CHAPTER 3

Number patterns

- This chapter covers investigating number patterns that involve a common difference and the general term is linear.
- Arithmetic sequences are only covered in grade 12 so do not use $T_n = a + (n-1)d$ here.
- The focus of this chapter is more about investigating patterns in numbers and diagrams rather than on formulae.

3.1 Introduction

3.2 Describing sequences

Some learners may see example 3 as $2^1; 2^2; 2^3; \ldots$ and see a pattern with the powers. You may choose to discuss this in class as a precursor to geometric series which will be introduced in Grade 12.

Common difference

Exercise 3 – 1:

1. Use the given pattern to complete the table below.

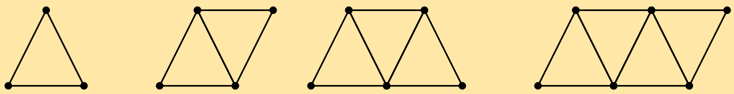


Figure number	1	2	3	4	n
Number of dots					
Number of lines					
Total					

Solution:

Figure number	1	2	3	4	n
Number of dots	3	4	5	6	$n+2$
Number of lines	3	5	7	9	$2n+1$
Total	6	9	12	15	$3(n+1)$

2. Consider the sequence shown here: $-4\ ;\ -1\ ;\ 2\ ;\ 5\ ;\ 8\ ;\ 11\ ;\ 14\ ;\ 17\ ;\ \ldots$
If $T_n = 2$ what is the value of T_{n-1}?
Solution:

$$\begin{aligned} T_3 &= 2 \\ \therefore T_{n-1} &= -1 \end{aligned}$$

3. Consider the sequence shown here: $C\ ;\ D\ ;\ E\ ;\ F\ ;\ G\ ;\ H\ ;\ I\ ;\ J\ ;\ \ldots$
If $T_n = G$ what is the value of T_{n-4}?
Solution:

$$\begin{aligned} T_5 &= G \\ \therefore T_{n-4} &= C \end{aligned}$$

4. For each of the following sequences determine the common difference. If the sequence is not linear, write "no common difference".

a) $9\ ;\ -7\ ;\ -8\ ;\ -25\ ;\ -34\ ;\ \ldots$
Solution:

$$d = T_2 - T_1 = (-7) - (9) = -16$$
$$d = T_3 - T_2 = (-8) - (-7) = -1$$
$$d = T_4 - T_3 = (-25) - (-8) = -17$$

You can see that the results are not the same - the difference is not 'common.' That means that this sequence of numbers in not linear, and it has no common difference.

b) $5\ ;\ 12\ ;\ 19\ ;\ 26\ ;\ 33\ ;\ \ldots$

Solution:

$$d = T_2 - T_1 = (12) - (5) = 7$$
$$d = T_3 - T_2 = (19) - (12) = 7$$
$$d = T_4 - T_3 = (26) - (19) = 7$$

All of the results are the same, which means we have found the **common** difference for these numbers: $d = 7$.

c) 2,93 ; 1,99 ; 1,14 ; 0,35 ; ...

Solution:

$$d = T_2 - T_1 = (1{,}99) - (2{,}93) = -0{,}94$$
$$d = T_3 - T_2 = (1{,}14) - (1{,}99) = -0{,}85$$

In this case the sequence is not linear. Therefore the final answer is that there is no common difference.

d) 2,53 ; 1,88 ; 1,23 ; 0,58 ; ...

Solution:

$$d = T_2 - T_1 = (1{,}88) - (2{,}53) = -0{,}65$$
$$d = T_3 - T_2 = (1{,}23) - (1{,}88) = -0{,}65$$

The common difference is $d = -0{,}65$.

5. Write down the next three terms in each of the following sequences:

a) $5\ ;\ 15\ ;\ 25\ ;\ \ldots$

Solution:

The common difference is:

$$\begin{aligned} d &= T_2 - T_1 \\ &= 15 - 5 \\ &= 10 \end{aligned}$$

Therefore we add 10 each time to get the next term in the sequence. The next three numbers are: 35, 45 and 55

and the sequence becomes:

$5\ ;\ 15\ ;\ 25\ ;\ 35\ ;\ 45\ ;\ 55\ ;\ \ldots$

b) $-8\ ;\ -3\ ;\ 2\ ;\ \ldots$

Solution:

The common difference is:

$$\begin{aligned} d &= T_2 - T_1 \\ &= -3 - (-8) \\ &= 5 \end{aligned}$$

Therefore we add 5 each time to get the next term in the sequence. The next three numbers are: 7, 12 and 17

and the sequence becomes:

$-8\ ;\ -3\ ;\ 2\ ;\ 7\ ;\ 12\ ;\ 17\ ;\ \ldots$

c) $30 \ ; \ 27 \ ; \ 24 \ ; \ \ldots$
Solution:
The common difference is:

$$\begin{aligned} d &= T_2 - T_1 \\ &= 27 - 30 \\ &= -3 \end{aligned}$$

Therefore we subtract 3 each time to get the next term in the sequence. The next three numbers are: 21, 18 and 15
and the sequence becomes:
$30 \ ; \ 27 \ ; \ 24 \ ; \ 21 \ ; \ 18 \ ; \ 15 \ ; \ \ldots$

d) $-13{,}1 \ ; \ -18{,}1 \ ; \ -23{,}1 \ ; \ldots$
Solution:

$$\begin{aligned} d &= T_2 - T_1 \text{ or } T_3 - T_2 \\ &= (-18{,}1) - (-13{,}1) \text{ or } (-23{,}1) - (-18{,}1) \\ &= -5 \\ \text{Therefore } T_4 &= -28{,}1 \\ T_5 &= -33{,}1 \\ T_6 &= -38{,}1 \end{aligned}$$

e) $-9x \ ; -19x \ ; -29x \ ; \ldots$
Solution:

$$\begin{aligned} d &= T_2 - T_1 \text{ or } T_3 - T_2 \\ &= (-19x) - (-9x) \text{ or } (-29x) - (-19x) \\ &= -10x \\ \text{Therefore } T_4 &= -39x \\ T_5 &= -49x \\ T_6 &= -59x \end{aligned}$$

f) $-15{,}8 \ ; \ 4{,}2 \ ; \ 24{,}2 \ ; \ldots$
Solution:

$$\begin{aligned} d &= T_2 - T_1 \text{ or } T_3 - T_2 \\ &= (4{,}2) - (-15{,}8) \text{ or } (24{,}2) - (4{,}2) \\ &= 20 \\ \text{Therefore } T_4 &= 44{,}2 \\ T_5 &= 64{,}2 \\ T_6 &= 84{,}2 \end{aligned}$$

g) $30b \ ; 34b \ ; 38b \ ; \ldots$
Solution:

$$\begin{aligned} d &= T_2 - T_1 \text{ or } T_3 - T_2 \\ &= (34b) - (30b) \text{ or } (38b) - (34b) \\ &= 4b \\ \text{Therefore } T_4 &= 42b \\ T_5 &= 46b \\ T_6 &= 50b \end{aligned}$$

6. Given a pattern which starts with the numbers: $3 \ ; \ 8 \ ; \ 13 \ ; \ 18 \ ; \ \ldots$ determine the values of T_6 and T_9.
Solution:

$$3\ ;\ 8\ ;\ 13\ ;\ 18\ ;\ 23\ ;\ \underline{28}\ ;\ 33\ ;\ 38\ ;\ \underline{43}\ ;\ \ldots$$

$T_6 = 28$ and $T_9 = 43$

7. Given a sequence which starts with the letters: $C\ ;\ D\ ;\ E\ ;\ F\ ;\ \ldots$ determine the values of T_5 and T_8.
Solution:

$$C\ ;\ D\ ;\ E\ ;\ F\ ;\ \underline{G}\ ;\ H\ ;\ I\ ;\ \underline{J}\ ;\ \ldots$$

$T_5 = G$ and $T_8 = J$

8. Given a pattern which starts with the numbers: $7\ ;\ 11\ ;\ 15\ ;\ 19\ ;\ \ldots$ determine the values of T_5 and T_8.
Solution:

$$7\ ;\ 11\ ;\ 15\ ;\ 19\ ;\ \underline{23}\ ;\ 27\ ;\ 31\ ;\ \underline{35}\ ;\ \ldots$$

$T_5 = 23$ and $T_8 = 35$

9. The general term is given for each sequence below. Calculate the missing terms (each missing term is represented by $\ldots$).

a) $0\ ;\ 3\ ;\ \ldots\ ;\ 15\ ;\ 24 \qquad T_n = n^2 - 1$
Solution:
The third term is:

$$\begin{aligned} T_n &= n^2 - 1 \\ T_3 &= (3)^2 - 1 \\ &= 9 - 1 \\ &= 8 \end{aligned}$$

The fourth term is:

$$\begin{aligned} T_n &= n^2 - 1 \\ T_4 &= (4)^2 - 1 \\ &= 16 - 1 \\ &= 15 \end{aligned}$$

Therefore the only missing term is the third one, which is 8. The full sequence is:
$0\ ;\ 8\ ;\ 15\ ;\ 24$

b) $3\ ;\ 2\ ;\ 1\ ;\ 0\ ;\ \ldots ; -2 \qquad T_n = -n + 4$
Solution:
The fifth term is:

$$\begin{aligned} T_n &= -n + 4 \\ T_5 &= -(5) + 4 \\ &= -1 \end{aligned}$$

The sixth term is:

$$\begin{aligned} T_n &= -n + 4 \\ T_6 &= -(6) + 4 \\ &= -2 \end{aligned}$$

Therefore the only missing term is the fifth one, which is -1. The full sequence is:
$3\ ;\ 2\ ;\ 1\ ;\ 0\ ;\ -1\ ;\ -2$

c) $-11\ ;\ \ldots\ ;\ -7\ ;\ \ldots\ ;\ -3 \qquad T_n = -13 + 2n$
Solution:
The second term is:

$$\begin{aligned} T_n &= -13 + 2n \\ T_2 &= -13 + 2(2) \\ &= -13 + 4 \\ &= -9 \end{aligned}$$

The third term is:

$$\begin{aligned} T_n &= -13 + 2n \\ T_3 &= -13 + 2(3) \\ &= -13 + 6 \\ &= -7 \end{aligned}$$

The fourth term is:

$$\begin{aligned} T_n &= -13 + 2n \\ T_4 &= -13 + 2(4) \\ &= -13 + 8 \\ &= -5 \end{aligned}$$

The fifth term is:

$$\begin{aligned} T_n &= -13 + 2n \\ T_5 &= -13 + 2(5) \\ &= -13 + 10 \\ &= -3 \end{aligned}$$

Therefore the two missing terms are the second and fourth ones, which are -9 and -5. The full sequence is: -11 ; -9 ; -7 ; -5 ; -3

d) 1 ; 10 ; 19 ; . . . ; 37 $\quad T_n = 9n - 8$
Solution:

$$\begin{aligned} T_n &= 9n - 8 \\ T_4 &= 9(4) - 8 \\ &= 28 \end{aligned}$$

e) 9 ; . . . ; 21 ; . . . ; 33 $\quad T_n = 6n + 3$
Solution:
To find the two missing terms, we use the equation for the general term:

$$\begin{aligned} T_n &= 6n+3 \\ T_2 &= 6(2)+3 \\ &= 15 \\ T_4 &= 6(4)+3 \\ &= 27 \end{aligned}$$

10. Find the general formula for the following sequences and then find T_{10}, T_{50} and T_{100}

a) $2; 5; 8; 11; 14; \ldots$
Solution:
We first need to find d:

$$\begin{aligned} d &= T_2 - T_1 \\ &= 5 - 2 \\ &= 3 \end{aligned}$$

Next we note that for each successive term we add d to the last term. We can express this as:

$$\begin{aligned} T_1 &= a = 2 \\ T_2 &= a + d = 2 + 3 \\ &= 2 + 1(3) \\ T_3 &= T_2 + d = 2 + 3 + 3 \\ &= 2 + 2(3) \\ T_4 &= T_3 + d = 2 + 3 + 3 + 3 \\ &= 2 + 3(3) \\ T_n &= T_{n-1} + d = 2 + 3(n-1) \\ &= 3n - 1 \end{aligned}$$

The general formula is $T_n = 3n - 1$.
T_{10}, T_{50} and T_{100} are:

$$\begin{aligned} T_{10} &= 3(10) - 1 \\ &= 29 \\ T_{50} &= 3(50) - 1 \\ &= 149 \\ T_{100} &= 3(100) - 1 \\ &= 299 \end{aligned}$$

b) $0; 4; 8; 12; 16; \ldots$

Solution:

We first need to find d:

$$\begin{aligned} d &= T_2 - T_1 \\ &= 4 - 0 \\ &= 4 \end{aligned}$$

Next we note that for each successive term we add d to the last term. We can express this as:

$$\begin{aligned} T_1 &= a = 0 \\ T_2 &= a + d = 0 + 4 \\ &= 4(1) \\ T_3 &= T_2 + d = 0 + 4 + 4 \\ &= 4(2) \\ T_4 &= T_3 + d = 0 + 4 + 4 + 4 \\ &= 4(3) \\ T_n &= T_{n-1} + d = 0 + 4(n-1) \\ &= 4n - 4 \end{aligned}$$

The general formula is $T_n = 4n - 4$.
T_{10}, T_{50} and T_{100} are:

$$\begin{aligned} T_{10} &= 4(10) - 4 \\ &= 36 \\ T_{50} &= 4(50) - 4 \\ &= 196 \\ T_{100} &= 4(100) - 4 \\ &= 396 \end{aligned}$$

c) $2; -1; -4; -7; -10; \ldots$

Solution:
We first need to find d:

$$\begin{aligned} d &= T_2 - T_1 \\ &= -1 - 2 \\ &= -3 \end{aligned}$$

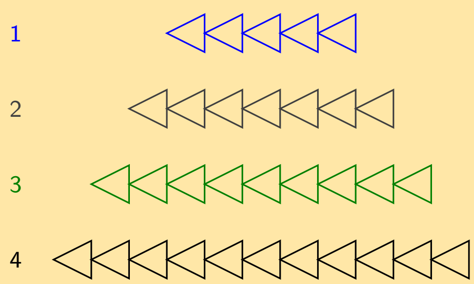
Next we note that for each successive term we add d to the last term. We can express this as:

$$\begin{aligned} T_1 &= a = 2 \\ T_2 &= a + d = 2 + (-3) \\ &= 2 + (-3)(1) \\ T_3 &= T_2 + d = 2 + (-3) + (-3) \\ &= 2 + (-3)(2) \\ T_4 &= T_3 + d = 2 + (-3) + (-3) + (-3) \\ &= 2 + (-3)(3) \\ T_n &= T_{n-1} + d = 2 + (-3)(n - 1) \\ &= 5 - 3n \end{aligned}$$

The general formula is $T_n = 5 - 3n$.
T_{10}, T_{50} and T_{100} are:

$$\begin{aligned} T_{10} &= 5 - 3(10) \\ &= -25 \\ T_{50} &= 5 - 3(50) \\ &= -145 \\ T_{100} &= 5 - 3(100) \\ &= -295 \end{aligned}$$

11. The diagram below shows pictures which follow a pattern.



a) How many triangles will there be in the 5th picture?
Solution:
$5 \; ; \; 7 \; ; \; 9 \; ; \; 11 \; ; \; \ldots$
Therefore two triangles are added each time and the fifth picture will have 13 triangles.
b) Determine the formula for the n^{th} term.
Solution:
The general term of the pattern is:

$$\begin{aligned} T_n &= T_1 + d = 5 + (2)(n - 1) \\ &= 2n + 3 \end{aligned}$$

c) Use the formula to find how many triangles are in the 25^{th} picture of the diagram.
Solution:

$$\begin{aligned} T_n &= 2n + 3 \\ T_{25} &= 2(25) + 3 \longleftarrow \text{substitute n } = 25 \\ &= 53 \end{aligned}$$

12. Study the following sequence: $15\ ;\ 23\ ;\ 31\ ;\ 39\ ;\ \ldots$

    a) Write down the next 3 terms.
    Solution:
    We note that we add 8 to each term to get the next term. Therefore the next three terms are $47\ ;\ 55\ ;\ 63$.

    b) Find the general formula for the sequence
    Solution:

    $$\begin{aligned} T_n &= T_1 + d(n-1) \\ &= 15 + 8(n-1) \\ &= 8n + 7 \end{aligned}$$

    c) Find the value of n if T_n is 191.
    Solution:

    $$\begin{aligned} 191 &= 8n + 7 \\ 184 &= 8n \\ n &= 23 \end{aligned}$$

13. Study the following sequence: $-44\ ;\ -14\ ;\ 16\ ;\ 46\ ;\ \ldots$

    a) Write down the next 3 terms.
    Solution:
    We note that we add 30 to each term to get the next term. Therefore the next three terms are $76\ ;\ 106\ ;\ 136$.

    b) Find the general formula for the sequence
    Solution:

    $$\begin{aligned} T_n &= T_1 + d(n-1) \\ &= -44 + 30(n-1) \\ &= 30n - 74 \end{aligned}$$

    c) Find the value of n if T_n is 406.
    Solution:

    $$\begin{aligned} 406 &= 30n - 74 \\ 480 &= 30n \\ n &= 16 \end{aligned}$$

14. Consider the following list:

    $$-z - 5\ ;\ -4z - 5\ ;\ -6z - 2\ ;\ -8z - 5\ ;\ -10z - 5\ ;\ \ldots$$

    a) Find the common difference for the terms of the list. If the sequence is not linear (if it does not have a common difference), write "no common difference".
    Solution:

    $$\begin{aligned} d = T_2 - T_1 &= (-4z - 5) - (-z - 5) = -3z \\ = T_3 - T_2 &= (-6z - 2) - (-4z - 5) = -2z + 3 \\ = T_4 - T_3 &= (-8z - 5) - (-6z - 2) = -2z - 3 \end{aligned}$$

    No common difference.

b) If you are now told that $z = -2$, determine the values of T_1 and T_2.
Solution:

$$\begin{aligned} T_1 &= -z - 5 \\ &= -(-2) - 5 \\ &= -3 \\ T_2 &= -4z - 5 \\ &= -4(-2) - 5 \\ &= 3 \end{aligned}$$

15. Consider the following pattern:

$$2n + 4\ ;\ 1\ ;\ -2n - 2\ ;\ -4n - 5\ ;\ -6n - 8\ ;\ \ldots$$

a) Find the common difference for the terms of the pattern. If the sequence is not linear (if it does not have a common difference), write "no common difference".
Solution:

$$\begin{aligned} d = T_2 - T_1 &= (1) - (2n + 4) = -2n - 3 \\ = T_3 - T_2 &= (-2n - 2) - (1) = -2n - 3 \\ = T_4 - T_3 &= (-4n - 5) - (-2n - 2) = -2n - 3 \end{aligned}$$

The common difference for these numbers: $d = -2n - 3$.

b) If you are now told that $n = -1$, determine the values of T_1 and T_3.
Solution:

$$\begin{aligned} T_1 &= 2n + 4 \\ &= 2(-1) + 4 \\ &= 2 \\ T_3 &= -2n - 2 \\ &= -2(-1) - 2 \\ &= 0 \end{aligned}$$

16. a) If the following terms make a linear sequence:

$$\frac{k}{3} - 1\ ;\ -\frac{5k}{3} + 2\ ;\ -\frac{2k}{3} + 10\ ;\ \ldots$$

Determine the value of k. If the answer is a non-integer, write the answer as a simplified fraction.
Solution:

$$\begin{aligned} T_2 - T_1 &= T_3 - T_2 \\ \left(-\frac{5k}{3} + 2\right) - \left(\frac{k}{3} - 1\right) &= \left(-\frac{2k}{3} + 10\right) - \left(-\frac{5k}{3} + 2\right) \\ 3\left(-\frac{5k}{3} + 2\right) - 3\left(\frac{k}{3} - 1\right) &= 3\left(-\frac{2k}{3} + 10\right) - 3\left(-\frac{5k}{3} + 2\right) \\ -5k + 6 - (k - 3) &= -2k + 30 - (-5k + 6) \\ -6k + 9 &= 3k + 24 \\ -15 &= 9k \\ k &= -\frac{5}{3} \end{aligned}$$

b) Now determine the numeric value of the first three terms. If the answers are not integers, write your answers as fractions.

Solution:

$$
\begin{aligned}
\text{First term: } T_1 &= \frac{k}{3} - 1 \\
&= \frac{\left(-\frac{5}{3}\right)}{3} - 1 \\
&= -\frac{14}{9} \\
\text{Second term: } T_2 &= -\frac{5k}{3} + 2 \\
&= -\frac{5\left(-\frac{5}{3}\right)}{3} + 2 \\
&= \frac{43}{9} \\
\text{Third term: } T_3 &= -\frac{2k}{3} + 10 \\
&= -\frac{2\left(-\frac{5}{3}\right)}{3} + 10 \\
&= \frac{100}{9}
\end{aligned}
$$

The first three terms of this sequence are: $-\frac{14}{9}$, $\frac{43}{9}$ and $\frac{100}{9}$.

17. a) If the following terms make a linear sequence:

$$y - \frac{3}{2}\ ;\ -y - \frac{7}{2}\ ;\ -7y - \frac{15}{2}\ ;\ \ldots$$

find y. If the answer is a non-integer, write the answer as a simplified fraction.

Solution:

$$
\begin{aligned}
T_2 - T_1 &= T_3 - T_2 \\
\left(-y - \frac{7}{2}\right) - \left(y - \frac{3}{2}\right) &= \left(-7y - \frac{15}{2}\right) - \left(-y - \frac{7}{2}\right) \\
2\left(-y - \frac{7}{2}\right) - 2\left(y - \frac{3}{2}\right) &= 2\left(-7y - \frac{15}{2}\right) - 2\left(-y - \frac{7}{2}\right) \\
-2y - 7 - (2y - 3) &= -14y - 15 - (-2y - 7) \\
-4y - 4 &= -12y - 8 \\
8y &= -4 \\
y &= -\frac{1}{2}
\end{aligned}
$$

b) Now determine the numeric value of the first three terms. If the answers are not integers, write your answers as fractions.

Solution:

$$\begin{aligned}
\text{First term: } T_1 &= y - \frac{3}{2} \\
&= \left(-\frac{1}{2}\right) - \frac{3}{2} \\
&= -2 \\
\text{Second term: } T_2 &= -y - \frac{7}{2} \\
&= -\left(-\frac{1}{2}\right) - \frac{7}{2} \\
&= -3 \\
\text{Third term: } T_3 &= -7y - \frac{15}{2} \\
&= -7\left(-\frac{1}{2}\right) - \frac{15}{2} \\
&= -4
\end{aligned}$$

The first three terms of this sequence are: $-2, -3$ and -4.

18. What is the 649^{th} letter of the sequence:
PATTERNPATTERNPATTERNPATTERNPATTERNPATTERNPATTE.............?
Solution:
The word "PATTERN" is 7 letters long, so:

$$\frac{649}{7} = 92 \text{ r } 5$$

The remainder of 5 shows us that the 649^{th} letter is the 5^{th} letter in the word, which is E

For more exercises, visit www.everythingmaths.co.za and click on 'Practise Maths'.

1. 2F73	2. 2F74	3. 2F75	4. 2F76	5a. 2F77	5b. 2F78
5c. 2F79	5d. 2F7B	5e. 2F7C	5f. 2F7D	5g. 2F7F	6. 2F7G
7. 2F7H	8. 2F7J	9a. 2F7K	9b. 2F7M	9c. 2F7N	9d. 2F7P
9e. 2F7Q	10a. 2F7R	10b. 2F7S	10c. 2F7T	11. 2F7V	12. 2F7W
13. 2F7X	14. 2F7Y	15. 2F7Z	16. 2F82	17. 2F83	18. 2F84

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3.3 Chapter summary

End of chapter Exercise 3 – 2:

1. Analyse the diagram and complete the table.

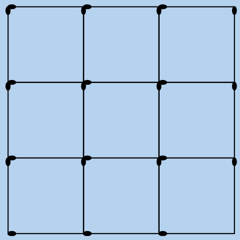


Figure number $(n \times n)$	1×1	2×2	3×3	4×4	$n \times n$
Number of horizontal matches					
Number of vertical matches					
Total number of matches					

Solution:

Figure number ($n \times n$)	1×1	2×2	3×3	4×4	$n \times n$
Number of horizontal matches	2	6	12	20	$n(n+1)$
Number of vertical matches	2	6	12	20	$n(n+1)$
Total number of matches	4	12	24	40	$2n(n+1)$

2. Given a list of numbers: $7\ ;\ 4\ ;\ 1\ ;\ -2\ ;\ -5\ ;\ \ldots$ determine the common difference for the list (if there is one).
Solution:

$$\begin{aligned} d &= T_2 - T_1 = (4) - (7) = -3 \\ &= T_3 - T_2 = (1) - (4) = -3 \\ &= T_4 - T_3 = (-2) - (1) = -3 \end{aligned}$$

All of the results are the same, which means we have found the **common** difference for these numbers: $d = -3$.

3. For the pattern here: $-0{,}55\ ;\ 0{,}99\ ;\ 2{,}49\ ;\ 3{,}91\ ;\ \ldots$ calculate the common difference.
If the pattern is not linear, write "no common difference". Otherwise, give your answer as a decimal.
Solution:

$$\begin{aligned} d &= T_2 - T_1 = (0{,}99) - (-0{,}55) = 1{,}54 \\ d &= T_3 - T_2 = (2{,}49) - (0{,}99) = 1{,}5 \end{aligned}$$

In this case the sequence is not linear. Therefore the final answer is that there is no common difference.

4. Consider the list shown here: $2\ ;\ 7\ ;\ 12\ ;\ 17\ ;\ 22\ ;\ 27\ ;\ 32\ ;\ 37\ ;\ \ldots$
If $T_5 = 22$ what is the value of T_{n-3}?
Solution:

$$\begin{aligned} T_5 &= 22 \\ \therefore T_{n-3} &= 7 \end{aligned}$$

5. Write down the next three terms in each of the following linear sequences:

   a) $-10{,}2\ ;\ -29{,}2\ ;\ -48{,}2\ ;\ \ldots$
   Solution:

$$\begin{aligned} d &= T_2 - T_1 \text{ or } T_3 - T_2 \\ &= (-29{,}2) - (-10{,}2) \text{ or } (-48{,}2) - (-29{,}2) \\ &= -19 \\ \text{Therefore } T_4 &= -67{,}2 \\ T_5 &= -86{,}2 \\ T_6 &= -105{,}2 \end{aligned}$$

   b) $50r\ ;\ 46r\ ;\ 42r\ ;\ \ldots$
   Solution:

$$\begin{aligned} d &= T_2 - T_1 \text{ or } T_3 - T_2 \\ &= (46r) - (50r) \text{ or } (42r) - (46r) \\ &= -4r \\ \text{Therefore } T_4 &= 38r \\ T_5 &= 34r \\ T_6 &= 30r \end{aligned}$$

6. Given a sequence which starts with the numbers: $6\ ;\ 11\ ;\ 16\ ;\ 21\ ;\ \ldots$ determine the values of T_6 and T_8.
Solution:

$$6;\ 11;\ 16;\ 21;\ 26;\ \underline{31};\ 36;\ \underline{41};\ \ldots$$

$T_6 = 31$ and $T_8 = 41$

7. Given a list which starts with the letters: $A\ ;\ B\ ;\ C\ ;\ D\ ;\ \ldots$ determine the values of T_6 and T_{10}.
   Solution:

$$A\ ;\ B\ ;\ C\ ;\ D\ ;\ E\ ;\ \underline{F}\ ;\ G\ ;\ H\ ;\ I\ ;\ \underline{J}\ ;\ \ldots$$

   $T_6 = F$ and $T_{10} = J$

8. Find the sixth term in each of the following sequences:
   a) $4\ ;\ 13\ ;\ 22\ ;\ 31\ ;\ \ldots$
      Solution:
      We first need to find d:

$$\begin{aligned} d &= T_2 - T_1 \\ &= 13 - 4 \\ &= 9 \end{aligned}$$

      Next we note that for each successive term we add d to the last term. We can express this as:

$$\begin{aligned} T_1 &= a = 4 \\ T_2 &= a + d = 4 + 9 \\ &= 4 + 9(1) \\ T_3 &= T_2 + d = 4 + 9 + 9 \\ &= 4 + 9(2) \\ T_n &= T_{n-1} + d = 4 + 9(n-1) \\ &= 9n - 5 \end{aligned}$$

      The general formula is $T_n = 9n - 5$.
      T_6 is:

$$\begin{aligned} T_6 &= 9(6) - 5 \\ &= 49 \end{aligned}$$

      $T_6 = 49$
   b) $5\ ;\ 2\ ;\ -1\ ;\ -4\ ;\ \ldots$
      Solution:
      We first need to find d:

$$\begin{aligned} d &= T_2 - T_1 \\ &= 2 - 5 \\ &= -3 \end{aligned}$$

      Next we note that for each successive term we add d to the last term. We can express this as:

$$\begin{aligned} T_1 &= a = 5 \\ T_2 &= a + d = 5 + (-3) \\ &= 5 + (-3)(1) \\ T_3 &= T_2 + d = 5 + (-3) + (-3) \\ &= 5 + (-3)(2) \\ T_n &= T_{n-1} + d = 5 + (-3)(n-1) \\ &= 7 - 3n \end{aligned}$$

      The general formula is $T_n = 7 - 3n$.
      T_6 is:

$$\begin{aligned} T_6 &= 7 - 3(6) \\ &= -11 \end{aligned}$$

      $T_6 = -11$

c) 7,4 ; 9,7 ; 12 ; 14,3 ; ...
Solution:
We first need to find d:

$$\begin{aligned} d &= T_2 - T_1 \\ &= 9{,}7 - 7{,}4 \\ &= 2{,}3 \end{aligned}$$

Next we note that for each successive term we add d to the last term. We can express this as:

$$\begin{aligned} T_1 &= a = 7{,}4 \\ T_2 &= a + d = 7{,}4 + 2{,}3 \\ &= 7{,}4 + 2{,}3(1) \\ T_3 &= T_2 + d = 7{,}4 + 2{,}3 + 2{,}3 \\ &= 7{,}4 + 2{,}3(2) \\ T_n &= T_{n-1} + d = 7{,}4 + 2{,}3(n-1) \\ &= 7{,}4 + 2{,}3n - 2{,}3 = 2{,}3n + 5{,}1 \end{aligned}$$

The general formula is $T_n = 2{,}3n + 5{,}1$.
T_6 is:

$$\begin{aligned} T_6 &= 2{,}3(6) + 5{,}1 \\ &= 18{,}9 \end{aligned}$$

$T_6 = 18{,}9$

9. Find the general formula for the following sequences and then find T_{10}, T_{15} and T_{30}

a) -18 ; -22 ; -26 ; -30 ; -34 ; ...
Solution:

$$\begin{aligned} d &= T_2 - T_1 \\ &= (-22) - (-18) \\ &= -4 \end{aligned}$$

Next we note that for each successive term we add d to the last term. We can express this as:

$$\begin{aligned} T_1 &= a = -18 \\ T_2 &= a + d = -18 + (-4) \\ &= -18 + (-4)(1) \\ T_3 &= T_2 + d = -18 + (-4) + (-4) \\ &= -18 + (-4)(2) \\ T_n &= T_{n-1} + d = -18 + (-4)(n-1) \\ &= -4n - 14 \end{aligned}$$

The general formula is $T_n = -4n - 14$.

$$\begin{aligned} T_{10} &= -4(10) - 14 \\ &= -54 \\ T_{15} &= -4(15) - 14 \\ &= -74 \\ T_{30} &= -4(30) - 14 \\ &= -134 \end{aligned}$$

b) $1; -6; -13; -20; -27; \ldots$
Solution:

$$\begin{aligned} d &= T_2 - T_1 \\ &= (-6) - (1) \\ &= -7 \end{aligned}$$

Next we note that for each successive term we add d to the last term. We can express this as:

$$\begin{aligned} T_1 &= a = 1 \\ T_2 &= a + d = 1 + (-7) \\ &= 1 + (-7)(1) \\ T_3 &= T_2 + d = 1 + (-7) + (-7) \\ &= 1 + (-7)(2) \\ T_n &= T_{n-1} + d = 1 + (-7)(n-1) \\ &= -7n + 8 \end{aligned}$$

The general formula is $T_n = -7n + 8$.

$$\begin{aligned} T_{10} &= -7(10) + 8 \\ &= -62 \\ T_{15} &= -7(15) + 8 \\ &= -97 \\ T_{30} &= -7(30) + 8 \\ &= -202 \end{aligned}$$

10. The general term is given for each sequence below. Calculate the missing terms (each missing term is represented by $\ldots$).

a) $10\ ;\ \ldots\ ;\ 14\ ;\ \ldots\ ;\ 18 \qquad T_n = 2n + 8$
Solution:

$$\begin{aligned} T_n &= 2n+8 \\ T_2 &= 2(2)+8 \\ &= 12 \\ T_4 &= 2(4)+8 \\ &= 16 \end{aligned}$$

The missing terms are 12 and 16

b) $2\ ;\ -2\ ;\ -6\ ;\ \ldots\ ;\ -14 \qquad T_n = -4n + 6$
Solution:

$$\begin{aligned} T_n &= -4n + 6 \\ T_4 &= -4(4)+6 \\ &= -10 \end{aligned}$$

c) $8\ ;\ \ldots\ ;\ 38\ ;\ \ldots\ ;\ 68 \qquad T_n = 15n - 7$
Solution:

$$\begin{aligned} T_n &= 15n - 7 \\ T_2 &= 15(2) - 7 \\ &= 23 \\ T_4 &= 15(4) - 7 \\ &= 53 \end{aligned}$$

11. Find the general term in each of the following sequences:

a) $3\ ;\ 7\ ;\ 11\ ;\ 15\ ;\ \ldots$

Solution:

We first need to find d:

$$\begin{aligned} d &= T_2 - T_1 \\ &= 7 - 3 \\ &= 4 \end{aligned}$$

Next we note that for each successive term we add d to the last term. We can express this as:

$$\begin{aligned} T_1 &= a = 3 \\ T_2 &= a + d = 3 + 4 \\ &= 3 + 4(1) \\ T_3 &= T_2 + d = 3 + 4 + 4 \\ &= 3 + 4(2) \\ T_n &= T_{n-1} + d = 3 + 4(n-1) \\ &= 4n - 1 \end{aligned}$$

The general formula is $T_n = 4n - 1$.

b) $-2\ ;\ 1\ ;\ 4\ ;\ 7\ ;\ \ldots$

Solution:

We first need to find d:

$$\begin{aligned} d &= T_2 - T_1 \\ &= 1 - (-2) \\ &= 3 \end{aligned}$$

Next we note that for each successive term we add d to the last term. We can express this as:

$$\begin{aligned} T_1 &= a = -2 \\ T_2 &= a + d = -2 + 3 \\ &= -2 + 3(1) \\ T_3 &= T_2 + d = -2 + 3 + 3 \\ &= -2 + 3(2) \\ T_n &= T_{n-1} + d = -2 + 3(n-1) \\ &= 3n - 5 \end{aligned}$$

The general formula is $T_n = 3n - 5$.

c) $11\ ;\ 15\ ;\ 19\ ;\ 23\ ;\ \ldots$

Solution:

We first need to find d:

$$\begin{aligned} d &= T_2 - T_1 \\ &= 15 - 11 \\ &= 4 \end{aligned}$$

Next we note that for each successive term we add d to the last term. We can express this as:

$$\begin{aligned} T_1 &= a = 11 \\ T_2 &= a + d = 11 + 4 \\ &= 11 + 4(1) \\ T_3 &= T_2 + d = 11 + 4 + 4 \\ &= 11 + 4(2) \\ T_n &= T_{n-1} + d = 11 + 4(n-1) \\ &= 4n + 7 \end{aligned}$$

The general formula is $T_n = 4n + 7$.

d) $\frac{1}{3}\ ;\ \frac{2}{3}\ ;\ 1\ ;\ 1\frac{1}{3}\ ;\ \ldots$

Solution:

We first need to find d:

$$\begin{aligned} d &= T_2 - T_1 \\ &= \frac{2}{3} - \frac{1}{3} \\ &= \frac{1}{3} \end{aligned}$$

Next we note that for each successive term we add d to the last term. We can express this as:

$$\begin{aligned} T_1 &= a = \frac{1}{3} \\ T_2 &= a + d = \frac{1}{3} + \frac{1}{3} \\ &= \frac{1}{3} + \frac{1}{3}(1) \\ T_3 &= T_2 + d = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} \\ &= \frac{1}{3} + \frac{1}{3}(2) \\ T_n &= T_{n-1} + d = \frac{1}{3} + \frac{1}{3}(n-1) \\ &= \frac{1}{3} + \frac{1}{3}n - \frac{1}{3} \\ &= \frac{1}{3}n \end{aligned}$$

The general formula is $T_n = \frac{1}{3}n$.

12. Study the following sequence $-7\ ;\ -21\ ;\ -35\ ;\ \ldots$

a) Write down the next 3 terms:

Solution:

$-49\ ;\ -63\ ;\ 77$

b) Find the general formula for the sequence.

Solution:

$$\begin{aligned} T_n &= -7 - 14(n-1) \\ T_n &= -7 - 14n + 14 \\ T_n &= -14n + 7 \end{aligned}$$

c) Find the value of n if T_n is -917.

Solution:

$$\begin{aligned} -917 &= 7 - 14n \\ -924 &= -14n \\ n &= 66 \end{aligned}$$

13. What is the 346^{th} letter of the sequence: COMMONCOMMON............?

Solution:

The word "COMMON" is 6 letters long, so:

$$\frac{346}{6} = 57 \text{ r } 4$$

The remainder of 4 shows us that the 346^{th} letter is the $4th^{\text{th}}$ letter in the word, which is M

14. What is the 1000^{th} letter of the sequence:
MATHEMATICSMATHEMATICSMATHEMATICSMATHE ....?
Solution:
The word "MATHEMATICS" is 11 letters long, so:

$$\frac{1000}{11} = 90 \text{ r } 10$$

The remainder of 10 shows us that the 1000^{th} letter is the tenth letter in the word, which is C

15. The seating of a sports stadium is arranged so that the first row has 15 seats, the second row has 19 seats, the third row has 23 seats and so on. Calculate how many seats are in the 25^{th} row.
Solution:
We start by writing the given information as a sequence:

$$15; 19; 23; \ldots$$

Now we can calculate d:

$$\begin{aligned} d &= T_2 - T_1 \\ &= 19 - 15 \\ &= 4 \end{aligned}$$

Next we note that for each successive term we add d to the last term. We can express this as:

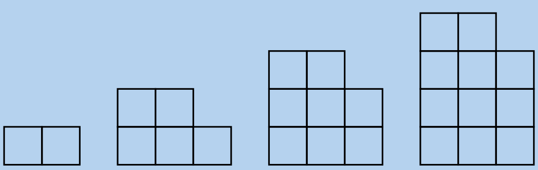
$$\begin{aligned} T_1 &= a = 15 \\ T_2 &= a + d = 15 + 4 \\ &= 15 + 4(1) \\ T_3 &= T_2 + d = 15 + 4 + 4 \\ &= 15 + 4(2) \\ T_n &= T_{n-1} + d = 15 + 4(n-1) \\ &= 4n + 11 \end{aligned}$$

The general formula is $T_n = 4n + 11$.
The 25^{th} row is represented by T_{25}. The number of seats in this row is:

$$\begin{aligned} T_{25} &= 4(25) + 11 \\ &= 111 \end{aligned}$$

There are 111 seats in the 25^{th} row.

16. The diagram below shows pictures which follow a pattern.



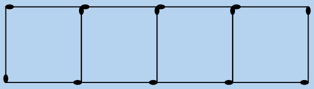
a) How many boxes will there be in the sixth picture?
Solution:
$2\ ;\ 5\ ;\ 8\ ;\ 11\ ;\ \ldots$
Therefore three boxes are added each time and the sixth picture will have 17 boxes

b) Determine the formula for the n^{th} term.
Solution:
The general term of the pattern is: $T_n = 3n - 1$.

c) Use the formula to find how many boxes are in the 30^{th} picture of the diagram.
Solution:

$$
\begin{aligned}
T_n &= 3n - 1 \\
T_{30} &= 3(30) - 1 \longleftarrow \text{ substitute n } = 30 \\
&= 89
\end{aligned}
$$

17. A single square is made from 4 matchsticks. Two squares in a row need 7 matchsticks and three squares need 10 matchsticks.



Answer the following questions for this sequence.

a) Determine the first term.
Solution:
We begin by writing a sequence to represent this:

$$4 \; ; \; 7 \; ; \; 10 \; ; \; \ldots$$

We see from this that the first term is 4.
$T_1 = 4$

b) Determine the common difference.
Solution:
The common difference (d) is:

$$
\begin{aligned}
d &= T_2 - T_1 \\
&= 7 - 4 \\
&= 3
\end{aligned}
$$

c) Determine the general formula.
Solution:
To determine the general formula we note that for each successive term we add d to the last term. We can express this as:

$$
\begin{aligned}
T_1 &= a = 4 \\
T_2 &= a + d = 4 + 3 \\
&= 4 + 3(1) \\
T_3 &= T_2 + d = 4 + 3 + 3 \\
&= 4 + 3(2) \\
T_n &= T_{n-1} + d = 4 + 3(n - 1) \\
&= 3n + 1
\end{aligned}
$$

The general formula is $T_n = 3n + 1$.

d) A row has twenty-five squares. How many matchsticks are there in this row?
Solution:
We note that a row with twenty-five squares is represented by T_{25}. The number of matchsticks in this row is:

$$
\begin{aligned}
T_{25} &= 3(25) + 1 \\
&= 76
\end{aligned}
$$

There are 76 matchsticks in the row with twenty-five squares.

18. You would like to start saving some money, but because you have never tried to save money before, you decide to start slowly. At the end of the first week you deposit R 5 into your bank account. Then at the end of the second week you deposit R 10 and at the end of the third week, R 15. After how many weeks will you deposit R 50 into your bank account?
Solution:
We begin by writing down a sequence to represent this:

$$5\ ;\ 10\ ;\ 15\ ;\ \ldots$$

Next we need to find d:

$$\begin{aligned} d &= T_2 - T_1 \\ &= 10 - 5 \\ &= 5 \end{aligned}$$

Now we note that for each successive term we add d to the last term. We can express this as:

$$\begin{aligned} T_1 &= a = 5 \\ T_2 &= a + d = 5 + 5 \\ &= 5 + 5(1) \\ T_3 &= T_2 + d = 5 + 5 + 5 \\ &= 5 + 5(2) \\ T_n &= T_{n-1} + d = 5 + 5(n-1) \\ &= 5n \end{aligned}$$

The general formula is $T_n = 5n$.
Now we need to find n such that $T_n = 50$:

$$\begin{aligned} T_n &= 5n \\ 50 &= 5n \\ \therefore n &= 10 \end{aligned}$$

After the 10$^{\text{th}}$ week you will deposit R 50 into your bank account.

19. Consider the following list:

$$-4y - 3\ ;\ -y\ ;\ 2y + 3\ ;\ 5y + 6\ ;\ 8y + 9\ ;\ \ldots$$

a) Find the common difference for the terms of the list. If the sequence is not linear (if it does not have a common difference), write "no common difference".
Solution:

$$\begin{aligned} d &= T_2 - T_1 = (-y) - (-4y - 3) = 3y + 3 \\ d &= T_3 - T_2 = (2y + 3) - (-y) = 3y + 3 \\ d &= T_4 - T_3 = (5y + 6) - (2y + 3) = 3y + 3 \end{aligned}$$

The common difference for these numbers: $d = 3y + 3$.

b) If you are now told that $y = 1$, determine the values of T_1 and T_2.
Solution:

$$\begin{aligned} T_1 &= -4y - 3 \\ &= -4(1) - 3 \\ &= -7 \\ T_2 &= -y \\ &= -(1) \\ &= -1 \end{aligned}$$

20. a) If the following terms make a linear sequence:

$$2n + \frac{1}{2}\ ;\ 3n + \frac{5}{2}\ ;\ 7n + \frac{11}{2}\ ;\ \ldots$$

Determine the value of n. If the answer is a non-integer, write the answer as a simplified fraction.
Solution:

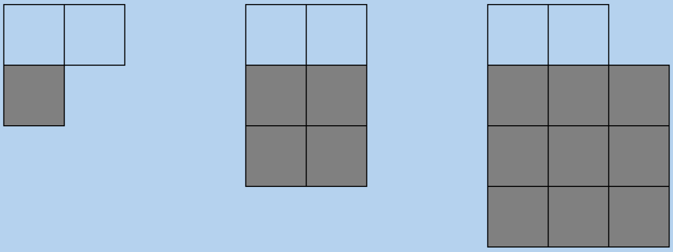
$$
\begin{aligned}
T_2 - T_1 &= T_3 - T_2 \\
\left(3n + \frac{5}{2}\right) - \left(2n + \frac{1}{2}\right) &= \left(7n + \frac{11}{2}\right) - \left(3n + \frac{5}{2}\right) \\
2\left(3n + \frac{5}{2}\right) - 2\left(2n + \frac{1}{2}\right) &= 2\left(7n + \frac{11}{2}\right) - 2\left(3n + \frac{5}{2}\right) \\
6n + 5 - (4n + 1) &= 14n + 11 - (6n + 5) \\
2n + 4 &= 8n + 6 \\
-2 &= 6n \\
n &= -\frac{1}{3}
\end{aligned}
$$

b) Now determine the numeric value of the first three terms. If the answers are not integers, write your answers as fractions.
Solution:

$$
\begin{aligned}
\text{First term: } T_1 &= 2n + \frac{1}{2} \\
&= 2\left(-\frac{1}{3}\right) + \frac{1}{2} \\
&= -\frac{1}{6} \\
\text{Second term: } T_2 &= 3n + \frac{5}{2} \\
&= 3\left(-\frac{1}{3}\right) + \frac{5}{2} \\
&= \frac{3}{2} \\
\text{Third term: } T_3 &= 7n + \frac{11}{2} \\
&= 7\left(-\frac{1}{3}\right) + \frac{11}{2} \\
&= \frac{19}{6}
\end{aligned}
$$

The first three terms of this sequence are: $-\frac{1}{6}$, $\frac{3}{2}$ and $\frac{19}{6}$.

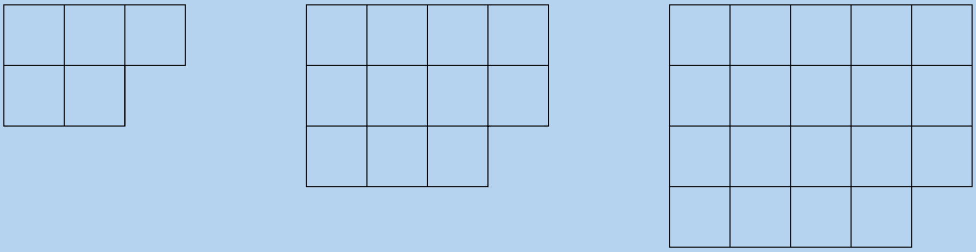
21. How many blocks will there be in the 85^{th} picture?
Hint: Use the grey blocks to help.



Solution:
The grey blocks can be represented by n^2 and there are always 2 white blocks.

$$
\begin{aligned}
T_n &= n^2 + 2 \\
T_{85} &= 85^2 + 2 \\
T_{85} &= 7227 \text{ blocks}
\end{aligned}
$$

22. Analyse the picture below:



a) How many blocks are there in the next picture?
Solution:
Picture 1: $2^2 + 1$
Picture 2: $3^2 + 2$
Picture 3: $4^2 + 3$
Picture 4: $5^2 + 4 = 29$ blocks

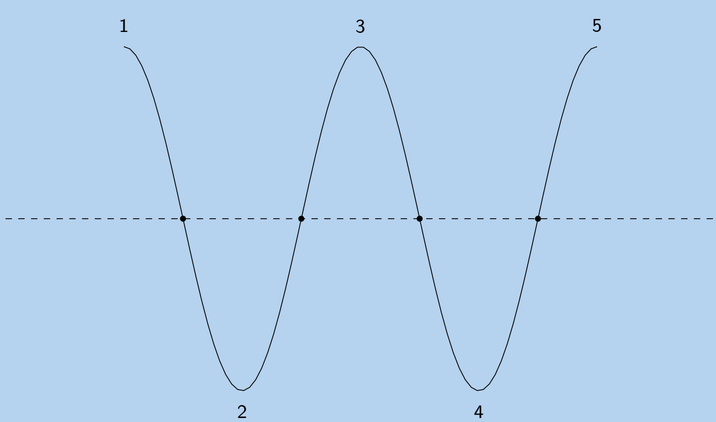
b) Write down the general formula for this pattern.
Solution:
Look at:
Picture 1: $2^2 + 1 \qquad (n = 1)$

$$T_n = (n+1)^2 + n$$

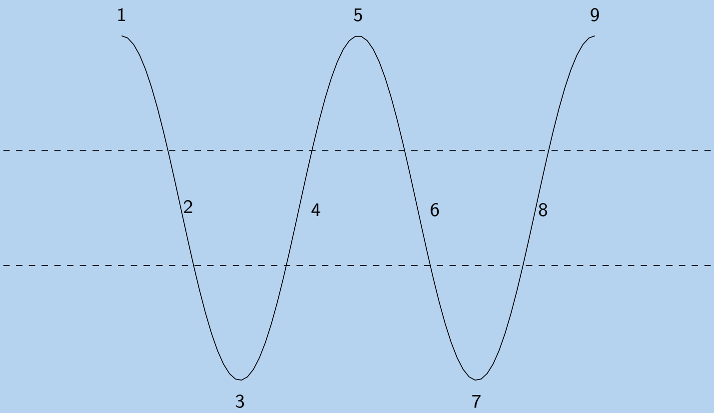
c) How many blocks will there be in the 14th picture?
Solution:

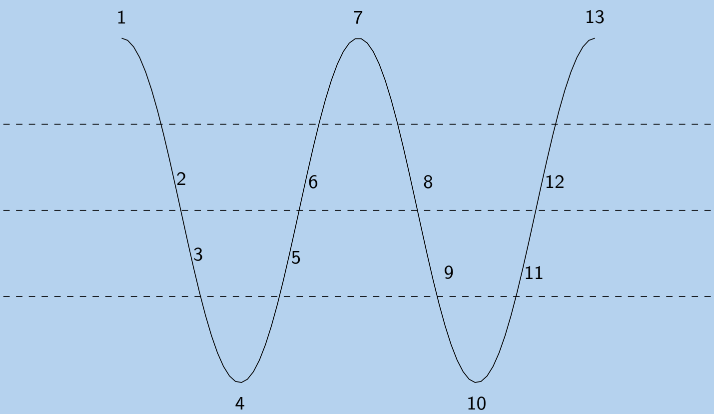
$$\begin{aligned} T_n &= (n+1)^2 + n \\ T_{14} &= (14+1)^2 + 14 \\ T_{14} &= 239 \text{ blocks} \end{aligned}$$

23. A horizontal line intersects a piece of string at 4 points and divides it into five parts, as shown below.



If the piece of string is intersected in this way by 19 parallel lines, each of which intersects it at 4 points, determine the number of parts into which the string will be divided.
Solution:
We need to determine a pattern for this scenario.
The first line divides the string into five parts. We can redraw the diagram to show the string with 2 and 3 lines:





From this we see that the two lines cut the string into 9 pieces. Three lines cut the string into 13 pieces. So for each line added we cut the line into 4 more pieces.
So we can write the following sequence:

$$5\ ;\ 9\ ;\ 13\ ;\ \ldots$$

The common difference is 4.
Next we note that for each successive term we add d to the last term. We can express this as:

$$\begin{aligned}
T_1 &= a = 5 \\
T_2 &= a + d = 5 + 4 \\
&= 5 + 4(1) \\
T_3 &= T_2 + d = 5 + 4 + 4 \\
&= 5 + 4(2) \\
T_n &= T_{n-1} + d = 5 + 4(n-1) \\
&= 4n + 1
\end{aligned}$$

The general formula is $T_n = 4n + 1$.
When there are 19 lines we are working with T_{19}:

$$\begin{aligned}
T_{19} &= 4(19) + 1 \\
&= 77
\end{aligned}$$

Therefore the string will be cut into 77 parts.

24. Use a calculator to explore and then generalise your findings to determine the:

 a) units digit of 3^{2007}
 Solution:

$$\left|\begin{array}{l|l|l} 3^1 = 3 & 3^5 = 243 & 3^9 = 19683 \\ 3^2 = 9 & 3^6 = 729 & 3^{10} = 59049 \\ 3^3 = 27 & 3^7 = 2187 & 3^{11} = 177147 \\ 3^4 = 81 & 3^8 = 6561 & 3^{12} = 531441 \end{array}\right|$$

$\frac{2007}{4} = 501$ r 3

Therefore 3^{2007} will follow the same pattern as the third row

therefore the units digit is 7

b) tens digit of 7^{2008}

Solution:

$$\left|\begin{array}{l|l|l} 7^1 = 07 & 7^5 = 16807 & 7^9 = 40353607 \\ 7^2 = 49 & 7^6 = 117649 & 7^{10} = 282475249 \\ 7^3 = 343 & 7^7 = 823543 & 7^{11} = 1977326743 \\ 7^4 = 2401 & 7^8 = 576801 & \end{array}\right|$$

$\frac{2008}{4} = 502$ r 0

Therefore 7^{2008} will follow the same pattern as the fourth row

therefore the tens digit is 0

c) remainder when 7^{250} is divided by 5

Solution:

$$\left|\begin{array}{l|l} \frac{7^1}{5} : \text{Remainder} = 2 & \frac{7^5}{5} : \text{Remainder} = 2 \\ \frac{7^2}{5} : \text{Remainder} = 4 & \frac{7^6}{5} : \text{Remainder} = 4 \\ \frac{7^3}{5} : \text{Remainder} = 3 & \frac{7^7}{5} : \text{Remainder} = 3 \\ \frac{7^4}{5} : \text{Remainder} = 1 & \frac{7^8}{5} : \text{Remainder} = 1 \end{array}\right|$$

$\frac{250}{4} = 62$ r 0

Therefore 2^{250} will follow the same pattern as the second row

therefore the remainder is 4

25. Analyse the diagram and complete the table.

The dots follow a triangular pattern and the formula is $T_n = \frac{n(n+1)}{2}$.

The general formula for the lines is $T_n = \frac{3n(n-1)}{2}$.

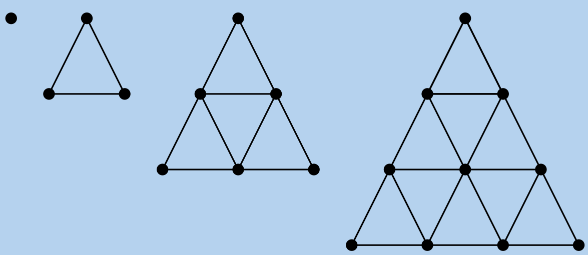


Figure number	1	2	3	4	5	20	n
Number of dots							
Number of lines							
Total							

Solution:

We are given the general formula for both the lines and the dots. We can determine the general formula for the sum of the lines and dots by adding the general formula for the lines to the general formula for the dots.

$$
\begin{aligned}
T_n &= \frac{n(n+1)}{2} + \frac{3n(n-1)}{2} \\
&= \frac{n^2+n+3n^2-3n}{2} \\
&= \frac{4n^2-2n}{2} \\
&= 2n^2 - n
\end{aligned}
$$

Figure number	1	2	3	4	5	20	n
Number of dots	1	3	6	10	15	210	$\frac{n(n+1)}{2}$
Number of lines	0	3	9	18	30	570	$\frac{3n(n-1)}{2}$
Total	1	6	15	28	45	780	$2n^2-n$

For more exercises, visit www.everythingmaths.co.za and click on 'Practise Maths'.

1. 2F86 2. 2F88 3. 2F89 4. 2F8B 5. 2F8C 6. 2F8D
7. 2F8F 8a. 2F8G 8b. 2F8H 8c. 2F8J 9. 2F8K 10. 2F8M
11a. 2F8N 11b. 2F8P 11c. 2F8Q 11d. 2F8R 12. 2F8S 13. 2F8T
14. 2F8V 15. 2F8W 16. 2F8X 17. 2F8Y 18. 2F8Z 19. 2F93
20. 2F94 21. 2F95 22. 2F96 23. 2F92 24a. 2F97 24b. 2F98
24c. 2F99 25. 2F87

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