTION

Find the product: $(3x-2)(5x+8)$

SOLUTION

$$\begin{aligned} (3x-2)(5x+8) &= (3x)(5x) + (3x)(8) + (-2)(5x) + (-2)(8) \\ &= 15x^2 + 24x - 10x - 16 \\ &= 15x^2 + 14x - 16 \end{aligned}$$

The product of two identical binomials is known as the square of the binomial and is written as:

$$(ax+b)^2 = a^2x^2 + 2abx + b^2$$

If the two terms are of the form $ax+b$ and $ax-b$ then their product is:

$$(ax+b)(ax-b) = a^2x^2 - b^2$$

This product yields the difference of two squares.

Multiplying a binomial and a trinomial EMAF

A trinomial is an expression with three terms, for example, ax^2+bx+c. Now we can learn how to multiply a binomial and a trinomial.

To find the product of a binomial and a trinomial, multiply out the brackets:

$$(A+B)(C+D+E) = A(C+D+E) + B(C+D+E)$$

VISIT:
This video shows some examples of multiplying a binomial and a trinomial.
See video: 2DFF at www.everythingmaths.co.za

Worked example 9: Multiplying a binomial and a trinomial

QUESTION

Find the product: $(x-1)\left(x^2-2x+1\right)$

SOLUTION

Step 1: Expand the bracket

$$(x-1)\left(x^2-2x+1\right) = x\left(x^2-2x+1\right) - 1\left(x^2-2x+1\right) = x^3-2x^2+x-x^2+2x-1$$

Step 2: Simplify

$$(x-1)\left(x^2-2x+1\right) = x^3-3x^2+3x-1$$

Exercise 1 – 4:

1. Expand the following products:

a) $2y(y+4)$
b) $(y+5)(y+2)$
c) $(2-t)(1-2t)$
d) $(x-4)(x+4)$
e) $-(4-x)(x+4)$
f) $-(a+b)(b-a)$
g) $(2p+9)(3p+1)$
h) $(3k-2)(k+6)$
i) $(s+6)^2$
j) $-(7-x)(7+x)$
k) $(3x-1)(3x+1)$
l) $(7k+2)(3-2k)$
m) $(1-4x)^2$
n) $(-3-y)(5-y)$
o) $(8-x)(8+x)$
p) $(9+x)^2$
q) $(-7y+11)(-12y+3)$
r) $(g-5)^2$
s) $(d+9)^2$
t) $(6d+7)(6d-7)$
u) $(5z+1)(5z-1)$
v) $(1-3h)(1+3h)$
w) $(2p+3)(2p+2)$
x) $(8a+4)(a+7)$
y) $(5r+4)(2r+4)$
z) $(w+1)(w-1)$

2. Expand the following products:

a) $(g+11)(g-11)$
b) $(4b-2)(2b-4)$
c) $(4b-3)(2b-1)$
d) $(6x-4)(3x+6)$
e) $(3w-2)(2w+7)$
f) $(2t-3)^2$

g) $(5p-8)^2$
h) $(4y+5)^2$
i) $(2y^6+3y^5)(-5y-12)$
j) $9(8y^2-2y+3)$
k) $(-2y^2-4y+11)(5y-12)$
l) $(7y^2-6y-8)(-2y+2)$
m) $(10y+3)(-2y^2-11y+2)$
n) $(-12y-3)(2y^2-11y+3)$
o) $(-10)(2y^2+8y+3)$
p) $(7y+3)(7y^2+3y+10)$
q) $(a+2b)(a^2+b^2+2ab)$
r) $(x+y)(x^2-xy+y^2)$
s) $3m(9m^2+2)+5m^2(5m+6)$
t) $4x^2(10x^3+4)+4x^3(2x^2+6)$
u) $3k^3(k^2+3)+2k^2(6k^3+7)$
v) $(3x+2)(3x-2)(9x^2-4)$
w) $(-6y^4+11y^2+3y)(y+4)(y-4)$
x) $(x+2)(x-3)(x^2+2x-3)$
y) $(a+2)^2-(2a-4)^2$

3. Expand the following products:

a) $(2x+3)^2-(x-2)^2$
b) $(2a^2-a-1)(a^2+3a+2)$
c) $(y^2+4y-1)(1-4y-y^2)$
d) $2(x-2y)(x^2+xy+y^2)$
e) $3(a-3b)(a^2+3ab-b^2)$
f) $(2a-b)(2a+b)(2a^2-3ab+b^2)$
g) $2(3x+y)(3x-y)-(3x-y)^2$
h) $(x+y)(x-3y)+(2x-y)^2$
i) $\left(\frac{x}{3}-\frac{3}{x}\right)\left(\frac{x}{4}+\frac{4}{x}\right)$
j) $\left(x-\frac{2}{x}\right)\left(\frac{x}{3}+\frac{4}{x}\right)$
k) $\frac{1}{2}(10x-12y)+\tfrac{1}{3}(15x-18y)$
l) $\frac{1}{2}a(4a+6b)+\frac{1}{4}(8a+12b)$

4. What is the value of b, in $(x+b)(x-1)=x^2+3x-4$
5. What is the value of g, in $(x-2)(x+g)=x^2-6x+8$
6. In $(x-4)(x+k)=x^2+bx+c$:
   a) For which of these values of k will b be positive?
      $-3; -1; 0; 3; 5$
   b) For which of these values of k will c be positive?
      $-3; -1; 0; 3; 5$
   c) For what real values of k will c be positive?
   d) For what real values of k will b be positive?
7. Answer the following:
   a) Expand $\left(x+\frac{4}{x}\right)^2$.
   b) Given that $\left(x+\frac{4}{x}\right)^2=14$, determine the value of $x^2+\frac{16}{x^2}$ without solving for x.
8. Answer the following:
   a) Expand: $\left(a+\frac{1}{a}\right)^2$
   b) Given that $\left(a+\frac{1}{a}\right)=3$, determine the value of $\left(a+\frac{1}{a}\right)^2$ without solving for a.
   c) Given that $\left(a-\frac{1}{a}\right)=3$, determine the value of $\left(a+\frac{1}{a}\right)^2$ without solving for a.

9. Answer the following:

   a) Expand: $\left(3y + \frac{1}{2y}\right)^2$

   b) Given that $3y + \frac{1}{2y} = 4$, determine the value of $\left(3y + \frac{1}{2y}\right)^2$ without solving for y.

10. Answer the following:

    a) Expand: $\left(a + \frac{1}{3a}\right)^2$

    b) Expand: $\left(a + \frac{1}{3a}\right)\left(a^2 - \frac{1}{3} + \frac{1}{9a^2}\right)$

    c) Given that $a + \frac{1}{3a} = 2$, determine the value of $a^3 + \frac{1}{27a^3}$ without solving for a.

For more exercises, visit www.everythingmaths.co.za and click on 'Practise Maths'.

1a. 2DFG	1b. 2DFH	1c. 2DFJ	1d. 2DFK	1e. 2DFM	1f. 2DFN
1g. 2DFP	1h. 2DFQ	1i. 2DFR	1j. 2DFS	1k. 2DFT	1l. 2DFV
1m. 2DFW	1n. 2DFX	1o. 2DFY	1p. 2DFZ	1q. 2DG3	1r. 2DG4
1s. 2DG5	1t. 2DG6	1u. 2DG7	1v. 2DG8	1w. 2DG9	1x. 2DGB
1y. 2DGC	1z. 2DGD	2a. 2DGF	2b. 2DGH	2c. 2DGJ	2d. 2DGK
2e. 2DGM	2f. 2DGN	2g. 2DGP	2h. 2DGQ	2i. 2DG2	2j. 2DGR
2k. 2DGS	2l. 2DGT	2m. 2DGV	2n. 2DGW	2o. 2DGX	2p. 2DGY
2q. 2DGZ	2r. 2DH2	2s. 2DH3	2t. 2DH4	2u. 2DH5	2v. 2DH6
2w. 2DH7	2x. 2DH8	2y. 2DH9	3a. 2DHB	3b. 2DHC	3c. 2DHD
3d. 2DHF	3e. 2DHG	3f. 2DHH	3g. 2DHJ	3h. 2DHK	3i. 2DHM
3j. 2DHN	3k. 2DHP	3l. 2DHQ	4. 2DHR	5. 2DHS	6. 2DHT
7. 2DHV	8. 2DHW	9. 2DHX	10. 2DHY		

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1.7 Factorisation EMAG

Factorisation is the opposite process of expanding brackets. For example, expanding brackets would require $2(x+1)$ to be written as $2x+2$. Factorisation would be to start with $2x+2$ and end up with $2(x+1)$.

$$\begin{tikzcd} 2(x+1) \arrow[rr, bend right, "\text{expanding}"'] & & 2x+2 \arrow[ll, bend right, "\text{factorising}"'] \end{tikzcd}$$

The two expressions $2(x+1)$ and $2x+2$ are equivalent; they have the same value for all values of x.

In previous grades, we factorised by taking out a common factor and using difference of squares.

Common factors EMAH

Factorising based on common factors relies on there being factors common to all the terms.

For example, $2x - 6x^2$ can be factorised as follows:

$$2x - 6x^2 = 2x(1 - 3x)$$

And $2(x - 1) - a(x - 1)$ can be factorised as follows:

$$(x - 1)(2 - a)$$

VISIT:
The following video shows an example of factorising by taking out a common factor.
See video: 2DHZ at www.everythingmaths.co.za

Worked example 10: Factorising using a switch around in brackets

QUESTION

Factorise:

$$5(a - 2) - b(2 - a)$$

SOLUTION

Use a "switch around" strategy to find the common factor.

Notice that $2 - a = -(a - 2)$

$$\begin{aligned} 5(a - 2) - b(2 - a) &= 5(a - 2) - [-b(a - 2)] \\ &= 5(a - 2) + b(a - 2) \\ &= (a - 2)(5 + b) \end{aligned}$$

Exercise 1 – 5:

Factorise:

1. $12x + 32y$
2. $-2ab^2 - 4a^2b$
3. $18ab - 3bc$
4. $12kj + 18kq$
5. $-12a + 24a^3$
6. $-2ab - 8a$
7. $24kj - 16k^2j$
8. $-a^2b - b^2a$
9. $72b^2q - 18b^3q^2$
10. $125x^6 - 5y^2$
11. $6x^2 + 2x + 10x^3$
12. $2xy^2 + xy^2z + 3xy$
13. $12k^2j + 24k^2j^2$
14. $3a^2 + 6a - 18$
15. $7a + 4$

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1. 2DJ2 2. 2DJ3 3. 2DJ4 4. 2DJ5 5. 2DJ6 6. 2DJ7 7. 2DJ8 8. 2DJ9
9. 2DJB 10. 2DJC 11. 2DJD 12. 2DJF 13. 2DJG 14. 2DJH 15. 2DJJ

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Difference of two squares

EMAJ

We have seen that $(ax+b)(ax-b)$ can be expanded to $a^2x^2-b^2$.

Therefore $a^2x^2-b^2$ can be factorised as $(ax+b)(ax-b)$.

For example, x^2-16 can be written as x^2-4^2 which is a difference of two squares. Therefore, the factors of x^2-16 are $(x-4)$ and $(x+4)$.

To spot a difference of two squares, look for expressions:

- consisting of two terms;
- with terms that have different signs (one positive, one negative);
- with each term a perfect square.

For example: a^2-1; $4x^2-y^2$; $-49+p^4$.

VISIT:
The following video explains factorising the difference of two squares.
See video: 2DJK at www.everythingmaths.co.za

Worked example 11: The difference of two squares

QUESTION

Factorise: $3a(a^2-4)-7(a^2-4)$.

SOLUTION

Step 1: Take out the common factor (a^2-4)

$$3a(a^2-4)-7(a^2-4)=(a^2-4)(3a-7)$$

Step 2: Factorise the difference of two squares (a^2-4)

$$(a^2-4)(3a-7)=(a-2)(a+2)(3a-7)$$

Exercise 1 – 6:

Factorise:

1. $4(y-3)+k(3-y)$
2. $a^2(a-1)-25(a-1)$
3. $bm(b+4)-6m(b+4)$
4. $a^2(a+7)+9(a+7)$
5. $3b(b-4)-7(4-b)$
6. $3g(z+6)+2(6+z)$
7. $4b(y+2)+5(2+y)$
8. $3d(r+5)+14(5+r)$
9. $(6x+y)^2-9$

10. $4x^2 - (4x - 3y)^2$
11. $16a^2 - (3b + 4c)^2$
12. $(b - 4)^2 - 9(b - 5)^2$
13. $4(a - 3)^2 - 49(4a - 5)$
14. $16k^2 - 4$
15. $a^2b^2c^2 - 1$
16. $\frac{1}{9}a^2 - 4b^2$
17. $\frac{1}{2}x^2 - 2$
18. $y^2 - 8$
19. $y^2 - 13$
20. $a^2(a - 2ab - 15b^2) - 9b^2(a^2 - 2ab - 15b^2)$

For more exercises, visit www.everythingmaths.co.za and click on 'Practise Maths'.

1. 2DJM 2. 2DJN 3. 2DJP 4. 2DJQ 5. 2DJR 6. 2DJS 7. 2DJT 8. 2DJV
9. 2DJW 10. 2DJX 11. 2DJY 12. 2DJZ 13. 2DK2 14. 2DK3 15. 2DK4 16. 2DK5
17. 2DK6 18. 2DK7 19. 2DK8 20. 2DK9

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Factorising by grouping in pairs EMAK

The taking out of common factors is the starting point in all factorisation problems. We know that the factors of $3x + 3$ are 3 and $(x + 1)$. Similarly, the factors of $2x^2 + 2x$ are $2x$ and $(x + 1)$. Therefore, if we have an expression:

$$2x^2 + 2x + 3x + 3$$

there is no common factor to all four terms, but we can factorise as follows:

$$\left(2x^2 + 2x\right) + (3x + 3) = 2x\,(x + 1) + 3\,(x + 1)$$

We can see that there is another common factor $(x + 1)$. Therefore, we can write:

$$(x + 1)\,(2x + 3)$$

We get this by taking out the $(x + 1)$ and seeing what is left over. We have $2x$ from the first group and $+3$ from the second group. This is called factorising by grouping.

Worked example 12: Factorising by grouping in pairs

QUESTION

Find the factors of $7x + 14y + bx + 2by$.

SOLUTION

Step 1: There are no factors common to all terms

Step 2: Group terms with common factors together

7 is a common factor of the first two terms and b is a common factor of the second two terms. We see that the ratio of the coefficients $7 : 14$ is the same as $b : 2b$.

$$\begin{aligned} 7x + 14y + bx + 2by &= (7x + 14y) + (bx + 2by) \\ &= 7\,(x + 2y) + b\,(x + 2y) \end{aligned}$$

Step 3: Take out the common factor $(x + 2y)$

$$7(x + 2y) + b(x + 2y) = (x + 2y)(7 + b)$$

OR

Step 4: Group terms with common factors together

x is a common factor of the first and third terms and $2y$ is a common factor of the second and fourth terms $(7 : b = 14 : 2b)$.

Step 5: Rearrange the equation with grouped terms together

$$\begin{aligned} 7x + 14y + bx + 2by &= (7x + bx) + (14y + 2by) \\ &= x(7 + b) + 2y(7 + b) \end{aligned}$$

Step 6: Take out the common factor $(7 + b)$

$$x(7 + b) + 2y(7 + b) = (7 + b)(x + 2y)$$

Step 7: Write the final answer

The factors of $7x + 14y + bx + 2by$ are $(7 + b)$ and $(x + 2y)$.

Exercise 1 – 7:

Factorise the following:

1. $6d - 9r + 2t^5d - 3t^5r$
2. $9z - 18m + b^3z - 2b^3m$
3. $35z - 10y + 7c^5z - 2c^5y$
4. $6x + a + 2ax + 3$
5. $x^2 - 6x + 5x - 30$
6. $5x + 10y - ax - 2ay$
7. $a^2 - 2a - ax + 2x$
8. $5xy - 3y + 10x - 6$
9. $ab - a^2 - a + b$
10. $14m - 4n + 7jm - 2jn$
11. $28r - 20x + 7gr - 5gx$
12. $25d - 15m + 5yd - 3ym$
13. $45q - 18z + 5cq - 2cz$
14. $6j - 15v + 2yj - 5yv$
15. $16a - 40k + 2za - 5zk$
16. $ax - bx + ay - by + 2a - 2b$
17. $3ax + bx - 3ay - by - 9a - 3b$

For more exercises, visit www.everythingmaths.co.za and click on 'Practise Maths'.

1. 2DKB 2. 2DKC 3. 2DKD 4. 2DKF 5. 2DKG 6. 2DKH 7. 2DKJ
8. 2DKK 9. 2DKM 10. 2DKN 11. 2DKP 12. 2DKQ 13. 2DKR 14. 2DKS
15. 2DKT 16. 2DKV 17. 2DKW

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Factorising is the reverse of calculating the product of factors. In order to factorise a quadratic, we need to find the factors which, when multiplied together, equal the original quadratic.

Consider a quadratic expression of the form $ax^2 + bx$. We see here that x is a common factor in both terms. Therefore $ax^2 + bx$ factorises as $x(ax + b)$. For example, $8y^2 + 4y$ factorises as $4y(2y + 1)$.

Another type of quadratic is made up of the difference of squares. We know that:

$$(a + b)(a - b) = a^2 - b^2$$

So $a^2 - b^2$ can be written in factorised form as $(a + b)(a - b)$.

This means that if we ever come across a quadratic that is made up of a difference of squares, we can immediately write down the factors. These types of quadratics are very simple to factorise. However, many quadratics do not fall into these categories and we need a more general method to factorise quadratics.

We can learn about factorising quadratics by looking at the opposite process, where two binomials are multiplied to get a quadratic. For example:

$$\begin{aligned}(x + 2)(x + 3) &= x^2 + 3x + 2x + 6\\ &= x^2 + 5x + 6\end{aligned}$$

We see that the x^2 term in the quadratic is the product of the x-terms in each bracket. Similarly, the 6 in the quadratic is the product of the 2 and 3 in the brackets. Finally, the middle term is the sum of two terms.

So, how do we use this information to factorise the quadratic?

Let us start with factorising $x^2 + 5x + 6$ and see if we can decide upon some general rules. Firstly, write down the two brackets with an x in each bracket and space for the remaining terms.

$$(x \quad\quad)(x \quad\quad)$$

Next, decide upon the factors of 6. Since the 6 is positive, possible combinations are: 1 and 6, 2 and 3, -1 and -6 or -2 and -3

Therefore, we have four possibilities:

Option 1	**Option 2**	**Option 3**	**Option 4**
$(x + 1)(x + 6)$	$(x - 1)(x - 6)$	$(x + 2)(x + 3)$	$(x - 2)(x - 3)$

Next, we expand each set of brackets to see which option gives us the correct middle term.

Option 1	**Option 2**	**Option 3**	**Option 4**
$(x + 1)(x + 6)$	$(x - 1)(x - 6)$	$(x + 2)(x + 3)$	$(x - 2)(x - 3)$
$x^2 + 7x + 6$	$x^2 - 7x + 6$	$\underline{x^2 + 5x + 6}$	$x^2 - 5x + 6$

We see that Option 3, $(x + 2)(x + 3)$, is the correct solution.

The process of factorising a quadratic is mostly trial and error but there are some strategies that can be used to ease the process.

General procedure for factorising a trinomial

EMAN

1. Take out any common factor in the coefficients of the terms of the expression to obtain an expression of the form $ax^2 + bx + c$ where a, b and c have no common factors and a is positive.
2. Write down two brackets with an x in each bracket and space for the remaining terms: $(x \quad)(x \quad)$
3. Write down a set of factors for a and c.
4. Write down a set of options for the possible factors for the quadratic using the factors of a and c.
5. Expand all options to see which one gives you the correct middle term bx.

IMPORTANT!

If c is positive, then the factors of c must be either both positive or both negative. If c is negative, it means only one of the factors of c is negative, the other one being positive. Once you get an answer, always multiply out your brackets again just to make sure it really works.

VISIT:
The following video summarises how to factorise expressions and shows some examples.
See video: 2DKX at www.everythingmaths.co.za

Worked example 13: Factorising a quadratic trinomial

QUESTION

Factorise: $3x^2 + 2x - 1$.

SOLUTION

Step 1: Check that the quadratic is in required form $ax^2 + bx + c$

Step 2: Write down a set of factors for a and c

$$(x \quad)(x \quad)$$

The possible factors for a are: 1 and 3

The possible factors for c are: -1 and 1

Write down a set of options for the possible factors of the quadratic using the factors of a and c. Therefore, there are two possible options.

Option 1	Option 2
$(x-1)(3x+1)$	$(x+1)(3x-1)$
$3x^2 - 2x - 1$	$\underline{3x^2 + 2x - 1}$

Step 3: Check that the solution is correct by multiplying the factors

$$\begin{aligned}(x+1)(3x-1) &= 3x^2 - x + 3x - 1\\ &= 3x^2 + 2x - 1\end{aligned}$$

Step 4: Write the final answer

$3x^2 + 2x - 1 = (x+1)(3x-1)$

Exercise 1 – 8:

Factorise the following:

1. $x^2 + 8x + 15$
2. $x^2 + 9x + 8$
3. $x^2 + 12x + 36$
4. $2h^2 + 5h - 3$
5. $3x^2 + 4x + 1$
6. $3s^2 + s - 10$
7. $x^2 - 2x - 15$
8. $x^2 + 2x - 3$
9. $x^2 + x - 20$
10. $x^2 - x - 20$
11. $2x^2 - 22x + 20$
12. $6a^2 + 14a + 8$
13. $6v^2 - 27v + 27$
14. $6g^2 - 15g - 9$
15. $3x^2 + 19x + 6$
16. $3x^2 + 17x - 6$
17. $7x^2 - 6x - 1$
18. $6x^2 - 15x - 9$
19. $a^2 - 7ab + 12b$
20. $3a^2 + 5ab - 12b^2$
21. $98x^4 + 14x^2 - 4$
22. $(x - 2)^2 - 7(x - 2) + 12$
23. $(a - 2)^2 - 4(a - 2) - 5$
24. $(y + 3)^2 - 3(y + 3) - 18$
25. $3(b^2 + 5b) + 12$
26. $6(a^2 + 3a) - 168$

For more exercises, visit www.everythingmaths.co.za and click on 'Practise Maths'.

1. 2DKY 2. 2DKZ 3. 2DM2 4. 2DM3 5. 2DM4 6. 2DM5
7. 2DM6 8. 2DM7 9. 2DM8 10. 2DM9 11. 2DMB 12. 2DMC
13. 2DMD 14. 2DMF 15. 2DMG 16. 2DMH 17. 2DMJ 18. 2DMK
19. 2DMM 20. 2DMN 21. 2DMP 22. 2DMQ 23. 2DMR 24. 2DMS
25. 2DMT 26. 2DMV

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Sum and difference of two cubes EMAP

We now look at two special results obtained from multiplying a binomial and a trinomial:

Sum of two cubes:

$$\begin{aligned}
(x+y)\left(x^2 - xy + y^2\right) &= x\left(x^2 - xy + y^2\right) + y\left(x^2 - xy + y^2\right) \\
&= \left[x\left(x^2\right) + x\left(-xy\right) + x\left(y^2\right)\right] + \left[y\left(x^2\right) + y\left(-xy\right) + y\left(y^2\right)\right] \\
&= x^3 - x^2y + xy^2 + x^2y - xy^2 + y^3 \\
&= x^3 + y^3
\end{aligned}$$

Difference of two cubes:

$$\begin{aligned}
(x-y)\left(x^2 + xy + y^2\right) &= x\left(x^2 + xy + y^2\right) - y\left(x^2 + xy + y^2\right) \\
&= \left[x\left(x^2\right) + x\left(xy\right) + x\left(y^2\right)\right] - \left[y\left(x^2\right) + y\left(xy\right) + y\left(y^2\right)\right] \\
&= x^3 + x^2y + xy^2 - x^2y - xy^2 - y^3 \\
&= x^3 - y^3
\end{aligned}$$

So we have seen that:

$$\begin{aligned}
x^3 + y^3 &= (x+y)\left(x^2 - xy + y^2\right) \\
x^3 - y^3 &= (x-y)\left(x^2 + xy + y^2\right)
\end{aligned}$$

We use these two basic identities to factorise more complex examples.

Worked example 14: Factorising a difference of two cubes

QUESTION

Factorise: $a^3 - 1$.

SOLUTION

Step 1: Take the cube root of terms that are perfect cubes

We are working with the difference of two cubes. We know that $x^3 - y^3 = (x-y)\left(x^2 + xy + y^2\right)$, so we need to identify x and y.

We start by noting that $\sqrt[3]{a^3} = a$ and $\sqrt[3]{1} = 1$. These give the terms in the first bracket. This also tells us that $x = a$ and $y = 1$.

Step 2: Find the three terms in the second bracket

We can replace x and y in the factorised form of the expression for the difference of two cubes with a and 1. Doing so we get the second bracket:

$$\left(a^3 - 1\right) = (a-1)\left(a^2 + a + 1\right)$$

Step 3: Expand the brackets to check that the expression has been correctly factorised

$$\begin{aligned}(a-1)\left(a^2+a+1\right) &= a\left(a^2+a+1\right) - 1\left(a^2+a+1\right)\\ &= a^3 + a^2 + a - a^2 - a - 1\\ &= a^3 - 1\end{aligned}$$

Worked example 15: Factorising a sum of two cubes

QUESTION

Factorise: $x^3 + 8$.

SOLUTION

Step 1: Take the cube root of terms that are perfect cubes

We are working with the sum of two cubes. We know that $x^3 + y^3 = (x+y)\left(x^2 - xy + y^2\right)$, so we need to identify x and y.

We start by noting that $\sqrt[3]{x^3} = x$ and $\sqrt[3]{8} = 2$. These give the terms in the first bracket. This also tells us that $x = x$ and $y = 2$.

Step 2: Find the three terms in the second bracket

We can replace x and y in the factorised form of the expression for the sum of two cubes with x and 2. Doing so we get the second bracket:

$$\left(x^3 + 8\right) = (x+2)\left(x^2 - 2x + 4\right)$$

Step 3: Expand the brackets to check that the expression has been correctly factorised

$$\begin{aligned}(x+2)\left(x^2-2x+4\right) &= x\left(x^2-2x+4\right) + 2\left(x^2-2x+4\right)\\ &= x^3 - 2x^2 + 4x + 2x^2 - 4x + 8\\ &= x^3 + 8\end{aligned}$$

Worked example 16: Factorising a difference of two cubes

QUESTION

Factorise: $16y^3 - 432$.

SOLUTION

Step 1: Take out the common factor 16

$$16y^3 - 432 = 16\left(y^3 - 27\right)$$

Step 2: Take the cube root of terms that are perfect cubes

We are working with the difference of two cubes. We know that $x^3 - y^3 = (x - y)\left(x^2 + xy + y^2\right)$, so we need to identify x and y.

We start by noting that $\sqrt[3]{y^3} = y$ and $\sqrt[3]{27} = 3$. These give the terms in the first bracket. This also tells us that $x = y$ and $y = 3$.

Step 3: Find the three terms in the second bracket

We can replace x and y in the factorised form of the expression for the difference of two cubes with y and 3. Doing so we get the second bracket:

$$16\left(y^3 - 27\right) = 16\,(y - 3)\left(y^2 + 3y + 9\right)$$

Step 4: Expand the brackets to check that the expression has been correctly factorised

$$\begin{aligned}16(y-3)(y^2+3y+9) &= 16[(y(y^2+3y+9) - 3(y^2+3y+9)]\\ &= 16[y^3+3y^2+9y-3y^2-9y-27]\\ &= 16y^3 - 432\end{aligned}$$

Worked example 17: Factorising a sum of two cubes

QUESTION

Factorise: $8t^3 + 125p^3$.

SOLUTION

Step 1: Take the cube root of terms that are perfect cubes

We are working with the sum of two cubes. We know that $x^3 + y^3 = (x + y)\left(x^2 - xy + y^2\right)$, so we need to identify x and y.

We start by noting that $\sqrt[3]{8t^3} = 2t$ and $\sqrt[3]{125p^3} = 5p$. These give the terms in the first bracket. This also tells us that $x = 2t$ and $y = 5p$.

Step 2: Find the three terms in the second bracket

We can replace x and y in the factorised form of the expression for the difference of two cubes with $2t$ and $5p$. Doing so we get the second bracket:

$$\begin{aligned}\left(8t^3 + 125p^3\right) &= (2t + 5p)\left[(2t)^2 - (2t)(5p) + (5p)^2\right] \\ &= (2t + 5p)\left(4t^2 - 10tp + 25p^2\right)\end{aligned}$$

Step 3: Expand the brackets to check that the expression has been correctly factorised

$$\begin{aligned}(2t + 5p)\left(4t^2 - 10tp + 25p^2\right) &= 2t\left(4t^2 - 10tp + 25p^2\right) + 5p\left(4t^2 - 10tp + 25p^2\right) \\ &= 8t^3 - 20pt^2 + 50p^2t + 20pt^2 - 50p^2t + 125p^3 \\ &= 8t^3 + 125p^3\end{aligned}$$

Exercise 1 – 9:

Factorise:

1. $w^3 - 8$
2. $g^3 + 64$
3. $h^3 + 1$
4. $x^3 + 8$
5. $27 - m^3$
6. $2x^3 - 2y^3$
7. $3k^3 + 81q^3$
8. $64t^3 - 1$
9. $64x^2 - 1$
10. $125x^3 + 1$
11. $25x^3 + 1$
12. $z - 125z^4$
13. $8m^6 + n^9$
14. $216n^3 - k^3$
15. $125s^3 + d^3$
16. $8k^3 + r^3$
17. $8j^3k^3l^3 - b^3$
18. $27x^3y^3 + w^3$
19. $128m^3 + 2f^3$
20. $p^{15} - \frac{1}{8}y^{12}$
21. $\frac{27}{t^3} - s^3$
22. $\frac{1}{64q^3} - h^3$
23. $72g^3 + \frac{1}{3}v^3$
24. $1 - (x - y)^3$
25. $h^4(8g^6 + h^3) - (8g^6 + h^3)$
26. $x(125w^3 - h^3) + y(125w^3 - h^3)$
27. $x^2(27p^3 + w^3) - 5x(27p^3 + w^3) - 6(27p^3 + w^3)$

For more exercises, visit www.everythingmaths.co.za and click on 'Practise Maths'.

1. 2DMW	2. 2DMX	3. 2DMY	4. 2DMZ	5. 2DN2	6. 2DN3	7. 2DN4
8. 2DN5	9. 2DN6	10. 2DN7	11. 2DN8	12. 2DN9	13. 2DNB	14. 2DNC
15. 2DND	16. 2DNF	17. 2DNG	18. 2DNH	19. 2DNJ	20. 2DNK	21. 2DNM
22. 2DNN	23. 2DNP	24. 2DNQ	25. 2DNR	26. 2DNS	27. 2DNT	

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1.8 Simplification of fractions EMAQ

We have studied procedures for working with fractions in earlier grades.

1. $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd} \qquad (b \neq 0; d \neq 0)$
2. $\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b} \qquad (b \neq 0)$
3. $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc} \qquad (b \neq 0; c \neq 0; d \neq 0)$

Note: dividing by a fraction is the same as multiplying by the reciprocal of the fraction.

In some cases of simplifying an algebraic expression, the expression will be a fraction. For example,

$$\frac{x^2 + 3x}{x + 3}$$

has a quadratic binomial in the numerator and a linear binomial in the denominator. We have to apply the different factorisation methods in order to factorise the numerator and the denominator before we can simplify the expression.

$$\begin{aligned} \frac{x^2 + 3x}{x + 3} &= \frac{x(x+3)}{x+3} \\ &= x \qquad (x \neq -3) \end{aligned}$$

If $x = -3$ then the denominator, $x + 3 = 0$ and the fraction is undefined.

VISIT:
This video shows some examples of simplifying fractions.
See video: 2DNV at www.everythingmaths.co.za

Worked example 18: Simplifying fractions

QUESTION

Simplify:

$$\frac{ax - b + x - ab}{ax^2 - abx}, \qquad (x \neq 0; x \neq b)$$

SOLUTION

Step 1: Use grouping to factorise the numerator and take out the common factor ax in the denominator

$$\frac{(ax - ab) + (x - b)}{ax^2 - abx} = \frac{a(x-b) + (x-b)}{ax(x-b)}$$

Step 2: Take out common factor $(x - b)$ in the numerator

$$= \frac{(x-b)(a+1)}{ax(x-b)}$$

Step 3: Cancel the common factor in the numerator and the denominator to give the final answer

$$= \frac{a+1}{ax}$$

Worked example 19: Simplifying fractions

QUESTION

Simplify:

$$\frac{x^2 - x - 2}{x^2 - 4} \div \frac{x^2 + x}{x^2 + 2x}, \quad (x \neq 0; x \neq \pm 2)$$

SOLUTION

Step 1: Factorise the numerator and denominator

$$= \frac{(x+1)(x-2)}{(x+2)(x-2)} \div \frac{x(x+1)}{x(x+2)}$$

Step 2: Change the division sign and multiply by the reciprocal

$$= \frac{(x+1)(x-2)}{(x+2)(x-2)} \times \frac{x(x+2)}{x(x+1)}$$

Step 3: Write the final answer

$$= 1$$

Worked example 20: Simplifying fractions

QUESTION

Simplify:

$$\frac{x-2}{x^2-4} + \frac{x^2}{x-2} - \frac{x^3+x-4}{x^2-4}, \quad (x \neq \pm 2)$$

SOLUTION

Step 1: Factorise the denominators

$$\frac{x-2}{(x+2)(x-2)} + \frac{x^2}{x-2} - \frac{x^3+x-4}{(x+2)(x-2)}$$

Step 2: Make all denominators the same so that we can add or subtract the fractions

The lowest common denominator is $(x-2)(x+2)$.

$$\frac{x-2}{(x+2)(x-2)}+\frac{\left(x^{2}\right)(x+2)}{(x+2)(x-2)}-\frac{x^{3}+x-4}{(x+2)(x-2)}$$

Step 3: Write as one fraction

$$\frac{x-2+\left(x^{2}\right)(x+2)-\left(x^{3}+x-4\right)}{(x+2)(x-2)}$$

Step 4: Simplify

$$\frac{x-2+x^{3}+2x^{2}-x^{3}-x+4}{(x+2)(x-2)}=\frac{2x^{2}+2}{(x+2)(x-2)}$$

Step 5: Take out the common factor and write the final answer

$$\frac{2\left(x^{2}+1\right)}{(x+2)(x-2)}$$

Worked example 21: Simplifying fractions

QUESTION

Simplify:

$$\frac{2}{x^{2}-x}+\frac{x^{2}+x+1}{x^{3}-1}-\frac{x}{x^{2}-1},\quad (x\neq 0;x\neq \pm 1)$$

SOLUTION

Step 1: Factorise the numerator and denominator

$$\frac{2}{x(x-1)}+\frac{\left(x^{2}+x+1\right)}{(x-1)\left(x^{2}+x+1\right)}-\frac{x}{(x-1)(x+1)}$$

Step 2: Simplify and find the common denominator

$$\frac{2(x+1)+x(x+1)-x^{2}}{x(x-1)(x+1)}$$

Step 3: Write the final answer

$$\frac{2x+2+x^{2}+x-x^{2}}{x(x-1)(x+1)}=\frac{3x+2}{x(x-1)(x+1)}$$

Exercise 1 – 10:

1. Simplify (assume all denominators are non-zero):

a) $\frac{3a}{15}$
b) $\frac{2a+10}{4}$
c) $\frac{5a+20}{a+4}$
d) $\frac{a^2-4a}{a-4}$
e) $\frac{3a^2-9a}{2a-6}$
f) $\frac{9a+27}{9a+18}$
g) $\frac{6ab+2a}{2b}$
h) $\frac{16x^2y-8xy}{12x-6}$
i) $\frac{4xyp-8xp}{12xy}$
j) $\frac{9x^2-16}{6x-8}$
k) $\frac{b^2-81a^2}{18a-2b}$
l) $\frac{t^2-s^2}{s^2-2st+t^2}$
m) $\frac{x^2-2x-15}{5x-25}$
n) $\frac{x^2+2x-15}{x^2+8x+15}$
o) $\frac{x^2-x-6}{x^3-27}$
p) $\frac{a^2+6a-16}{a^3-8}$
q) $\frac{a^2-4ab-12b^2}{a^2+4ab+4b^2}$
r) $\frac{6a^2-7a-3}{3ab+b}$
s) $\frac{2x^2-x-1}{x^3-x}$
t) $\frac{qz+qr+16z+16r}{z+r}$
u) $\frac{pz-pq+5z-5q}{z-q}$
v) $\frac{hx-hg+13x-13g}{x-g}$
w) $\frac{f^2a-fa^2}{f-a}$

2. Simplify (assume all denominators are non-zero):

a) $\frac{b^2+10b+21}{3(b^2-9)} \div \frac{2b^2+14b}{30b^2-90b}$
b) $\frac{x^2+17x+70}{5(x^2-100)} \div \frac{3x^2+21x}{45x^2-450x}$
c) $\frac{z^2+17z+66}{3(z^2-121)} \div \frac{2z^2+12z}{24z^2-264z}$
d) $\frac{3a+9}{14} \div \frac{7a+21}{a+3}$
e) $\frac{a^2-5a}{2a+10} \times \frac{4a}{3a+15}$
f) $\frac{3xp+4p}{8p} \div \frac{12p^2}{3x+4}$
g) $\frac{24a-8}{12} \div \frac{9a-3}{6}$
h) $\frac{a^2+2a}{5} \div \frac{2a+4}{20}$
i) $\frac{p^2+pq}{7p} \times \frac{21q}{8p+8q}$
j) $\frac{5ab-15b}{4a-12} \div \frac{6b^2}{a+b}$
k) $\frac{16-x^2}{x^2-x-12} \times \frac{x+3}{x+4}$
l) $\frac{a^3+b^3}{a^3} \times \frac{5a+5b}{a^2+2ab+b^2}$
m) $\frac{a-4}{a+5a+4} \times \frac{a^2+2a+1}{a^2-3a-4}$
n) $\frac{3x+2}{x^2-6x+8} \times \frac{x-2}{3x^2+8x+4}$
o) $\frac{a^2-2a+8}{a^2+6a+8} \times \frac{a^2+a-12}{3} - \frac{3}{2}$
p) $\frac{4x^2-1}{3x^2+10x+3} \div \frac{6x^2+5x+1}{4x^2+7x-3} \times \frac{9x^2+6x+1}{8x^2-6x+1}$
q) $\frac{x+4}{3} - \frac{x-2}{2}$
r) $\frac{p^3+q^3}{p^2} \times \frac{3p-3q}{p^2-q^2}$

3. Simplify (assume all denominators are non-zero):

a) $\frac{x-3}{3} - \frac{x+5}{4}$
b) $\frac{2x-4}{9} - \frac{x-3}{4} + 1$
c) $1 + \frac{3x-4}{4} - \frac{x+2}{3}$
d) $\frac{11}{a+11} + \frac{8}{a-8}$
e) $\frac{12}{x-12} - \frac{6}{x-6}$
f) $\frac{12}{r+12} + \frac{8}{r-8}$

g) $\dfrac{2}{xy} + \dfrac{4}{xz} + \dfrac{3}{yz}$

h) $\dfrac{5}{t-2} - \dfrac{1}{t-3}$

i) $\dfrac{k+2}{k^2+2} - \dfrac{1}{k+2}$

j) $\dfrac{t+2}{3q} + \dfrac{t+1}{2q}$

k) $\dfrac{3}{p^2-4} + \dfrac{2}{(p-2)^2}$

l) $\dfrac{x}{x+y} + \dfrac{x^2}{y^2-x^2}$

m) $\dfrac{1}{m+n} + \dfrac{3mn}{m^3+n^3}$

n) $\dfrac{h}{h^3-f^3} - \dfrac{1}{h^2+hf+f^2}$

o) $\dfrac{x^2-1}{3} \times \dfrac{1}{x-1} - \dfrac{1}{2}$

p) $\dfrac{x^2-2x+1}{(x-1)^3} - \dfrac{x^2+x+1}{x^3-1}$

q) $\dfrac{1}{(x-1)^2} - \dfrac{2x}{x^3-1}$

r) $\dfrac{t^2+2t-8}{t^2+t-6} + \dfrac{1}{t^2-9} + \dfrac{t+1}{t-3}$

s) $\dfrac{x^2-3x+9}{x^3+27} + \dfrac{x-2}{x^2+4x+3} - \dfrac{1}{x-2}$

t) $\dfrac{1}{a^2-4ab+4b^2} + \dfrac{a^2+2ab+b^2}{a^3-8b^3} - \dfrac{1}{a^2-4b^2}$

4. What are the restrictions in the following:

a) $\dfrac{1}{x-2}$

b) $\dfrac{3x-9}{4x+4}$

c) $\dfrac{3}{x} - \dfrac{1}{x^2-1}$

For more exercises, visit www.everythingmaths.co.za and click on 'Practise Maths'.

1a. 2DNW	1b. 2DNX	1c. 2DNY	1d. 2DNZ	1e. 2DP2	1f. 2DP3	1g. 2DP4	1h. 2DP5
1i. 2DP6	1j. 2DP7	1k. 2DP8	1l. 2DP9	1m. 2DPB	1n. 2DPC	1o. 2DPD	1p. 2DPF
1q. 2DPG	1r. 2DPH	1s. 2DPJ	1t. 2DPK	1u. 2DPM	1v. 2DPN	1w. 2DPP	2a. 2DPQ
2b. 2DPR	2c. 2DPS	2d. 2DPT	2e. 2DPV	2f. 2DPW	2g. 2DPX	2h. 2DPY	2i. 2DPZ
2j. 2DQ2	2k. 2DQ3	2l. 2DQ4	2m. 2DQ5	2n. 2DQ6	2o. 2DQ7	2p. 2DQ8	2q. 2DQ9
2r. 2DQB	3a. 2DQC	3b. 2DQD	3c. 2DQF	3d. 2DQG	3e. 2DQH	3f. 2DQJ	3g. 2DQK
3h. 2DQM	3i. 2DQN	3j. 2DQP	3k. 2DQQ	3l. 2DQR	3m. 2DQS	3n. 2DQT	3o. 2DQV
3p. 2DQW	3q. 2DQX	3r. 2DQY	3s. 2DQZ	3t. 2DR2	4a. 2DR3	4b. 2DR4	4c. 2DR5

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1.9 Chapter summary EMAR

See presentation: 2DR6 at www.everythingmaths.co.za

- - $\mathbb{N}$: natural numbers are $\{1; 2; 3; \ldots\}$
  - $\mathbb{N}_0$: whole numbers are $\{0; 1; 2; 3; \ldots\}$
  - $\mathbb{Z}$: integers are $\{\ldots; -3; -2; -1; 0; 1; 2; 3; \ldots\}$
- A rational number is any number that can be written as $\frac{a}{b}$ where a and b are integers and $b \neq 0$.
- The following are rational numbers:
  - Fractions with both numerator and denominator as integers
  - Integers
  - Decimal numbers that terminate
  - Decimal numbers that recur (repeat)

- Irrational numbers are numbers that cannot be written as a fraction with the numerator and denominator as integers.
- If the n^{th} root of a number cannot be simplified to a rational number, it is called a surd.
- If a and b are positive whole numbers, and $a < b$, then $\sqrt[n]{a} < \sqrt[n]{b}$.
- A binomial is an expression with two terms.
- The product of two identical binomials is known as the square of the binomial.
- We get the difference of two squares when we multiply $(ax + b)(ax - b)$
- Factorising is the opposite process of expanding the brackets.
- The product of a binomial and a trinomial is:

$$(A + B)(C + D + E) = A(C + D + E) + B(C + D + E)$$

- Taking out a common factor is the basic factorisation method.
- We often need to use grouping to factorise polynomials.
- To factorise a quadratic we find the two binomials that were multiplied together to give the quadratic.
- The sum of two cubes can be factorised as:

$$x^3 + y^3 = (x + y)\left(x^2 - xy + y^2\right)$$

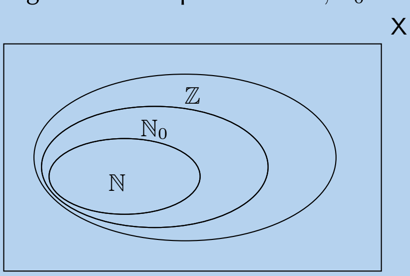
- The difference of two cubes can be factorised as:

$$x^3 - y^3 = (x - y)\left(x^2 + xy + y^2\right)$$

- We can simplify fractions by incorporating the methods we have learnt to factorise expressions.
- Only factors can be cancelled out in fractions, never terms.
- To add or subtract fractions, the denominators of all the fractions must be the same.

End of chapter Exercise 1 – 11:

1. The figure here shows the Venn diagram for the special sets $\mathbb{N}, \mathbb{N}_0$ and $\mathbb{Z}$.



   a) Where does the number 2,13 belong in the diagram?
   b) In the following list, there are two false statements and one true statement. Which of the statements is **true**?
      - Every natural number is an integer.
      - Every whole number is a natural number.
      - There are fractions in the integers.

2. State whether the following numbers are real, non-real or undefined.

   a) $-\sqrt{-5}$ b) $\frac{\sqrt{8}}{0}$ c) $-\sqrt{15}$ d) $-\sqrt{7}$ e) $\sqrt{-1}$ f) $\sqrt{2}$

3. State whether each of the following numbers are rational or irrational.
   a) $\sqrt[3]{4}$ b) 45π c) $\sqrt{9}$ d) $\sqrt[3]{8}$
4. If a is an integer, b is an integer and c is irrational, which of the following are rational numbers?
   a) $\frac{-b}{a}$ b) $c \div c$ c) $\frac{a}{c}$ d) $\frac{1}{c}$
5. Consider the following list of numbers:

$$\sqrt[3]{26}\ ;\ \frac{3}{2}\ ;\ \sqrt{-24}\ ;\ \sqrt{39}\ ;\ 7{,}\dot{1}\dot{1}\ ;\ \pi^2\ ;\ \frac{\pi}{2}\ ;\ 7{,}12\ ;\ -\sqrt{24}\ ;\ \frac{\sqrt{2}}{0}\ ;\ 3\pi\ ;\ \sqrt{78}\ ;\ 9\ ;\ \pi$$

   a) Which of the numbers are non-real numbers?
   b) Without using a calculator, rank all the real numbers in ascending order.
   c) Which of the numbers are irrational numbers?
   d) Which of the numbers are rational numbers?
   e) Which of the numbers are integers?
   f) Which of the numbers are undefined?
6. Write each decimal as a simple fraction.
   a) 0,12 b) 0,006 c) $4{,}\overline{14}$ d) 1,59
   e) $12{,}27\dot{7}$ f) $0{,}8\dot{2}$ g) $7{,}\overline{36}$
7. Show that the decimal $3{,}21\dot{1}\dot{8}$ is a rational number.
8. Write the following fractions as decimal numbers:
   a) $\frac{1}{18}$ b) $1\frac{1}{2}$
9. Express $0{,}\overline{78}$ as a fraction $\frac{a}{b}$ where $a, b \in \mathbb{Z}$ (show all working).
10. For each of the following numbers:
    - write the next three digits;
    - state whether the number is rational or irrational.

    a) 1,11235...
    b) $1{,}\dot{1}$
11. Write the following rational numbers to 2 decimal places.
    a) $\frac{1}{2}$ b) 1 c) $0{,}11111\bar{1}$ d) $0{,}99999\bar{1}$
12. Round off the following irrational numbers to 3 decimal places.
    a) 3,141592654... b) 1,618033989...
    c) 1,41421356... d) 2,71828182845904523536...
13. Round off the number 1523,00195593 to 4 decimal places.
14. Round off the number 1982,94028996 to 6 decimal places.
15. Round off the number 101,52378984 to 4 decimal places.
16. Use your calculator and write the following irrational numbers to 3 decimal places.
    a) $\sqrt{2}$ b) $\sqrt{3}$ c) $\sqrt{5}$ d) $\sqrt{6}$
17. Use your calculator (where necessary) and write the following numbers to 5 decimal places. State whether the numbers are irrational or rational.
    a) $\sqrt{8}$ b) $\sqrt{768}$ c) $\sqrt{0{,}49}$ d) $\sqrt{0{,}0016}$ e) $\sqrt{0{,}25}$
    f) $\sqrt{36}$ g) $\sqrt{1960}$ h) $\sqrt{0{,}0036}$ i) $-8\sqrt{0{,}04}$ j) $5\sqrt{80}$
18. Round off:
    a) $\frac{\sqrt{2}}{2}$ to the nearest 2 decimal places.
    b) $\sqrt{14}$ to the nearest 3 decimal places.

19. Write the following irrational numbers to 3 decimal places and then write each one as a rational number to get an approximation of the irrational number.
    a) 3,141592654...
    b) 1,618033989...
    c) 1,41421356...
    d) 2,71828182845904523536...
20. Determine between which two consecutive integers the following irrational numbers lie, without using a calculator.
    a) $\sqrt{5}$
    b) $\sqrt{10}$
    c) $\sqrt{20}$
    d) $\sqrt{30}$
    e) $\sqrt[3]{5}$
    f) $\sqrt[3]{10}$
    g) $\sqrt[3]{20}$
    h) $\sqrt[3]{30}$
    i) $\sqrt{90}$
    j) $\sqrt{72}$
    k) $\sqrt[3]{58}$
    l) $\sqrt[3]{118}$
21. Estimate the following surds to the nearest 1 decimal place, without using a calculator.
    a) $\sqrt{14}$
    b) $\sqrt{110}$
    c) $\sqrt{48}$
    d) $\sqrt{57}$
22. Expand the following products:
    a) $(a+5)^2$
    b) $(n+12)^2$
    c) $(d-4)^2$
    d) $(7w+2)(7w-2)$
    e) $(12q+1)(12q-1)$
    f) $-(-x-2)(x+2)$
    g) $(5k-4)(5k+4)$
    h) $(5f+4)(2f+2)$
    i) $(3n+6)(6n+5)$
    j) $(2g+6)(g+6)$
    k) $(4y+1)(4y+8)$
    l) $(d-3)(7d+2)$
    m) $(6z-4)(z-2)$
    n) $(5w-11)^2$
    o) $(5s-1)^2$
    p) $(3d-8)^2$
    q) $5f^2(3f+5)+7f(3f^2+7)$
    r) $8d(4d^3+2)+6d^2(7d^2+4)$
    s) $5x^2(2x+2)+7x(7x^2+7)$
23. Expand the following:
    a) $(y^4+3y^2+y)(y+1)(y-2)$
    b) $(x+1)^2-(x-1)^2$
    c) $(x^2+2x+1)(x^2-2x+1)$
    d) $(4a-3b)(16a^2+12ab+9b^2)$
    e) $2(x+3y)(x^2-xy-y^2)$
    f) $(3a-5b)(3a+5b)(a^2+ab-b^2)$
    g) $\left(y-\frac{1}{y}\right)\left(y+\frac{1}{y}\right)$
    h) $\left(\frac{a}{3}-\frac{3}{a}\right)\left(\frac{a}{3}+\frac{3}{a}\right)$
    i) $\frac{1}{3}(12x-9y)+\frac{1}{6}(12x+18y)$
    j) $(x+2)(x-2)-(x+2)^2$
24. What is the value of e in $(x-4)(x+e)=x^2-16$?
25. In $(x+2)(x+k)=x^2+bx+c$:
    a) For which of these values of k will b be positive?
       $-6\ ;\ -1\ ;\ 0\ ;\ 1\ ;\ 6$
    b) For which of these values of k will c be positive?
       $-6\ ;\ -1\ ;\ 0\ ;\ 1\ ;\ 6$
    c) For what values of k will c be positive?
    d) For what values of k will b be positive?
26. Answer the following:
    a) Expand: $\left(3a-\frac{1}{2a}\right)^2$
    b) Expand: $\left(3a-\frac{1}{2a}\right)\left(9a^2+\frac{3}{2}+\frac{1}{4a^2}\right)$
    c) Given that $3a-\frac{1}{2a}=7$, determine the value of $27a^3-\frac{1}{8a^3}$ without solving for a.

27. Solve by factorising:

a) $17^2 - 15^2$ b) $13^2 - 12^2$ c) $120045^2 - 120035^2$ d) $26^2 - 24^2$

28. Represent the following as a product of its prime factors:

a) 143 b) 168 c) 899 d) 99 e) 1599

29. Factorise:

a) $a^2 - 9$
b) $9b^2 - 81$
c) $m^2 - \frac{1}{9}$
d) $5 - 5a^2b^6$
e) $16ba^4 - 81b$
f) $a^2 - 10a + 25$
g) $16b^2 + 56b + 49$
h) $-4b^2 - 144b^8 + 48b^5$
i) $16 - x^4$
j) $7x^2 - 14x + 7xy - 14y$
k) $y^2 - 7y - 30$
l) $1 - x - x^2 + x^3$
m) $-3(1 - p^2) + p + 1$
n) $x^2 - 2x + 1 - y^4$
o) $4b(x^3 - 1) + x(1 - x^3)$
p) $3m(v - 7) + 19(-7 + v)$
q) $3f(z + 3) + 19(3 + z)$
r) $3p^3 - \frac{1}{9}$
s) $8x^6 - 125y^9$
t) $(2 + p)^3 - 8(p + 1)^3$
u) $\dfrac{1}{3}a^3 - a^2b + 2a^2b - 6ab^2 + 3ab^2 - 9b^3$
v) $6a^2 - 17a + 5$
w) $s^2 + 2s - 15$
x) $16v + 24h + 2j^5v + 3j^5h$
y) $18h - 45g + 2m^3h - 5m^3g$
z) $63d - 18s + 7u^2d - 2u^2s$

30. Factorise the following:

a) $6a^2 + 14a + 8$
b) $6g^2 - 15g - 9$
c) $125g^3 - r^3$
d) $8r^3 + z^3$
e) $14m - 4n + 7jm - 2jn$
f) $25d - 15m + 5yd - 3ym$
g) $g^3 - 27$
h) $z^3 + 125$
i) $b^2 - (3a - 2b)^2$
j) $9y^2 - (4x + 2y)^2$
k) $16x^6 - 3y^8$
l) $\dfrac{1}{6}a^2 - 24b^4$
m) $4(a - 3) - 81x^2(a - 3)$
n) $(2 + b)^2 - 11(2 + b) - 12$
o) $2x^2 + 7xy + 5y^2$
p) $x^2 - 2xy - 15y^2$
q) $4x^4 + 11x^2 + 6$
r) $6x^4 - 38x^2 + 40$
s) $9a^2x + 9a^2y + 27a^2 - b^2x - b^2y - 3b^2$
t) $2(2y^2 - 5y) - 24$
u) $\dfrac{1}{2}x^3 - \frac{9}{2}x - 2x^2 + 18$
v) $27r^3s^3 - 1$
w) $\dfrac{1}{125h^3} + r^3$
x) $j(64n^3 - b^3) + k(64n^3 - b^3)$

31. Simplify the following:

a) $(a - 2)^2 - a(a + 4)$
b) $(5a - 4b)(25a^2 + 20ab + 16b^2)$
c) $(2m - 3)(4m^2 + 9)(2m + 3)$
d) $(a + 2b - c)(a + 2b + c)$

e) $\frac{m^2+11m+18}{4(m^2-4)} \div \frac{3m^2+27m}{24m^2-48m}$

f) $\frac{t^2+9t+18}{5(t^2-9)} \div \frac{4t^2+24t}{100t^2-300t}$

g) $\frac{4-b^2}{3b-6}$

h) $\frac{x^2+2x+4}{x^3-8}$

i) $\frac{x^2-5x-14}{3x+6}$

j) $\frac{d^2+23d+132}{5(d^2-121)} \div \frac{4d^2+48d}{100d^2-1100d}$

k) $\frac{a-2}{a^2+4a+3} \div \frac{(a-1)(a+1)}{a-1} \times \frac{a^2-2a-15}{a-2}$

l) $\frac{a+6}{a^2+12a+11} \times \frac{a^2+14a+33}{a+3} \div \frac{a^3+216}{a+1}$

m) $2 \div \frac{a+b}{a+2b} \times \frac{b^2-ba-6a^2}{a^2-4b^2} \times \frac{a^2-b-2b^2}{3a-b}$

n) $\frac{st+sb+31t+31b}{t+b}$

o) $\frac{ny+nq+8y+8q}{y+q}$

p) $\frac{p^2-q^2}{p} \div \frac{p+q}{p^2-pq}$

q) $\frac{2}{x}+\frac{x}{2}-\frac{2x}{3}$

r) $\frac{1}{a+7}-\frac{a+7}{a^2-49}$

s) $\frac{x+2}{2x^3}+16$

t) $\frac{1-2a}{4a^2-1}-\frac{a-1}{2a^2-3a+1}-\frac{1}{1-a}$

u) $\frac{1}{2}x+\frac{x-2}{3}+4$

v) $\frac{1}{x^2+2x}+\frac{4x^2-x-3}{x^2+2x-3}$

w) $\frac{b^2+6b+9}{b^2-9}+\frac{b^2-6b+8}{(b-2)(b+3)}+\frac{1}{b+3}$

x) $\frac{x^2+2x}{x^2+x+6} \times \frac{x^2+2x+1}{x^2+3x+2}$

y) $\frac{12}{z+12}+\frac{5}{z-5}$

z) $\frac{11}{w-11}-\frac{4}{w-4}$

32. Show that $(2x-1)^2-(x-3)^2$ can be simplified to $(x+2)(3x-4)$.
33. What must be added to x^2-x+4 to make it equal to $(x+2)^2$?
34. Evaluate $\frac{x^3+1}{x^2-x+1}$ if $x = 7{,}85$ without using a calculator. Show your work.
35. With what expression must $(a-2b)$ be multiplied to get a product of (a^3-8b^3)?
36. With what expression must $27x^3+1$ be divided to get a quotient of $3x+1$?
37. What are the restrictions on the following?

a) $\frac{4}{3x^2+2x-1}$

b) $\frac{a}{3(b-a)+ab-a^2}$

For more exercises, visit www.everythingmaths.co.za and click on 'Practise Maths'.

1. 2DR7	2a. 2DR8	2b. 2DR9	2c. 2DRB	2d. 2DRC	2e. 2DRD	2f. 2DRF
3a. 2DRG	3b. 2DRH	3c. 2DRJ	3d. 2DRK	4. 2DRM	5. 2DRN	6a. 2DRP
6b. 2DRQ	6c. 2DRR	6d. 2DRS	6e. 2DRT	6f. 2DRV	6g. 2DRW	7. 2DRX
8a. 2DRY	8b. 2DRZ	9. 2DS2	10a. 2DS3	10b. 2DS4	11a. 2DS5	11b. 2DS6
11c. 2DS7	11d. 2DS8	12a. 2DS9	12b. 2DSB	12c. 2DSC	12d. 2DSD	13. 2DSF
14. 2DSG	15. 2DSH	16a. 2DSJ	16b. 2DSK	16c. 2DSM	16d. 2DSN	17a. 2DSP
17b. 2DSQ	17c. 2DSR	17d. 2DSS	17e. 2DST	17f. 2DSV	17g. 2DSW	17h. 2DSX
17i. 2DSY	17j. 2DSZ	18a. 2DT2	18b. 2DT3	19a. 2DT4	19b. 2DT5	19c. 2DT6
19d. 2DT7	20a. 2DT8	20b. 2DT9	20c. 2DTB	20d. 2DTC	20e. 2DTD	20f. 2DTF
20g. 2DTG	20h. 2DTH	20i. 2DTJ	20j. 2DTK	20k. 2DTM	20l. 2DTN	21a. 2DTP
21b. 2DTQ	21c. 2DTR	21d. 2DTS	22a. 2DTT	22b. 2DTW	22c. 2DTX	22d. 2DTY
22e. 2DTZ	22f. 2DV2	22g. 2DV3	22h. 2DV4	22i. 2DV5	22j. 2DV6	22k. 2DV7
22l. 2DV8	22m. 2DV9	22n. 2DVB	22o. 2DVC	22p. 2DVD	22q. 2DVF	22r. 2DVG
22s. 2DVH	23a. 2DVJ	23b. 2DVK	23c. 2DVM	23d. 2DVN	23e. 2DVP	23f. 2DVQ
23g. 2DVR	23h. 2DVS	23i. 2DVT	23j. 2DVV	24. 2DVW	25. 2DVX	26. 2DVY

27a. 2DVZ	27b. 2DW2	27c. 2DW3	27d. 2DW4	28a. 2DW5	28b. 2DW6	28c. 2DW7
28d. 2DW8	28e. 2DW9	29a. 2DWB	29b. 2DWC	29c. 2DWD	29d. 2DWF	29e. 2DWG
29f. 2DWH	29g. 2DWJ	29h. 2DWK	29i. 2DWM	29j. 2DWN	29k. 2DWP	29l. 2DWQ
29m. 2DWR	29n. 2DWS	29o. 2DWT	29p. 2DWV	29q. 2DWW	29r. 2DWX	29s. 2DWY
29t. 2DWZ	29u. 2DX2	29v. 2DX3	29w. 2DX4	29x. 2DX5	29y. 2DX6	29z. 2DX7
30a. 2DX8	30b. 2DX9	30c. 2DXB	30d. 2DXC	30e. 2DXD	30f. 2DXF	30g. 2DXG
30h. 2DXH	30i. 2DXJ	30j. 2DXK	30k. 2DXM	30l. 2DXN	30m. 2DXP	30n. 2DXQ
30o. 2DXR	30p. 2DXS	30q. 2DXT	30r. 2DXV	30s. 2DXW	30t. 2DXX	30u. 2DXY
30v. 2DXZ	30w. 2DY2	30x. 2DY3	31a. 2DY4	31b. 2DY5	31c. 2DY6	31d. 2DY7
31e. 2DY8	31f. 2DY9	31g. 2DYB	31h. 2DYC	31i. 2DYD	31j. 2DYF	31k. 2DYG
31l. 2DYH	31m. 2DYJ	31n. 2DYK	31o. 2DYM	31p. 2DYN	31q. 2DYP	31r. 2DYQ
31s. 2DYR	31t. 2DYS	31u. 2DYT	31v. 2DYV	31w. 2DYW	31x. 2DYX	31y. 2DYY
31z. 2DYZ	32. 2DZ2	33. 2DZ3	34. 2DZ4	35. 2DZ5	36. 2DZ6	37a. 2DZ7
37b. 2DZ8						

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