CHAPTER 5

Trigonometry

5.1 Introduction

- Content covered in this chapter includes defining the trigonometric ratios and extending these definitions to any angle. Also covered is the definitions of the reciprocals of the trigonometric ratios. Both the trigonometric ratios and their reciprocals are solved for several special angles. In addition simple trigonometric equations are covered.
- Solving problems in two-dimensions using trigonometry is only covered later in the year and the content for this can be found in chapter 11.
- Similarity of triangles is fundamental to the trigonometric ratios
- Trigonometric ratios are independent of the lengths of the sides and instead depend only on the angles
- Doubling a ratio has a different effect from doubling an angle.
- Emphasise the value and importance of making sketches, where appropriate.
- Remind learners that angles in the Cartesian plane are always measured from the positive x-axis.
- When working with angles on the Cartesian plane remind learners to check that their answers are within the correct quadrant.
- Calculator skills are very important in this chapter. Methods for CASIO calculators are shown but practical demonstration may be required. For a SHARP calculator the keys are generally the same although the SHIFT key is now the 2ndF key.
- We will refer to sine, cosine, tangent, secant, cosecant and cotangent as trigonometric ratios rather than as trigonometric functions. Both these terms are correct though but for the nature of the content in this chapter the term ratio better captures the content and is likely to be more accessible to learners at this stage.

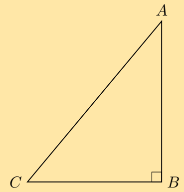
Fabumaths has some useful links and content for trigonometry.

5.2 Similarity of triangles

5.3 Defining the trigonometric ratios

Exercise 5 – 1:

1. Complete each of the following:



a) $\sin \hat{A} =$

Solution:

First find the right angle, the hypotenuse is always directly opposite the right angle. The opposite and adjacent sides depend on the angle we are looking at. The opposite side relative to the angle $\hat{A}$ is directly opposite the angle $\hat{A}$. Finally the adjacent side is the remaining side of the triangle and must be one of the sides that forms the angle $\hat{A}$.

$$\sin \hat{A} = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{CB}{AC}$$

b) $\cos \hat{A} =$

Solution:

First find the right angle, the hypotenuse is always directly opposite the right angle. The opposite and adjacent sides depend on the angle we are looking at. The opposite side relative to the angle $\hat{A}$ is directly opposite the

angle $\hat{A}$. Finally the adjacent side is the remaining side of the triangle and must be one of the sides that forms the angle $\hat{A}$.

$$\cos \hat{A} = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{AB}{AC}$$

c) $\tan \hat{A} =$

Solution:

First find the right angle, the hypotenuse is always directly opposite the right angle. The opposite and adjacent sides depend on the angle we are looking at. The opposite side relative to the angle $\hat{A}$ is directly opposite the angle $\hat{A}$. Finally the adjacent side is the remaining side of the triangle and must be one of the sides that forms the angle $\hat{A}$.

$$\tan \hat{A} = \frac{\text{opposite}}{\text{adjacent}} = \frac{CB}{AB}$$

d) $\sin \hat{C} =$

Solution:

First find the right angle, the hypotenuse is always directly opposite the right angle. The opposite and adjacent sides depend on the angle we are looking at. The opposite side relative to the angle $\hat{C}$ is directly opposite the angle $\hat{C}$. Finally the adjacent side is the remaining side of the triangle and must be one of the sides that forms the angle $\hat{C}$.

$$\sin \hat{C} = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{AB}{AC}$$

e) $\cos \hat{C} =$

Solution:

First find the right angle, the hypotenuse is always directly opposite the right angle. The opposite and adjacent sides depend on the angle we are looking at. The opposite side relative to the angle $\hat{C}$ is directly opposite the angle $\hat{C}$. Finally the adjacent side is the remaining side of the triangle and must be one of the sides that forms the angle $\hat{C}$.

$$\cos \hat{C} = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{CB}{AC}$$

f) $\tan \hat{C} =$

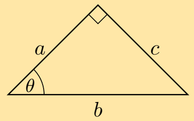
Solution:

First find the right angle, the hypotenuse is always directly opposite the right angle. The opposite and adjacent sides depend on the angle we are looking at. The opposite side relative to the angle $\hat{C}$ is directly opposite the angle $\hat{C}$. Finally the adjacent side is the remaining side of the triangle and must be one of the sides that forms the angle $\hat{C}$.

$$\tan \hat{C} = \frac{\text{opposite}}{\text{adjacent}} = \frac{AB}{CB}$$

2. In each of the following triangles, state whether a, b and c are the hypotenuse, opposite or adjacent sides of the triangle with respect to θ.

a)



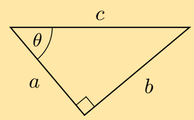
Solution:

First find the right angle, the hypotenuse is always directly opposite the right angle. The opposite and adjacent sides depend on the angle we are looking at. The opposite side relative to the angle θ is directly opposite the angle θ. Finally the adjacent side is the remaining side of the triangle and must be one of the sides that forms the angle θ.

- a is the adjacent side

- b is the hypotenuse
- c is the opposite side

b)

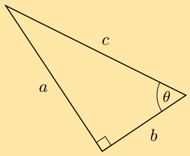


Solution:

First find the right angle, the hypotenuse is always directly opposite the right angle. The opposite and adjacent sides depend on the angle we are looking at. The opposite side relative to the angle θ is directly opposite the angle θ. Finally the adjacent side is the remaining side of the triangle and must be one of the sides that forms the angle θ.

- a is the adjacent side
- b is the opposite side
- c is the hypotenuse

c)

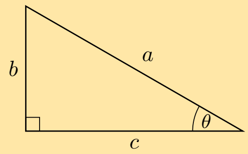


Solution:

First find the right angle, the hypotenuse is always directly opposite the right angle. The opposite and adjacent sides depend on the angle we are looking at. The opposite side relative to the angle θ is directly opposite the angle θ. Finally the adjacent side is the remaining side of the triangle and must be one of the sides that forms the angle θ.

- a is the opposite side
- b is the adjacent side
- c is the hypotenuse

d)

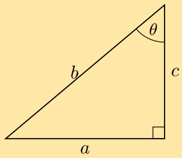


Solution:

First find the right angle, the hypotenuse is always directly opposite the right angle. The opposite and adjacent sides depend on the angle we are looking at. The opposite side relative to the angle θ is directly opposite the angle θ. Finally the adjacent side is the remaining side of the triangle and must be one of the sides that forms the angle θ.

- a is the hypotenuse
- b is the opposite side
- c is the adjacent side

e)

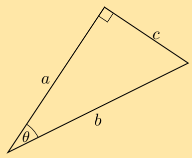


Solution:

First find the right angle, the hypotenuse is always directly opposite the right angle. The opposite and adjacent sides depend on the angle we are looking at. The opposite side relative to the angle θ is directly opposite the angle θ. Finally the adjacent side is the remaining side of the triangle and must be one of the sides that forms the angle θ.

- a is the opposite side
- b is the hypotenuse
- c is the adjacent side

f)

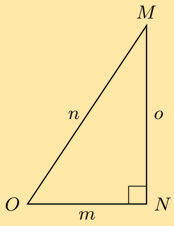


Solution:

First find the right angle, the hypotenuse is always directly opposite the right angle. The opposite and adjacent sides depend on the angle we are looking at. The opposite side relative to the angle θ is directly opposite the angle θ. Finally the adjacent side is the remaining side of the triangle and must be one of the sides that forms the angle θ.

- a is the adjacent side
- b is the hypotenuse
- c is the opposite side

3. Consider the following diagram:



Without using a calculator, answer each of the following questions.

a) Write down $\cos \hat{O}$ in terms of m, n and o.

Solution:

- m is the adjacent side
- n is the hypotenuse
- o is the opposite side

$$\cos \hat{O} = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{m}{o}$$

b) Write down $\tan \hat{M}$ in terms of m, n and o.

Solution:

- m is the opposite side
- n is the hypotenuse
- o is the adjacent side

$$\tan \hat{M} = \frac{\text{opposite}}{\text{adjacent}} = \frac{m}{o}$$

c) Write down $\sin \hat{O}$ in terms of m, n and o.

Solution:

- m is the adjacent side
- n is the hypotenuse
- o is the opposite side

$$\sin \hat{O} = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{o}{n}$$

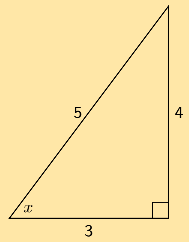
d) Write down $\cos \hat{M}$ in terms of m, n and o.

Solution:

- m is the opposite side
- n is the hypotenuse
- o is the adjacent side

$\cos \hat{M} = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{o}{n}$

4. Find x in the diagram in three different ways. You do not need to calculate the value of x, just write down the appropriate ratio for x.



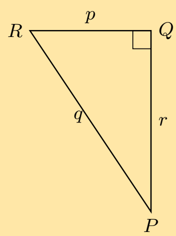
Solution:

- Side of length 4 is the opposite side
- Side of length 5 is the hypotenuse
- Side of length 3 is the adjacent side

Notice that the hypotenuse is the longest side as we would expect.

$$\sin x = \frac{4}{5}$$
$$\cos x = \frac{3}{5}$$
$$\tan x = \frac{4}{3}$$

5. Which of these statements is true about $\triangle PQR$?



a) $\sin \hat{R} = \frac{p}{q}$ b) $\tan \hat{Q} = \frac{r}{p}$ c) $\cos \hat{P} = \frac{r}{q}$ d) $\sin \hat{P} = \frac{p}{r}$

Solution:

We first find the opposite and adjacent sides with respect to $\hat{P}$ and $\hat{R}$:

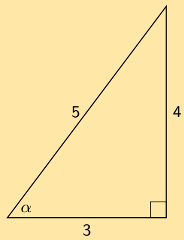
- p is the opposite side to $\hat{P}$ and the adjacent side to $\hat{R}$
- q is the hypotenuse
- r is the adjacent side to $\hat{P}$ and the opposite side to $\hat{R}$

We also note that:

- $\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$
- $\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$
- $\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$

Looking at each of the given ratios we can see that only $\cos \hat{P} = \frac{r}{q}$ is correct.

6. Sarah wants to find the value of α in the triangle below. Which statement is a correct line of working?



a) $\sin\alpha = \frac{4}{5}$
b) $\cos\left(\frac{3}{5}\right) = \alpha$
c) $\tan\alpha = \frac{5}{4}$
d) $\cos 0{,}8 = \alpha$

Solution:

Sarah first needs to identify the hypotenuse, opposite and adjacent sides in the triangle. She then needs to write down a trigonometric ratio that will allow her to find α.

$\sin\alpha = \frac{4}{5}$ is one such ratio that will help her find α. From the given list of options this is the only correct line of reasoning.

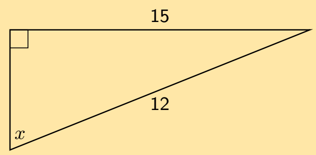
$\cos\left(\frac{3}{5}\right) = \alpha$ has the angle and the lengths of the sides switched around.

$\tan\alpha = \frac{3}{4}$ uses the wrong sides with respect to α for $\tan$.

$\cos 0{,}8 = \alpha$ uses the wrong sides with respect to α for $\cos$. Note that you can reduce the fraction to a decimal number but you need to first write the correct fraction.

7. Explain what is wrong with each of the following diagrams.

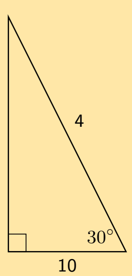
a)



Solution:

The hypotenuse is too small. The hypotenuse is the longest side of the right-angled triangle and in this case one side of the triangle is given as being larger than the hypotenuse.

b)



Solution:

The hypotenuse is too small. The hypotenuse is the longest side of the right-angled triangle and in this case one side of the triangle is given as being larger than the hypotenuse.

For more exercises, visit www.everythingmaths.co.za and click on 'Practise Maths'.

1. 2FN9 2a. 2FNB 2b. 2FNH 2c. 2FNC 2d. 2FND 2e. 2FNF
2f. 2FNG 3a. 2FNJ 3b. 2FNK 3c. 2FNM 3d. 2FNN 4. 2FNP
5. 2FNQ 6. 2FNR 7a. 2FNT 7b. 2FNS

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5.4 Reciprocal ratios

5.5 Calculator skills

Exercise 5 – 2:

1. Use your calculator to determine the value of the following (correct to 2 decimal places):

   a) $\tan 65°$
   Solution:

$$\begin{aligned} \tan 65° &= 2{,}1445069... \\ &\approx 2{,}14 \end{aligned}$$

   b) $\sin 38°$
   Solution:

$$\begin{aligned} \sin 38° &= 0{,}615661... \\ &\approx 0{,}62 \end{aligned}$$

   c) $\cos 74°$
   Solution:

$$\begin{aligned} \cos 74° &= 0{,}275637... \\ &\approx 0{,}28 \end{aligned}$$

   d) $\sin 12°$
   Solution:

$$\begin{aligned} \sin 12° &= 0{,}20791... \\ &\approx 0{,}21 \end{aligned}$$

   e) $\cos 26°$
   Solution:

$$\begin{aligned} \cos 26° &= 0{,}898794... \\ &\approx 0{,}90 \end{aligned}$$

   f) $\tan 49°$
   Solution:

$$\begin{aligned} \tan 49° &= 1{,}150368... \\ &\approx 1{,}15 \end{aligned}$$

   g) $\sin 305°$
   Solution:

$$\begin{aligned} \sin 305° &= -0{,}81915... \\ &\approx -0{,}82 \end{aligned}$$

   h) $\tan 124°$
   Solution:

$$\begin{aligned} \tan 124° &= -1{,}482560... \\ &\approx -1{,}48 \end{aligned}$$

i) $\sec 65°$
Solution:

$$\begin{aligned}\sec 65° &= \frac{1}{\cos 65°} \\ &= 2{,}36620... \\ &\approx 2{,}37\end{aligned}$$

j) $\sec 10°$
Solution:

$$\begin{aligned}\sec 10° &= \frac{1}{\cos 10°} \\ &= 1{,}01542... \\ &\approx 1{,}02\end{aligned}$$

k) $\sec 48°$
Solution:

$$\begin{aligned}\sec 48° &= \frac{1}{\cos 48°} \\ &= 1{,}49447... \\ &\approx 1{,}49\end{aligned}$$

l) $\cot 32°$
Solution:

$$\begin{aligned}\cot 32° &= \frac{1}{\tan 32°} \\ &= 1{,}6003334... \\ &\approx 1{,}60\end{aligned}$$

m) $\text{cosec } 140°$
Solution:

$$\begin{aligned}\text{cosec } 140° &= \frac{1}{\sin 140°} \\ &= 1{,}555724... \\ &\approx 1{,}56\end{aligned}$$

n) $\text{cosec } 237°$
Solution:

$$\begin{aligned}\text{cosec } 237° &= \frac{1}{\sin 237°} \\ &= -1{,}192363... \\ &\approx -1{,}19\end{aligned}$$

o) $\sec 231°$
Solution:

$$\begin{aligned}\sec 231° &= \frac{1}{\cos 231°} \\ &= -1{,}589016... \\ &\approx -1{,}59\end{aligned}$$

p) cosec $226°$

Solution:

$$\begin{aligned}\text{cosec } 226° &= \frac{1}{\sin 226°}\\ &= -1{,}390164...\\ &\approx -1{,}39\end{aligned}$$

q) $\frac{1}{4}\cos 20°$

Solution:

$$\begin{aligned}\frac{1}{4}\cos 20° &= \frac{1}{4}(0{,}939692...)\\ &= 0{,}234923...\\ &\approx 0{,}23\end{aligned}$$

r) $3\tan 40°$

Solution:

$$\begin{aligned}3\tan 40° &= 3(0{,}83909963...)\\ &= 2{,}517298894...\\ &\approx 2{,}52\end{aligned}$$

s) $\frac{2}{3}\sin 90°$

Solution:

$$\begin{aligned}\frac{2}{3}\sin 90° &= \frac{2}{3}(1)\\ &= 0{,}66666...\\ &\approx 0{,}67\end{aligned}$$

t) $\frac{5}{\cos 4{,}3°}$

Solution:

$$\begin{aligned}\frac{5}{\cos 4{,}3°} &= \frac{5}{0{,}9971...}\\ &\approx 5{,}01\end{aligned}$$

u) $\sqrt{\sin 55°}$

Solution:

$$\begin{aligned}\sqrt{\sin 55°} &= \sqrt{0{,}81915...}\\ &\approx 0{,}91\end{aligned}$$

v) $\frac{\sin 90°}{\cos 90°}$

Solution:

$$\begin{aligned}\frac{\sin 90°}{\cos 90°} &= \frac{1}{0}\\ &\text{undefined}\end{aligned}$$

w) $\tan 35° + \cot 35°$

Solution:

$$\begin{aligned} \tan 35° + \cot 35° &= 0{,}7002... + \frac{1}{\tan 35°} \\ &= 0{,}7002... + 1{,}4281... \\ &\approx 2{,}13 \end{aligned}$$

x) $\dfrac{2 + \cos 310°}{2 + \sin 87°}$

Solution:

$$\begin{aligned} \frac{2 + \cos 310°}{2 + \sin 87°} &= \frac{2{,}64278...}{2{,}99862...} \\ &\approx 0{,}88 \end{aligned}$$

y) $\sqrt{4 \sec 99°}$

Solution:

$$\begin{aligned} \sqrt{4 \sec 99°} &= \sqrt{\frac{4}{\cos 99°}} \\ &= \sqrt{-25{,}5698...} \\ &\text{non-real} \end{aligned}$$

z) $\sqrt{\dfrac{\cot 103° + \sin 1090°}{\sec 10° + 5}}$

Solution:

$$\begin{aligned} \sqrt{\frac{\cot 85° + \sin 1090°}{\sec 10° + 5}} &= \sqrt{\frac{\frac{1}{\tan 85°} + \sin 1090°}{\frac{1}{\cos 10°} + 5}} \\ &= \sqrt{\frac{0{,}2611...}{6{,}015...}} \\ &= \sqrt{0{,}043411...} \\ &\approx 0{,}21 \end{aligned}$$

2. If $x = 39°$ and $y = 21°$, use a calculator to determine whether the following statements are true or false:

a) $\cos x + 2\cos x = 3\cos x$

Solution:

LHS:

$$\begin{aligned} \cos x + 2\cos x &= \cos 39° + 2\cos 39° \\ &= 0{,}7771... + 1{,}55429... \\ &= 2{,}3314... \\ &\approx 2{,}33 \end{aligned}$$

RHS:

$$\begin{aligned} 3\cos x &= 3\cos 39° \\ &= 2{,}3314... \\ &\approx 2{,}33 \end{aligned}$$

Therefore the statement is true.

b) $\cos 2y = \cos y + \cos y$

Solution:

LHS:

$$\begin{aligned}\cos 2y &= \cos 2(21°)\\ &= 0{,}7431...\\ &\approx 0{,}74\end{aligned}$$

RHS:

$$\begin{aligned}\cos y + \cos y &= \cos 21° + \cos 21°\\ &= 0{,}93358... + 0{,}93358...\\ &= 1{,}86716...\\ &\approx 1{,}86\end{aligned}$$

Therefore the statement is false.

c) $\tan x = \frac{\sin x}{\cos x}$

Solution:

LHS:

$$\begin{aligned}\tan x &= \tan 39°\\ &= 0{,}809784...\\ &\approx 0{,}81\end{aligned}$$

RHS:

$$\begin{aligned}\frac{\sin x}{\cos x} &= \frac{\sin 39°}{\cos 39°}\\ &= \frac{0{,}62932...}{0{,}777145...}\\ &= 0{,}80978...\\ &\approx 0{,}81\end{aligned}$$

Therefore the statement is true.

d) $\cos(x + y) = \cos x + \cos y$

Solution:

LHS:

$$\begin{aligned}\cos(x + y) &= \cos 39° + 21°\\ &\approx 0{,}5\end{aligned}$$

RHS:

$$\begin{aligned}\cos x + \cos y &= \cos 39° + \cos 21°\\ &= 0{,}777145... + 0{,}933358...\\ &= 1{,}71072...\\ &\approx 1{,}71\end{aligned}$$

Therefore the statement is false.

3. Solve for x in $5^{\tan x} = 125$.

Solution:

To solve this problem we need to recall from exponents that if $a^x = a^y$ then $x = y$. Then we note that $125 = 5^3$. Now we can solve the problem:

$$\begin{aligned}5^{\tan x} &= 5^3\\ \therefore \tan x &= 3\\ x &= 71{,}56505...\\ &\approx 71{,}57\end{aligned}$$

For more exercises, visit www.everythingmaths.co.za and click on 'Practise Maths'.

1a. 2FNW	1b. 2FNX	1c. 2FNY	1d. 2FNZ	1e. 2FP2	1f. 2FP3
1g. 2FP4	1h. 2FP5	1i. 2FP6	1j. 2FP7	1k. 2FP8	1l. 2FP9
1m. 2FPB	1n. 2FPC	1o. 2FPD	1p. 2FPF	1q. 2FPG	1r. 2FPH
1s. 2FPJ	1t. 2FPK	1u. 2FPM	1v. 2FPN	1w. 2FPP	1x. 2FPQ
1y. 2FPR	1z. 2FPS	2. 2FPT	3. 2FPV		

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5.6 Special angles

Exercise 5 – 3:

1. Select the closest answer for each expression from the list provided:

a) $\cos 45°$

$$\frac{1}{2} \quad 1 \quad \sqrt{2} \quad \frac{1}{\sqrt{3}} \quad \frac{1}{\sqrt{2}}$$

Solution:

$$\cos 45° = \frac{1}{\sqrt{2}}$$

b) $\sin 45°$

$$\sqrt{2} \quad \frac{1}{2} \quad \frac{1}{\sqrt{2}} \quad \frac{\sqrt{3}}{1} \quad 1$$

Solution:

$$\sin 45° = \frac{1}{\sqrt{2}}$$

c) $\tan 30°$

$$\frac{1}{2} \quad \frac{\sqrt{3}}{1} \quad \frac{1}{\sqrt{2}} \quad \frac{\sqrt{3}}{2} \quad \frac{1}{\sqrt{3}}$$

Solution:

$$\tan 30° = \frac{1}{\sqrt{3}}$$

d) $\tan 60°$

$$\frac{\sqrt{3}}{2} \quad \frac{1}{\sqrt{2}} \quad \frac{1}{\sqrt{3}} \quad \frac{\sqrt{3}}{1} \quad \frac{1}{1}$$

Solution:

$$\tan 60° = \frac{\sqrt{3}}{1}$$

e) $\cos 45°$

$$\frac{\sqrt{3}}{2} \quad \frac{1}{2} \quad \frac{1}{\sqrt{3}} \quad \frac{1}{\sqrt{2}} \quad \sqrt{2}$$

Solution:

$$\cos 45° = \frac{1}{\sqrt{2}}$$

f) $\tan 30°$

$\frac{1}{\sqrt{2}}$ $\frac{1}{2}$ $\frac{\sqrt{3}}{2}$ $\frac{1}{1}$ $\frac{1}{\sqrt{3}}$

Solution:

$$\tan 30° = \frac{1}{\sqrt{3}}$$

g) $\tan 30°$

$\frac{1}{\sqrt{3}}$ $\frac{1}{2}$ $\frac{1}{\sqrt{2}}$ $\frac{\sqrt{3}}{2}$ $\frac{\sqrt{3}}{1}$

Solution:

$$\tan 30° = \frac{1}{\sqrt{3}}$$

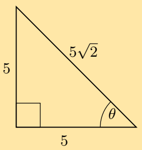
h) $\cos 60°$

$\frac{1}{\sqrt{3}}$ $\frac{1}{2}$ $\frac{\sqrt{3}}{2}$ $\frac{\sqrt{3}}{1}$ $\frac{1}{\sqrt{2}}$

Solution:

$$\cos 60° = \frac{1}{2}$$

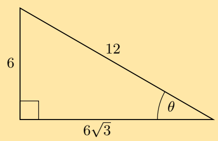
2. Solve for $\cos\theta$ in the following triangle, in surd form:



Solution:

$$\begin{aligned}\cos\theta &= \frac{\text{opposite}}{\text{hypotenuse}}\\ &= \frac{5}{5\sqrt{2}}\\ &= \frac{1}{\sqrt{2}}\end{aligned}$$

3. Solve for $\tan\theta$ in the following triangle, in surd form:



Solution:

$$\begin{aligned}\tan\theta &= \frac{\text{opposite}}{\text{adjacent}}\\ &= \frac{6}{6\sqrt{3}}\\ &= \frac{1}{\sqrt{3}}\end{aligned}$$

4. Calculate the value of the following without using a calculator:

a) $\sin 45° \times \cos 45°$

Solution:

For both ratios the angle given is 45°. This is one of the special angles. We note that $\sin 45° = \cos 45° = \frac{1}{\sqrt{2}}$ using special angles.

$$\begin{aligned} \sin 45° \times \cos 45° &= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right) \\ &= \frac{1}{2} \end{aligned}$$

b) $\cos 60° + \tan 45°$

Solution:

We are given angles of 45° and 60°. These are both special angles. We note that $\cos 60° = \frac{1}{2}$ and $\tan 45° = 1$ using special angles.

$$\begin{aligned} \cos 60° + \tan 45° &= \frac{1}{2} + 1 \\ &= \frac{3}{2} \end{aligned}$$

c) $\sin 60° - \cos 60°$

Solution:

For both ratios the angle given is 60°. This is one of the special angles. We note that $\sin 60° = \frac{\sqrt{3}}{2}$ and $\cos 60° = \frac{1}{2}$ using special angles.

$$\begin{aligned} \sin 60° - \cos 60° &= \frac{\sqrt{3}}{2} - \frac{1}{2} \\ &= \frac{\sqrt{3} - 1}{2} \end{aligned}$$

5. Evaluate the following without using a calculator. Select the closest answer from the list provided.

a) $\tan 45° \div \sin 60°$

$\frac{2}{\sqrt{3}}$ $\frac{\sqrt{3}}{1}$ $\frac{\sqrt{2}}{\sqrt{3}}$ $\frac{1}{1}$ $\frac{1}{2}$

Solution:

We need to use special angles to help us solve this problem. First write down each ratio using special angles and then simplify the answer.

$$\begin{aligned} \tan 45° \div \sin 60° &= \frac{1}{1} \div \frac{\sqrt{3}}{2} \\ &= 1 \times \frac{2}{\sqrt{3}} \\ &= \frac{2}{\sqrt{3}} \end{aligned}$$

b) $\tan 30° - \sin 60°$

0 $\frac{1}{2}$ $-\frac{\sqrt{3}}{2}$ $-\frac{1}{2\sqrt{3}}$ $\frac{\sqrt{3}}{2}$

Solution:

We need to use special angles to help us solve this problem. First write down each ratio using special angles and then simplify the answer.

$$\begin{aligned}
\tan 30^\circ - \sin 60^\circ &= \frac{1}{\sqrt{3}} - \frac{\sqrt{3}}{2} \\
&= \frac{2 - \left(\sqrt{3}\right)\left(\sqrt{3}\right)}{(2)\left(\sqrt{3}\right)} \\
&= \frac{2-3}{2\sqrt{3}} \\
&= \frac{-1}{2\sqrt{3}}
\end{aligned}$$

c) $\sin 30^\circ - \tan 45^\circ - \sin 30^\circ$

$$-\tfrac{\sqrt{3}}{2} \quad -1 \quad -\tfrac{1}{\sqrt{3}} \quad -\tfrac{\sqrt{3}}{1} \quad -\tfrac{7}{2\sqrt{3}}$$

Solution:

We need to use special angles to help us solve this problem. First write down each ratio using special angles and then simplify the answer.

$$\begin{aligned}
\sin 30^\circ - \tan 45^\circ - \sin 30^\circ &= \frac{1}{2} - \frac{1}{1} - \frac{1}{2} \\
&= \frac{1-2-1}{2} \\
&= -1
\end{aligned}$$

d) $\tan 30^\circ \div \tan 30^\circ \div \sin 45^\circ$

$$\tfrac{\sqrt{3}}{1} \quad \tfrac{2\sqrt{3}}{\sqrt{2}} \quad \tfrac{2}{\sqrt{3}} \quad \tfrac{\sqrt{2}}{1} \quad \tfrac{2\sqrt{2}}{\sqrt{3}}$$

Solution:

We need to use special angles to help us solve this problem. First write down each ratio using special angles and then simplify the answer.

$$\begin{aligned}
\tan 30^\circ \div \tan 30^\circ \div \sin 45^\circ &= \frac{1}{\sqrt{3}} \div \frac{1}{\sqrt{3}} \div \frac{1}{\sqrt{2}} \\
&= \left(\frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{1}\right) \div \frac{1}{\sqrt{2}} \\
&= 1 \times \frac{\sqrt{2}}{1} \\
&= \frac{\sqrt{2}}{1}
\end{aligned}$$

e) $\sin 45^\circ \div \sin 30^\circ \div \cos 45^\circ$

$$\tfrac{\sqrt{2}}{\sqrt{3}} \quad \tfrac{1}{\sqrt{2}} \quad \tfrac{4}{\sqrt{3}} \quad 2 \quad \tfrac{2\sqrt{2}}{\sqrt{3}}$$

Solution:

We need to use special angles to help us solve this problem. First write down each ratio using special angles and then simplify the answer.

$$\begin{aligned}
\sin 45^\circ \div \sin 30^\circ \div \cos 45^\circ &= \frac{1}{\sqrt{2}} \div \frac{1}{2} \div \frac{1}{\sqrt{2}} \\
&= \left(\frac{1}{\sqrt{2}} \times \frac{2}{1}\right) \div \frac{1}{\sqrt{2}} \\
&= \frac{2}{\sqrt{2}} \times \frac{\sqrt{2}}{1} \\
&= 2
\end{aligned}$$

f) $\tan 60° - \tan 60° - \sin 60°$

$$-\frac{1}{\sqrt{3}} \quad -\frac{1}{2} \quad -\frac{1}{\sqrt{2}} \quad -\frac{1}{1} \quad \frac{-\sqrt{3}}{2}$$

Solution:

We need to use special angles to help us solve this problem. First write down each ratio using special angles and then simplify the answer.

$$\begin{aligned}
\tan 60° - \tan 60° - \sin 60° &= \frac{\sqrt{3}}{1} - \frac{\sqrt{3}}{1} - \frac{\sqrt{3}}{2} \\
&= \frac{(2)\left(\sqrt{3}\right) - (2)\left(\sqrt{3}\right) - \sqrt{3}}{2} \\
&= \frac{-\sqrt{3}}{2}
\end{aligned}$$

g) $\cos 45° - \sin 60° - \sin 45°$

$$-\frac{1}{2} \quad -\frac{1}{\sqrt{2}} \quad -\frac{7}{2\sqrt{3}} \quad \frac{-\sqrt{3}}{2} \quad -\frac{1}{\sqrt{3}}$$

Solution:

We need to use special angles to help us solve this problem. First write down each ratio using special angles and then simplify the answer.

$$\begin{aligned}
\cos 45° - \sin 60° - \sin 45° &= \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \\
&= \frac{2 - \left(\sqrt{3}\right)\left(\sqrt{2}\right) - 2}{(2)\left(\sqrt{2}\right)} \\
&= \frac{-\sqrt{3}\sqrt{2}}{2\sqrt{2}} \\
&= \frac{-\sqrt{3}}{2}
\end{aligned}$$

6. Use special angles to show that:

a) $\dfrac{\sin 60°}{\cos 60°} = \tan 60°$

Solution:

We are told to use special angles, so we first write each ratio using special angles and then simplify each side of the equation.

LHS:

$$\begin{aligned}
\frac{\sin 60°}{\cos 60°} &= \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} \\
&= \frac{\sqrt{3}}{2} \times \frac{2}{1} \\
&= \sqrt{3}
\end{aligned}$$

RHS:

$$\tan 60° = \sqrt{3}$$

Therefore the equation is true.

b) $\sin^2 45° + \cos^2 45° = 1$

Solution:

We are told to use special angles, so we first write each ratio using special angles and then simplify each side of the equation.

LHS:

$$\sin^2 45° + \cos^2 45° = \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right)$$
$$= \frac{1}{2} + \frac{1}{2}$$
$$= 1$$

RHS $= 1$
Therefore the equation is true.

c) $\cos 30° = \sqrt{1 - \sin^2 30°}$
Solution:
We are told to use special angles, so we first write each ratio using special angles and then simplify each side of the equation.
LHS:

$$\cos 30° = \frac{\sqrt{3}}{2}$$

RHS:

$$\sqrt{1 - \sin^2 30°} = \sqrt{1 - \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)}$$
$$= \sqrt{1 - \frac{1}{4}}$$
$$= \sqrt{\frac{3}{4}}$$
$$= \frac{\sqrt{3}}{2}$$

Therefore the equation is true.

7. Use the definitions of the trigonometric ratios to answer the following questions:

a) Explain why $\sin \alpha \leq 1$ for all values of α.
Solution:
The sine ratio is defined as $\frac{\text{opposite}}{\text{hypotenuse}}$. In any right-angled triangle, the hypotenuse is the side of longest length. Therefore the maximum length of the opposite side is equal to the length of the hypotenuse. The maximum value of the sine ratio is then $\frac{\text{hypotenuse}}{\text{hypotenuse}} = 1$.

b) Explain why $\cos \alpha$ has a maximum value of 1.
Solution:
The cosine ratio is defined as $\frac{\text{adjacent}}{\text{hypotenuse}}$. In any right-angled triangle, the hypotenuse is the side of longest length. Therefore the maximum length of the adjacent side is equal to the length of the hypotenuse. The maximum value of the cosine ratio is then $\frac{\text{hypotenuse}}{\text{hypotenuse}} = 1$.

c) Is there a maximum value for $\tan \alpha$?
Solution:
The tangent ratio is defined as $\frac{\text{opposite}}{\text{adjacent}}$. Since the opposite and adjacent sides can have any value (so long as the length of the side is less than or equal to the length of the hypotenuse), there is no maximum value for the tangent ratio.

For more exercises, visit www.everythingmaths.co.za and click on 'Practise Maths'.

1a. 2FPW	1b. 2FPX	1c. 2FPY	1d. 2FPZ	1e. 2FQ2	1f. 2FQ3
1g. 2FQ4	1h. 2FQ5	2. 2FQ6	3. 2FQ7	4a. 2FQ8	4b. 2FQ9
4c. 2FQB	5a. 2FQC	5b. 2FQD	5c. 2FQF	5d. 2FQG	5e. 2FQH
5f. 2FQJ	5g. 2FQK	6a. 2FQM	6b. 2FQN	6c. 2FQP	7a. 2FQQ
7b. 2FQR	7c. 2FQS				

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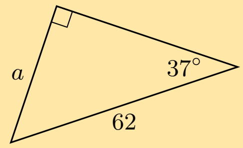
Finding lengths

Exercise 5 – 4:

1. In each triangle find the length of the side marked with a letter. Give your answers correct to 2 decimal places.

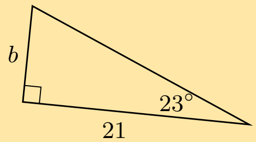
   a)



Solution:

$$\begin{aligned}
\sin\theta &= \frac{\text{opposite}}{\text{hypotenuse}} \\
\sin 37° &= \frac{a}{62} \\
62(0{,}6018\ldots) &= a \\
a &= 36{,}10890\ldots \\
&\approx 36{,}11
\end{aligned}$$

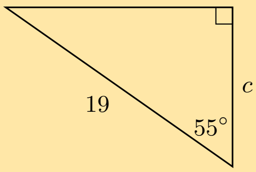
   b)



Solution:

$$\begin{aligned}
\tan\theta &= \frac{\text{opposite}}{\text{adjacent}} \\
\tan 23° &= \frac{b}{21} \\
21(0{,}42447\ldots) &= b \\
b &= 8{,}91397\ldots \\
&\approx 8{,}91
\end{aligned}$$

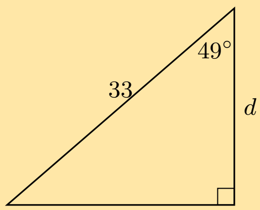
   c)



Solution:

$$\begin{aligned}
\cos\theta &= \frac{\text{adjacent}}{\text{hypotenuse}}\\
\cos 55° &= \frac{c}{19}\\
19(0{,}5735...) &= c\\
c &= 10{,}89795...\\
&\approx 10{,}90
\end{aligned}$$

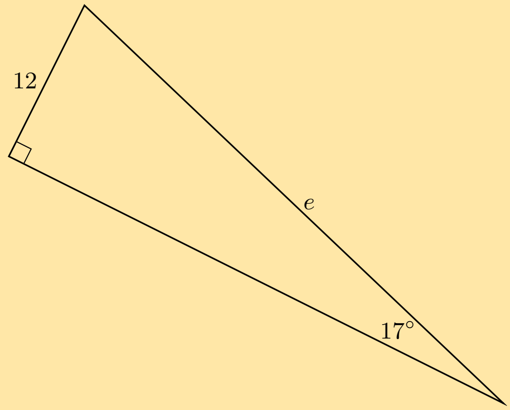
d)



Solution:

$$\begin{aligned}
\cos\theta &= \frac{\text{adjacent}}{\text{hypotenuse}}\\
\cos 49° &= \frac{d}{33}\\
33(0{,}65605...) &= d\\
d &= 21{,}64994...\\
&\approx 21{,}65
\end{aligned}$$

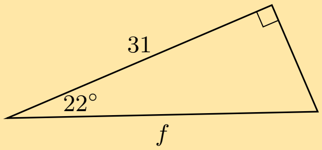
e)



Solution:

$$\begin{aligned}
\sin\theta &= \frac{\text{opposite}}{\text{hypotenuse}}\\
\sin 17° &= \frac{e}{12}\\
12(0{,}29237...) &= e\\
e &= 3{,}50846...\\
&\approx 3{,}51
\end{aligned}$$

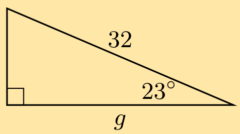
f)



Solution:

$$\begin{aligned}
\cos\theta &= \frac{\text{adjacent}}{\text{hypotenuse}}\\
\cos 22° &= \frac{31}{f}\\
f(0{,}92718...) &= 31\\
f &= 33{,}434577...\\
&\approx 33{,}43
\end{aligned}$$

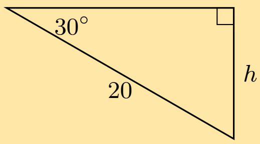
g)



Solution:

$$\begin{aligned}
\cos\theta &= \frac{\text{adjacent}}{\text{hypotenuse}}\\
\cos 23° &= \frac{g}{32}\\
32(0{,}92050...) &= g\\
g &= 29{,}4561...\\
&\approx 29{,}46
\end{aligned}$$

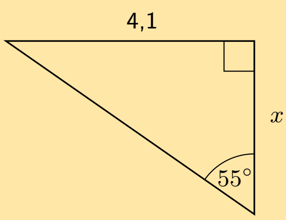
h)



Solution:

$$\begin{aligned}
\sin\theta &= \frac{\text{opposite}}{\text{hypotenuse}}\\
\sin 30° &= \frac{h}{20}\\
20(0{,}5) &= h\\
h &\approx 10
\end{aligned}$$

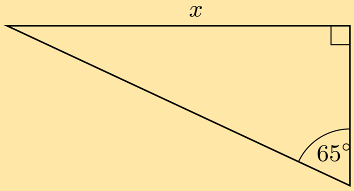
i)



Solution:

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$
$$\tan 55° = \frac{4{,}1}{x}$$
$$x = 2{,}87$$

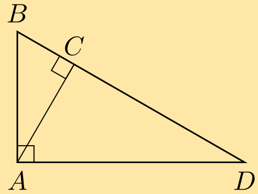
j)



Solution:

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$
$$\tan 65° = \frac{x}{4{,}23}$$
$$x = 9{,}06$$

2. Write down two ratios for each of the following in terms of the sides: AB; BC; BD; AD; DC and AC.



a) $\sin \hat{B}$
Solution:
We note that triangles ABC and ABD both contain angle B so we can use these triangles to write down the ratios:

$$\sin \hat{B} = \frac{AC}{AB} = \frac{AD}{BD}$$

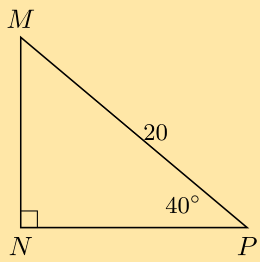
b) $\cos \hat{D}$
Solution:
We note that triangles ACD and ABD both contain angle D so we can use these triangles to write down the ratios:

$$\cos \hat{D} = \frac{AD}{BD} = \frac{CD}{AD}$$

c) $\tan \hat{B}$
Solution:
We note that triangles ABC and ABD both contain angle B so we can use these triangles to write down the ratios:

$$\tan \hat{B} = \frac{AC}{BC} = \frac{AD}{AB}$$

3. In $\triangle MNP$, $\hat{N} = 90°$, $MP = 20$ and $\hat{P} = 40°$. Calculate NP and MN (correct to 2 decimal places).
Solution:
Sketch the triangle:



To find MN we use the sine ratio:

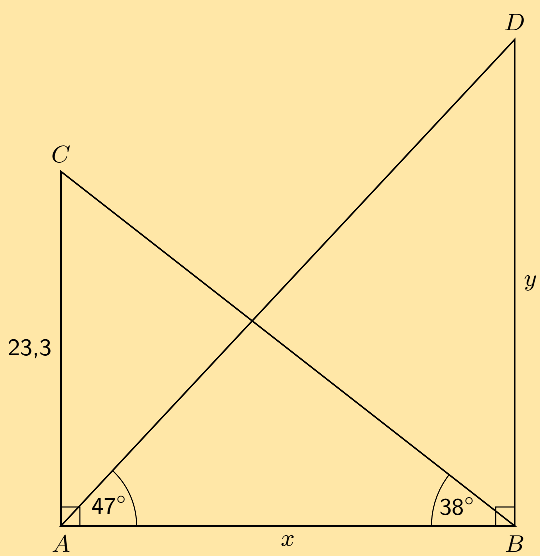
$$\begin{aligned}
\sin \hat{P} &= \frac{MN}{MP} \\
\sin 40° &= \frac{MN}{20} \\
20(0{,}642787...) &= MN \\
MN &= 12{,}8557... \\
&\approx 12{,}86
\end{aligned}$$

To find NP we can use the cosine ratio:

$$\begin{aligned}
\cos \hat{P} &= \frac{NP}{MP} \\
\cos 40° &= \frac{NP}{20} \\
20(0{,}76604...) &= NP \\
NP &= 15{,}32088... \\
&\approx 15{,}32
\end{aligned}$$

Therefore $MN = 12{,}86$ and $NP = 15{,}32$

4. Calculate x and y in the following diagram.



Solution:
To find x we use $\triangle ABC$ and the tangent ratio. To find y we use $\triangle ABD$ and the tangent ratio.

$$\tan 38° = \frac{23{,}3}{x}$$
$$x = \frac{23{,}3}{\tan 38°}$$
$$= 29{,}82264...$$
$$\approx 29{,}82$$

$$\tan 47° = \frac{y}{29{,}82264...}$$
$$y = 29{,}82264... \tan 47°$$
$$= 31{,}98086...$$
$$\approx 31{,}98$$

Therefore $x = 29{,}82$ and $y = 31{,}98$.

For more exercises, visit www.everythingmaths.co.za and click on 'Practise Maths'.

1a. 2FQV 1b. 2FQW 1c. 2FQX 1d. 2FQY 1e. 2FQZ 1f. 2FR2
1g. 2FR3 1h. 2FR4 1i. 2FR5 1j. 2FR6 2. 2FR7 3. 2FR8
4. 2FR9

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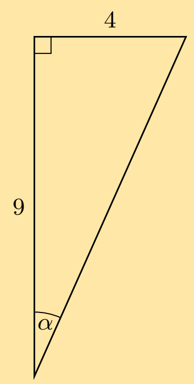
Finding an angle

Exercise 5 – 5:

Determine α in the following right-angled triangles:

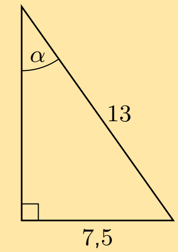
1.



Solution:

$$\tan\alpha = \frac{4}{9}$$
$$= 0{,}4444...$$
$$\alpha = 23{,}9624...$$
$$\approx 23{,}96°$$

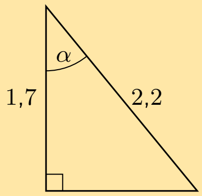
2.



Solution:

$$\begin{aligned}\sin\alpha &= \frac{7{,}5}{13}\\ &= 0{,}5769\ldots\\ \alpha &= 35{,}2344\ldots\\ &\approx 35{,}23°\end{aligned}$$

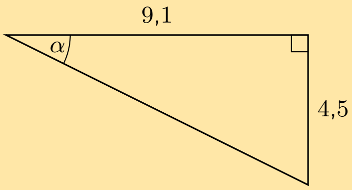
3.



Solution:

$$\begin{aligned}\sin\alpha &= \frac{1{,}7}{2{,}2}\\ \alpha &= 39{,}4005\ldots\\ &\approx 39{,}40°\end{aligned}$$

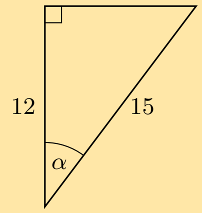
4.



Solution:

$$\begin{aligned}\tan\alpha &= \frac{4{,}5}{9{,}1}\\ &= 0{,}49450\ldots\\ \alpha &= 26{,}3126\ldots\\ &\approx 26{,}31°\end{aligned}$$

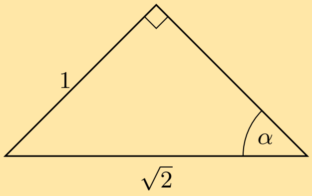
5.



Solution:

$$\begin{aligned}\cos\alpha &= \frac{12}{15}\\ &= 0{,}8\\ \alpha &= 36{,}869897...\\ &\approx 36{,}87°\end{aligned}$$

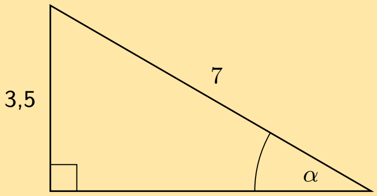
6.



Solution:

$$\begin{aligned}\sin\alpha &= \frac{1}{\sqrt{2}}\\ &= 0{,}7071...\\ \alpha &= 45°\end{aligned}$$

7.



Solution:

$$\begin{aligned}\sin\alpha &= \frac{3{,}5}{7}\\ &= 0{,}5\\ \alpha &= 30°\end{aligned}$$

For more exercises, visit www.everythingmaths.co.za and click on 'Practise Maths'.

1. 2FRB 2. 2FRC 3. 2FRD 4. 2FRF 5. 2FRG 6. 2FRH 7. 2FRJ

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If learners get a math error on their calculator encourage them to think about what might have happened. It is also important to ensure that they know they must write down no solution rather than math error when this happens.

Exercise 5 – 6:

1. Determine the angle (correct to 1 decimal place):

a) $\tan\theta = 1{,}7$

Solution:

$$\begin{aligned}\tan\theta &= 1{,}7\\ \theta &= 59{,}5344...\\ &\approx 59{,}5^\circ\end{aligned}$$

b) $\sin\theta = 0{,}8$

Solution:

$$\begin{aligned}\sin\theta &= 0{,}8\\ \theta &= 53{,}1301...\\ &\approx 53{,}1^\circ\end{aligned}$$

c) $\cos\alpha = 0{,}32$

Solution:

$$\begin{aligned}\cos\alpha &= 0{,}32\\ \alpha &= 71{,}3370...\\ &\approx 71{,}3^\circ\end{aligned}$$

d) $\tan\beta = 4{,}2$

Solution:

$$\begin{aligned}\tan\beta &= 4{,}2\\ \beta &= 76{,}60750...\\ &\approx 76{,}6^\circ\end{aligned}$$

e) $\tan\theta = 5\frac{3}{4}$

Solution:

$$\begin{aligned}\tan\theta &= 5\frac{3}{4}\\ &= 5{,}75\\ \theta &= 80{,}13419...\\ &\approx 80{,}1^\circ\end{aligned}$$

f) $\sin\theta = \frac{2}{3}$

Solution:

$$\begin{aligned}\sin\theta &= \frac{2}{3}\\ &= 0{,}666...\\ \theta &= 41{,}8103...\\ &\approx 41{,}8^\circ\end{aligned}$$

g) $\cos\beta = 1{,}2$

Solution:

$$\begin{aligned}\cos\beta &= 1{,}2\\ &\text{no solution}\end{aligned}$$

h) $4\cos\theta = 3$
Solution:

$$\begin{aligned} 4\cos\theta &= 3 \\ \cos\theta &= \frac{3}{4} \\ &= 0{,}75 \\ \theta &= 41{,}40962\ldots \\ &\approx 41{,}4^\circ \end{aligned}$$

i) $\cos 4\theta = 0{,}3$
Solution:

$$\begin{aligned} \cos 4\theta &= 0{,}3 \\ 4\theta &= 72{,}54239\ldots \\ \theta &= 18{,}135599\ldots \\ &\approx 18{,}1^\circ \end{aligned}$$

j) $\sin\beta + 2 = 2{,}65$
Solution:

$$\begin{aligned} \sin\beta + 2 &= 2{,}65 \\ \sin\beta &= 0{,}65 \\ \beta &= 40{,}54160\ldots \\ &\approx 40{,}5^\circ \end{aligned}$$

k) $2\sin\theta + 5 = 0{,}8$
Solution:

$$\begin{aligned} 2\sin\theta + 5 &= 0{,}8 \\ 2\sin\theta &= -4{,}2 \\ \sin\theta &= -2{,}1 \\ &\text{no solution} \end{aligned}$$

l) $3\tan\beta = 1$
Solution:

$$\begin{aligned} 3\tan\beta &= 1 \\ \tan\beta &= \frac{1}{3} \\ &= 0{,}3333\ldots \\ \beta &= 18{,}434948\ldots \\ &\approx 18{,}4^\circ \end{aligned}$$

m) $\sin 3\alpha = 1{,}2$
Solution:

$$\begin{aligned} \sin 3\alpha &= 1{,}2 \\ &\text{no solution} \end{aligned}$$

n) $\tan\frac{\theta}{3} = \sin 48^\circ$
Solution:

$$\begin{aligned}
\tan\frac{\theta}{3} &= \sin 48° \\
&= 0{,}7431... \\
\frac{\theta}{3} &= 36{,}61769... \\
\theta &= 109{,}8530... \\
&\approx 109{,}9°
\end{aligned}$$

o) $\frac{1}{2}\cos 2\beta = 0{,}3$

Solution:

$$\begin{aligned}
\frac{1}{2}\cos 2\beta &= 0{,}3 \\
\cos 2\beta &= 0{,}6 \\
2\beta &= 53{,}1301... \\
\beta &= 26{,}56505... \\
&\approx 26{,}6°
\end{aligned}$$

p) $2\sin 3\theta + 1 = 2{,}6$

Solution:

$$\begin{aligned}
2\sin 3\theta + 1 &= 2{,}6 \\
2\sin 3\theta &= 1{,}6 \\
\sin 3\theta &= 0{,}8 \\
3\theta &= 53{,}1301... \\
\theta &= 17{,}71003... \\
&\approx 17{,}7°
\end{aligned}$$

2. If $x = 16°$ and $y = 36°$, use your calculator to evaluate each of the following, correct to 3 decimal places.

a) $\sin(x - y)$

Solution:

$$\begin{aligned}
\sin(x - y) &= \sin(16 - 36) \\
&= \sin(-20) \\
&= -0{,}3420201... \\
&\approx -0{,}342
\end{aligned}$$

b) $3\sin x$

Solution:

$$\begin{aligned}
3\sin x &= 3\sin(16) \\
&= 0{,}826912... \\
&\approx 0{,}827
\end{aligned}$$

c) $\tan x - \tan y$

Solution:

$$\begin{aligned}
\tan x - \tan y &= \tan(16) - \tan(36) \\
&= -0{,}439797... \\
&\approx -0{,}440
\end{aligned}$$

d) $\cos x + \cos y$

Solution:

$$\begin{aligned}\cos x + \cos y &= \cos(16) + \cos(36) \\ &= 1{,}77027\ldots \\ &\approx 1{,}770\end{aligned}$$

e) $\frac{1}{3}\tan y$
Solution:

$$\begin{aligned}\frac{1}{3}\tan y &= \frac{1}{3}\tan(36) \\ &= 0{,}24218\ldots \\ &\approx 0{,}242\end{aligned}$$

f) cosec $(x - y)$
Solution:

$$\begin{aligned}\text{cosec }(x - y) &= \text{cosec }(16 - 36) \\ &= \text{cosec }(-20) \\ &= \frac{1}{\sin(-20)} \\ &= -2{,}92380\ldots \\ &\approx -2{,}924\end{aligned}$$

g) $2\cos x + \cos 3y$
Solution:

$$\begin{aligned}2\cos x + \cos 3y &= 2\cos(16) + \cos(3(36)) \\ &= 2\cos 16 + \cos 108 \\ &= 1{,}61350\ldots \\ &\approx 1{,}614\end{aligned}$$

h) $\tan(2x - 5y)$
Solution:

$$\begin{aligned}\tan(2x - 5y) &= \tan(2(16) - 5(36)) \\ &= \tan(-148) \\ &= 0{,}624869\ldots \\ &\approx 0{,}625\end{aligned}$$

3. In each of the following find the value of x correct to two decimal places.

a) $\sin x = 0{,}814$
Solution:

$$\begin{aligned}\sin x &= 0{,}814 \\ x &= 54{,}48860\ldots \\ &\approx 54{,}49°\end{aligned}$$

b) $\sin x = \tan 45°$
Solution:

$$\begin{aligned}\sin x &= \tan 45° \\ &= 1 \\ x &= 90°\end{aligned}$$

c) $\tan 2x = 3{,}123$
Solution:

$$\begin{aligned} \tan 2x &= 3{,}123 \\ 2x &= 72{,}244677... \\ x &= 36{,}12233... \\ &\approx 36{,}12° \end{aligned}$$

d) $\tan x = 3 \sin 41°$
Solution:

$$\begin{aligned} \tan x &= 3 \sin 41° \\ &= 1{,}96817... \\ x &= 63{,}06558... \\ &\approx 63{,}07° \end{aligned}$$

e) $\sin(2x + 45) = 0{,}123$
Solution:

$$\begin{aligned} \sin(2x + 45°) &= 0{,}123 \\ 2x + 45 &= 7{,}06527... \\ 2x &= -37{,}9347... \\ x &= -18{,}9673... \\ &\approx -18{,}97° \end{aligned}$$

f) $\sin(x - 10°) = \cos 57°$
Solution:

$$\begin{aligned} \sin(x - 10°) &= \cos 57° \\ &= 0{,}54463... \\ x - 10 &= 33 \\ x &= 43° \end{aligned}$$

For more exercises, visit www.everythingmaths.co.za and click on 'Practise Maths'.

1a. 2FRK	1b. 2FRM	1c. 2FRN	1d. 2FRP	1e. 2FRQ	1f. 2FRR
1g. 2FRS	1h. 2FRT	1i. 2FRV	1j. 2FRW	1k. 2FRX	1l. 2FRY
1m. 2FRZ	1n. 2FS2	1o. 2FS3	1p. 2FS4	2a. 2FS5	2b. 2FS6
2c. 2FS7	2d. 2FS8	2e. 2FS9	2f. 2FSB	2g. 2FSC	2h. 2FSD
3a. 2FSF	3b. 2FSG	3c. 2FSH	3d. 2FSJ	3e. 2FSK	3f. 2FSM

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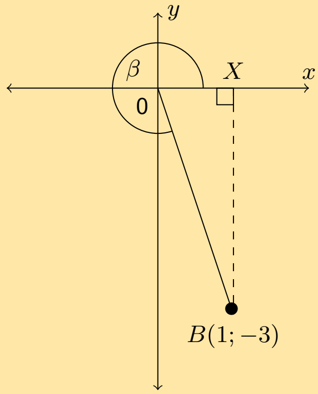
5.8 Defining ratios in the Cartesian plane

Exercise 5 – 7:

1. B is a point in the Cartesian plane. Determine without using a calculator:



a) OB
Solution:
OB is the hypotenuse of $\triangle BOX$. We can calculate the length of OB using the theorem of Pythagoras:

$$\begin{aligned} OB^2 &= OX^2 + XB^2 \\ &= (1)^2 + (3)^2 \\ &= 10 \\ OB &= \sqrt{10} \end{aligned}$$

b) $\cos\beta$
Solution:
From the diagram and the first question we know that $x = 1$, $y = -3$ and $r = \sqrt{10}$.

$$\begin{aligned} \cos\beta &= \frac{x}{r} \\ &= \frac{1}{\sqrt{10}} \end{aligned}$$

c) cosec β
Solution:
From the diagram and the first question we know that $x = 1$, $y = -3$ and $r = \sqrt{10}$.

$$\begin{aligned} \text{cosec } \beta &= \frac{r}{y} \\ &= \frac{\sqrt{10}}{-3} \end{aligned}$$

d) $\tan\beta$
Solution:
From the diagram and the first question we know that $x = 1$, $y = -3$ and $r = \sqrt{10}$.

$$\begin{aligned} \tan\beta &= \frac{y}{x} \\ &= \frac{-3}{1} \\ &= -3 \end{aligned}$$

2. If $\sin\theta = 0{,}4$ and θ is an obtuse angle, determine:

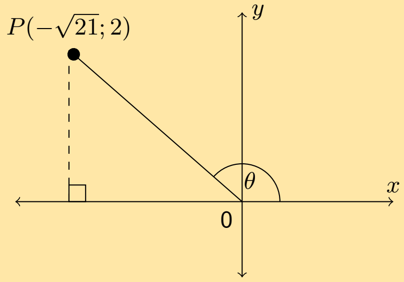
a) $\cos\theta$
Solution:
We first need to determine x, y and r.

$$\begin{aligned}\sin\theta &= 0{,}4\\ &= \frac{4}{10}\\ &= \frac{2}{5}\\ &= \frac{y}{r}\end{aligned}$$

Therefore $y = 2$ and $r = 5$.

$$\begin{aligned}x^2 &= r^2 - y^2\\ &= (5)^2 - (2)^2\\ &= 21\\ x &= \pm\sqrt{21}\end{aligned}$$

We are told that the angle is obtuse. An obtuse angle is greater than 90° but less than 180°. Therefore the angle is in the second quadrant and x is negative. Therefore $x = -\sqrt{21}$.
Next draw a sketch:



Now we can determine $\cos\theta$:

$$\begin{aligned}\cos\theta &= \frac{x}{r}\\ &= \frac{-\sqrt{21}}{5}\end{aligned}$$

b) $\sqrt{21}\tan\theta$

Solution:

From the first question we have a sketch of the angle and x, y and r.

$$\begin{aligned}\sqrt{21}\tan\theta &= \sqrt{21}\left(\frac{y}{x}\right)\\ &= \sqrt{21}\left(\frac{2}{-\sqrt{21}}\right)\\ &= -2\end{aligned}$$

3. Given $\tan\theta = \frac{t}{2}$, where $0° \leq \theta \leq 90°$. Determine the following in terms of t:

a) $\sec\theta$

Solution:

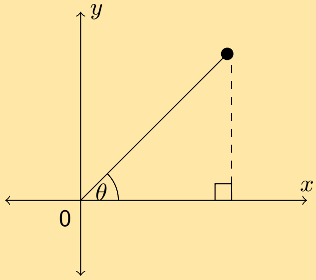
We first need to determine x, y and r. We are given $\tan\theta = \frac{t}{2}$ and so we can use this to find x and y.

$$\begin{aligned}\tan\theta &= \frac{t}{2}\\ \frac{y}{x} &= \frac{t}{2}\end{aligned}$$

Therefore $y = t$ and $x = 2$.

$$
\begin{aligned}
r^2 &= x^2 + y^2 \\
&= (2)^2 + (t)^2 \\
&= 4 + t^2 \\
r &= \sqrt{4 + t^2}
\end{aligned}
$$

We are told that $0° \leq \theta \leq 90°$. Therefore the angle is in the first quadrant. Even though we do not know the value of t we can draw a rough sketch:



Now we can determine $\sec\theta$:

$$
\begin{aligned}
\sec\theta &= \frac{r}{x} \\
&= \frac{\sqrt{t^2 + 4}}{2}
\end{aligned}
$$

b) $\cot\theta$

Solution:

From the first question we have a sketch of the angle and x, y and r.

$$
\begin{aligned}
\cot\theta &= \frac{x}{y} \\
&= \frac{2}{t}
\end{aligned}
$$

c) $\cos^2\theta$

Solution:

From the first question we have a sketch of the angle and x, y and r.

$$
\begin{aligned}
\cos^2\theta &= \left(\frac{x}{r}\right)^2 \\
&= \left(\frac{2}{\sqrt{t^2 + 4}}\right)^2 \\
&= \frac{4}{t^2 + 4}
\end{aligned}
$$

d) $\tan^2\theta - \sec^2\theta$

Solution:

From the first question we have a sketch of the angle and x, y and r.

$$
\begin{aligned}
\tan^2\theta - \sec^2\theta &= \left(\frac{y}{x}\right)^2 + \left(\frac{x}{y}\right)^2 \\
&= \left(\frac{t}{2}\right)^2 - \left(\frac{\sqrt{t^2 + 4}}{2}\right)^2 \\
&= \frac{t^2}{4} - \frac{t^2 + 4}{4} \\
&= \frac{t^2 - t^2 - 4}{4} \\
&= -1
\end{aligned}
$$

4. Given: $10\cos\beta + 8 = 0$ and $180° < \beta < 360°$. Determine the value of:

a) $\cos\beta$

Solution:

We are given an equation with $\cos\beta$ in it. We can therefore rearrange this equation to find $\cos\beta$:

$$\begin{aligned} 10\cos\beta + 8 &= 0 \\ \cos\beta &= \frac{-8}{10} \\ &= \frac{-4}{5} \end{aligned}$$

b) $\dfrac{3}{\tan\beta} + 2\sin^2\beta$

Solution:

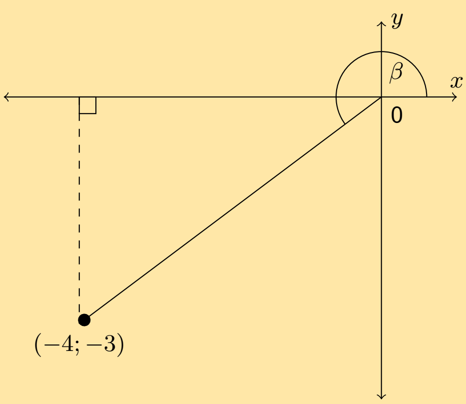
We first need to determine x, y and r. In the first question we found that $\cos\beta = \frac{-4}{5}$ and so we can use this to find x and r.

$$\begin{aligned} \cos\beta &= \frac{-4}{5} \\ \frac{x}{r} &= \frac{-4}{5} \end{aligned}$$

Therefore $x = -4$ and $r = 5$.

$$\begin{aligned} y^2 &= r^2 - x^2 \\ &= (5)^2 - (-4)^2 \\ &= 25 - 16 \\ y &= \pm 3 \end{aligned}$$

We are told that $180° < \beta < 360°$. Therefore the angle is in the third quadrant and $y = -3$. We can draw a rough sketch of the angle:



We can now find $\frac{3}{\tan\beta} + 2\sin^2\beta$:

$$\begin{aligned}\frac{3}{\tan\beta} + 2\sin^2\beta &= \frac{3}{\frac{y}{x}} + 2\left(\frac{y}{r}\right)^2 \\ &= \frac{3x}{y} + \frac{2y^2}{r^2} \\ &= \frac{3(-4)}{-3} + \frac{2(-3)^2}{(5)^2} \\ &= \frac{-12}{3} + \frac{18}{25} \\ &= -4 + \frac{18}{25} \\ &= \frac{-100+18}{25} \\ &= \frac{-82}{25}\end{aligned}$$

5. If $\sin\theta = -\frac{15}{17}$ and $\cos\theta < 0$ find the following, without the use of a calculator:

   a) $\cos\theta$

   Solution:

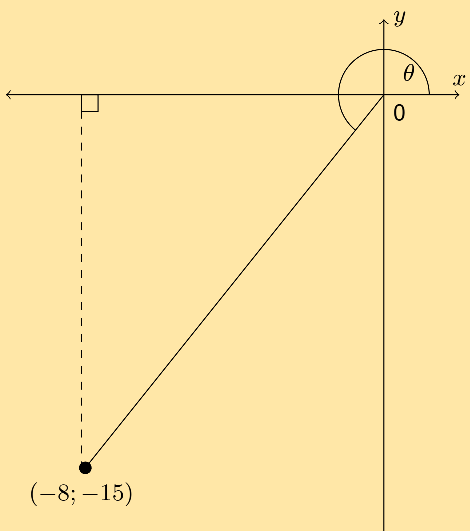
   We first need to determine x, y and r. We are given $\sin\theta = -\frac{15}{17}$ and so we can use this to find y and r.

$$\begin{aligned}\sin\theta &= -\frac{15}{17} \\ \frac{y}{r} &= -\frac{15}{17}\end{aligned}$$

   Therefore $y = -15$ and $r = 17$ (remember that r cannot be negative).

$$\begin{aligned}x^2 &= r^2 - y^2 \\ &= (17)^2 - (-15)^2 \\ &= 289 - 225 \\ x &= \pm 8\end{aligned}$$

   We are told that $\cos\theta < 0$. Therefore the angle is in either the second or the third quadrant. From the value of y we see that the angle must lie in the third quadrant and $x = -8$.



   Now we can determine $\cos\theta$:

$$\begin{aligned}\cos\theta &= \frac{x}{r} \\ &= \frac{-8}{17}\end{aligned}$$

b) $\tan\theta$

Solution:

From the first part we have $x = -8$, $y = -15$ and $r = 17$ so we can find $\tan\theta$.

$$\begin{aligned}\tan\theta &= \frac{y}{x}\\ &= \frac{-15}{-8}\\ &= \frac{15}{8}\end{aligned}$$

c) $\cos^2\theta + \sin^2\theta$

Solution:

From the first part we have $x = -8$, $y = -15$ and $r = 17$ so we can find $\cos^2\theta + \sin^2\theta$.

$$\begin{aligned}\cos^2\theta + \sin^2\theta &= \left(\frac{x}{r}\right)^2 + \left(\frac{y}{r}\right)^2\\ &= \frac{x^2}{r^2} + \frac{y^2}{r^2}\\ &= \frac{x^2 + y^2}{r^2}\\ &= \frac{(-8)^2 + (-15)^2}{(17)^2}\\ &= \frac{64 + 225}{289}\\ &= 1\end{aligned}$$

6. Find the value of $\sin A + \cos A$ without using a calculator, given that $13\sin A - 12 = 0$, where $\cos A < 0$.

Solution:

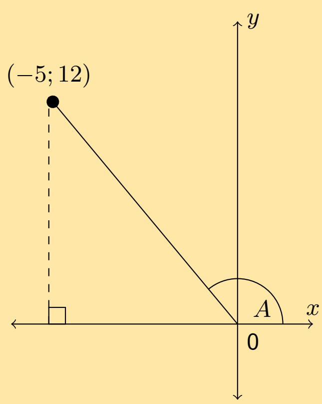
We first need to determine x, y and r. We are given $13\sin A - 12 = 0$ and so we can use this to find y and r.

$$\begin{aligned}13\sin A - 12 &= 0\\ \sin A &= \frac{12}{13}\end{aligned}$$

Therefore $y = 12$ and $r = 13$.

$$\begin{aligned}x^2 &= r^2 - y^2\\ &= (13)^2 - (12)^2\\ &= 169 - 144\\ x &= \pm 5\end{aligned}$$

We are told that $\cos A < 0$. Therefore the angle is in either the second or the third quadrant. From the value of y we see that the angle must lie in the second quadrant.



Now we can determine $\sin A + \cos A$:

$$\begin{aligned}
\sin A + \cos A &= \frac{y}{r} + \frac{x}{r} \\
&= \frac{y + x}{r} \\
&= \frac{12 - 5}{13} \\
&= \frac{7}{13}
\end{aligned}$$

7. If $17 \cos \theta = -8$ and $\tan \theta > 0$ determine the following with the aid of a diagram (not a calculator):

   a) $\dfrac{\cos \theta}{\sin \theta}$

   Solution:

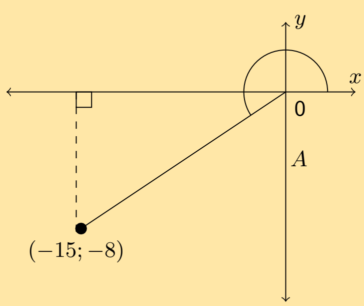
   We first need to determine x, y and r. We are given $17 \cos \theta = -8$ and so we can use this to find x and r.

$$\begin{aligned}
17 \cos \theta &= -8 = 0 \\
\cos \theta &= \frac{-8}{17}
\end{aligned}$$

   Therefore $x = -8$ and $r = 17$.

$$\begin{aligned}
y^2 &= r^2 - x^2 \\
&= (17)^2 - (8)^2 \\
y &= \pm 15
\end{aligned}$$

   We are told that $\tan \theta > 0$. Therefore the angle is in either the first or the third quadrant. From the value of x we see that the angle must lie in the third quadrant.



   Now we can determine $\dfrac{\cos \theta}{\sin \theta}$:

$$\begin{aligned}
\frac{\cos \theta}{\sin \theta} &= \cos \theta \times \frac{1}{\sin \theta} \\
&= \frac{y}{r} \times \frac{1}{\frac{x}{r}} \\
&= \frac{y}{r} \times \frac{r}{x} \\
&= \frac{y}{x} \\
&= \frac{-8}{-15} \\
&= \frac{8}{15}
\end{aligned}$$

   b) $17 \sin \theta - 16 \tan \theta$

   Solution:

   From the first part we have that $x = -15$, $y = -8$ and $r = 17$.

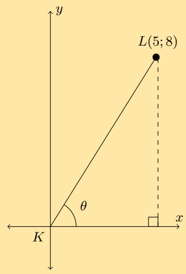
$$\begin{aligned} 17\sin\theta - 16\tan\theta &= 17\frac{y}{r} - 16\frac{y}{x} \\ &= 17\left(\frac{-15}{17}\right) - 16\left(\frac{-15}{-8}\right) \\ &= -15 - 2(15) \\ &= -45 \end{aligned}$$

8. L is a point with co-ordinates $(5;8)$ on a Cartesian plane. LK forms an angle, θ, with the positive x-axis. Set up a diagram and use it to answer the following questions.

   a) Find the distance LK.

   Solution:

   We are given $L(5;8)$. Therefore the angle lies in the first quadrant. We can sketch this and use our sketch to find x, y and r.



   Therefore $x = 5$ and $y = 8$. We can calculate r using the theorem of Pythagoras. From the diagram we note that $LK = r$.

$$\begin{aligned} LK^2 &= 5^2 + 8^2 \\ &= 89 \\ LK &= \sqrt{89} \end{aligned}$$

   b) $\sin\theta$

   Solution:

   From the previous question we have that $x = 5$, $y = 8$ and $r = \sqrt{89}$.

$$\begin{aligned} \sin\theta &= \frac{y}{r} \\ &= \frac{8}{\sqrt{89}} \end{aligned}$$

   c) $\cos\theta$

   Solution:

   From the previous question we have that $x = 5$, $y = 8$ and $r = \sqrt{89}$.

$$\begin{aligned} \cos\theta &= \frac{x}{r} \\ &= \frac{5}{\sqrt{89}} \end{aligned}$$

   d) $\tan\theta$

   Solution:

   From the previous question we have that $x = 5$, $y = 8$ and $r = \sqrt{89}$.

$$\begin{aligned} \tan\theta &= \frac{y}{x} \\ &= \frac{8}{5} \end{aligned}$$

e) cosec θ
Solution:
From the previous question we have that $x = 5$, $y = 8$ and $r = \sqrt{89}$.

$$\begin{aligned}\text{cosec }\theta &= \frac{r}{y}\\ &= \frac{\sqrt{89}}{8}\end{aligned}$$

f) $\sec\theta$
Solution:
From the previous question we have that $x = 5$, $y = 8$ and $r = \sqrt{89}$.

$$\begin{aligned}\sec\theta &= \frac{r}{x}\\ &= \frac{\sqrt{89}}{5}\end{aligned}$$

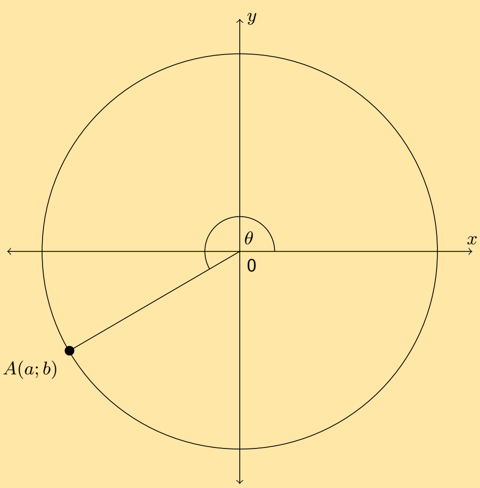
g) $\cot\theta$
Solution:
From the previous question we have that $x = 5$, $y = 8$ and $r = \sqrt{89}$.

$$\begin{aligned}\cot\theta &= \frac{x}{y}\\ &= \frac{5}{8}\end{aligned}$$

h) $\sin^2\theta + \cos^2\theta$
Solution:
From the previous question we have that $x = 5$, $y = 8$ and $r = \sqrt{89}$.

$$\begin{aligned}\sin^2\theta + \cos^2\theta &= \left(\frac{y}{r}\right)^2 + \left(\frac{x}{r}\right)^2\\ &= \frac{y^2}{r^2} + \frac{x^2}{r^2}\\ &= \frac{y^2 + x^2}{r^2}\\ &= \frac{64 + 25}{89}\\ &= 1\end{aligned}$$

9. Given the following diagram and that $\cos\theta = -\frac{24}{25}$.



a) State two sets of possible values of a and b.

Solution:

We first need to use the given information to find a possible set of values for a and b.

Using $\cos\theta = -\frac{24}{25}$ and the fact that $\cos\theta = \frac{x}{r}$ we can determine that $x = -24$ and $r = 25$. Now we can find y:

$$\begin{aligned} y^2 &= r^2 - x^2 \\ &= (25)^2 - (24)^2 \\ &= 625 - 576 \\ &= 49 \\ y &= \pm 7 \end{aligned}$$

From the diagram we see that y must be negative.

This gives us one possible set of values for a and b: $a = 24$ and $b = -7$.

Now we note that we can simply double the size of the circle and the trigonometric ratios will stay the same. We could even multiply the radius of the circle by any integer and the trigonometric ratios will still remain the same.

Therefore the possible sets of values for $A(a, b)$ are multiples of $(-24; -7)$. Two possible sets are $(-24; -7)$ and $(-48; -14)$.

b) If $OA = 100$, state the values of a and b.

Solution:

First note that in the original diagram $OA = 25$. Now we are multiplying OA by 4. This also means that the x and y values must be multiplied by 4.

Therefore $a = 4(-24) = -96$ and $b = 4(-7) = -28$.

c) Hence determine without the use of a calculator the value of $\sin\theta$.

Solution:

The question states: "hence". This means we must use the scaled values for a and b not the original values. We know that $x = -96$, $y = -28$ and $r = 100$.

$$\begin{aligned} \sin\theta &= \frac{y}{r} \\ &= \frac{-28}{100} \\ &= \frac{-7}{25} \end{aligned}$$

Notice how the answer reduced to the original values of y and r as we would expect from the first question.

10. If $\tan\alpha = \frac{5}{-12}$ and $0° \leq \alpha \leq 180°$, determine **without the use of a calculator** the value of $\frac{12}{\cos\alpha}$

Solution:

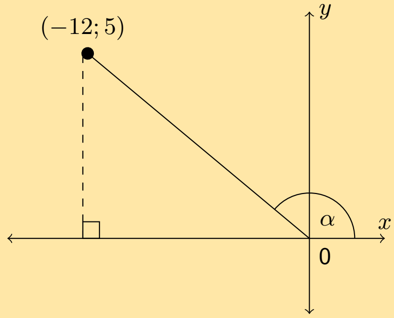
We first need to determine x, y and r. We are given $\tan\alpha = \frac{5}{-12}$ and so we can use this to find x and y.

$$\begin{aligned} \tan\alpha &= \frac{5}{-12} \\ \frac{y}{x} &= \frac{5}{-12} \end{aligned}$$

Therefore $y = 5$ and $x = -12$.

$$\begin{aligned} r^2 &= x^2 + y^2 \\ &= (-12)^2 + (5)^2 \\ &= 144 + 25 \\ r &= 13 \end{aligned}$$

We are told that $0° \leq \alpha \leq 180°$. Therefore the angle is in either the first or the second quadrant. From the values of x and y we see that the angle must lie in the second quadrant.



Now we can determine $\frac{12}{\cos\alpha}$:

$$\begin{aligned}\frac{12}{\cos\alpha} &= \frac{12}{\frac{x}{r}} \\ &= \frac{12r}{x} \\ &= \frac{12(13)}{-12} \\ &= -13\end{aligned}$$

For more exercises, visit www.everythingmaths.co.za and click on 'Practise Maths'.

1. 2FSP 2. 2FSQ 3. 2FSR 4. 2FSS 5. 2FST 6. 2FSV
7. 2FSW 8. 2FSX 9. 2FSY 10. 2FSZ

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5.9 Chapter summary

End of chapter Exercise 5 – 8:

1. State whether each of the following trigonometric ratios has been written correctly.

   a) $\sin\theta = \frac{\text{hypotenuse}}{\text{adjacent}}$

   Solution:

   We recall the definition of the sine ratio: $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$. Therefore this trigonometric ratio has not been written correctly.

   b) $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$

   Solution:

   We recall the definition of the tangent ratio: $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$. Therefore this trigonometric ratio has been written correctly.

   c) $\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}}$

   Solution:

   We recall the definition of the secant ratio: $\sec\theta = \frac{\text{hypotenuse}}{\text{opposite}}$. Therefore this trigonometric ratio has not been written correctly.

2. Use your calculator to evaluate the following expressions to two decimal places:

a) $\tan 80°$

Solution:

$$\begin{aligned} \tan 80° &= 5{,}6712\ldots \\ &\approx 5{,}67 \end{aligned}$$

b) $\cos 73°$

Solution:

$$\begin{aligned} \cos 73° &= 0{,}29237\ldots \\ &\approx 0{,}29 \end{aligned}$$

c) $\sin 17°$

Solution:

$$\begin{aligned} \sin 17° &= 0{,}2923\ldots \\ &\approx 0{,}29 \end{aligned}$$

d) $\tan 313°$

Solution:

$$\begin{aligned} \tan 313° &= -1{,}07236\ldots \\ &\approx -1{,}07 \end{aligned}$$

e) $\cos 138°$

Solution:

$$\begin{aligned} \cos 138° &= -0{,}743144\ldots \\ &\approx -0{,}74 \end{aligned}$$

f) $\sec 56°$

Solution:

$$\begin{aligned} \sec 56° &= \frac{1}{\cos 56°} \\ &= \frac{1}{0{,}5591\ldots} \\ &= 1{,}78829\ldots \\ &\approx 1{,}79 \end{aligned}$$

g) $\cot 18°$

Solution:

$$\begin{aligned} \cot 18° &= \frac{1}{\tan 18°} \\ &= \frac{1}{0{,}32491\ldots} \\ &= 3{,}07768\ldots \\ &\approx 3{,}08 \end{aligned}$$

h) $\operatorname{cosec} 37°$

Solution:

$$\begin{aligned} \operatorname{cosec} 37° &= \frac{1}{\sin 37°} \\ &= \frac{1}{0{,}6018\ldots} \\ &= 1{,}66164\ldots \\ &\approx 1{,}66 \end{aligned}$$

i) $\sec 257°$
Solution:

$$\begin{aligned}
\sec 257° &= \frac{1}{\cos 257°} \\
&= \frac{1}{-0{,}224951...} \\
&= -4{,}445411... \\
&\approx -4{,}45
\end{aligned}$$

j) $\sec 304°$
Solution:

$$\begin{aligned}
\sec 304° &= \frac{1}{\cos 304°} \\
&= \frac{1}{0{,}559193...} \\
&= 1{,}788292... \\
&\approx 1{,}79
\end{aligned}$$

k) $3\sin 51°$
Solution:

$$\begin{aligned}
3\sin 51° &= 2{,}3314... \\
&\approx 2{,}33
\end{aligned}$$

l) $4\cot 54° + 5\tan 44°$
Solution:

$$\begin{aligned}
4\cot 54° + 5\tan 44° &= \frac{4}{\tan 54°} + 5\tan 44° \\
&= 7{,}7346... \\
&\approx 7{,}73
\end{aligned}$$

m) $\dfrac{\cos 205°}{4}$
Solution:

$$\begin{aligned}
\frac{\cos 205°}{4} &= -0{,}22657... \\
&\approx -0{,}23
\end{aligned}$$

n) $\sqrt{\sin 99°}$
Solution:

$$\begin{aligned}
\sqrt{\sin 99°} &= \sqrt{0{,}98768...} \\
&= 0{,}9938... \\
&\approx 0{,}99
\end{aligned}$$

o) $\sqrt{\cos 687° + \sin 120°}$
Solution:

$$\begin{aligned}
\sqrt{\cos 687° + \sin 120°} &= \sqrt{1{,}7046...} \\
&= 1{,}3056... \\
&\approx 1{,}31
\end{aligned}$$

p) $\dfrac{\tan 70^\circ}{\text{cosec } 1^\circ}$

Solution:

$$\begin{aligned}\frac{\tan 70^\circ}{\text{cosec } 1^\circ} &= \tan 70^\circ \times \frac{1}{\frac{1}{\sin 1^\circ}} \\ &= \tan 70^\circ \times \sin 1^\circ \\ &= 0{,}04795... \\ &\approx 0{,}05\end{aligned}$$

q) $\sec 84^\circ + 4 \sin 0{,}4^\circ \times 50 \cos 50^\circ$

Solution:

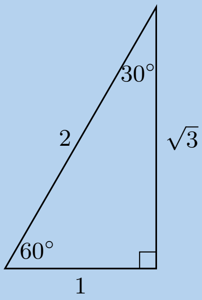
$$\begin{aligned}\sec 84^\circ + 4 \sin 0{,}4^\circ \times 50 \cos 50^\circ &= \frac{1}{\cos 84^\circ} + 4 \sin 0{,}4^\circ \times 50 \cos 50^\circ \\ &= 9{,}56677... + 0{,}89749... \\ &= 10{,}46426... \\ &\approx 10{,}46\end{aligned}$$

r) $\dfrac{\cos 40^\circ}{\sin 35^\circ} + \tan 38^\circ$

Solution:

$$\begin{aligned}\frac{\cos 40^\circ}{\sin 35^\circ} + \tan 38^\circ &= 1{,}3355... + 0{,}7812... \\ &= 2{,}1168... \\ &\approx 2{,}12\end{aligned}$$

3. Use the triangle below to complete the following:



a) $\sin 60^\circ =$

Solution:

Remember to first identify the hypotenuse, opposite and adjacent sides for the given angle. Then write down the correct fraction for each ratio. You can confirm your answer by using your calculator to find the value of the ratio for that angle.

$$\sin 60^\circ = \frac{\sqrt{3}}{2}$$

b) $\cos 60^\circ =$

Solution:

Remember to first identify the hypotenuse, opposite and adjacent sides for the given angle. Then write down the correct fraction for each ratio. You can confirm your answer by using your calculator to find the value of the ratio for that angle.

$$\cos 60^\circ = \frac{1}{2}$$

c) $\tan 60^\circ =$

Solution:

Remember to first identify the hypotenuse, opposite and adjacent sides for the given angle. Then write down the correct fraction for each ratio. You can confirm your answer by using your calculator to find the value of the ratio for that angle.

$$\tan 60° = \frac{\sqrt{3}}{1} = \sqrt{3}$$

d) $\sin 30° =$

Solution:

Remember to first identify the hypotenuse, opposite and adjacent sides for the given angle. Then write down the correct fraction for each ratio. You can confirm your answer by using your calculator to find the value of the ratio for that angle.

$$\sin 30° = \frac{1}{2}$$

e) $\cos 30° =$

Solution:

Remember to first identify the hypotenuse, opposite and adjacent sides for the given angle. Then write down the correct fraction for each ratio. You can confirm your answer by using your calculator to find the value of the ratio for that angle.

$$\cos 30° = \frac{\sqrt{3}}{2}$$

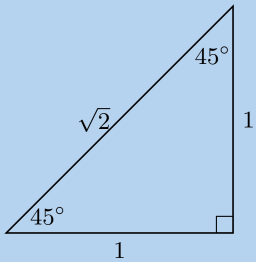
f) $\tan 30° =$

Solution:

Remember to first identify the hypotenuse, opposite and adjacent sides for the given angle. Then write down the correct fraction for each ratio. You can confirm your answer by using your calculator to find the value of the ratio for that angle.

$$\tan 30° = \frac{1}{\sqrt{3}}$$

4. Use the triangle below to complete the following:



a) $\sin 45° =$

Solution:

Remember to first identify the hypotenuse, opposite and adjacent sides for the given angle. Then write down the correct fraction for each ratio. You can confirm your answer by using your calculator to find the value of the ratio for that angle.

$$\sin 45° = \frac{1}{\sqrt{2}}$$

b) $\cos 45° =$

Solution:

Remember to first identify the hypotenuse, opposite and adjacent sides for the given angle. Then write down the correct fraction for each ratio. You can confirm your answer by using your calculator to find the value of the ratio for that angle.

$$\cos 45° = \frac{1}{\sqrt{2}}$$

c) $\tan 45° =$

Solution:

Remember to first identify the hypotenuse, opposite and adjacent sides for the given angle. Then write down the correct fraction for each ratio. You can confirm your answer by using your calculator to find the value of the ratio for that angle.

$$\tan 45° = \frac{1}{1} = 1$$

5. Evaluate the following without using a calculator. Select the closest answer from the list provided.

a) $\sin 60° - \tan 60°$

$$0 \quad -\frac{1}{2} \quad \frac{2}{\sqrt{3}} \quad -\frac{\sqrt{3}}{2} \quad -\frac{2}{\sqrt{3}}$$

Solution:

$$\begin{aligned}\sin 60° - \tan 60° &= \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{1} \\ &= \frac{\sqrt{3} - 2\sqrt{3}}{2} \\ &= -\frac{\sqrt{3}}{2}\end{aligned}$$

b) $\tan 30° - \cos 30°$

$$0 \quad -\frac{1}{2\sqrt{3}} \quad \frac{\sqrt{3}}{2} \quad -\frac{2}{\sqrt{3}} \quad -\frac{\sqrt{3}}{2}$$

Solution:

$$\begin{aligned}\tan 30° - \cos 30° &= \frac{1}{\sqrt{3}} - \frac{\sqrt{3}}{2} \\ &= \frac{2 - (\sqrt{3})(\sqrt{3})}{2\sqrt{3}} \\ &= -\frac{1}{2\sqrt{3}}\end{aligned}$$

c) $\tan 60° - \sin 60° - \tan 60°$

$$-\frac{\sqrt{3}}{2} \quad -\frac{\sqrt{3}}{1} \quad -\frac{1}{2} \quad -\frac{1}{1} \quad -\frac{1}{\sqrt{2}}$$

Solution:

$$\begin{aligned}\tan 60° - \sin 60° - \tan 60° &= \frac{\sqrt{3}}{1} - \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{1} \\ &= \frac{2\sqrt{3} - \sqrt{3} - 2\sqrt{3}}{2} \\ &= -\frac{\sqrt{3}}{2}\end{aligned}$$

d) $\sin 30° \times \sin 30° \times \sin 30°$

$$\frac{1}{2} \quad \frac{1}{2\sqrt{3}} \quad \frac{1}{8} \quad \frac{1}{4} \quad \frac{\sqrt{3}}{4\sqrt{2}}$$

Solution:

$$\begin{aligned}\sin 30° \times \sin 30° \times \sin 30° &= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \\ &= \frac{1}{8}\end{aligned}$$

e) $\sin 45° \times \tan 45° \times \tan 60°$

$$\frac{3}{2\sqrt{2}} \quad \frac{\sqrt{3}}{8} \quad \frac{3}{4} \quad \frac{\sqrt{3}}{\sqrt{2}} \quad \frac{1}{4}$$

Solution:

$$\begin{aligned}\sin 45° \times \tan 45° \times \tan 60° &= \frac{1}{\sqrt{2}} \times \frac{1}{1} \times \frac{\sqrt{3}}{1} \\ &= \frac{\sqrt{3}}{\sqrt{2}}\end{aligned}$$

f) $\cos 60° \times \cos 45° \times \tan 60°$

$$\frac{\sqrt{3}}{2\sqrt{2}} \quad \frac{\sqrt{3}}{4} \quad \frac{3}{4\sqrt{2}} \quad \frac{1}{2} \quad \frac{1}{4\sqrt{3}}$$

Solution:

$$\begin{aligned}\cos 60° \times \cos 45° \times \tan 60° &= \frac{1}{2} \times \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{1} \\ &= \frac{\sqrt{3}}{2\sqrt{2}}\end{aligned}$$

g) $\tan 45° \times \sin 60° \times \tan 45°$

$$\frac{\sqrt{3}}{2} \quad \frac{3}{8} \quad \frac{1}{3} \quad \frac{\sqrt{3}}{2\sqrt{2}} \quad \frac{1}{4\sqrt{3}}$$

Solution:

$$\begin{aligned}\tan 45° \times \sin 60° \times \tan 45° &= \frac{1}{1} \times \frac{\sqrt{3}}{2} \times \frac{1}{1} \\ &= \frac{\sqrt{3}}{2}\end{aligned}$$

h) $\cos 30° \times \cos 60° \times \sin 60°$

$$\frac{3}{8} \quad \frac{3}{2\sqrt{2}} \quad \frac{\sqrt{3}}{4\sqrt{2}} \quad \frac{1}{2\sqrt{3}} \quad \frac{1}{4\sqrt{3}}$$

Solution:

$$\begin{aligned}\cos 30° \times \cos 60° \times \sin 60° &= \frac{\sqrt{3}}{2} \times \frac{1}{2} \times \frac{\sqrt{3}}{2} \\ &= \frac{3}{8}\end{aligned}$$

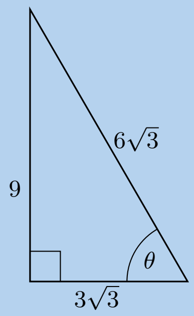
6. Without using a calculator, determine the value of:

$$\sin 60° \cos 30° - \cos 60° \sin 30° + \tan 45°$$

Solution:
These are all special angles.

$$\begin{aligned}\sin 60° \cos 30° - \cos 60° \sin 30° + \tan 45° &= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + 1 \\ &= \frac{3}{4} - \frac{1}{4} + 1 \\ &= \frac{2}{4} + 1 \\ &= \frac{3}{2}\end{aligned}$$

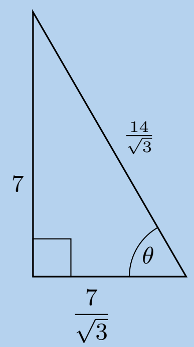
7. Solve for $\sin\theta$ in the following triangle, in surd form:



Solution:

$$\begin{aligned}\sin\theta &= \frac{9}{6\sqrt{3}} \\ &= \frac{3}{2\sqrt{3}}\end{aligned}$$

8. Solve for $\tan\theta$ in the following triangle, in surd form:



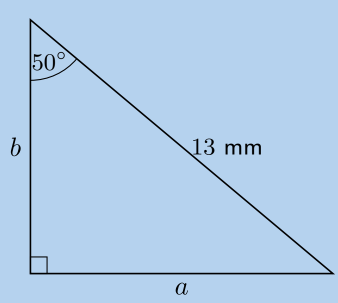
Solution:

$$\begin{aligned}\tan\theta &= \frac{7}{\frac{7}{\sqrt{3}}} \\ &= 7 \times \frac{\sqrt{3}}{7} \\ &= \sqrt{3}\end{aligned}$$

9. A right-angled triangle has hypotenuse 13 mm. Find the length of the other two sides if one of the angles of the triangle is 50°.

Solution:

First draw a diagram:



Next we get:

$$\begin{aligned}
\sin\theta &= \frac{\text{opposite}}{\text{hypotenuse}} \\
\sin 50° &= \frac{a}{13} \\
a &= 13\sin 50° \\
&= 9{,}9585... \\
&\approx 9{,}96 \text{ mm}
\end{aligned}$$

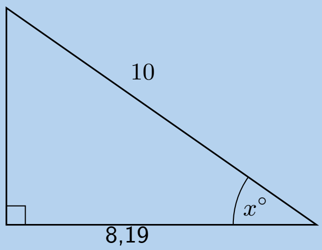
Now we can use the theorem of Pythagoras to find the other side:

$$\begin{aligned}
b^2 &= c^2 - a^2 \\
&= (13)^2 - (9{,}9585...)^2 \\
&= 69{,}8267... \\
b &= 8{,}3562... \\
&= 8{,}36 \text{ mm}
\end{aligned}$$

Therefore the other two sides are 9,96 mm and 8,35 mm.

10. Solve for x to the nearest integer.

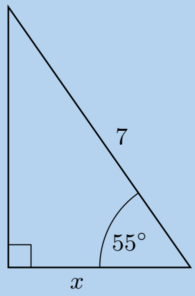
a)



Solution:

$$\begin{aligned}
\cos\theta &= \frac{\text{adjacent}}{\text{hypotenuse}} \\
\cos x &= \frac{8{,}19}{10} \\
&= 0{,}819 \\
x &= 35{,}0151... \\
&\approx 35°
\end{aligned}$$

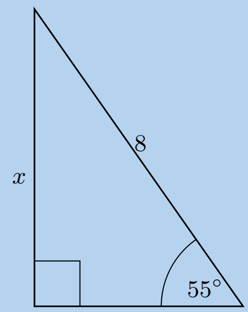
b)



Solution:

$$\begin{aligned}
\cos\theta &= \frac{\text{adjacent}}{\text{hypotenuse}} \\
\cos 55^\circ &= \frac{x}{7} \\
7\cos 55^\circ &= x \\
x &= 4{,}01503\ldots \\
&\approx 4
\end{aligned}$$

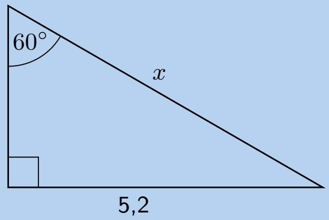
c)



Solution:

$$\begin{aligned}
\sin\theta &= \frac{\text{opposite}}{\text{hypotenuse}} \\
\sin 55^\circ &= \frac{x}{8} \\
8\sin 55^\circ &= x \\
x &= 6{,}55321\ldots \\
&\approx 7
\end{aligned}$$

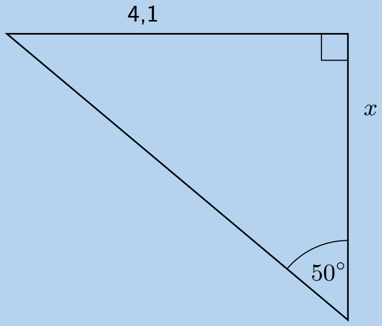
d)



Solution:

$$\begin{aligned}
\sin\theta &= \frac{\text{opposite}}{\text{hypotenuse}} \\
\sin 60^\circ &= \frac{5{,}2}{x} \\
x &= \frac{5{,}2}{\sin 60^\circ} \\
&= 6{,}00444\ldots \\
&\approx 6
\end{aligned}$$

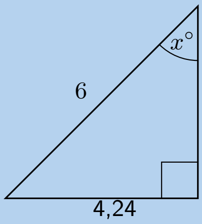
e)



Solution:

$$\begin{aligned} \tan\theta &= \frac{\text{opposite}}{\text{adjacent}} \\ \tan 50^\circ &= \frac{4{,}1}{x} \\ x &= \frac{4{,}1}{\tan 50^\circ} \\ &= 3{,}4403... \\ &\approx 3 \end{aligned}$$

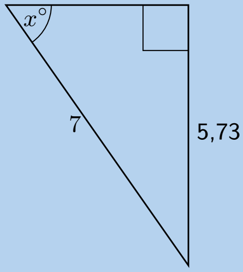
f)



Solution:

$$\begin{aligned} \sin\theta &= \frac{\text{opposite}}{\text{hypotenuse}} \\ \sin x &= \frac{4{,}24}{6} \\ &= 0{,}7067... \\ x &= 44{,}96434... \\ &\approx 45^\circ \end{aligned}$$

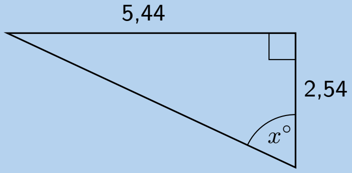
g)



Solution:

$$\begin{aligned}\sin\theta &= \frac{\text{opposite}}{\text{hypotenuse}}\\ \sin x &= \frac{5{,}73}{7}\\ &= 0{,}81857\ldots\\ x &= 54{,}9420\ldots\\ &\approx 55^{\circ}\end{aligned}$$

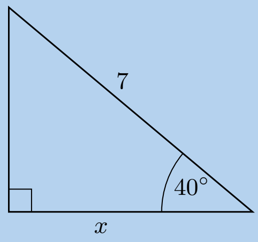
h)



Solution:

$$\begin{aligned}\tan\theta &= \frac{\text{opposite}}{\text{adjacent}}\\ \tan x &= \frac{5{,}44}{2{,}54}\\ &= 2{,}14173\ldots\\ x &= 64{,}9715\ldots\\ &\approx 65^{\circ}\end{aligned}$$

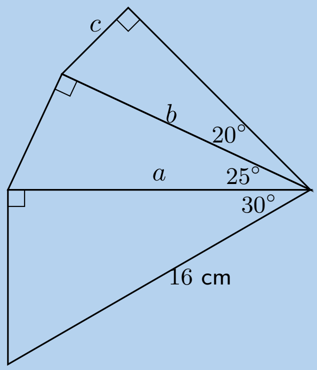
i)

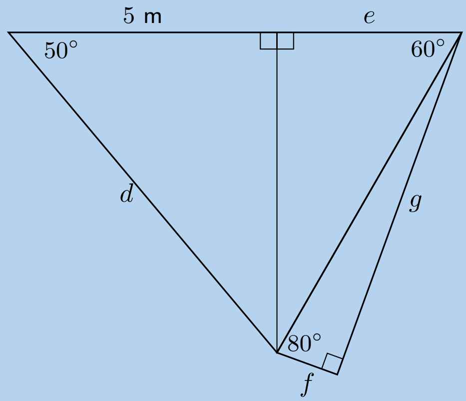


Solution:

$$\begin{aligned}\cos\theta &= \frac{\text{adjacent}}{\text{hypotenuse}}\\ \cos 40^{\circ} &= \frac{x}{7}\\ 7\cos 40^{\circ} &= x\\ x &= 5{,}36231\ldots\\ &\approx 5\end{aligned}$$

11. Calculate the unknown lengths in the diagrams below:





Solution:

For all of these we use the appropriate trigonometric ratio or the theorem of Pythagoras to solve.

To find a and b we use $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$:

$$\begin{aligned}\cos 30° &= \frac{a}{16}\\ a &= 16\cos 30°\\ &\approx 13{,}86 \text{ cm}\end{aligned}$$

$$\begin{aligned}\cos 25° &= \frac{b}{13{,}86}\\ b &= 13{,}86\cos 25°\\ &\approx 12{,}56 \text{ cm}\end{aligned}$$

To find c we use $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$:

$$\begin{aligned}\sin 20° &= \frac{c}{12{,}56}\\ c &= 12{,}56\sin 20°\\ &\approx 4{,}30 \text{ cm}\end{aligned}$$

To find d we use $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$

$$\begin{aligned}\cos 50° &= \frac{5}{d}\\ d\cos 50 &= 5\\ d &= \frac{5}{\cos 50°}\\ &\approx 7{,}78 \text{ cm}\end{aligned}$$

Next we use the theorem of Pythagoras to find the third side, so we can use trig functions to find e:

$$\begin{aligned}(5)^2 + (7{,}78)^2 &= 85{,}5284\\ \sqrt{85{,}5284} &\approx 9{,}25\end{aligned}$$

We use $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$ to find e:

$$\begin{aligned}\tan 60° &= \frac{9{,}25}{e}\\ e\tan 60° &= 9{,}25\\ e &= \frac{9{,}25}{\tan 60°}\\ &\approx 5{,}34 \text{ cm}\end{aligned}$$

Next we use the theorem of Pythagoras to find the third side, so we can use trig functions to find f and g:

$$(5{,}34)^2 + (7{,}78)^2 = 89{,}0044...$$
$$\sqrt{89{,}0044...} \approx 9{,}44$$

We use $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$ to find g:

$$\begin{aligned}
\tan 80° &= \frac{9{,}44}{g} \\
g \tan 80° &= 9{,}44 \\
g &= \frac{9{,}44}{\tan 80°} \\
&\approx 1{,}66 \text{ cm}
\end{aligned}$$

And finally we find f using the theorem of Pythagoras:

$$\begin{aligned}
f^2 &= (9{,}44)^2 - (1{,}65)^2 \\
f &= \sqrt{86{,}39} \\
&\approx 9{,}29 \text{ cm}
\end{aligned}$$

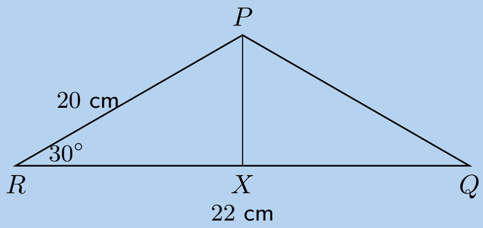
The final answers are: $a = 13{,}86$, $b = 12{,}56$, $c = 4{,}30$, $d = 7{,}78$, $e = 5{,}34$, $f = 9{,}29$ and $g = 1{,}66$.

12. In $\triangle PQR$, $PR = 20$ cm, $QR = 22$ cm and $P\hat{R}Q = 30°$. The perpendicular line from P to QR intersects QR at X. Calculate:

    a) the length XR

    Solution:

    First draw a sketch:

    Since we are told that $PX \perp QR$ we can use $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$ to find XR.

    $$\begin{aligned}
    \cos 30° &= \frac{XR}{20} \\
    XR &= 20 \cos 30° \\
    &= 17{,}3205... \\
    &\approx 17{,}32 \text{ cm}
    \end{aligned}$$

    b) the length PX

    Solution:

    We can use $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$ to find PX.

    $$\begin{aligned}
    \sin 30° &= \frac{PX}{20} \\
    PX &= 20 \sin 30° \\
    &= 9{,}999... \\
    &\approx 10 \text{ cm}
    \end{aligned}$$

    c) the angle $Q\hat{P}X$

    Solution:

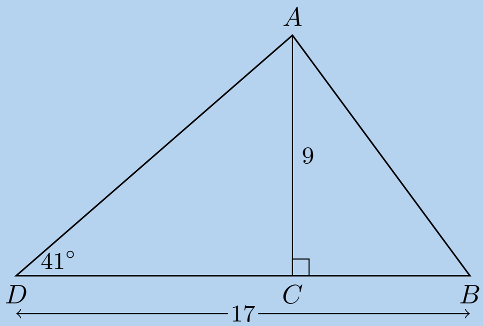
    We know the length of QR and we have found the length of XR, so we can work out the length of QX:

$$
\begin{aligned}
QX &= QR - XR \\
&= (22) - (17{,}32) \\
&= 4{,}68
\end{aligned}
$$

Since we know two sides and an angle we can use $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$ to find the angle:

$$
\begin{aligned}
\tan(Q\hat{P}X) &= \frac{4{,}68}{10} \\
&= 0{,}468 \\
Q\hat{P}X &= 25{,}0795... \\
&\approx 25{,}08°
\end{aligned}
$$

13. In the following triangle find the size of $A\hat{B}C$.



Solution:
We use $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$ to find DC:

$$
\begin{aligned}
\tan 41° &= \frac{9}{DC} \\
DC &= 9\tan 41° \\
&= 7{,}8235...
\end{aligned}
$$

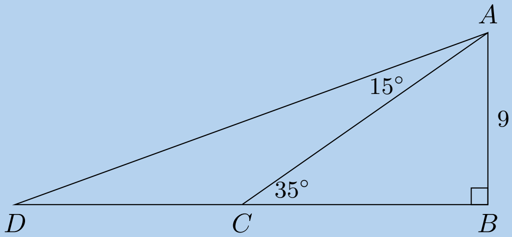
Next we find BC:

$$
\begin{aligned}
BC &= BD - DC \\
&= 17 - 7{,}8235... \\
&= 9{,}1764...
\end{aligned}
$$

And then we use $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$ to find the angle:

$$
\begin{aligned}
\tan A\hat{B}C &= \frac{9}{9{,}1764...} \\
&= 0{,}98077... \\
A\hat{B}C &= 44{,}439... \\
&\approx 44{,}44°
\end{aligned}
$$

14. In the following triangle find the length of side CD:



Solution:
We use the angles in a triangle to find $C\hat{A}B$:

$$C\hat{A}B = 180° - 90° - 35° = 55°$$

Then we find $D\hat{A}B$:

$$D\hat{A}B = 15° + 55° = 70°$$

Now we can use $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$ to find BC:

$$\begin{aligned} \tan 35° &= \frac{9}{BC} \\ BC &= \frac{9}{\tan 35°} \\ BC &= 12{,}85 \end{aligned}$$

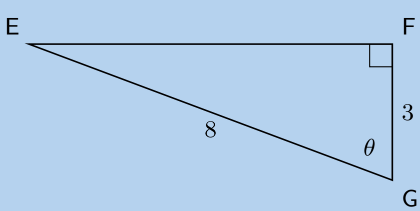
Then we find BD also using $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$:

$$\begin{aligned} \tan 70° &= \frac{BD}{9} \\ BD &= 9\tan 70° \\ BD &= 24{,}73 \end{aligned}$$

Finally we can find CD:

$$\begin{aligned} CD &= BD - BC \\ &= 24{,}73 - 12{,}85 \\ &= 11{,}88 \end{aligned}$$

15. Determine



a) The length of EF
Solution:

$$\begin{aligned} GE^2 &= EF^2 + FG^2 \\ EF^2 &= GE^2 - FG^2 \\ EF &= \sqrt{GE^2 - FG^2} \\ &= \sqrt{8^2 - 3^2} \\ &= \sqrt{64 - 9} \\ &= \sqrt{55} \end{aligned}$$

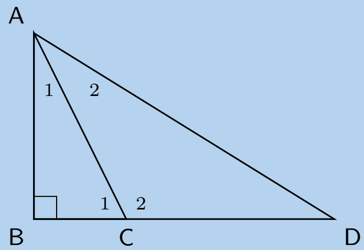
b) $\tan(90° - \theta)$
Solution:
We note that $\hat{G} = \theta$ and $\hat{F} = 90°$, therefore $\hat{E} = 90° - \theta$. So we need to find $\tan \hat{E}$:

$$\tan(90° - \theta) = \frac{3}{\sqrt{55}}$$

c) The value of θ
Solution:

$$\begin{aligned} \cos\theta &= \frac{3}{8} \\ \theta &= \cos^{-1}\frac{3}{8} \\ \theta &= 67{,}976° \end{aligned}$$

16. Given that $\hat{D} = x,\ \hat{C}_1 = 2x,\ BC = 12{,}2$ cm, $AB = 24{,}6$ cm. Calculate CD.



Solution:
We first calculate $\hat{C}_1$ by using the given information about AB and BC.

$$\begin{aligned} \tan\hat{C}_1 &= \frac{AB}{BC} \\ &= \frac{24{,}6}{12{,}2} \\ \hat{C}_1 &= 63{,}62257... \end{aligned}$$

Next we find $\hat{D}$:

$$\begin{aligned} \hat{D} &= \frac{\hat{C}_1}{2} \\ &= \frac{63{,}62257...}{2} \\ &= 31{,}8107... \end{aligned}$$

Now we can calculate BD:

$$
\begin{aligned}
\tan \hat{D} &= \frac{AB}{BD} \\
BD &= \frac{AB}{\tan \hat{D}} \\
&= \frac{24{,}6}{\tan 31{,}8107...} \\
&= 39{,}65906...
\end{aligned}
$$

Finally we can calculate CD:

$$
\begin{aligned}
CD &= BD - BC \\
&= 39{,}65906... - 12{,}2 \\
&= 27{,}45906... \\
&\approx 27{,}46 \text{ cm}
\end{aligned}
$$

17. Solve for θ if θ is a positive, acute angle:

a) $2\sin\theta = 1{,}34$

Solution:

$$
\begin{aligned}
2\sin\theta &= 1{,}34 \\
\sin\theta &= 0{,}67 \\
\theta &= 42{,}06706\ldots \\
&= 42{,}07°
\end{aligned}
$$

b) $1 - \tan\theta = -1$

Solution:

$$
\begin{aligned}
1 - \tan\theta &= -1 \\
-\tan\theta &= -2 \\
\tan\theta &= 2 \\
\theta &= 63{,}43494\ldots \\
&= 63{,}43°
\end{aligned}
$$

c) $\cos 2\theta = \sin 40°$

Solution:

$$
\begin{aligned}
\cos 2\theta &= \sin 40° \\
&= 0{,}64278\ldots \\
2\theta &= 50 \\
\theta &= 25°
\end{aligned}
$$

d) $\sec\theta = 1{,}8$

Solution:

$$
\begin{aligned}
\sec\theta &= 1{,}8 \\
\frac{1}{\cos\theta} &= 1{,}8 \\
1 &= 1{,}8\cos\theta \\
\frac{1}{1{,}8} &= \cos\theta \\
\theta &= 56{,}25101... \\
&\approx 56{,}25°
\end{aligned}
$$

e) $\cot 4\theta = \sin 40°$
Solution:

$$\begin{aligned}
\cot 4\theta &= \sin 40° \\
\cot 4\theta &= 0{,}642787... \\
\frac{1}{\tan 4\theta} &= 0{,}642787... \\
\frac{1}{0{,}642787...} &= \tan 4\theta \\
4\theta &= 57{,}2675... \\
\theta &= 14{,}3168... \\
&\approx 14{,}32°
\end{aligned}$$

f) $\sin 3\theta + 5 = 4$
Solution:

$$\begin{aligned}
\sin 3\theta + 5 &= 4 \\
\sin 3\theta &= -1 \\
3\theta &= 90 \\
\theta &= 30°
\end{aligned}$$

g) $\cos(4 + \theta) = 0{,}45$
Solution:

$$\begin{aligned}
\cos(4 + \theta) &= 0{,}45 \\
4 + \theta &= 63{,}25631... \\
\theta &= 59{,}25631... \\
&\approx 59{,}26
\end{aligned}$$

h) $\frac{\sin \theta}{\cos \theta} = 1$
Solution:
First we note that:

$$\begin{aligned}
\frac{\sin \theta}{\cos \theta} &= \sin \theta \times \frac{1}{\cos \theta} \\
&= \frac{\text{opposite}}{\text{hypotenuse}} \times \frac{\text{hypotenuse}}{\text{adjacent}} \\
&= \frac{\text{opposite}}{\text{adjacent}} \\
&= \tan \theta
\end{aligned}$$

Now we can solve for θ:

$$\begin{aligned}
\frac{\sin \theta}{\cos \theta} &= 1 \\
\tan \theta &= 1 \\
\theta &= 45°
\end{aligned}$$

18. If $a = 29°$, $b = 38°$ and $c = 47°$, use your calculator to evaluate each of the following, correct to 2 decimal places.

a) $\tan(a + c)$
Solution:

$$\begin{aligned}
\tan(a + c) &= \tan(29 + 47) \\
&= \tan 76 \\
&= 4{,}0107... \\
&\approx 4{,}01
\end{aligned}$$

b) cosec $(c - b)$
Solution:

$$\begin{aligned}\text{cosec } (c - b) &= \sin(47 - 38)\\ &= \text{cosec } 9\\ &= \frac{1}{\sin 9}\\ &= 6{,}3924...\\ &\approx 6{,}39\end{aligned}$$

c) $\sin(a \times b \times c)$
Solution:

$$\begin{aligned}\sin(a \times b \times c) &= \sin((29)(38)(47))\\ &= \sin(114)\\ &= 0{,}9135...\\ &\approx 0{,}91\end{aligned}$$

d) $\tan a + \sin b + \cos c$
Solution:

$$\begin{aligned}\tan a + \sin b + \cos c &= \tan 29 + \sin 38 + \cos 47\\ &= 1{,}8519...\\ &\approx 1{,}85\end{aligned}$$

19. If $3 \tan \alpha = -5$ and $0° < \alpha < 270°$, use a sketch to determine:

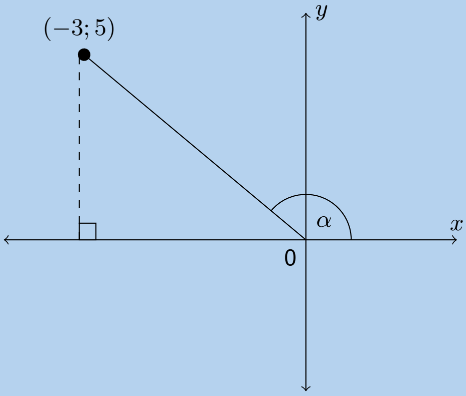
a) $\cos \alpha$
Solution:
Find x, y and r

$$\begin{aligned}3 \tan \alpha &= -5\\ \tan \alpha &= \frac{-5}{3}\end{aligned}$$

Therefore $x = -3$ and $y = 5$.

$$\begin{aligned}r^2 &= x^2 + y^2\\ &= (-3)^2 + (5)^2\\ &= 34\\ r &= \sqrt{34}\end{aligned}$$

Draw a sketch:



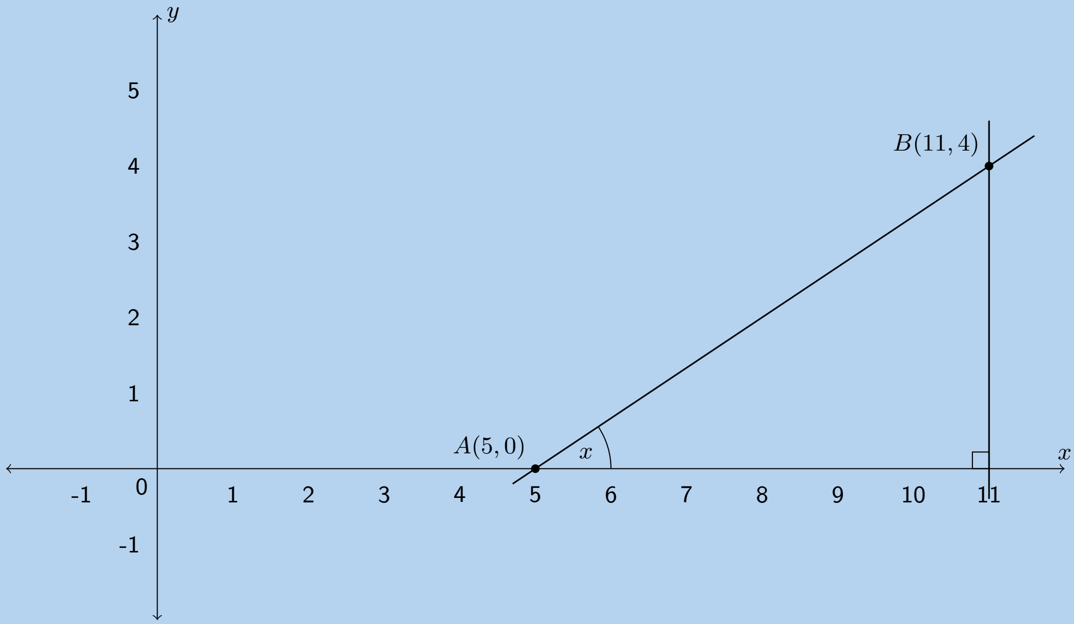
Now we can find $\cos\alpha$:

$$\begin{aligned}\cos\alpha &= \frac{x}{r}\\ &= \frac{-3}{\sqrt{34}}\end{aligned}$$

b) $\tan^2\alpha - \sec^2\alpha$
Solution:
We have x, y and r from the first question.

$$\begin{aligned}\tan^2\alpha - \sec^2\alpha &= \left(\frac{y}{x}\right)^2 - \left(\frac{r}{x}\right)^2\\ &= \left(\frac{5}{-3}\right)^2 - \left(\frac{\sqrt{34}}{-3}\right)^2\\ &= \frac{25}{9} - \frac{34}{9}\\ &= \frac{-9}{9}\\ &= -1\end{aligned}$$

20. Given $A(5;0)$ and $B(11;4)$, find the angle between the line through A and B and the x-axis.
Solution:
First draw a diagram:

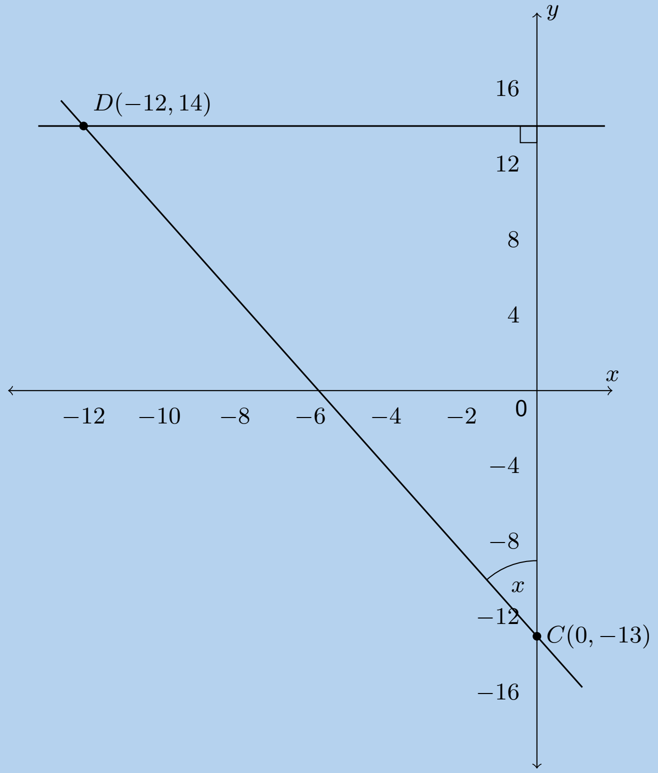


Next we note that the distance from B to the x-axis is 4 (B is 4 units up from the x-axis) and that the distance from A to C is $11 - 5 = 6$ units.
We use the tangent ratio to find the angle:

$$\begin{aligned}\tan x &= \frac{4}{6}\\ \tan x &= 0{,}66666\ldots\\ x &= 33{,}69°\end{aligned}$$

Therefore the angle between line AB and the x-axis is 33,69°.

21. Given $C(0;-13)$ and $D(-12;14)$, find the angle between the line through C and D and the y-axis.
Solution:
First draw a diagram:

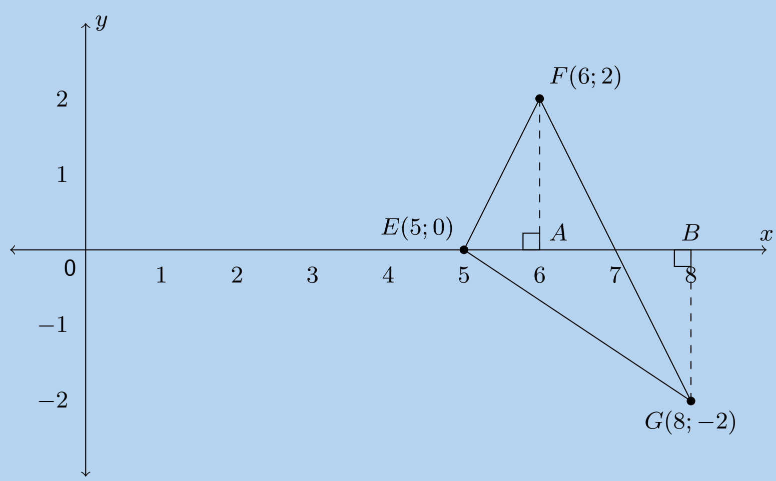


Next we note that the distance from D to the x-axis is 12 (although D is $(-12; 14)$ the distance is positive). The distance from C to the point where the perpendicular line from D intercepts the y-axis is $14 - (-13) = 27$ units. We use the tangent ratio to find the angle:

$$\begin{aligned} \tan x &= \frac{12}{27} \\ \tan x &= 0{,}4444\ldots \\ x &= 23{,}96° \end{aligned}$$

Therefore the angle between line CD and the x-axis is 23,96°.

22. Given the points $E(5; 0)$, $F(6; 2)$ and $G(8; -2)$. Find the angle $F\hat{E}G$.
Solution:
First draw a sketch:



To find $F\hat{E}G$ we look at $\triangle FEA$ and $\triangle GEB$ in turn. These two triangles will each give one part of the angle that we want.

In triangle FEA we can use the tangent ratio. FA is 2 units and EA is 1 unit.

$$\begin{aligned} \tan F\hat{E}X &= \frac{2}{1} \\ F\hat{E}X &= 63{,}43° \end{aligned}$$

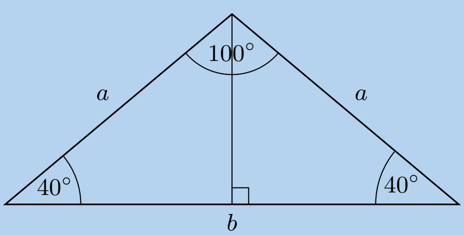
In triangle GEB we also use the tangent ratio. GB is 2 units and EB is 3 units.

$$\begin{aligned} \tan G\hat{E}X &= \frac{2}{3} \\ G\hat{E}X &= 33{,}69° \end{aligned}$$

Now we add these two angles together to get the angle we want to find:

$$\begin{aligned} G\hat{E}X + F\hat{E}X &= F\hat{E}G \\ F\hat{E}G &= 33{,}69° + 63{,}43° \\ &= 97{,}12° \end{aligned}$$

23. A triangle with angles 40°, 40° and 100° has a perimeter of 20 cm. Find the length of each side of the triangle.
Solution:
First draw a sketch:



We construct a perpendicular bisector and now we have a right-angled triangle to work with. We can use either of these two triangles.
We know $2a + b = 20$. Rearranging gives: $b = 2(10 - a)$. We can use the cos ratio to find a:

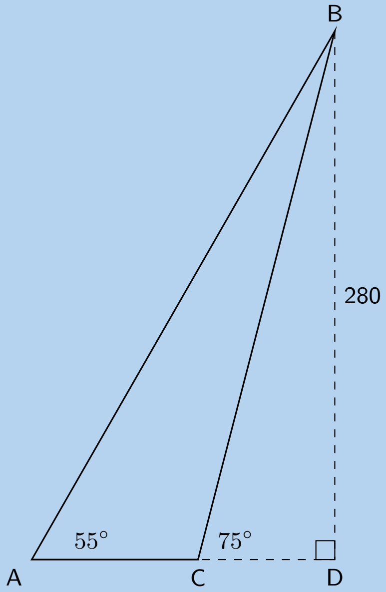
$$\begin{aligned} \cos 40° &= \frac{\frac{b}{2}}{a} \\ 0{,}77 &= \frac{\frac{2(10-a)}{2}}{a} \\ &= \frac{10-a}{a} \\ 0{,}77a &= 10 - a \\ a &= 5{,}65 \text{ cm} \end{aligned}$$

From the perimeter we get:

$$b = 2(10 - 5{,}65) = 8{,}7 \text{ cm}$$

Therefore the lengths of the sides are 8,7 cm, 5,65 cm and 5,65 cm.

24. Determine the area of $\triangle ABC$.



Solution:

Let the right angled vertex be D

$$\tan 55 = \frac{280}{AD}$$
$$AD = \frac{280}{\tan 55}$$
$$AD = 196{,}058$$

$$\tan 75 = \frac{280}{CD}$$
$$CD = \frac{280}{\tan 75}$$
$$CD = 75{,}026$$

$$AC = AD - CD$$
$$AC = 121{,}032$$

$$\text{Area of } \triangle ABC = \frac{1}{2}\text{base} \times \text{height}$$
$$\text{Area of } \triangle ABC = \frac{1}{2} \times 121{,}032 \times 280$$
$$\therefore \text{Area of } \triangle ABC = 16944 \text{ units}^2$$

For more exercises, visit www.everythingmaths.co.za and click on 'Practise Maths'.

1a. 2FT3	1b. 2FT4	1c. 2FT5	2a. 2FT6	2b. 2FT7	2c. 2FT8
2d. 2FT9	2e. 2FTB	2f. 2FTC	2g. 2FTD	2h. 2FTF	2i. 2FTG
2j. 2FTH	2k. 2FTJ	2l. 2FTK	2m. 2FTM	2n. 2FTN	2o. 2FTP
2p. 2FTQ	2q. 2FTR	2r. 2FTS	3. 2FTT	4. 2FTV	5a. 2FTW
5b. 2FTX	5c. 2FTY	5d. 2FTZ	5e. 2FV2	5f. 2FV3	5g. 2FV4
5h. 2FV5	6. 2FV6	7. 2FV7	8. 2FV8	9. 2FV9	10a. 2FVB
10b. 2FVC	10c. 2FVD	10d. 2FVF	10e. 2FVG	10f. 2FVH	10g. 2FVJ
10h. 2FVK	10i. 2FVM	11. 2FVN	12. 2FVP	13. 2FVQ	14. 2FVR
15. 2FVS	16. 2FVT	17a. 2FVV	17b. 2FVW	17c. 2FVX	17d. 2FVY
17e. 2FVZ	17f. 2FW2	17g. 2FW3	17h. 2FW4	18. 2FW5	19. 2FW6
20. 2FW7	21. 2FW8	22. 2FW9	23. 2FWB	24. 2FWC	

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