CHAPTER 7

Euclidean geometry

- Content covered in this chapter includes revision of lines, angles and triangles. The mid-point theorem is introduced. Kites, parallelograms, rectangle, rhombus, square and trapezium are investigated.
- Solving problems and proving riders is only covered later in the year. The focus of this chapter is on introducing the special quadrilaterals and revising content from earlier grades.
- Revision of triangles should focus on similar and congruent triangles.
- Sketches are valuable and important tools. Encourage learners to draw accurate diagrams to solve problems.
- It is important to stress to learners that proportion gives no indication of actual length. It only indicates the ratio between lengths.
- Notation - emphasise to learners the importance of the correct ordering of letters, as this indicates which angles are equal and which sides are in the same proportion.

GeoGebra is a useful tool to use for sketching out the worked examples and activities.

7.1 Introduction

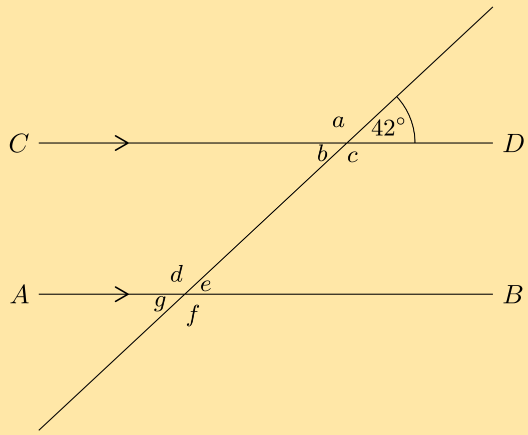
Angles

Properties and notation

Parallel lines and transversal lines

Exercise 7 – 1:

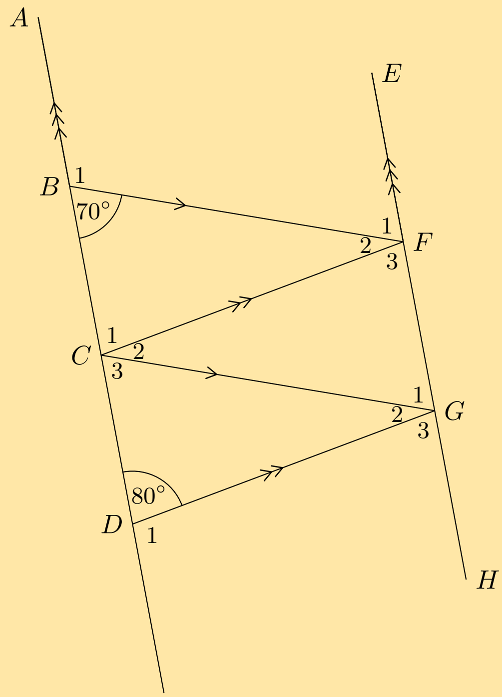
1. Use adjacent, corresponding, co-interior and alternate angles to fill in all the angles labelled with letters in the diagram:



Solution:
You can redraw the diagram and fill in the angles as you find them.

$$\begin{aligned}
a &= 180° - 42° = 138° && (\angle\text{s on a str line})\\
b &= 42° && (\text{vert opp } \angle\text{s } =)\\
c &= 138° && (\text{vert opp } \angle\text{s } =)\\
d &= 138° && (\text{co-int } \angle\text{s; } AB \parallel CD)\\
e &= 180° - 138° = 42° && (\angle\text{s on a str line})\\
f &= 138° && (\text{vert opp } \angle\text{s } =)\\
g &= 42° && (\text{vert opp } \angle\text{s } =)
\end{aligned}$$

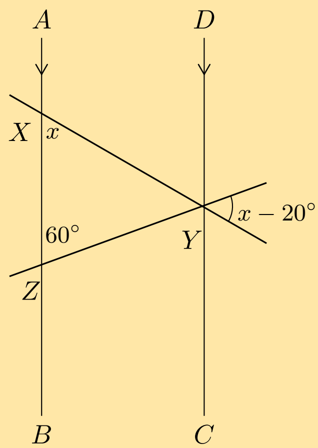
2. Find all the unknown angles in the figure:



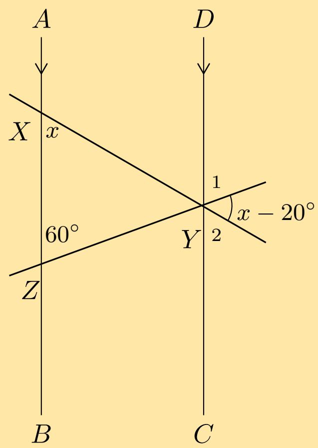
Solution:

$$\begin{array}{lll}
\hat{B}_1 & = 180° - 70° = 110° & (\angle\text{s on a str line}) \\
\hat{D}_1 & = 180° - 80° = 100° & (\angle\text{s on a str line}) \\
\hat{F}_1 & = 70° & (\text{co-int } \angle\text{s; } AD \parallel EH) \\
\hat{G}_3 & = 80° & (\text{co-int } \angle\text{s; } AD \parallel EH) \\
\hat{C}_3 & = 70° & (\text{corresp } \angle\text{s; } BF \parallel CG) \\
\hat{G}_1 & = 70° & (\text{corresp } \angle\text{s; } BF \parallel CG) \\
\hat{G}_2 & = 180° - 70° - 80° = 30° & (\angle\text{s on a str line}) \\
\hat{C}_2 & = 30° & (\text{alt } \angle\text{s; } CF \parallel DG) \\
\hat{F}_2 & = 30° & (\text{alt } \angle\text{s; } BF \parallel CG) \\
\hat{F}_3 & = 80° & (\text{sum of } \angle\text{'s str. line}) \\
\hat{C}_1 & = 80° & (\angle\text{s on a str line})
\end{array}$$

3. Find the value of x in the figure:



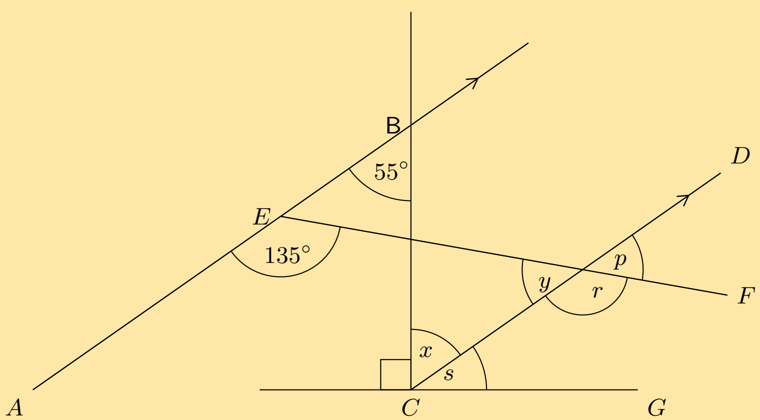
Solution:



$\hat{Y}_1 = 60°$ (corresp $\angle$s; $AB \parallel DC$).
$\hat{Y}_2 = x$ (corresp $\angle$s; $AB \parallel DC$).

$$\begin{aligned} \therefore x + 60° + (x - 20°) &= 180° \qquad (\angle\text{s on a str line}) \\ 2x &= 180° - 40° \\ 2x &= 140° \\ \therefore x &= 70° \end{aligned}$$

4. Find each of the unknown angles marked in the figure below. Find a reason that leads to the answer in a single step.



a) $\hat{x}$
Solution:
$\hat{x}$ and $A\hat{B}C$ are alternate interior angles on transversal BC. Therefore, they must be equal in size since $AB \parallel CD$.
Therefore $\hat{x} = 55°$.

b) $\hat{s}$
Solution:
We have just found that $\hat{x} = 55°$. $\hat{x} + \hat{s} + 90° = 180°$ ($\angle$s on a str line)

$$\begin{aligned} \hat{s} &= 90° - 55° \\ &= 35° \end{aligned}$$

c) $\hat{r}$
Solution:
$\angle A\hat{E}F$ and $\hat{r}$ are corresponding angles ($AB \parallel CD$).
Therefore: $\hat{r} = 135°$.

d) $\hat{y}$
Solution:
$\hat{r} + \hat{y} = 180°$ ($\angle$s on a str line)

$$\begin{aligned}\hat{y} &= 180° - 135° \\ &= 45°\end{aligned}$$

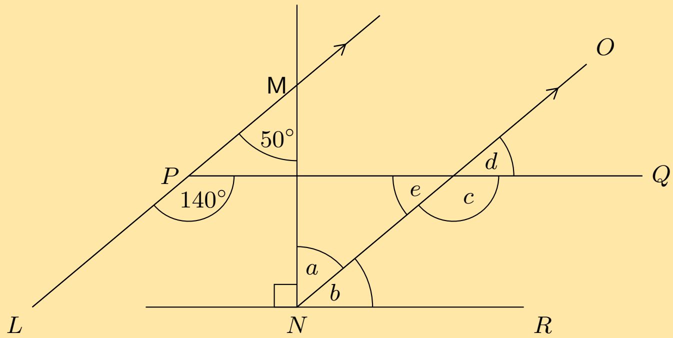
e) $\hat{p}$
Solution:
$\hat{p} = \hat{y}$ (vert opp $\angle$s $=$)
Therefore: $\hat{p} = 45°$.

f) Based on the results for the angles above, is $EF \parallel CG$?
Solution:
To prove $EF \parallel CG$ we need to show that one of the following is true:

- $\hat{s} = \hat{p}$ (corresp $\angle$s)
- $\hat{s} = \hat{y}$ (alt $\angle$s)
- $\hat{s} + \hat{r} = 180°$ (co-int $\angle$s)

However $\hat{s} \neq \hat{p}$, therefore EF is not parallel to CG.

5. Find each of the unknown angles marked in the figure below. Find a reason that leads to the answer in a single step.



a) $\hat{a}$
Solution:
$\hat{a}$ and $L\hat{M}N$ are alternate interior angles on transversal MN. Since $LM \parallel NO$ they must be equal in size.
Therefore $\hat{a} = 50°$.

b) $\hat{b}$
Solution:
We have just found that $\hat{a} = 50°$. $\hat{a} + \hat{b} + 90° = 180°$ ($\angle$s on a str line)

$$\begin{aligned}\hat{b} &= 90° - 50° \\ &= 40°\end{aligned}$$

c) $\hat{c}$
Solution:
$\angle L\hat{P}Q$ and $\hat{c}$ are corresponding angles ($LM \parallel NO$).
Therefore: $\hat{c} = 140°$.

d) $\hat{e}$
Solution:
$\hat{c} + \hat{e} = 180°$ ($\angle$s on a str line)

$$\begin{aligned}\hat{e} &= 180° - 140° \\ &= 40°\end{aligned}$$

e) $\hat{d}$

Solution:

$\hat{d} = \hat{e}$ (vert opp $\angle$s $=$)

Therefore: $\hat{d} = 40°$.

f) Based on the results for the angles above, is $PQ \parallel NR$?

Solution:

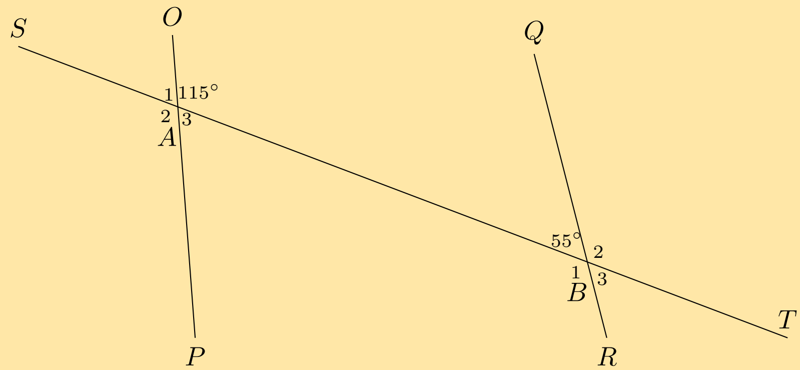
To prove $PQ \parallel NR$ we need to show that one of the following is true:

- $\hat{b} = \hat{d}$ (corresp $\angle$s)
- $\hat{b} = \hat{e}$ (alt $\angle$s)
- $\hat{b} + \hat{c} = 180°$ (co-int $\angle$s)

$\hat{b} = \hat{d}$ (corresp $\angle$s), therefore $PQ \parallel NR$. We also note that $\hat{b} = \hat{e}$ and $\hat{b} + \hat{c} = 180°$.

6. Determine whether the pairs of lines in the following figures are parallel:

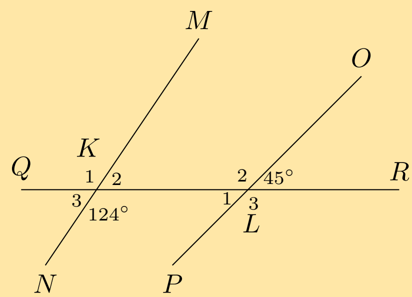
a)



Solution:

If $OP \parallel QR$ then $O\hat{A}B + Q\hat{B}A = 180°$ (co-int $\angle$s). But $O\hat{A}B + Q\hat{B}A = 115° + 55° = 170°$. Therefore there are no parallel lines, OP is not parallel to QR. Note that we do not consider ST as this is a transversal.

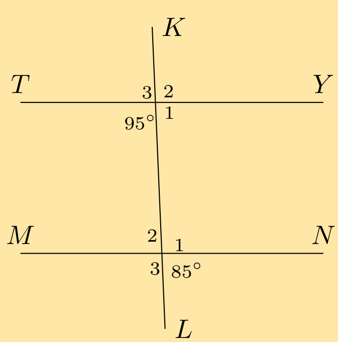
b)



Solution:

$K_2 = 180° - 124° = 56°$ ($\angle$s on a str line). If $MN \parallel OP$ then $\hat{K}_2$ would be equal to $\hat{L}$, $\therefore$ MN is not parallel to OP. Note that QR is a transversal.

c)

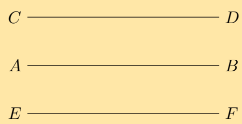


Solution:
Let U be point of intersection of lines KL and TY and V be the point of intersection of lines KL and MN.

$$\begin{aligned} \hat{U}_4 &= 95° \\ \hat{U}_1 &= 180° - 95° \qquad (\angle\text{s on a str line}) \\ \hat{U}_1 &= 85° \\ \hat{V}_4 &= 85° \qquad (\text{given}) \\ \therefore \hat{V}_4 &= \hat{U}_1 \end{aligned}$$

These are corresponding angles $\therefore TY \parallel MN$.

7. If AB is parallel to CD and AB is parallel to EF, explain why CD must be parallel to EF.



Solution:
If $a = 2$ and $b = a$ then we know that $b = 2$.
Similarly if $AB \parallel CD$ and $EF \parallel AB$ then we know that $EF \parallel CD$.

For more exercises, visit www.everythingmaths.co.za and click on 'Practise Maths'.
1. 2G5Y 2. 2G5Z 3. 2G62 4. 2G63 5. 2G64 6a. 2G65 6b. 2G66 6c. 2G67 7. 2G68

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7.2 Triangles

Classification of triangles

Congruency

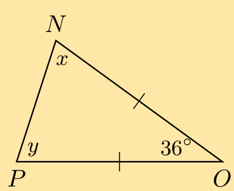
Similarity

The theorem of Pythagoras

Exercise 7 – 2:

1. Calculate the unknown variables in each of the following figures.

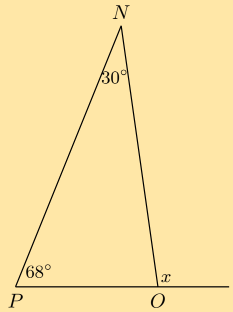
a)



Solution:
The triangle is isosceles therefore $x = y$ ($\angle$s opp equal sides).

$$\begin{aligned} 180° &= 36° - 2x && \text{(sum of } \angle\text{s in } \triangle) \\ 2x &= 144° \\ \therefore x &= 72° = y \end{aligned}$$

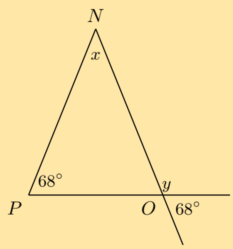
b)



Solution:
x is an exterior angle, therefore $P\hat{N}O + O\hat{P}N = x$ (ext $\angle$ of $\triangle$).

$$\begin{aligned} x &= 30° + 68° \\ &= 98° \end{aligned}$$

c)

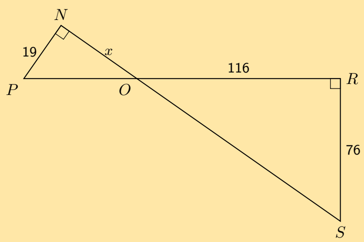


Solution:
First find y. $y + 68° = 180°$ ($\angle$s on a str line). Therefore $y = 112°$.
y is an exterior angle, therefore $P\hat{N}O + O\hat{P}N = y$ (ext $\angle$ of $\triangle$).

$$\begin{aligned} 112° &= x + 68° \\ x &= 112° - 68° \\ &= 44° \end{aligned}$$

Therefore $y = 112°$ and $x = 44°$.

d)



Solution:

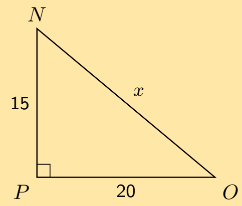
$$\begin{aligned} N\hat{P}O &= 180° - P\hat{N}O - N\hat{O}P \qquad \text{(sum of } \angle\text{s in } \triangle) \\ &= 180° - 90° - N\hat{O}P \\ &= 90° - N\hat{O}P \end{aligned}$$

$$\begin{aligned} R\hat{S}O &= 180° - O\hat{R}S - R\hat{O}S \qquad \text{(sum of } \angle\text{s in } \triangle) \\ &= 180° - 90° - R\hat{O}S \\ &= 90° - R\hat{O}S \end{aligned}$$

$N\hat{O}P = R\hat{O}S$ (vert opp $\angle$s).
$\therefore N\hat{P}O = R\hat{S}O$.
Therefore $\triangle NPO$ and $\triangle ROS$ are similar because they have the same angles.
Similar triangles have proportional sides:

$$\begin{aligned} \therefore \frac{NP}{RS} &= \frac{NO}{OR} \\ \frac{19}{76} &= \frac{x}{116} \\ \therefore x &= 29 \end{aligned}$$

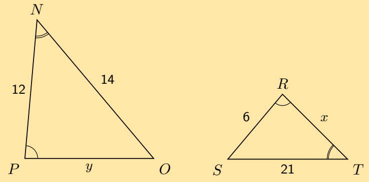
e)



Solution:
From the theorem of Pythagoras we have:

$$\begin{aligned} x^2 &= 15^2 + 20^2 \\ \therefore x &= \sqrt{625} \\ &= 25 \end{aligned}$$

f)



Solution:
We note that:

$$\begin{aligned} N\hat{P}O &= S\hat{R}T \qquad \text{(given)} \\ P\hat{N}O &= R\hat{T}S \qquad \text{(given)} \\ \therefore P\hat{N}O &= R\hat{T}S \qquad \text{(sum of} \angle\text{s in } \triangle) \\ \therefore \triangle NPO &\ ||| \ \triangle TSR \text{ (AAA)} \end{aligned}$$

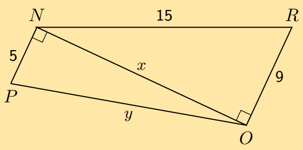
Now we can use the fact that the sides are in proportion to find x and y:

$$\begin{aligned}\frac{NO}{OP} &= \frac{TS}{TR}\\ \frac{14}{12} &= \frac{21}{x}\\ x &= \frac{21\times 12}{14}\\ &= 18\end{aligned}$$

$$\begin{aligned}\frac{OP}{NP} &= \frac{SR}{TR}\\ \frac{y}{12} &= \frac{6}{18}\\ 18y &= 72\\ y &= 4\end{aligned}$$

Therefore $x = 18$ and $y = 4$.

g)



Solution:
From the theorem of Pythagoras:

$$\begin{aligned}x^2 &= 15^2 - 9^2\\ x &= \sqrt{144}\\ &= 12\end{aligned}$$

$$\begin{aligned}y^2 &= x^2 + 5^2\\ y^2 &= 144 + 25\\ y &= \sqrt{169}\\ y &= 13\end{aligned}$$

Therefore $x = 12$ and $y = 13$.

2. Given the following diagrams:
Diagram A

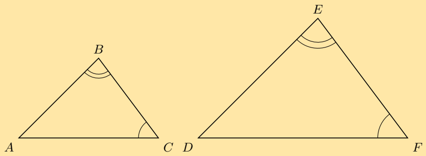
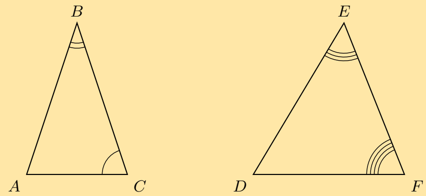


Diagram B



Which diagram correctly gives a pair of similar triangles?

Solution:

Diagram A shows a pair of triangles with all pairs of corresponding angles equal (the same three angle markers are shown in both triangles). Diagram B shows a pair of triangles with different angles in each triangle. All six angles are different and there are no pairs of corresponding angles that are equal.

Therefore diagram A gives a pair of triangles that are similar.

3. Given the following diagrams:

   Diagram A

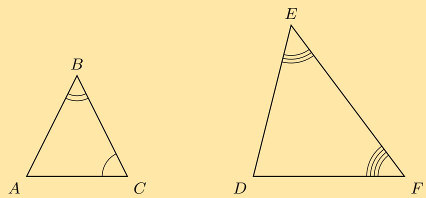
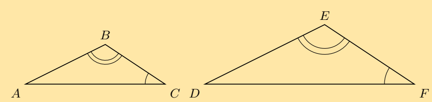


   Diagram B



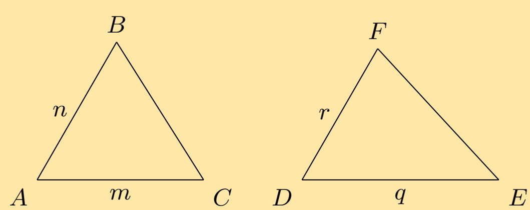
   Which diagram correctly gives a pair of similar triangles?

   Solution:

   Diagram A shows a pair of triangles with different angles in each triangle. All six angles are different and there are no pairs of corresponding angles that are equal. Diagram B shows a pair of triangles with all pairs of corresponding angles equal (the same two angle markers are shown in both triangles and the third angle in each triangle must be equal).

   Therefore diagram B gives a pair of triangles that are similar.

4. Have a look at the following triangles, which are drawn to scale:

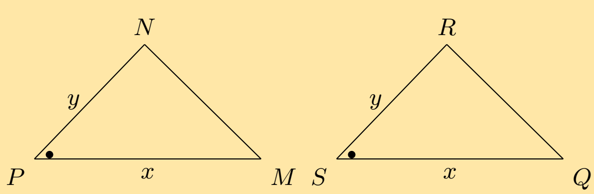


   Are the two triangles congruent? If so state the reason and use the correct notation to state that they are congruent.

   Solution:

   We are not told if $n = r$ and $m = q$ or $n = q$ and $m = r$ therefore we cannot say that the sides are the same length. Also we are not given any information about the angles of the two triangles. Therefore we cannot say if the two triangles are congruent.

5. Have a look at the following triangles, which are drawn to scale:



Are the two triangles congruent? If so state the reason and use the correct notation to state that they are congruent.

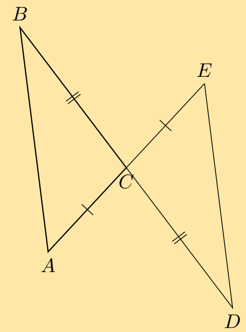
Solution:

Note that the two pairs of sides are equal, as indicated by the x and y. In addition, the angle between those two sides are marked as equal (this is the included angle).

Therefore, these two triangles are congruent. $\triangle PNM \equiv \triangle QSR$, reason: SAS.

6. State whether the following pairs of triangles are congruent or not. Give reasons for your answers. If there is not enough information to make a decision, explain why.

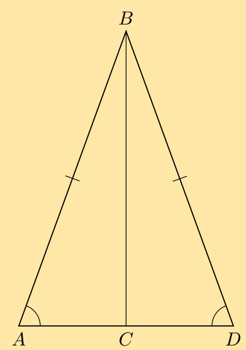
a)



Solution:

$$\begin{aligned} AC &= CE \qquad \text{(given)} \\ BC &= CD \qquad \text{(given)} \\ A\hat{C}B &= D\hat{C}E \qquad \text{(vert opp } \angle\text{s} = \text{)} \\ \therefore \triangle ABC &\equiv \triangle EDC \qquad \text{SAS} \end{aligned}$$

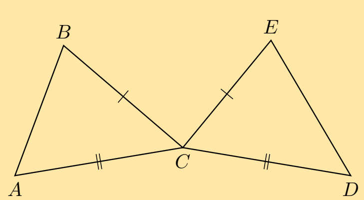
b)



Solution:

We have two equal sides ($AB = BD$ and BC is common to both triangles) and one equal angle ($\hat{A} = \hat{D}$) but the sides do not include the known angle. The triangles therefore do not have a SAS and are therefore not congruent. (Note: $A\hat{C}B$ is not necessarily equal to $D\hat{C}B$ because it is not given that $BC \perp AD$).

c)

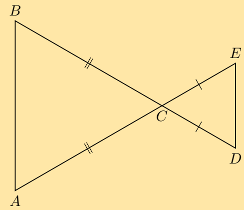


Solution:

There is not enough information given. We need at least three facts about the triangles and in this example we only know two sides in each triangle.

Note that BCD and ECA are not straight lines and so we cannot use vertically opposite angles.

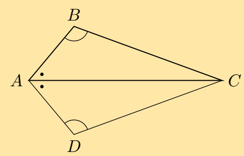
d)



Solution:

There is not enough information given. Although we can work out which angles are equal we are not given any sides as equal. All we know is that we have two isosceles triangles. Note how this question differs from part a). In part a) we were given equal sides in both triangles, in this question we are only given that sides in the same triangle are equal.

e)



Solution:

$$
\begin{aligned}
AC &= AC && \text{(common side)}\\
B\hat{A}C &= D\hat{A}C && \text{(given)}\\
A\hat{B}C &= A\hat{D}C && \text{(given)}\\
\therefore \triangle ABC &\equiv \triangle ADC && \text{AAS}
\end{aligned}
$$

For more exercises, visit www.everythingmaths.co.za and click on 'Practise Maths'.

1a. 2G6G 1b. 2G6H 1c. 2G6J 1d. 2G6K 1e. 2G6M 1f. 2G6N 1g. 2G6P 2. 2G6Q
3. 2G6R 4. 2G6S 5. 2G6T 6a. 2G6V 6b. 2G6W 6c. 2G6X 6d. 2G6Y 6e. 2G6Z

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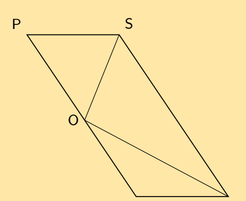
7.3 Quadrilaterals

Mathopenref has some useful simulations on different types of quadrilaterals. Clicking on any of the named quadrilaterals will take you to a page specific to that quadrilateral.

Parallelogram

Exercise 7 – 3:

1. $PQRS$ is a parallelogram. $PS = OS$ and $QO = QR$. $S\hat{O}R = 96°$ and $Q\hat{O}R = x$.



a) Find with reasons, two other angles equal to x.

Solution:

$S\hat{R}O = Q\hat{O}R = x$ (alt $\angle$s; $SR \parallel OQ$).

$O\hat{R}Q = Q\hat{O}R = x$ ($\angle$s opp equal sides).

Therefore $S\hat{R}O$ and $O\hat{R}Q$ are both equal to x.

b) Write $\hat{P}$ in terms of x.

Solution:

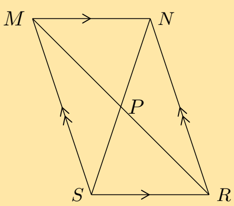
$$\begin{aligned} \hat{P} &= Q\hat{R}S \qquad (\text{ opp } \angle\text{s of } \parallel \text{m}) \\ &= S\hat{R}O + O\hat{R}Q \\ \therefore \hat{P} &= 2x \end{aligned}$$

c) Calculate the value of x.

Solution:

$$\begin{aligned} S\hat{O}R &= 96^\circ \qquad (\text{given}) \\ S\hat{O}P &= \hat{P} \qquad (\angle\text{s opp equal sides}) \\ 180^\circ &= \hat{P} + 96^\circ + Q\hat{O}R \qquad (\text{sum of } \angle\text{s on a str line}) \\ 84^\circ &= 2x + x \\ 3x &= 84^\circ \\ \therefore x &= 28^\circ \end{aligned}$$

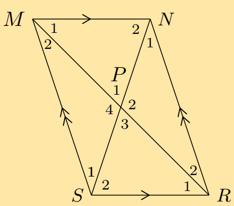
2. Prove that the diagonals of parallelogram $MNRS$ bisect one another at P.



Hint: Use congruency.

Solution:

First number each angle on the given diagram:



In $\triangle MNP$ and $\triangle RSP$:

$$\begin{aligned} \hat{M}_1 &= \hat{R}_1 \quad (\text{alt } \angle\text{s; } MN \parallel SR) \\ \hat{P}_1 &= \hat{P}_3 \quad (\text{vert opp } \angle\text{s } =) \\ MN &= RS \ (\text{opp sides of } \parallel \text{ m}) \end{aligned}$$

Therefore $\triangle MNP \equiv \triangle RSP$ (AAS).

Now we know that $MP = RP$ and therefore P is the mid-point of MR.

Similarly, in $\triangle MSP$ and $\triangle RNP$:

$$\hat{M}_2 = \hat{R}_2 \quad \text{(alt } \angle\text{s; } MS \parallel NR)$$
$$\hat{P}_4 = \hat{P}_2 \quad \text{(vert opp } \angle\text{s } =)$$
$$MS = RN \text{ (opp sides of } \parallel \text{m)}$$

Therefore $\triangle MSP \equiv \triangle RNP$ (AAS).
Now we know that $NP = SP$ and therefore P is the mid-point of NS.
Therefore the diagonals of a parallelogram bisect each other.

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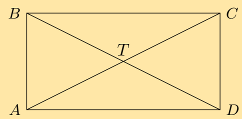
Rectangle

Exercise 7 – 4:

1. $ABCD$ is a quadrilateral. Diagonals AC and BD intersect at T. $AC = BD$, $AT = TC$, $DT = TB$. Prove that:



a) $ABCD$ is a parallelogram
Solution:
$AT = TC$ (given)
$\therefore DB$ bisects AC at T
and $DT = TB$ (given)
$\therefore AC$ bisects DB at T
therefore quadrilateral $ABCD$ is a parallelogram (diag of $\parallel$m)

b) $ABCD$ is a rectangle
Solution:
$AC = BD$ (given).
Therefore $ABCD$ is a rectangle (diags of rectangle).

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Rhombus

Square

Trapezium

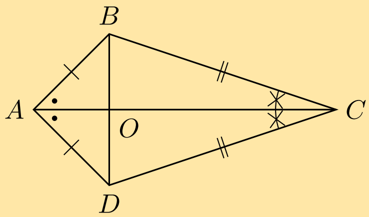
In British English a trapezium is used to indicate a quadrilateral with one pair of opposite sides parallel while in American English a trapezium is a quadrilateral with no pairs of opposite sides parallel. We will use the British English definition of trapezium in this book.

In British English a trapezoid is used to indicate a quadrilateral with no pairs of opposite sides parallel while in American English a trapezoid is a quadrilateral with one pair of opposite sides parallel.

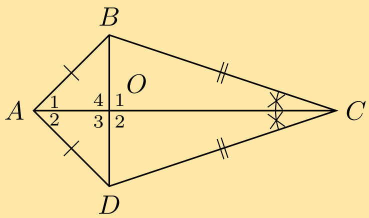
Kite

Exercise 7 – 5:

1. Use the sketch of quadrilateral $ABCD$ to prove the diagonals of a kite are perpendicular to each other.



Solution:
First number the angles:



In $\triangle ADO$ and $\triangle ABO$:

$$\begin{aligned} AD &= AB && \text{(given)} \\ AO & && \text{(common side)} \\ B\hat{A}O &= D\hat{A}O && \text{(given)} \\ \therefore \triangle ADO &\equiv \triangle ABO && \text{(SAS)} \\ \therefore A\hat{B}O &= A\hat{D}O && \end{aligned}$$

In $\triangle ADB$:

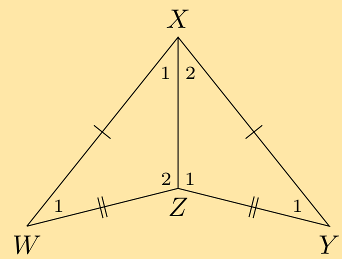
$$\begin{aligned} \text{let } \hat{A}_1 = \hat{A}_2 &= t \\ \text{and let } A\hat{D}O = A\hat{B}O &= p \\ 2t + 2p &= 180^\circ \quad \text{(sum of } \angle\text{s in } \triangle) \\ \therefore t + p &= 90^\circ \end{aligned}$$

Next we note that:

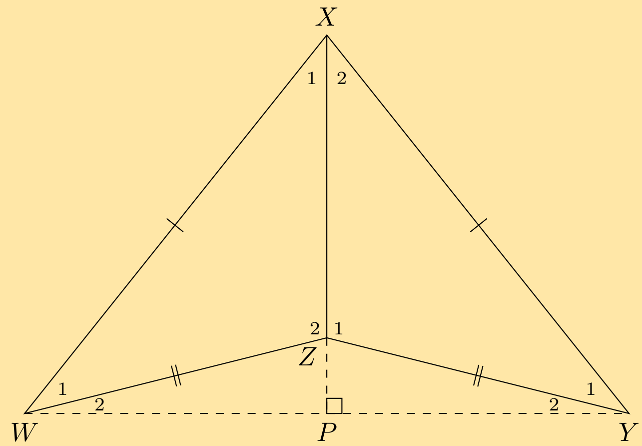
$$\begin{aligned} \hat{O}_1 &= A\hat{B}O + \hat{A}_1 \text{ (ext } \angle \text{ of } \triangle) \\ \hat{O}_1 &= p + t \\ &= 90^\circ \\ \therefore AC &\perp BD \end{aligned}$$

Therefore the diagonals of a kite are perpendicular to each other.

2. Explain why quadrilateral $WXYZ$ is a kite. Write down all the properties of quadrilateral $WXYZ$.



Solution:



Quadrilateral $WXYZ$ is a kite because is has two pairs of adjacent sides that are equal in length.

- Diagonal between equal sides bisects the other diagonal: $WP = PY$.
- One pair of opposite angles are equal: $\hat{W}_1 = \hat{Y}_1$.
- Diagonal between equal sides bisects the interior angles and is an axis of symmetry: $\hat{X}_1 = \hat{X}_2$.
- Diagonals intersect at $90°$: $WY \perp PX$.

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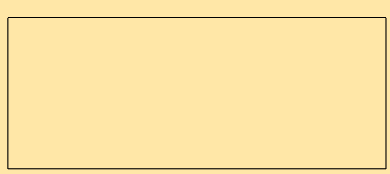
Exercise 7 – 6:

1. The following shape is drawn **to scale** :



Give the most specific name for the shape.

Solution:

We start by counting the number of sides. There are four sides in this figure and so it is either just a quadrilateral or one of the special types of quadrilateral.

Next we ask ourselves if there are any parallel lines in the figure. You can look at the figure to see if any of the lines look parallel or make a quick sketch of the image and see if any pairs of opposite lines meet at a point.

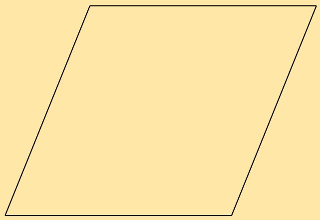
Both pairs of opposite sides are parallel. This means that the figure can only be one of the following: parallelogram, rectangle, rhombus or square.

Next we ask ourselves if all the interior angles are 90°. All the interior angles are 90° and so this must be a square or a rectangle. Finally we check to see if all the sides are equal in length. In this figure the sides are not equal in length and so it is a rectangle.

Therefore this is a rectangle.

The shape is also a parallelogram and a quadrilateral. This question, however, asked for the most specific name for the shape.

2. The following shape is drawn **to scale** :



Give the most specific name for the shape.

Solution:

We start by counting the number of sides. There are four sides in this figure and so it is either just a quadrilateral or one of the special types of quadrilateral.

Next we ask ourselves if there are any parallel lines in the figure. You can look at the figure to see if any of the lines look parallel or make a quick sketch of the image and see if any pairs of opposite lines meet at a point.

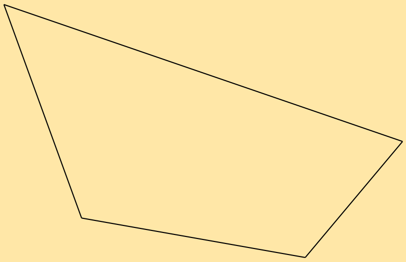
Both pairs of opposite sides are parallel. This means that the figure can only be one of the following: parallelogram, rectangle, rhombus or square.

Next we ask ourselves if all the interior angles are 90°. All the interior angles are not 90° and so this must be a parallelogram or a rhombus. Finally we check to see if all the sides are equal in length. In this figure the sides are equal in length and so it is a rhombus.

Therefore this is a rhombus.

The shape is also a parallelogram and a quadrilateral. This question, however, asked for the most specific name for the shape.

3. Based on the shape that you see list the all the names of the shape. The figure is drawn to scale

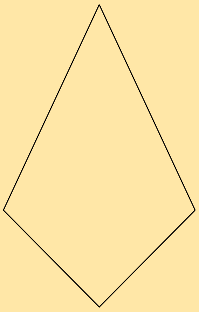


Solution:

Both pairs of opposite sides are not parallel. This means that the figure can only be some combination of the following: trapezium, kite, or quadrilateral.

The shape is definitely a quadrilateral because it has four sides. It does not have any special properties: it does not have parallel sides, or right angles, or sides which are equal in length. Therefore it is only a quadrilateral.

4. Based on the shape that you see list the all the names of the shape. The figure is drawn to scale



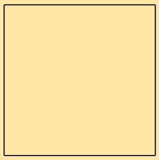
Solution:

Both pairs of opposite sides are not parallel. This means that the figure can only be some combination of the following: trapezium, kite, or quadrilateral.

The shape is definitely a quadrilateral because it has four sides. It is also a kite because it has two pairs of adjacent sides which are the same lengths. It cannot be a square or a rectangle because it does not have right angles. It cannot be a parallelogram or a trapezium because it does not have any parallel sides. And it is not a rhombus because the four sides are not all the same length.

Therefore the correct answer is: kite and quadrilateral.

5. Based on the shape that you see list the all the names of the shape. The figure is drawn to scale



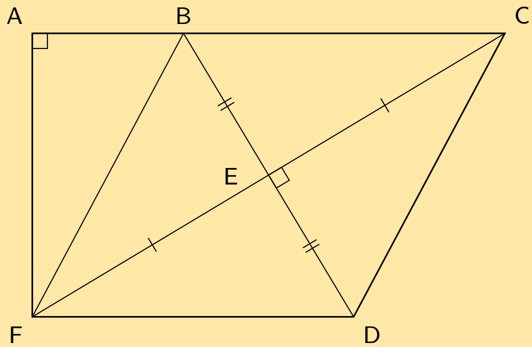
Solution:

Both pairs of opposite sides are parallel. That means that this shape can belong to one or more of these groups: square, rhombus, rectangle or parallelogram.

The given shape is a square. However, it is also a rectangle. A square is also a parallelogram, because it has parallel sides; and it is a rhombus as well, it just happens to have right angles. A square is also a kite, a trapezium and of course a quadrilateral.

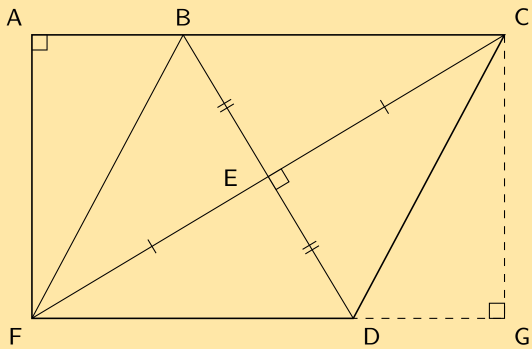
Therefore the correct answer is: square, rectangle, rhombus, parallelogram, kite, trapezium and quadrilateral.

6. Find the area of $ACDF$ if $AB = 8,\ BF = 17,\ FE = EC,\ BE = ED,\ \hat{A} = 90°,\ C\hat{E}D = 90°$



Solution:

Construct G such that $AC = FG$



$BCDF$ is a rhombus (diagonals bisect at right angles)

Since $BCDF$ is a rhombus $BC = DF$. We constructed G such that $AC = FG$. Therefore $AB = DG$.

In $\triangle ABF$ and $\triangle CGD$:

$$B\hat{A}F = C\hat{G}D = 90^\circ \quad \text{(given and by construction)}$$
$$AB = DG \quad \text{(by construction)}$$
$$BF = CD \quad (BCDF \text{ is a rhombus})$$

Therefore $ABF \equiv CGD$ (RHS)
Therefore $AF = CG$ and so $ACGF$ is a rectangle (both pairs of opposite sides equal in length and all interior angles are 90°).
We are given the length of AB and BF. Since $\triangle ABF$ is right-angled we can use the theorem of Pythagoras to find the length of AF:

$$\begin{aligned} BF^2 &= AB^2 + AF^2 \\ (17)^2 &= (8)^2 + AF^2 \\ AF^2 &= 225 \\ AF &= 15 \end{aligned}$$

We also know that $FD = BF = 17$ and so $AC = 17 + 8 = 25$.
Therefore the area of rectangle $ACGF$ is:

$$\begin{aligned} A_{\text{rectangle}} &= l \times b \\ &= (25)(15) \\ &= 375 \end{aligned}$$

We are almost there. We now need to calculate the area of triangle CDG and subtract this from the area of the rectangle to get the area of $ACDF$.
The area of triangle CDG is:

$$\begin{aligned} A_{\text{triangle}} &= \frac{1}{2} DG \times CG \\ &= \frac{1}{2}(8 \times 15) \\ &= 60 \end{aligned}$$

Therefore the area of $ACDF$ is $375 - 60 = 315$.

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1. 2G78 2. 2G79 3. 2G7B 4. 2G7C 5. 2G7D 6. 2G7F

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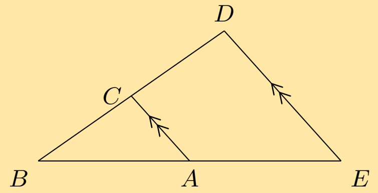
7.4 The mid-point theorem

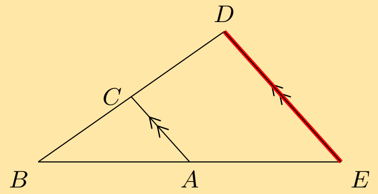
Exercise 7 – 7:

1. Points C and A are the mid-points on lines BD and BE. Study $\triangle EDB$ carefully. Identify the third side of this triangle, using the information as shown, together with what you know about the mid-point theorem. Name the third side by its endpoints, e.g., FG.



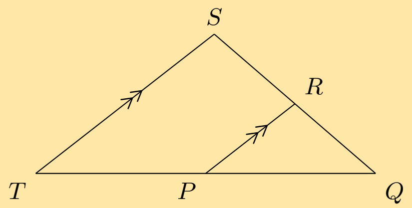
Solution:

The red line, ED or DE, indicates the third side of the triangle. According to the mid-point theorem, the line joining the mid-points of two sides of a triangle is parallel to the third side of the triangle.



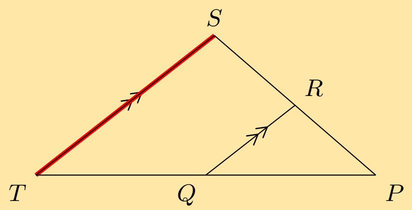
The third side is: ED or DE.

2. Points R and P are the mid-points on lines QS and QT. Study $\triangle TSQ$ carefully. Identify the third side of this triangle, using the information as shown, together with what you know about the mid-point theorem. Name the third side by its endpoints, e.g., FG.



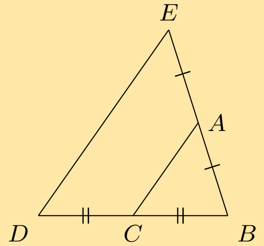
Solution:

The red line, TS or ST, indicates the third side of the triangle. According to the mid-point theorem, the line joining the mid-points of two sides of a triangle is parallel to the third side of the triangle.



The third side is: TS or ST.

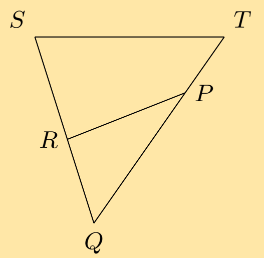
3. Points C and A are given on the lines BD and BE. Study the triangle carefully, then identify and name the parallel lines.



Solution:

The lines ED and AC are parallel according to the mid-point theorem because AC is bisecting the lines EB and DB.

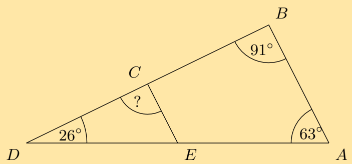
4. Points R and P are given on the lines QS and QT. Study the triangle carefully, then identify and name the parallel lines.



Solution:

The lines TS and PR are not parallel according to the mid-point theorem because line PR does not bisect TQ and SQ. Therefore there are no parallel lines in the triangle.

5. The figure below shows a large triangle with vertices A, B and D, and a smaller triangle with vertices at C, D and E. Point C is the mid-point of BD and point E is the mid-point of AD.

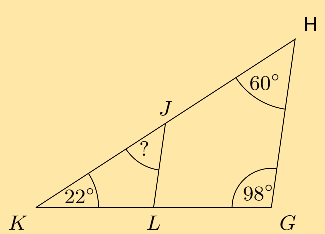


a) Three angles are given: $\hat{A} = 63°$, $\hat{B} = 91°$ and $\hat{D} = 26°$; determine the value of $D\hat{C}E$.
Solution:

$$\begin{aligned} AB &\parallel EC && \text{(Midpt Theorem)} \\ \therefore D\hat{C}E &= \hat{B} && \text{(corresp } \angle\text{s; } AB \parallel EC) \\ D\hat{C}E &= 91° \end{aligned}$$

b) The two triangles in this question are similar triangles. Complete the following statement correctly by giving the three vertices in the correct order (there is only one correct answer).
$\triangle DEC \ ||| \ \triangle ?$
Solution:
Angle D corresponds to angle D; angle E corresponds to angle A; and angle C corresponds to angle B. Therefore, $\triangle DEC \ ||| \ \triangle DAB$.

6. The figure below shows a large triangle with vertices G, H and K, and a smaller triangle with vertices at J, K and L. Point J is the mid-point of HK and point L is the mid-point of GK.



a) Three angles are given: $\hat{G} = 98°$, $\hat{H} = 60°$, and $\hat{K} = 22°$; determine the value of $K\hat{J}L$.
Solution:

$$\begin{aligned} GH &\parallel LJ && \text{(Midpt Theorem)} \\ \therefore K\hat{J}L &= \hat{H} && \text{(corresp } \angle\text{s; } GH \parallel LJ) \\ K\hat{J}L &= 60° \end{aligned}$$

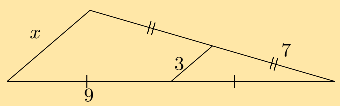
b) The two triangles in this question are similar triangles. Complete the following statement correctly by giving the three vertices in the correct order (there is only one correct answer).

$\triangle HKG \ ||| \ \triangle ?$

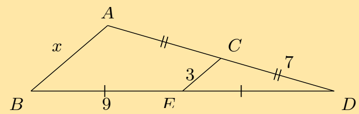
Solution:

Angle H corresponds to angle J; angle K corresponds to angle K ; and angle G corresponds to angle L. Therefore, $\triangle HKG \ ||| \ \triangle JKL$.

7. Consider the triangle in the diagram below. There is a line crossing through a large triangle. Notice that some lines in the figure are marked as equal to each other. One side of the triangle has a given length of 3. Some information is also given about the lengths of other lines along the edges of the triangle.



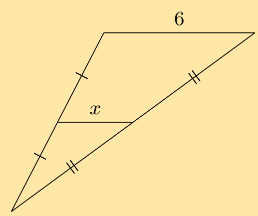
Determine the value of x.
Solution:



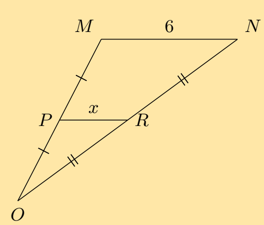
From the mid-point theorem we know:

$$\begin{aligned} AB &= 2 \times CE \\ x &= 2(3) \\ &= 6 \end{aligned}$$

8. Consider the triangle in the diagram below. There is a line crossing through a large triangle. Notice that some lines in the figure are marked as equal to each other. One side of the triangle has a given length of 6.



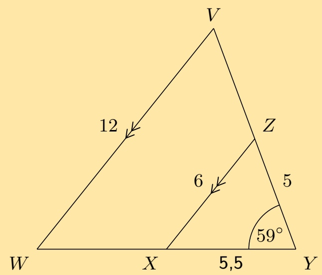
Determine the value of x.
Solution:



From the mid-point theorem we know:

$$\begin{aligned} MN &= 2 \times PR \\ (6) &= 2x \\ \frac{1}{2}(6) &= x \\ 3 &= x \end{aligned}$$

9. In the figure below, $VW \parallel ZX$, as labelled. Furthermore, the following lengths and angles are given: $VW = 12$; $ZX = 6$; $XY = 5{,}5$; $YZ = 5$ and $\hat{V} = 59°$. The figure is drawn to scale.



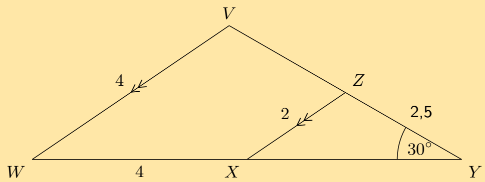
Determine the length of WY.

Solution:

X is the mid-point of WY and Z is the mid-point of VY ($VW \parallel ZX$, also it is given that $XZ = \frac{1}{2}VW$).

$$\begin{aligned} WY &= 2 \times XY && \text{definition of mid-point} \\ &= 2(5{,}5) \\ &= 11 \end{aligned}$$

10. In the figure below, $VW \parallel ZX$, as labelled. Furthermore, the following lengths and angles are given: $VW = 4$; $ZX = 2$; $WX = 4$; $YZ = 3{,}5$ and $\hat{Y} = 30°$. The figure is drawn to scale.



Determine the length of XY.

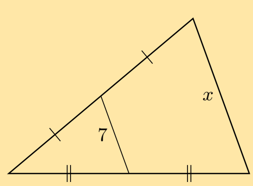
Solution:

X is the mid-point of WY and Z is the mid-point of VY ($VW \parallel ZX$, also it is given that $XZ = \frac{1}{2}VW$).

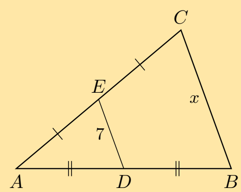
$$\begin{aligned} XY &= WX && \text{definition of mid-point} \\ &= 4 \end{aligned}$$

11. Find x and y in the following:

a)



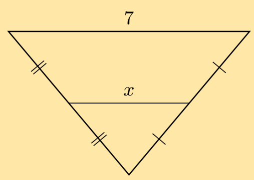
Solution:



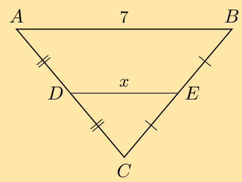
From the mid-point theorem we know:

$$\begin{aligned} BC &= 2 \times DE \\ x &= 2(7) \\ &= 14 \end{aligned}$$

b)



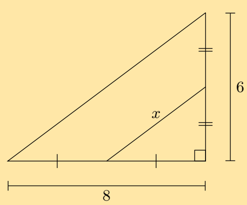
Solution:



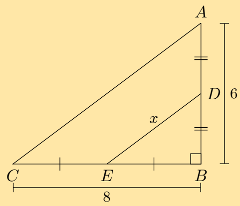
From the mid-point theorem we know:

$$\begin{aligned} AB &= 2 \times DE \\ 7 &= 2x \\ 3{,}5 &= x \end{aligned}$$

c)



Solution:



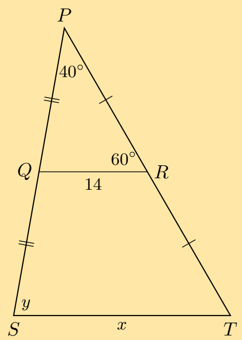
We can use the theorem of Pythagoras to find AC:

$$\begin{aligned} AC^2 &= BC^2 + AB^2 \\ &= (8)^2 + (6)^2 \\ &= 64 + 36 \\ &= 100 \\ AC &= 10 \end{aligned}$$

From the mid-point theorem we know:

$$
\begin{aligned}
AC &= 2 \times DE \\
10 &= 2x \\
5 &= x
\end{aligned}
$$

d)



Solution:

From the mid-point theorem we know:

$$
\begin{aligned}
ST &= 2 \times QR \\
x &= 2(14) \\
&= 28
\end{aligned}
$$

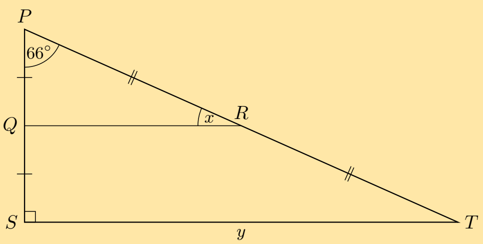
To find y we note the following:

- $P\hat{Q}R = 180° - 60° - 40° = 100°$ (sum of $\angle$s in $\triangle$).
- From the mid-point theorem we also know that $QR \parallel ST$.

Therefore $y = 100°$ (corresp $\angle$s; $QR \parallel ST$).

The final answer is: $x = 28$ units and $y = 100°$.

e) In the following diagram $PQ = 2{,}5$ and $RT = 6{,}5$.



Solution:

From the mid-point theorem we know that $QR \parallel ST$. Therefore $P\hat{Q}R = P\hat{S}T = 90°$ (corresp $\angle$s; $QR \parallel ST$).

Therefore $x = 180° - 90° - 66° = 24°$ (sum of $\angle$s in $\triangle$).

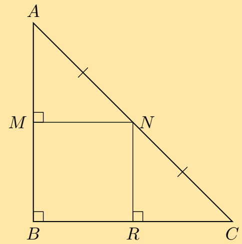
To find y we note that $PQ + QS = PS$ and $PQ = QS$, therefore $PS = 2PQ$. Similarly $PT = 2RT$.

We can use the theorem of Pythagoras to find ST:

$$
\begin{aligned}
ST^2 &= PS^2 + PT^2 \\
&= 2PQ + 2RT \\
&= (2(2{,}5))^2 + (2(6{,}5))^2 \\
&= 25 + 169 \\
&= 194 \\
ST &= 13{,}93
\end{aligned}
$$

Therefore: $x = 24°$ and $y = 13{,}93$.

12. Show that M is the mid-point of AB and that $MN = RC$.



Solution:

We are given that $AN = NC$.

We are also given that $\hat{B} = \hat{M} = 90°$, therefore $MN \parallel BR$ ($\hat{B}$ and $\hat{M}$ are equal, corresponding angles).

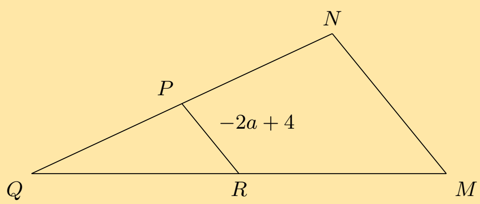
Therefore M is the mid-point of AB (converse of mid-point theorem).

Similarly we can show that R is the mid-point of BC.

We also know that $MN = BR$ ($MB \parallel NR$ and parallel lines are a constant distance apart).

But $BR = RC$ (R is the mid-point of BC), therefore $MN = RC$.

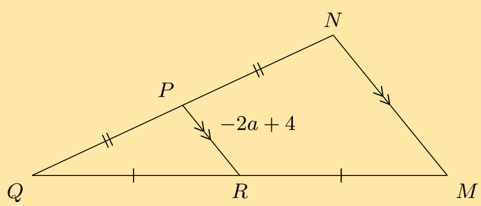
13. In the diagram below, P is the mid-point of NQ and R is the mid-point of MQ. The segment inside of the large triangle is labelled with a length of $-2a + 4$.



a) Calculate the value of MN in terms of a.

Solution:

Use the mid-point theorem to fill in known information on the diagram:



Remember that the mid-point theorem tells us that the segments MN and PR have a ratio of $2 : 1$ (MN is twice as long as PR).

$$
\begin{aligned}
MN &= 2 \times PR \\
&= 2(-2a + 4) \\
&= -4a + 8
\end{aligned}
$$

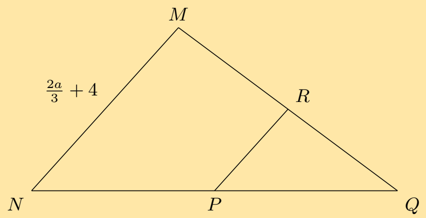
The final answer is $MN = -4a + 8$ (twice as long as PR).

b) You are now told that MN has a length of 18. What is the value of a? Give your answer as a fraction.

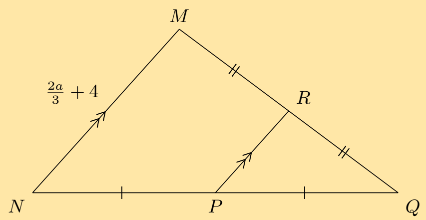
Solution:

$$
\begin{aligned}
-4a + 8 &= 18 \\
-4a &= 10 \\
\left(-\frac{1}{4}\right)(-4a) &= (10)\left(-\frac{1}{4}\right) \\
a &= -\frac{5}{2}
\end{aligned}
$$

14. In the diagram below, P is the mid-point of NQ and R is the mid-point of MQ. One side of the triangle has a given length of $\frac{2a}{3} + 4$.



a) Find the value of PR in terms of a.
Solution:
Use the mid-point theorem to fill in known information on the diagram:



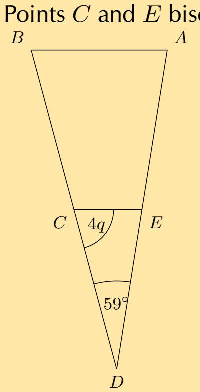
Remember that the mid-point theorem tells us that the segments MN and PR have a ratio of $2 : 1$ (MN is twice as long as PR).

$$
\begin{aligned}
MN &= 2 \times PR \\
\left(\frac{2a}{3} + 4\right) &= 2\,(PR) \\
\frac{1}{2}\left(\frac{2a}{3} + 4\right) &= PR \\
\frac{a}{3} + 2 &= PR
\end{aligned}
$$

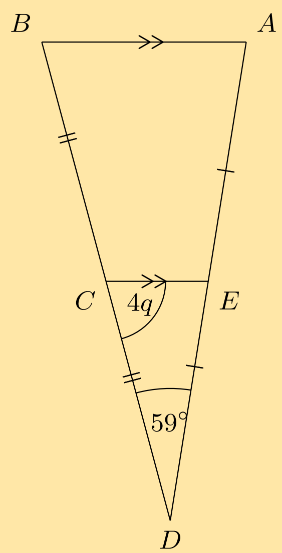
b) You are now told that PR has a length of 8. What is the value of a?
Solution:

$$
\begin{aligned}
\frac{a}{3} + 2 &= 8 \\
\frac{a}{3} &= 6 \\
(3)\left(\frac{a}{3}\right) &= (6)\,(3) \\
a &= 18
\end{aligned}
$$

15. The figure below shows $\triangle ABD$ crossed by EC. Points C and E bisect their respective sides of the triangle.



a) The angles $\hat{D} = 59°$ and $E\hat{C}D = 4q$ are given; determine the value of $\hat{A}$ in terms of q.
Solution:
We note the following from the mid-point theorem:



Also $\hat{A} = D\hat{E}C$

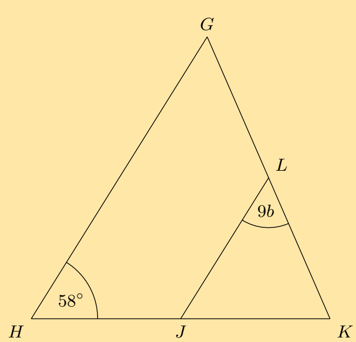
$$\begin{aligned} \hat{A} + 4q + 59° &= 180° \qquad \text{(sum of } \angle\text{s in } \triangle) \\ \hat{A} &= 180° - (4q + 59°) \\ &= -4q + 121° \end{aligned}$$

In terms of q, the answer is: $\hat{A} = -4q + 121°$.

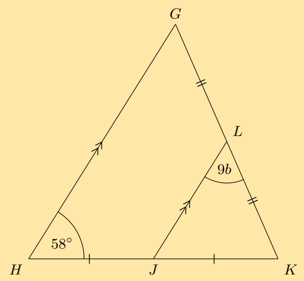
b) You are now told that $E\hat{C}D$ has a measure of $72°$. Calculate for the value of q.
Solution:

$$\begin{aligned} E\hat{C}D &= 72° \\ 4q &= 72° \\ q &= 18° \end{aligned}$$

16. The figure below shows $\triangle GHK$ crossed by LJ. Points J and L bisect their respective sides of the triangle.



a) Given the angles $\hat{H} = 58°$ and $K\hat{L}J = 9b$, determine the value of $\hat{K}$ in terms of b.
Solution:
Using the mid-point theorem we can add the following information to the diagram:



Also: $\hat{H} = K\hat{J}L = 58°$

$$\begin{aligned} \hat{K} + 9b + 58° &= 180° \qquad \text{(sum of } \angle\text{s in } \triangle) \\ \hat{K} &= 180° - (9b + 58°) \\ &= -9b + 122° \end{aligned}$$

In terms of b, the answer is: $\hat{K} \ = -9b + 122°$.

b) You are now told that $\hat{K}$ has a measure of $74°$. Solve for the value of b. Give your answer as a fraction.

Solution:

$$\begin{aligned} \hat{K} &= 74° \\ -9b + 122° &= 74° \\ b &= \frac{16}{3} \end{aligned}$$

For more exercises, visit www.everythingmaths.co.za and click on 'Practise Maths'.

1. 2G7G	2. 2G7H	3. 2G7J	4. 2G7K	5. 2G7M	6. 2G7N	7. 2G7P	8. 2G7Q
9. 2G7R	10. 2G7S	11a. 2G7T	11b. 2G7V	11c. 2G7W	11d. 2G7X	11e. 2G7Y	12. 2G7Z
13. 2G82	14. 2G83	15. 2G84	16. 2G85				

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7.5 Chapter summary

End of chapter Exercise 7 – 8:

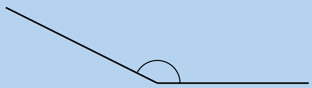
1. Identify the types of angles shown below:

a)

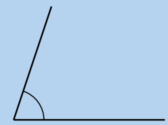
Solution:

straight angle

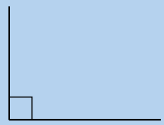
b)



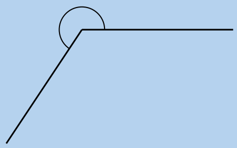
Solution:
obtuse angle

c) 

Solution:
acute angle

d) 

Solution:
right angle

e) 

Solution:
Reflex angle

f) An angle of 91°
Solution:
obtuse angle

g) An angle of 180°
Solution:
straight angle

h) An angle of 210°
Solution:
reflex angle

2. Assess whether the following statements are true or false. If the statement is false, explain why:
   a) A trapezium is a quadrilateral with two pairs of opposite sides that are parallel.
   Solution:
   False, a trapezium only has one pair of opposite parallel sides.
   b) Both diagonals of a parallelogram bisect each other.
   Solution:
   True
   c) A rectangle is a parallelogram that has all interior angles equal to 90°.
   Solution:
   True
   d) Two adjacent sides of a rhombus have different lengths.
   Solution:
   False, two adjacent sides of a rhombus are equal in length.
   e) The diagonals of a kite intersect at right angles.
   Solution:
   True
   f) All squares are parallelograms.
   Solution:
   True

g) A rhombus is a kite with a pair of equal, opposite sides.
Solution:
True

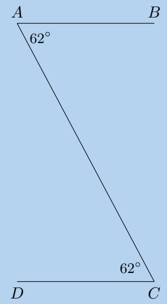
h) The diagonals of a parallelogram are axes of symmetry.
Solution:
True

i) The diagonals of a rhombus are equal in length.
Solution:
False, the diagonals of a rhombus are not equal in length.

j) Both diagonals of a kite bisect the interior angles.
Solution:
False, only one diagonal of a kite bisects one pair of interior angles.

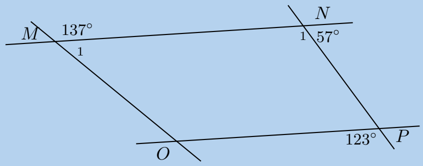
3. Find all pairs of parallel lines in the following figures, giving reasons in each case.

a)



Solution:
$AB \parallel CD$ (alt $\angle$s equal)

b)

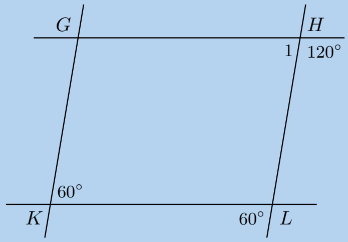


Solution:
Using the sum of angles on a straight line we can state the following:

- $\hat{M}_1 = 180° - 137° = 43°$
- $\hat{N}_1 = 180° - 57° = 123°$

NP not $\parallel$ MO (corresp $\angle$s not equal).
$MN \parallel OP$ (corresp $\angle$s equal).

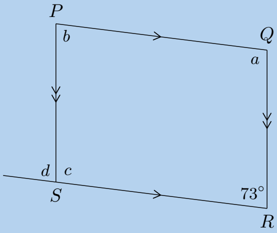
c)



Solution:
$\hat{H}_1 = 180° - 120° = 60°$ ($\angle$s on str line).
Therefore $GH \parallel KL$ (corresp $\angle$s equal).
And $GK \parallel HL$ (alt $\angle$s equal).
The pairs of parallel lines are $GH \parallel KL$ and $GK \parallel HL$.

4. Find angles a, b, c and d in each case, giving reasons:

a)

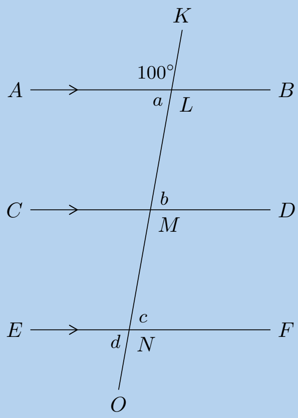


Solution:

$$a = 180° - 73° = 107° \qquad \text{(co-int } \angle\text{s; } PQ \parallel SR)$$
$$b = 180° - 107° = 73° \qquad \text{(co-int } \angle\text{s; } PS \parallel QR)$$
$$c = 180° - 73° = 107° \qquad \text{(co-int } \angle\text{s; } PQ \parallel SR)$$
$$d = 73° \qquad \text{(corresp } \angle\text{s; } PS \parallel QR)$$

Therefore $a = 107°, b = 73°, c = 107°, d = 73°$.

b)

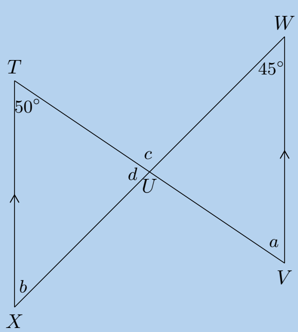


Solution:

$$a = 80° \qquad \text{(sum of } \angle\text{s on str line)}$$
$$b = 80° \qquad \text{(alt } \angle\text{s; } AB \parallel CD)$$
$$c = 80° \qquad \text{(corresp } \angle\text{s; } CD \parallel EF)$$
$$d = 80° \qquad \text{(vert opp } \angle\text{s } =)$$

Therefore $a = b = c = d = 80°$.

c)

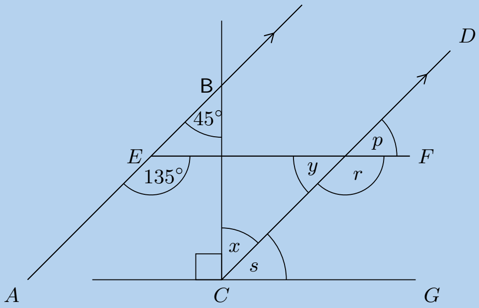


Solution:

$$\begin{aligned} a &= 50° && \text{(alt } \angle\text{s; } TX \parallel WV) \\ b &= 45° && \text{(alt } \angle\text{s; } TX \parallel WV) \\ c &= 95° && \text{(ext } \angle \text{ of } \triangle) \\ d &= 85° && \text{(sum of } \angle\text{'s in } \triangle) \end{aligned}$$

Therefore $a = 50°, b = 45°, c = 95°, d = 85°$.

5. Find each of the unknown angles marked in the figure below. Find a reason that leads to the answer in a single step.



a) $\hat{x}$
   Solution:
   $\hat{x}$ and $A\hat{B}C$ are alternate interior angles on transversal BC. $AB \parallel CD$, therefore they must be equal in size.
   Therefore $\hat{x} = 45°$.

b) $\hat{s}$
   Solution:

$$\begin{aligned} \hat{s} &= 90° - 45° \\ &= 45° \end{aligned}$$

c) $\hat{r}$
   Solution:
   $A\hat{E}F$ corresponds to (matches) $\hat{r}$; and corresponding angles are equal in size since $AB \parallel CD$.
   Therefore: $\hat{r} = 135°$.

d) $\hat{y}$
   Solution:
   $\hat{r} + \hat{y} = 180°$ ($\angle$s on str line):

$$\begin{aligned} \hat{y} &= 180° - 135° \\ &= 45° \end{aligned}$$

e) $\hat{p}$
   Solution:
   $\hat{p}$ and $\hat{y}$ are vertically opposite angles and vertically opposite angles have the same measure (equal sizes).
   Therefore: $\hat{p} = 45°$.

f) Based on the results for the angles above, is $EF \parallel CG$?
   Solution:
   If EF is parallel to CG, then the following things must all be true:
   - $\hat{s} = \hat{p}$ (corresponding angles)
   - $\hat{s} = \hat{y}$ (alternate interior angles)
   - $\hat{s} + \hat{r} = 180°$ (co-interior angles)

   All the above is true, therefore the lines are parallel.

6. Given the following diagrams:
   Diagram A

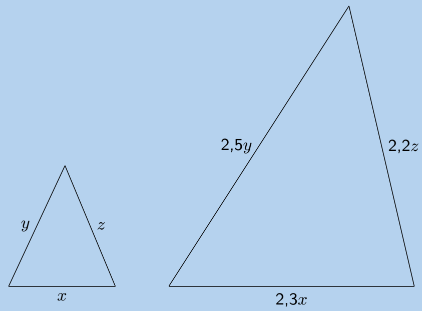
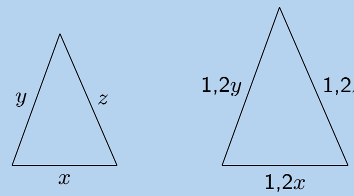


Diagram B



Which diagram correctly gives a pair of similar triangles?

Solution:

We look at the side labels. In diagram A we note that the three pairs of corresponding sides are in different proportions. In diagram B we note the three pairs of corresponding sides are in proportion.

Therefore diagram B gives a pair of triangles that are similar.

7. Given the following diagrams:

   Diagram A

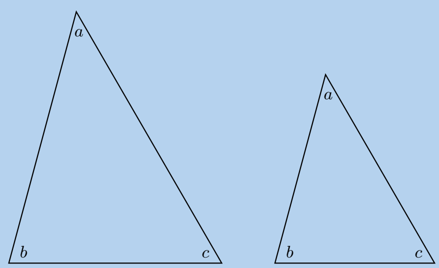
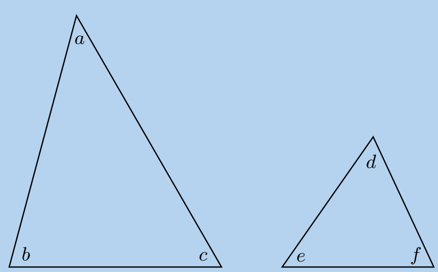


Diagram B



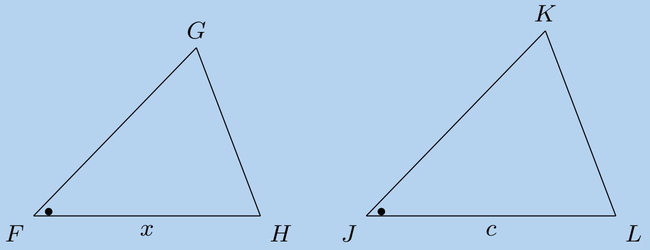
Which diagram correctly gives a pair of similar triangles?

Solution:

Diagram A shows a pair of triangles with all pairs of corresponding angles equal (the same three angle markers are shown in both triangles). Diagram B shows a pair of triangles with different angles in each triangle. All six angles are different and there are no pairs of corresponding angles that are equal.

Therefore diagram A gives a pair of triangles that are similar.

8. Have a look at the following triangles, which are drawn to scale:



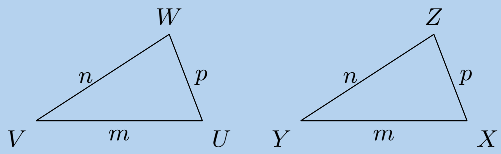
Are the triangles congruent? If so state the reason and use correct notation to state that they are congruent.

Solution:

We are given one angle that is equal. We are not given any equal sides (we do not know if $x = c$). To determine if two triangles are congruent we need to have three pieces of information (recall that the reasons for congruent triangles are: SSS, SAS, AAS and RHS). Therefore we cannot state whether or not the triangles are congruent.

Therefore, there is not enough information to determine if the two triangles are congruent.

9. Have a look at the following triangles, which are drawn to scale:



Are the triangles congruent? If so state the reason and use correct notation to state that they are congruent.

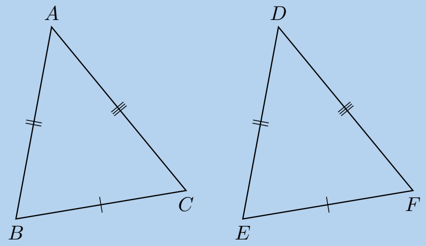
Solution:

The sides of both triangles are labelled with m, n and p. This means that there are three pairs of corresponding and equal sides.

Therefore, these two triangles are congruent ($\triangle VWU \equiv \triangle YZX$), reason: SSS.

10. Say which of the following pairs of triangles are congruent with reasons.

a)

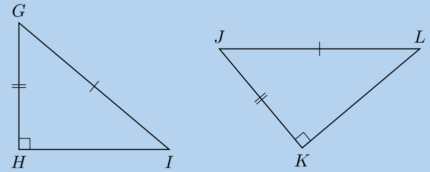


Solution:

We are given $CB = FE$, $AB = DE$ and $AC = DF$.

Therefore $\triangle ABC \equiv \triangle DEF$ by SSS.

b)

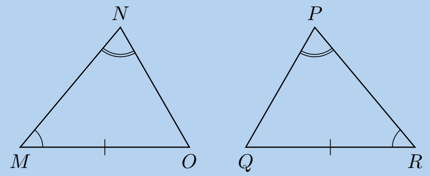


Solution:

We are given $GI = JL$, $GH = JK$ and $G\hat{H}I = J\hat{K}L = 90°$.

Therefore $\triangle GHI \equiv \triangle JKL$ by RHS.

c)

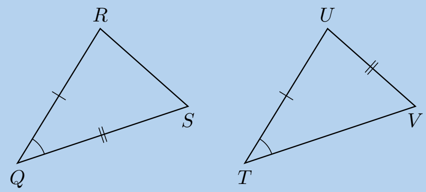


Solution:

We are given $MO = QR$, $O\hat{M}N = P\hat{R}Q$ and $M\hat{N}O = Q\hat{P}R$.

Therefore $\triangle MNO \equiv \triangle RPQ$ by AAS.

d)



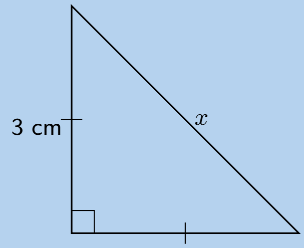
Solution:

We are given $QR = TU$, $QS = UV$ and $S\hat{Q}R = V\hat{T}U$. But $V\hat{T}U$ is not the included angle between sides UV and TU.

Therefore $\triangle QRS$ not congruent $\triangle TUV$.

11. Using the theorem of Pythagoras, calculate the length x:

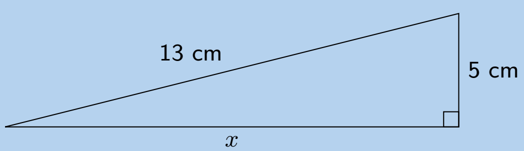
a)



Solution:

$$\begin{aligned} x^2 &= (3)^2 + (3)^2 \\ &= 18 \\ x &= \sqrt{18} \\ &= 4{,}24 \text{ cm} \end{aligned}$$

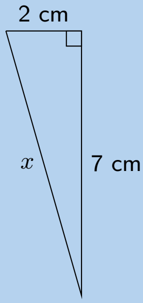
b)



Solution:

$$\begin{aligned} x^2 &= (13)^2 - (5)^2 \\ &= 144 \\ x &= 12 \text{ cm} \end{aligned}$$

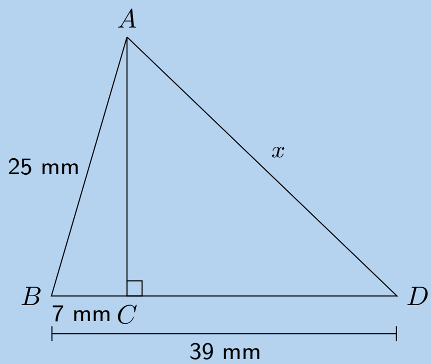
c)



Solution:

$$
\begin{aligned}
x^2 &= (2)^2 + (7)^2 \\
&= 53 \\
x &= \sqrt{53} \\
&= 7{,}28 \text{ cm}
\end{aligned}
$$

d)



Solution:
First find AC:

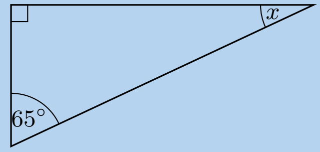
$$
\begin{aligned}
AC^2 &= (25)^2 - (7)^2 \\
&= 576 \\
AC &= \sqrt{576}
\end{aligned}
$$

Now we note that $CD = 39 - 7 = 32$ and then we find x:

$$
\begin{aligned}
x^2 &= (\sqrt{576})^2 + (32)^2 \\
x^2 &= 1600 \\
x &= 40 \text{ mm}
\end{aligned}
$$

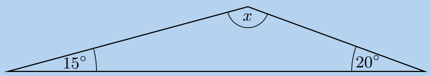
12. Calculate x and y in the diagrams below:

a)



Solution:
$x = 180° - 90° - 65° = 25°$ (sum of $\angle$s in $\triangle$).

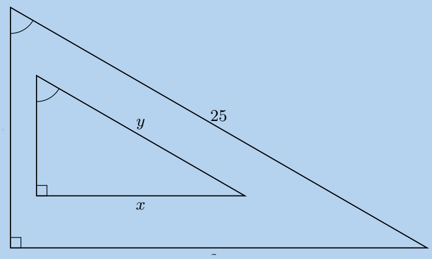
b)



Solution:

$x = 180° - 20° - 15° = 145°$ (sum of $\angle$s in $\triangle$).

c)



Solution:

We can find x using the theorem of Pythagoras:

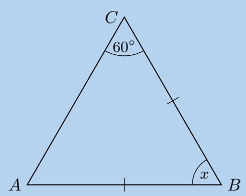
$$\begin{aligned} 25^2 &= 15^2 + (2x)^2 \\ 4x^2 &= 400 \\ x^2 &= 100 \\ \therefore x &= 10 \end{aligned}$$

We note that the triangles are similar by AAA. Therefore the sides must be in proportion. Therefore y is:

$$\begin{aligned} \frac{x}{2x} &= \frac{y}{25} \\ \therefore y &= 12{,}5 \end{aligned}$$

Therefore $x = 10$ and $y = 12{,}5$.

d)

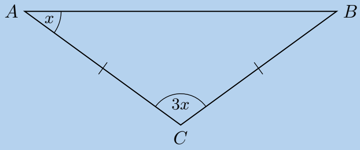


Solution:

This is an isosceles triangle so $\hat{C} = \hat{A} = 60°$.

Therefore $x = 180° - 60° - 60° = 60°$ (sum of $\angle$s in $triangle$).

e)

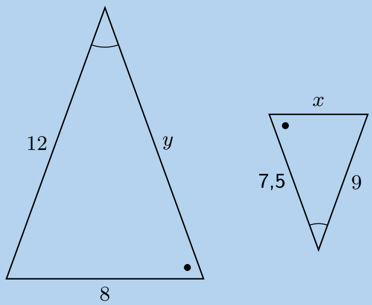


Solution:

This is an isosceles triangle so $\hat{A} = \hat{B} = x$.

$$\begin{aligned} x + x + 3x &= 180° \qquad \text{(sum of } \angle\text{s in } \triangle) \\ \therefore 5x &= 180° \\ x &= 36° \end{aligned}$$

f)



Solution:

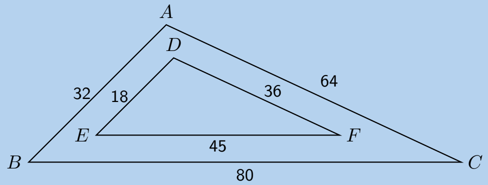
The two triangles are similar by AAA. Therefore the sides are in proportion.

$$\frac{x}{9} = \frac{8}{12}$$
$$\therefore x = 6$$

$$\frac{y}{12} = \frac{7{,}5}{9}$$
$$\therefore y = 10$$

Therefore $x = 6$ and $y = 10$.

13. Consider the diagram below. Is $\triangle ABC \ ||| \ \triangle DEF$? Give reasons for your answer.

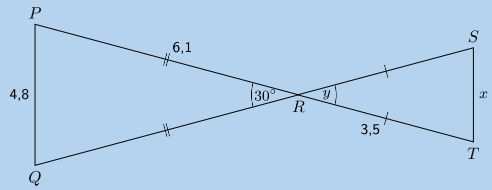


Solution:

$$\frac{ED}{BA} = \frac{18}{32} = \frac{9}{16}$$
$$\frac{DF}{AC} = \frac{32}{64} = \frac{9}{16}$$
$$\frac{EF}{BC} = \frac{45}{80} = \frac{9}{16}$$

All three pairs of sides are in proportion, $\therefore \triangle ABC \ ||| \ \triangle DEF$.

14. Explain why $\triangle PQR$ is similar to $\triangle TSR$ and calculate the values of x and y.



Solution:

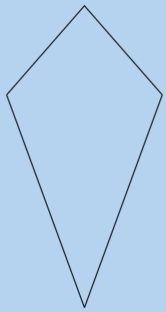
$$y = 30° \qquad \text{(vert opp } \angle\text{s} = \text{)}$$
$$\hat{P} = \hat{Q} \qquad (\angle\text{s opp equal sides})$$
$$\text{and } \hat{S} = \hat{T} \qquad (\angle\text{s opp equal sides})$$

However $\hat{P} + \hat{Q} + 30° = 180°$ (sum of $\angle$s in $\triangle$). Therefore $\hat{P} + \hat{Q} = 150°$.
Similarly $\hat{S} + \hat{T} = 150°$.
But $\hat{P} = \hat{Q}$ so $2\hat{P} = 150°$ and $\hat{S} = \hat{T}$ so $2\hat{S} = 150°$. Therefore $\hat{P} = \hat{S}$.
Therefore $\triangle PQR \ ||| \ \triangle TRS$ (AAA).
Now we can use the fact that the sides are in proportion to find x:

$$\frac{x}{4,8} = \frac{3,5}{6,1}$$
$$\therefore x = 2,75$$

Therefore $x = 2,75$ and $y = 30°$.

15. The following shape is drawn to scale:



Give the most specific name for the shape.

Solution:

We start by counting the number of sides. There are four sides in this figure and so it is either just a quadrilateral or one of the special types of quadrilateral.

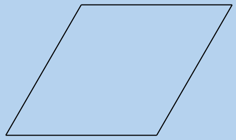
Next we ask ourselves if there are any parallel lines in the figure. You can look at the figure to see if any of the lines look parallel or make a quick sketch of the image and see if any pairs of opposite lines meet at a point.

Both pairs of opposite sides are not parallel. This means that the figure can only be one of the following: trapezium, kite or quadrilateral.

Next we ask ourselves if one of the pairs of opposite sides is parallel, while the other is not. Neither of the two pairs of opposite sides is parallel so we must now look to see if both pairs of adjacent sides are equal in length. Both pairs of adjacent sides are equal in length. So this is a kite.

Therefore this is a kite.

16. Based on the shape that you see list the all the names of the shape. The figure is drawn to scale.



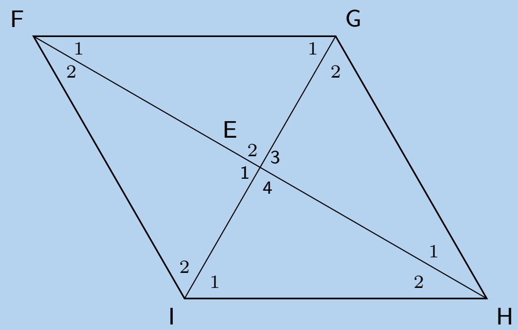
Solution:

Both pairs of opposite sides are parallel. That means that this shape can belong to one or more of these groups: square, rhombus, rectangle, and/or parallelogram.

The shape shown is a rhombus. It is certainly a quadrilateral (because it has four sides). It is also a parallelogram, because the opposite sides are parallel to each other. The rhombus is not a rectangle or a square because it does not have right angles. However, the rhombus is a kite, because it has two pairs of adjacent sides which are equal in length. And finally, it is a trapezium because it has a pair of opposite sides which are parallel.

Therefore the correct answer is: rhombus, parallelogram, kite, trapezium and quadrilateral.

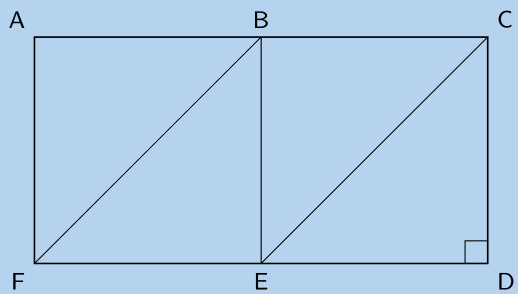
17. $FGHI$ is a rhombus. $\hat{F}_1 = 3x + 20°$; $\hat{G}_1 = x + 10°$. Determine the value of x.



Solution:
$\hat{E}_2 = 90°$ (diagonals of a rhombus bisect at right angles)

$$\begin{aligned} \hat{F}_1 + \hat{G}_1 + 90° &= 180° && \text{(sum of } \angle\text{s in } \triangle) \\ 3x + 20° + x + 10° &= 90° \\ 4x &= 60° \\ \therefore x &= 15° \end{aligned}$$

18. In the diagram below, $AB = BC = CD = DE = EF = FA = BE$.



Name:

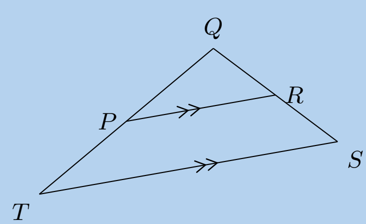
a) 3 rectangles
Solution:
$ACDF$, $ABEF$ and $BCDE$

b) 4 parallelograms
Solution:
$ACDF$, $ABEF$, $BCDE$ and $BCEF$

c) 2 trapeziums
Solution:
$ACEF$ and $BCDF$

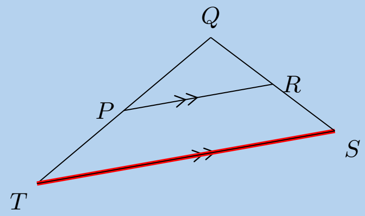
d) 2 rhombi
Solution:
$ABEF$ and $BCDE$

19. Points R and P are the mid-points on lines QS and QT. Study $\triangle TSQ$ carefully. Identify the third side of this triangle, using the information as shown, together with what you know about the mid-point theorem. (Name the third side by its endpoints, e.g., FG.)



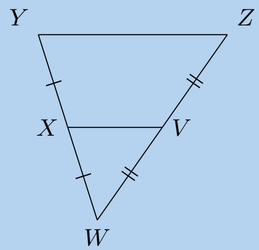
Solution:

The red line, TS or ST, indicates the third side of the triangle. According to the mid-point theorem, the line joining the mid-points of two sides of a triangle is parallel to the third side of the triangle.



The third side is: TS or ST.

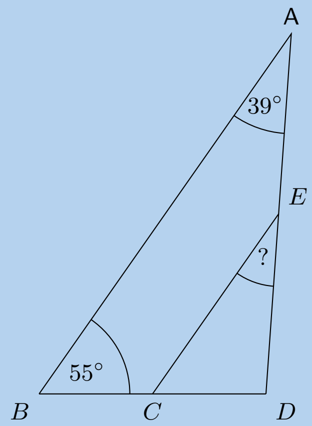
20. Points X and V are given on the segments WY and WZ. Study the triangle carefully, then identify and name the parallel line segments.



Solution:

The line segments YZ and VX are parallel according to the mid-point theorem because segment VX is bisecting the line segments WZ and WY.

21. The figure below shows a large triangle with vertices A, B and D, and a smaller triangle with vertices at C, D and E. Point C is the mid-point of BD and point E is the mid-point of AD.



a) The angles $\hat{A} = 39°$ and $\hat{B} = 55°$ are given; determine the value of $D\hat{E}C$.

Solution:

$$\begin{aligned} AB &\parallel EC && \text{(Midpt Theorem)} \\ D\hat{E}C &= \hat{A} && (\text{corresp } \angle\text{s; } AB \parallel EC) \\ D\hat{E}C &= 39° \end{aligned}$$

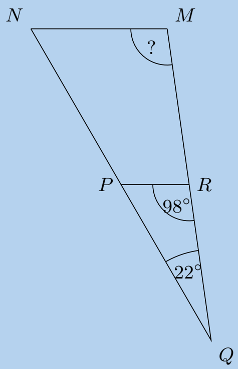
b) The two triangles in this question are similar triangles. Complete the following statement correctly by giving the three vertices in the correct order (there is only one correct answer).

$\triangle DEC \ ||| \ \triangle ?$

Solution:

Angle D corresponds to angle D; angle E corresponds to angle A ; and angle C corresponds to angle B. Therefore, $\triangle DEC \ ||| \ \triangle DAB$.

22. The figure below shows a large triangle with vertices M, N and Q, and a smaller triangle with vertices at P, Q and R. Point P is the mid-point of NQ and point R is the mid-point of MQ.



a) With the two angles given, $\hat{Q} = 22°$ and $Q\hat{R}P = 98°$, determine the value of $\hat{M}$.

Solution:

$$MN \parallel RP \qquad \text{(Midpt Theorem)}$$
$$\hat{M} = Q\hat{R}P \qquad \text{(corresp } \angle\text{s; } MN \parallel PR\text{)}$$
$$\hat{M} = 98°$$

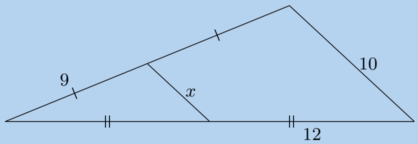
b) The two triangles in this question are similar triangles. Complete the following statement correctly by giving the three vertices in the correct order (there is only one correct answer).

$\triangle QMN \ ||| \ \triangle ?$

Solution:

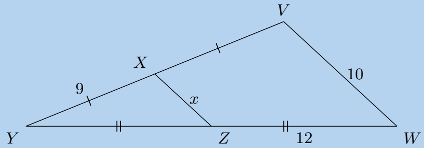
Angle Q corresponds to angle Q; angle M corresponds to angle R ; and angle N corresponds to angle P. Therefore, $\triangle QMN \ ||| \ \triangle QRP$.

23. Consider the triangle in the diagram below. There is a line segment crossing through a large triangle. Notice that some segments in the figure are marked as equal to each other. One side of the triangle has a given length of 10. Some information is also given about the lengths of other segments along the edges of the triangle.



Determine the value of x.

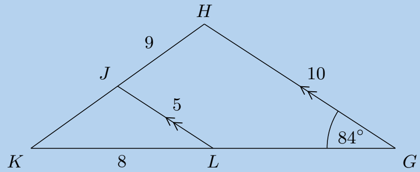
Solution:



From the mid-point theorem we know:

$$VW = 2 \times XZ$$
$$(10) = 2x$$
$$\frac{1}{2}(10) = x$$
$$5 = x$$

24. In the figure below, $GH \parallel LJ$, as labelled. Furthermore, the following lengths and angles are given: $GH = 10$; $LJ = 5$; $HJ = 9$; $KL = 8$ and $\hat{G} = 84°$. The figure is drawn to scale.

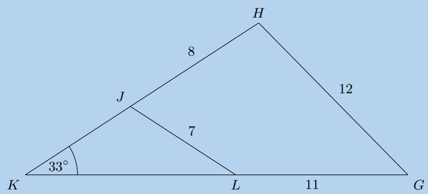


Calculate the length of JK.

Solution:

We are given that $GH \parallel LJ$. The length of JL is 5 and the length of GH is 10, therefore $JL = \frac{1}{2}GH$.
Therefore we know from the mid-point theorem that L is the mid-point of GK and J is the mid-point of HK.
Therefore $HJ = JK = 9$.

25. The figure below shows triangle GHK with the smaller triangle JKL sitting inside of it. Furthermore, the following lengths and angles are given: $GH = 12$; $LJ = 7$; $HJ = 8$; $LG = 11$; $\hat{K} = 33°$. The figure is drawn to scale.

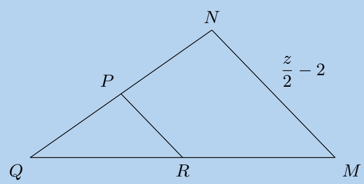


Find the length of KL.

Solution:

You can see in the figure that the segment LJ is not parallel to GH. This means that the mid-point theorem cannot apply to this triangle. There are no other options to use either: this question cannot be solved. There is no solution.

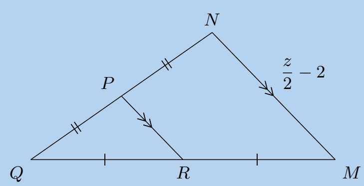
26. In the diagram below, P is the mid-point of NQ and R is the mid-point of MQ. One side of the triangle has a given length of $\frac{z}{2} - 2$.



a) Determine the value of PR in terms of z.

Solution:

Fill in information on the diagram using the mid-point theorem:



Remember that the mid-point theorem tells us that the segments MN and PR have a ratio of $2 : 1$ (MN is twice as long as PR).

$$\begin{aligned} MN &= 2 \times PR \\ \left(\frac{z}{2} - 2\right) &= 2\,(PR) \\ \frac{1}{2}\left(\frac{z}{2} - 2\right) &= PR \\ \frac{z}{4} - 1 &= PR \end{aligned}$$

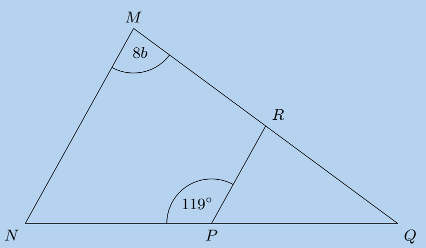
The final answer is $PR = \frac{z}{4} - 1$ (half the size as MN).

b) You are now told that PR has a length of 2. What is the value of z?
Solution:

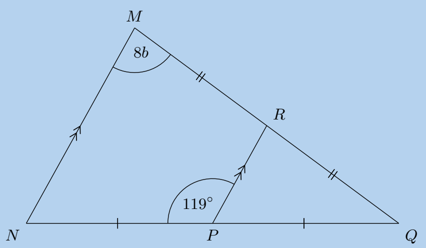
$$\begin{aligned}
\frac{z}{4} - 1 &= 2 \\
\frac{z}{4} &= 3 \\
(4)\left(\frac{z}{4}\right) &= (3)(4) \\
z &= 12
\end{aligned}$$

The final answer is $z = 12$.

27. The figure below shows $\triangle MNQ$ crossed by RP. Points P and R bisect their respective sides of the triangle.



a) With the two angles given, $\hat{M} = 8b$ and $\angle NPR = 119°$, determine the value of $\hat{Q}$ in terms of b.
Solution:
Redraw the diagram and fill in the known information using the mid-point theorem:



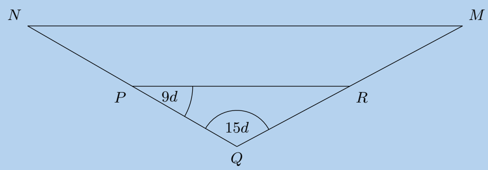
$Q\hat{P}R = 180° - R\hat{P}N = 180° - 119° = 61°$ ($\angle$s on str line).
$Q\hat{R}P = 8b$ (corresp $\angle$s; $MN \parallel RP$).
Therefore:

$$\begin{aligned}
\hat{Q} + 8b + 61° &= 180° \qquad \text{(sum of } \angle\text{s in } \triangle) \\
\hat{Q} &= 180° - (8b + 61°) \\
&= -8b + 119°
\end{aligned}$$

b) You are now told that $\hat{M}$ has a measure of $76°$. Determine for the value of b. Give your answer as an exact fractional value.
Solution:

$$\begin{aligned}
\hat{M} &= 76° \\
8b &= 76° \\
b &= \frac{19}{2}
\end{aligned}$$

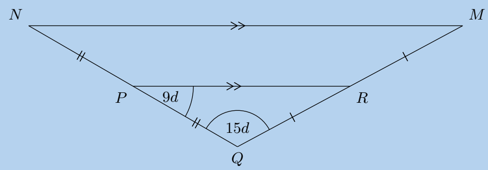
28. The figure below shows $\triangle MNQ$ crossed by RP. Points P and R bisect their respective sides of the triangle.



a) The angles $\hat{Q} = 15d$ and $R\hat{P}Q = 9d$ are given in the large triangle; determine the value of $\hat{M}$ in terms of d.

Solution:

Redraw the diagram and fill in known information using the mid-point theorem:



$P\hat{R}Q = \hat{M}$ (corresp $\angle$s; $MN \parallel RP$).

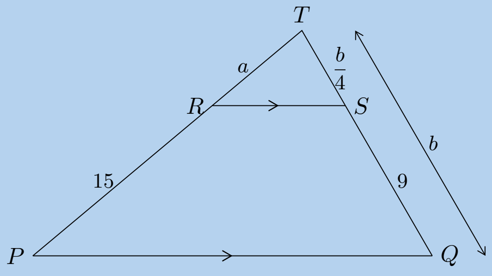
$$\hat{M} + 9d + 15d = 180^\circ \qquad \text{(sum of } \angle\text{s in } \triangle\text{)}$$
$$\hat{M} = 180^\circ - (9d + 15d)$$
$$= -24d + 180^\circ$$

b) You are now told that $R\hat{P}Q$ has a measure of 60°. Solve for the value of d. Give your answer as an exact fractional value.

Solution:

$$R\hat{P}Q = 60^\circ$$
$$9d = 60^\circ$$
$$d = \frac{20}{3}$$

29. Calculate a and b:



Solution:

In $\triangle TRS$ and $\triangle PTQ$:

$$\hat{T} = \hat{T} \qquad \text{(common } \angle\text{)}$$
$$T\hat{R}S = \hat{P} \qquad \text{(corresp } \angle\text{s; } RS \parallel PQ\text{)}$$
$$T\hat{S}R = \hat{Q} \qquad \text{(corresp } \angle\text{s; } RS \parallel PQ\text{)}$$

Therefore $\triangle TRS \ ||| \ \triangle PTQ$ (AAA).

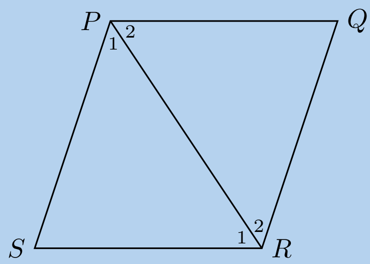
Therefore the sides are in proportion.

$$\frac{TR}{TP} = \frac{TS}{TQ}$$
$$\frac{a}{a+15} = \frac{\frac{b}{4}}{b}$$
$$\frac{a}{a+15} = \frac{1}{4}$$
$$a = (a+15)\left(\frac{1}{4}\right)$$
$$4a = a + 15$$
$$3a = 15$$
$$\therefore a = 5$$

$$b = \frac{b}{4} + 9$$
$$4b = b + 36$$
$$3b = 36$$
$$\therefore b = 12$$

Therefore: $a = 5$ and $b = 12$.

30. $\triangle PQR$ and $\triangle PSR$ are equilateral triangles. Prove that $PQRS$ is a rhombus.



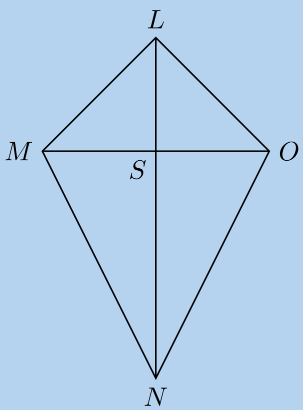
Solution:

We are given two equilateral triangles, therefore in $\triangle PSR$: $PS = SR = PR$ and in $\triangle PQR$: $PQ = QR = PR$. But PR is a common side and so $PR = PS = SR = PQ = QR$.

Also in each triangle all the interior angles are equal to 60°. Therefore $\hat{P}_1 = \hat{R}_2$ and $\hat{P}_2 = \hat{R}_2$. Therefore $PQ \parallel SR$ and $PS \parallel QR$ (alt. int. $\angle$'s equal).

$\therefore$ $PQRS$ is a rhombus (all sides are equal in length, both pairs of opposite sides parallel).

31. $LMNO$ is a quadrilateral with $LM = LO$ and diagonals that intersect at S such that $MS = SO$. Prove that:



a) $M\hat{L}S = S\hat{L}O$

Solution:

$$\begin{aligned}
&\text{In } \triangle LMS \text{ and } \triangle LOS \\
&LM = LO \text{ (given)} \\
&MS = SO \text{ (given)} \\
&LS \text{ is a common side} \\
&\therefore \triangle LMS \equiv \triangle LOS \text{ (SSS)} \\
&\therefore M\hat{L}S = S\hat{L}O
\end{aligned}$$

b) $\triangle LON \equiv \triangle LMN$

Solution:

$$\begin{aligned}
&\text{In } \triangle LON \text{ and } \triangle LMN \\
&LO = LM \text{ (given)} \\
&M\hat{L}S = S\hat{L}O \text{ (proved above)} \\
&LN \text{ is a common side} \\
&\therefore \triangle LON \equiv \triangle LMN \text{ (SAS)}
\end{aligned}$$

c) $MO \perp LN$

Solution:

We need to show that one of $L\hat{S}M$ or $L\hat{S}O$ or $M\hat{S}N$ or $O\hat{S}N$ is equal to 90°.

We have already proved that $M\hat{L}S = O\hat{L}S$ and that $L\hat{M}S = L\hat{O}S$ (using congruent triangles).

We also note that $M\hat{L}O = M\hat{L}S + O\hat{L}S$.

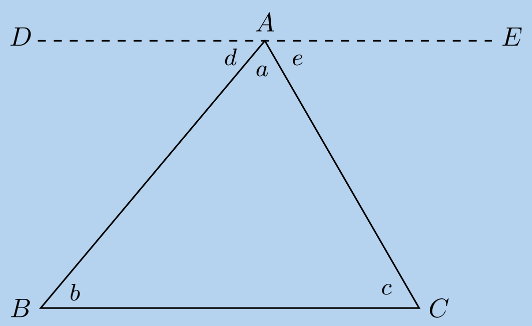
Next we note that:

$$\begin{aligned}
M\hat{L}S + O\hat{L}S + L\hat{M}S = L\hat{O}S &= 180° \text{ (sum of } \angle\text{s in } \triangle) \\
\therefore 2(M\hat{L}S) + 2(L\hat{M}S) &= 180° \\
2(M\hat{L}S + L\hat{M}S) &= 180° \\
M\hat{L}S + L\hat{M}S &= 90°
\end{aligned}$$

Now we note that:

$$\begin{aligned}
&L\hat{S}O = M\hat{L}S + L\hat{M}S \text{ (ext } \angle \text{ of } \triangle) \\
&\therefore L\hat{S}O = 90° \\
&\therefore MO \perp LN
\end{aligned}$$

32. Using the figure below, show that the sum of the three angles in a triangle is 180°. Line DE is parallel to BC.



Solution:

$DE \parallel BC$ (given).

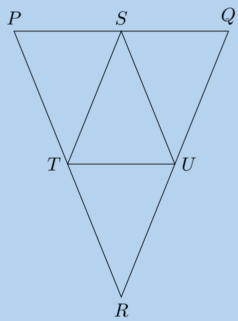
$e = c$ (alt $\angle$s; $DE \parallel BC$).

$d = b$ (alt $\angle$s; $DE \parallel BC$).

$d + a + e = 180°$ ($\angle$s on str line).

And we have shown that $e = c$ and $d = b$ therefore we can replace d with b and e with c to get:
$a + b + c = 180°$.
Therefore the angles in a triangle add up to $180°$.

33. PQR is an isosceles triangle with $PR = QR$. S is the mid-point of PQ, T is the mid-point of PR and U is the mid-point of RQ.



a) Prove $\triangle STU$ is also isosceles.
Solution:

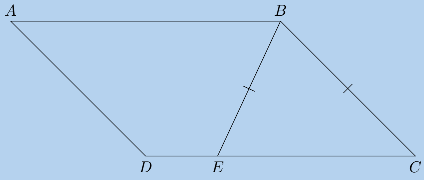
$$
\begin{aligned}
PT &= \frac{1}{2}PR \text{ (given)}\\
&S \text{ mid-point of } PQ\\
&U \text{ mid-point of } RQ\\
SU &= \frac{1}{2}PR\\
\therefore SU &= PT\\
&S \text{ mid-point of } PQ\\
&T \text{ mid-point of } PR\\
\therefore ST &= \frac{1}{2}QR = QU\\
\text{But } PR &= QR \text{ (given)}\\
\therefore SU &= ST\\
\therefore \triangle STU &\text{ is isosceles.}
\end{aligned}
$$

b) What type of quadrilateral is $STRU$? Motivate your answer.
Solution:
$STRU$ is a rhombus. It is a parallelogram since $SU \parallel TR$ and $ST \parallel UR$ (from the mid-point theorem) with four equal sides: $US = ST = TR = RU$ (given and proved above).

c) If $R\hat{T}U = 68°$ calculate, with reasons, the size of $T\hat{S}U$.
Solution:

$$
\begin{aligned}
R\hat{T}U &= 68°\\
\therefore T\hat{U}S &= 68° \qquad \text{(alt } \angle\text{s; } TR \parallel SU\text{)}\\
\therefore S\hat{T}U &= 68° \qquad \text{(}\angle\text{s opp equal sides}\\
\therefore T\hat{S}U &= 180° - 2(68)° \qquad \text{(sum of } \angle\text{s in } \triangle\text{)}\\
\therefore T\hat{S}U &= 180° - 136°\\
&= 44°
\end{aligned}
$$

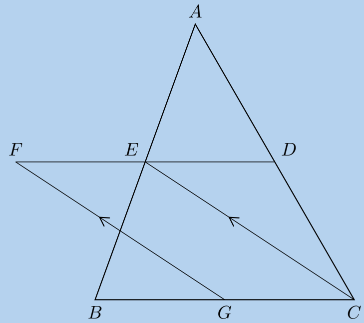
34. $ABCD$ is a parallelogram. $BE = BC$. Prove that $A\hat{B}E = B\hat{C}D$.



Solution:

$$\begin{aligned} B\hat{C}D &= B\hat{E}C \qquad (\angle\text{s opp equal sides} \\ A\hat{B}E &= B\hat{E}C \text{ (alt } \angle\text{s; } AB \parallel DC) \\ \therefore A\hat{B}E &= B\hat{C}D \end{aligned}$$

35. In the diagram below, D, E and G are the mid-points of AC, AB and BC respectively. $EC \parallel FG$.



a) Prove that $FECG$ is a parallelogram.
Solution:

$$\begin{aligned} AE &= EB \text{ (} E \text{ is mid-point)} \\ AD &= DC \text{ (} D \text{ is mid-point)} \\ FD &\parallel BC \text{ (Midpt Theorem)} \\ EC &\parallel FG \text{ (given)} \\ \therefore FECG & \text{ is a parallelogram (opp sides of quad are } \parallel \text{)} \end{aligned}$$

b) Prove that $FE = ED$.
Solution:

$$\begin{aligned} ED &= \frac{1}{2}BC \text{ (Midpt Theorem)} \\ GC &= \frac{1}{2}BC \text{(definition of mid-point)} \\ \therefore ED &= GC \\ FE &= GC \text{ (opp sides of } \parallel \text{m)} \\ \therefore ED &= FE \end{aligned}$$

For more exercises, visit www.everythingmaths.co.za and click on 'Practise Maths'.

1a. 2G87	1b. 2G88	1c. 2G89	1d. 2G8B	1e. 2G8C	1f. 2G8D
1g. 2G8F	1h. 2G8G	2a. 2G8H	2b. 2G8J	2c. 2G8K	2d. 2G8M
2e. 2G8N	2f. 2G8P	2g. 2G8Q	2h. 2G8R	2i. 2G8S	2j. 2G8T
3a. 2G8V	3b. 2G8W	3c. 2G8X	4a. 2G8Y	4b. 2G8Z	4c. 2G92
5. 2G93	6. 2G94	7. 2G95	8. 2G96	9. 2G97	10a. 2G98
10b. 2G99	10c. 2G9B	10d. 2G9C	11a. 2G9D	11b. 2G9F	11c. 2G9G
11d. 2G9H	12a. 2G9J	12b. 2G9K	12c. 2G9M	12d. 2G9N	12e. 2G9P
12f. 2G9Q	13. 2G9R	14. 2G9S	15. 2G9T	16. 2G9V	17. 2G9W
18. 2G9X	19. 2G9Y	20. 2G9Z	21. 2GB2	22. 2GB3	23. 2GB4
24. 2GB5	25. 2GB6	26. 2GB7	27. 2GB8	28. 2GB9	29. 2GBB
30. 2GBC	31. 2GBD	32. 2GBF	33. 2GBG	34. 2GBH	35. 2GBJ

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