CHAPTER 1

Algebraic expressions

1.1 Introduction

- Content covered in this chapter includes understanding how numbers are classified as rational or irrational, estimating surds, rounding off, factorisation and simplification.
- This chapter provides a lot of core skills that learners will apply to the rest of mathematics. Ensure that learners are sufficiently proficient in the skills covered in this chapter.
- Rounding real numbers is an important skill that learners will use often. Ensure that learners are completely comfortable with this skill.
- Factorisation forms the groundwork for solving equations. Learners should be comfortable factorising trinomials and binomials.
- Factorisation should include types covered in grade 9 as well as trinomials, grouping in pairs and sum and difference of two cubes.

1.2 The real number system

1.3 Rational and irrational numbers

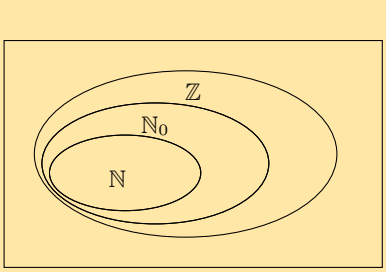
Decimal numbers

Converting terminating decimals into rational numbers

Converting recurring decimals into rational numbers

Exercise 1 – 1:

1. The figure here shows the Venn diagram for the special sets $\mathbb{N}, \mathbb{N}_0$ and $\mathbb{Z}$.



a) Where does the number $-\frac{12}{3}$ belong in the diagram?
Solution:
First simplify the fraction: $-\frac{12}{3} = -4$
-4 is an integer, so it falls into the set $\mathbb{Z}$.

b) In the following list, there are two false statements and one true statement. Which of the statements is **true**?

i. Every integer is a natural number.
ii. Every natural number is a whole number.
iii. There are no decimals in the whole numbers.

Solution:

Consider each option carefully:

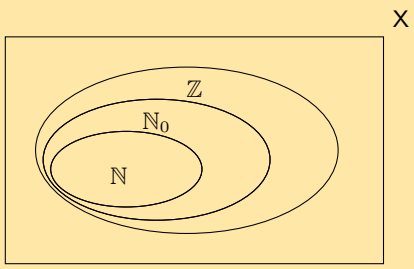
i. There are integers which do not fall into the natural numbers (all negative numbers), so this is false.

ii. The natural numbers are $\{1; 2; 3; \ldots\}$ and whole numbers are $\{0; 1; 2; 3; \ldots\}$ (the circle $\mathbb{N}$ is inside $\mathbb{N}_0$) so if a number is a natural number it must be a whole number. This is true.

iii. Whole numbers $\{0; 1; 2; 3; \ldots\}$ only go up in steps of 1, so there cannot be any decimal numbers in the whole numbers, making this false.

So only (ii) is true.

2. The figure here shows the Venn diagram for the special sets $\mathbb{N}, \mathbb{N}_0$ and $\mathbb{Z}$.



a) Where does the number $-\frac{1}{2}$ belong in the diagram?

Solution:

$-\frac{1}{2}$ is in its simplest form, therefore it is not in $\mathbb{N}$, $\mathbb{N}_0$ or $\mathbb{Z}$. It is in the space between the rectangle and $\mathbb{Z}$.

b) In the following list, there are two false statements and one true statement. Which of the statements is **true**?

i. Every integer is a natural number.

ii. Every whole number is an integer.

iii. There are no decimals in the whole numbers.

Solution:

Consider each option carefully:

i. There are integers which do not fall into the natural numbers (all negative numbers), so this is false.

ii. The integers are $\{\ldots; -3; -2; -1; 0; 1; 2; 3; \ldots\}$ and the whole numbers are $\{0; 1; 2; 3; \ldots\}$ (the circle $\mathbb{Z}$ is inside $\mathbb{N}_0$) so if a number is an integer it must be a whole number. This is true.

iii. Whole numbers $\{0; 1; 2; 3; 4; \ldots\}$ only go up in steps of 1, so there cannot be any decimal numbers in the whole numbers, making this false.

So only (ii) is true.

3. State whether the following numbers are real, non-real or undefined.

a) $-\sqrt{3}$

Solution:

$-\sqrt{3}$ has no minus sign under the square root (the minus is outside the root) and is not divided by zero, so it is real.

b) $\dfrac{0}{\sqrt{2}}$

Solution:

$\dfrac{0}{\sqrt{2}}$ has no minus sign under the square root (the minus is outside the root) and is not divided by zero, so it is real.

c) $\sqrt{-9}$

Solution:

$\sqrt{-9}$ has a minus sign under the square root so it is non-real.

d) $\dfrac{-\sqrt{7}}{0}$

Solution:

$\dfrac{-\sqrt{7}}{0}$ has division by zero so it is undefined.

e) $-\sqrt{-16}$

Solution:

$-\sqrt{-16}$ has a negative number under the square root so it is non-real.

f) $\sqrt{2}$

Solution:

$\sqrt{2}$ has no minus under the square root (the minus is outside the root), is not divided by zero, so it is real.

4. State whether the following numbers are rational or irrational. If the number is rational, state whether it is a natural number, whole number or an integer.

a) $-\frac{1}{3}$

Solution:

$-\frac{1}{3}$ is rational. A fraction of integers is a rational number.

b) 0,651268962154862...

Solution:

0,651268962154862... is irrational. It cannot be simplified to a fraction of integers.

c) $\frac{\sqrt{9}}{3}$

Solution:

$\frac{\sqrt{9}}{3}$ is rational, an integer, a whole number and a natural number. An integer is a rational number.

d) π^2

Solution:

π^2 is irrational. It cannot be simplified to a fraction of integers.

e) π^4

Solution:

π^4 is irrational. It cannot be simplified to a fraction of integers.

f) $\sqrt[3]{19}$

Solution:

$\sqrt[3]{19}$ is irrational. It cannot be simplified to a fraction of integers.

g) $\left(\sqrt[3]{1}\right)^7$

Solution:

$\left(\sqrt[3]{1}\right)^7$ is rational, an integer, a whole number and a natural number. It can be written as an integer.

h) $\pi + 3$

Solution:

π is irrational. 3 is rational (it is an integer). Any rational number added to any irrational number is irrational.

Therefore $\pi + 3$ is irrational.

i) π + 0,858408346

Solution:

π is irrational. 0,858408346 is rational (it is a terminating decimal). Any rational number added to any irrational number is irrational.

Therefore π + 0,858408346 is irrational.

5. If a is an integer, b is an integer and c is irrational, which of the following are rational numbers?

a) $\frac{5}{6}$

Solution:

$\frac{5}{6}$ is rational.

b) $\frac{a}{3}$

Solution:

Since a is an integer, $\frac{a}{3}$ is rational.

c) $\frac{-2}{b}$

Solution:

Since b is an integer, $\frac{-2}{b}$ is rational.

Note that b cannot be 0 as that makes the fraction undefined.

d) $\frac{1}{c}$

Solution:

Since c is irrational, $\frac{1}{c}$ is irrational.

6. For each of the following values of a state whether $\frac{a}{14}$ is rational or irrational.

   a) 1
   Solution:
   $\frac{a}{14} = \frac{1}{14}$ is rational.

   b) -10
   Solution:
   $\frac{a}{14} = \frac{-10}{14}$ is rational.

   c) $\sqrt{2}$
   Solution:
   $\frac{a}{14} = \frac{\sqrt{2}}{14}$ is irrational.

   d) 2,1
   Solution:
   $\frac{a}{14} = \frac{2,1}{14}$ is rational.

7. Consider the following list of numbers:

$$-3\ ;\ 0\ ;\ \sqrt{-1}\ ;\ -8\frac{4}{5}\ ;\ -\sqrt{8}\ ;\ \frac{22}{7}\ ;\ \frac{14}{0}\ ;\ 7\ ;\ 1,\overline{34}\ ;\ 3,3231089...\ ;\ 3+\sqrt{2}\ ;\ 9\frac{7}{10}\ ;\ \pi\ ;\ 11$$

   Which of the numbers are:

   a) natural numbers
   Solution:
   Check which of the numbers are in the set $\{1; 2; 3; 4; \ldots\}$. Therefore 7 and 11 are natural numbers.

   b) irrational numbers
   Solution:
   Remember that rational numbers can be written as $\frac{a}{b}$ where a and b are integers. Also remember that rational numbers include terminating decimal numbers. Therefore $-\sqrt{8}\ ;\ 3,3231089...\ ;\ 3+\sqrt{2}\ ;\ \pi$ are all irrational.

   c) non-real numbers
   Solution:
   Any number that is a square root of a negative number is non-real. Therefore only $\sqrt{-1}$ is non-real.

   d) rational numbers
   Solution:
   Remember that rational numbers can be written as $\frac{a}{b}$ where a and b are integers. Also remember that rational numbers include terminating decimal numbers. Therefore $-3\ ;\ 0\ ;\ -8\frac{4}{5}\ ;\ \frac{22}{7}\ ;\ 7\ ;\ 1,\overline{34}\ ;\ 9\frac{7}{10}\ ;\ 11$ are all rational numbers.

   e) integers
   Solution:
   Check which of the numbers are in the set $\{\ldots; -3; -2; -1; 0; 1; 2; 3; \ldots\}$. Therefore $-3\ ;\ 7\ ;\ 11$ are integers.

   f) undefined
   Solution:
   Any fraction divided by 0 is undefined. Therefore only $\frac{14}{0}$ is undefined.

8. For each of the following numbers:

   - write the next three digits and
   - state whether the number is rational or irrational.

   a) $1,1\dot{5}$
   Solution:

   - Since there is a dot over the 5 we know that the 5 repeats. The next three digits are: 555
   - Rational, there is a repeating pattern of digits.

   b) 2,121314...
   Solution:

   - The number does not terminate (this is shown by the . . .). There is also no indication of a repeating pattern of digits since there is not dot or bar over any of the numbers. The next three digits could be any numbers. Note that while it looks like there is a pattern in the digits we do not know if this pattern continues on.
   - Irrational, there is no repeating pattern.

c) 1,242244246...

Solution:

- The number does not terminate (this is shown by the . . .). There is also no indication of a repeating pattern of digits since there is not dot or bar over any of the numbers. The next three digits could be any numbers. Note that while it looks like there is a pattern in the digits we do not know if this pattern continues on.
- Irrational, there is no repeating pattern.

d) 3,324354...

Solution:

- The number does not terminate (this is shown by the . . .). There is also no indication of a repeating pattern of digits since there is not dot or bar over any of the numbers. The next three digits could be any numbers. Note that while it looks like there is a pattern in the digits we do not know if this pattern continues on.
- Irrational, there is no repeating pattern.

e) $3,32435\dot{4}$

Solution:

- Since there is a dot over both the 5 and the 4 we know that the pattern 54 repeats. The next three digits are: 545
- Rational, there is a repeating pattern.

9. Write the following as fractions:

a) 0,1

Solution:

$0,1 = \frac{1}{10}$

b) 0,12

Solution:

$$\begin{aligned} 0,12 &= \frac{1}{10} + \frac{2}{100} \\ &= \frac{10}{100} + \frac{2}{100} \\ &= \frac{12}{100} \\ &= \frac{3}{25} \end{aligned}$$

c) 0,58

Solution:

$$\begin{aligned} 0,58 &= \frac{5}{10} + \frac{8}{100} \\ &= \frac{50}{100} + \frac{8}{100} \\ &= \frac{58}{100} \\ &= \frac{29}{50} \end{aligned}$$

d) 0,2589

Solution:

$$\begin{aligned} 0,2589 &= \frac{2}{10} + \frac{5}{100} + \frac{8}{1000} + \frac{9}{10\,000} \\ &= \frac{2000}{10\,000} + \frac{500}{10\,000} + \frac{80}{10\,000} + \frac{9}{10\,000} \\ &= \frac{2589}{10\,000} \end{aligned}$$

10. Write the following using the recurring decimal notation:

a) 0,1111111...

Solution:

We see that only the digit 1 is repeated and so we can write this as: $0,\dot{1}$.

b) 0,1212121212...
Solution:
There is a repeating pattern of 12 and so we can write this number as: $0,\overline{12}$

c) 0,123123123123...
Solution:
There is a repeating pattern of 123 and so we can write this number as: $0,\overline{123}$

d) 0,11414541454145...
Solution:
The pattern 4145 repeats and so we can write this number as: $0,11\overline{4145}$.

11. Write the following in decimal form, using the recurring decimal notation:

a) $\frac{25}{45}$
Solution:

$$\begin{aligned} 45|\overline{25,0000} &= 0 \text{ remainder } 25 \\ 45|\overline{25,^{25}0000} &= 5 \text{ remainder } 25 \\ 45|\overline{25,^{25}0^{25}000} &= 5 \text{ remainder } 25 \\ 45|\overline{25,^{25}0^{25}0^{25}00} &= 5 \text{ remainder } 25 \\ \frac{25}{45} &= 0,5555\ldots \\ &= 0,\dot{5} \end{aligned}$$

b) $\frac{10}{18}$
Solution:

$$\begin{aligned} 18|\overline{10,0000} &= 0 \text{ remainder } 10 \\ 18|\overline{10,^{10}0000} &= 5 \text{ remainder } 10 \\ 18|\overline{10,^{10}0^{10}000} &= 5 \text{ remainder } 10 \\ 18|\overline{10,^{10}0^{10}0^{10}00} &= 5 \text{ remainder } 10 \\ \frac{10}{18} &= 0,5555\ldots \\ &= 0,\dot{5} \end{aligned}$$

c) $\frac{7}{33}$
Solution:

$$\begin{aligned} 33|\overline{7,0000} &= 0 \text{ remainder } 7 \\ 33|\overline{7,^{7}0000} &= 2 \text{ remainder } 4 \\ 33|\overline{7,^{4}0^{4}000} &= 1 \text{ remainder } 7 \\ 33|\overline{7,^{7}0^{4}0^{7}00} &= 2 \text{ remainder } 4 \\ \frac{7}{33} &= 0,2121\ldots \\ &= 0,\dot{2}\dot{1} \end{aligned}$$

d) $\frac{2}{3}$
Solution:

$$\begin{aligned}\frac{2}{3} &= 2\left(\frac{1}{3}\right)\\ &= 2(0{,}333333...)\\ &= 0{,}666666...\\ &= 0{,}\dot{6}\end{aligned}$$

e) $1\frac{3}{11}$

Solution:

$$\begin{aligned}1\frac{3}{11} &= 1 + 3\left(\frac{1}{11}\right)\\ &= 1 + 3(0{,}090909...)\\ &= 1 + 0{,}27272727...\\ &= 1{,}\overline{27}\end{aligned}$$

f) $4\frac{5}{6}$

Solution:

$$\begin{aligned}4\frac{5}{6} &= 4 + 5\left(\frac{1}{6}\right)\\ &= 4 + 5(0{,}1666666...)\\ &= 4 + 0{,}833333...\\ &= 4{,}8\dot{3}\end{aligned}$$

g) $2\frac{1}{9}$

Solution:

$$\begin{aligned}2\frac{1}{9} &= 2 + 0{,}1111111...\\ &= 2{,}\dot{1}\end{aligned}$$

12. Write the following decimals in fractional form:

a) $0{,}\dot{5}$

Solution:

$$\begin{aligned}x &= 0{,}55555... \text{ and}\\ 10x &= 5{,}55555...\\ 10x - x &= (5{,}55555...) - (0{,}55555...)\\ 9x &= 5\\ \therefore x &= \frac{5}{9}\end{aligned}$$

b) $0{,}6\dot{3}$

Solution:

$$\begin{aligned}10x &= 6{,}3333... \text{ and}\\ 100x &= 63{,}3333...\\ 100x - 10x &= (63{,}3333...) - (6{,}3333...)\\ 99x &= 57\\ \therefore x &= \frac{57}{90}\end{aligned}$$

c) $0,\dot{4}$

Solution:

$$\begin{aligned} x &= 0,4444\ldots \text{ and} \\ 10x &= 4,4444\ldots \\ 10x - x &= (4,4444\ldots) - (0,4444\ldots) \\ 9x &= 4 \\ \therefore x &= \frac{4}{9} \end{aligned}$$

d) $5,\overline{31}$

Solution:

$$\begin{aligned} x &= 5,313131\ldots \text{ and} \\ 100x &= 531,313131\ldots \\ 100x - x &= (531,313131\ldots) - (5,313131\ldots) \\ 99x &= 526 \\ \therefore x &= \frac{526}{99} \end{aligned}$$

e) $4,\overline{93}$

Solution:

$$\begin{aligned} x &= 4,939393\ldots \text{ and} \\ 100x &= 493,939393\ldots \\ 100x - x &= (493,939393\ldots) - (4,939393\ldots) \\ 99x &= 489 \\ \therefore x &= \frac{163}{33} \end{aligned}$$

f) $3,\overline{93}$

Solution:

$$\begin{aligned} x &= 3,939393\ldots \text{ and} \\ 100x &= 393,939393\ldots \\ 100x - x &= (393,939393\ldots) - (3,939393\ldots) \\ 99x &= 390 \\ \therefore x &= \frac{130}{33} \end{aligned}$$

For more exercises, visit www.everythingmaths.co.za and click on 'Practise Maths'.

1. 2DBM	2. 2DBN	3a. 2DBP	3b. 2DBQ	3c. 2DBR	3d. 2DBS	3e. 2DBT	3f. 2DBV
4a. 2DBX	4b. 2DBY	4c. 2DC2	4d. 2DC3	4e. 2DC4	4f. 2DC5	4g. 2DC6	4h. 2DBZ
4i. 2DBW	5. 2DC7	6. 2DC8	7. 2DC9	8a. 2DCB	8b. 2DCC	8c. 2DCD	8d. 2DCF
8e. 2DCG	9a. 2DCH	9b. 2DCJ	9c. 2DCK	9d. 2DCM	10a. 2DCN	10b. 2DCP	10c. 2DCQ
10d. 2DCR	11a. 2DCS	11b. 2DCT	11c. 2DCV	11d. 2DCW	11e. 2DCX	11f. 2DCY	11g. 2DCZ
12a. 2DD2	12b. 2DD3	12c. 2DD4	12d. 2DD5	12e. 2DD6	12f. 2DD7		

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Exercise 1 – 2:

1. Round off the following to 3 decimal places:
   a) 12,56637061...
   Solution:
   Mark off the required number of decimal places: 12,566|37061 . . . . The next digit is a 3 and so we round down: 12,566.
   b) 3,31662479...
   Solution:
   Mark off the required number of decimal places: 3,316|62479 . . . . The next digit is a 6 and so we round up: 3,317.
   c) 0,2666666...
   Solution:
   Mark off the required number of decimal places: 0,266|6666 . . . . The next digit is a 6 and so we round up: 0,267.
   d) 1,912931183...
   Solution:
   Mark off the required number of decimal places: 1,912|931183 . . . . The next digit is a 9 and so we round up: 1,913.
   e) 6,32455532...
   Solution:
   Mark off the required number of decimal places: 6,324|55532 . . . . The next digit is a 5 and so we round up: 6,325.
   f) 0,05555555...
   Solution:
   Mark off the required number of decimal places: 0,055|55555 . . . . The next digit is a 5 and so we round up: 0,056.
2. Round off each of the following to the indicated number of decimal places:
   a) 345,04399906 to 4 decimal places.
   Solution:

$$345{,}04399906 \approx 345{,}0440$$

   b) 1361,72980445 to 2 decimal places.
   Solution:

$$1361{,}72980445 \approx 1361{,}73$$

   c) 728,00905239 to 6 decimal places.
   Solution:

$$728{,}00905239 \approx 728{,}009052$$

   d) $\frac{1}{27}$ to 4 decimal places.
   Solution:
   We first write the fraction as a decimal and then we can round off.

$$\begin{aligned}\frac{1}{27} &= 0{,}037037... \\ &\approx 0{,}0370\end{aligned}$$

   e) $\frac{45}{99}$ to 5 decimal places.
   Solution:

We first write the fraction as a decimal and then we can round off.

$$\begin{aligned}\frac{45}{99} &= 0{,}45454545... \\ &\approx 0{,}45455\end{aligned}$$

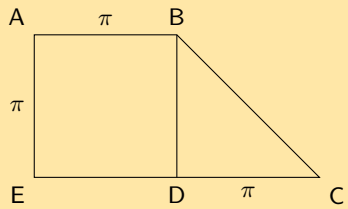
f) $\frac{1}{12}$ to 2 decimal places.

Solution:

We first write the fraction as a decimal and then we can round off.

$$\begin{aligned}\frac{1}{12} &= 0{,}08333... \\ &\approx 0{,}08\end{aligned}$$

3. Study the diagram below



a) Calculate the area of $ABDE$ to 2 decimal places.

Solution:

$ABDE$ is a square and so the area is just the length squared.

$$\begin{aligned}A &= l^2 \\ &= \pi^2 \\ &= 9{,}86904... \\ &\approx 9{,}87\end{aligned}$$

b) Calculate the area of BCD to 2 decimal places.

Solution:

BCD is a right-angled triangle and so we have the perpendicular height. The area is:

$$\begin{aligned}A &= \frac{1}{2}bh \\ &= \frac{1}{2}\pi^2 \\ &= 4{,}934802... \\ &\approx 4{,}93\end{aligned}$$

c) Using you answers in (a) and (b) calculate the area of $ABCDE$.

Solution:

The area of $ABCDE$ is the sum of the areas of $ABDE$ and BCD.

$$\begin{aligned}A &= 9{,}87 + 4{,}93 \\ &\approx 14{,}80\end{aligned}$$

d) Without rounding off, what is the area of $ABCDE$?

Solution:

$$\begin{aligned} A_{ABCDE} &= A_{ABDE} + A_{BCD} \\ &= l^2 + \frac{1}{2}bh \\ &= \pi^2 + \frac{1}{2}\pi^2 \\ &= 14{,}8044... \end{aligned}$$

4. Given $i = \frac{r}{600}$; $r = 7{,}4$; $n = 96$; $P = 200\ 000$.

   a) Calculate i correct to 2 decimal places.
   Solution:

$$\begin{aligned} i &= \frac{r}{600} \\ &= \frac{7{,}4}{600} \\ &= 0{,}01233 \\ &\approx 0{,}01 \end{aligned}$$

   b) Using you answer from (a), calculate A in $A = P(1+i)^n$.
   Solution:

$$\begin{aligned} A &= P(1+i)^n \\ &= 200\ 000\,(1+0{,}01)^{96} \\ &= 519\ 854{,}59 \end{aligned}$$

   c) Calculate A without rounding off your answer in (a), compare this answer with your answer in (b).
   Solution:

$$\begin{aligned} A &= P(1+i)^n \\ A &= 200\ 000\left(1+\frac{7{,}4}{600}\right)^{96} \\ &= 648\ 768{,}22 \end{aligned}$$

   There is a 128 913,63 difference between the answer in (b) and the one calculated without rounding until the final step.

5. If it takes 1 person to carry 3 boxes, how many people are needed to carry 31 boxes?
   Solution:
   Each person can carry 3 boxes. So we need to divide 31 by 3 to find out how many people are needed to carry 31 boxes.

$$\frac{31}{3} = 10{,}3333...$$

   Therefore 11 people are needed to carry 31 boxes. We cannot have 0,333 of a person so we round up to the nearest whole number.

6. If 7 tickets cost R 35,20, how much does one ticket cost?
   Solution:
   Since 7 tickets cost R 35,20, 1 ticket must cost R 35,20 divided by 7.

$$\frac{35{,}20}{7} = 5{,}028571429$$

   Therefore one ticket costs R 5,03. Money should be rounded off to 2 decimal places.

For more exercises, visit www.everythingmaths.co.za and click on 'Practise Maths'.

1a. 2DD9 1b. 2DDB 1c. 2DDC 1d. 2DDD 1e. 2DDF 1f. 2DDG 2a. 2DDH 2b. 2DDJ
2c. 2DDK 2d. 2DDM 2e. 2DDN 2f. 2DDP 3. 2DDQ 4. 2DDR 5. 2DDS 6. 2DDT

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Exercise 1 – 3:

1. Determine between which two consecutive integers the following numbers lie, without using a calculator:

   a) $\sqrt{18}$
   Solution:
   4 and 5 ($4^2 = 16$ and $5^2 = 25$)

   b) $\sqrt{29}$
   Solution:
   5 and 6 ($5^2 = 25$ and $6^2 = 36$)

   c) $\sqrt[3]{5}$
   Solution:
   1 and 2 ($1^3 = 1$ and $2^3 = 8$)

   d) $\sqrt[3]{79}$
   Solution:
   4 and 5 ($4^3 = 64$ and $5^3 = 125$)

   e) $\sqrt{155}$
   Solution:
   12 and 13 ($12^2 = 144$ and $13^2 = 169$)

   f) $\sqrt{57}$
   Solution:
   7 and 8 ($7^2 = 49$ and $8^2 = 64$)

   g) $\sqrt{71}$
   Solution:
   8 and 9 ($8^2 = 64$ and $9^2 = 81$)

   h) $\sqrt[3]{123}$
   Solution:
   4 and 5 ($4^3 = 64$ and $5^3 = 125$)

   i) $\sqrt[3]{90}$
   Solution:
   4 and 5 ($4^3 = 64$ and $5^3 = 125$)

   j) $\sqrt[3]{81}$
   Solution:
   4 and 5 ($4^3 = 64$ and $5^3 = 125$)

2. Estimate the following surds to the nearest 1 decimal place, without using a calculator.

   a) $\sqrt{10}$
   Solution:
   Since $3^2 = 9$ and $4^2 = 16$, $\sqrt{10}$ must lie between 3 and 4. But we note that 10 is closer to 9 than to 16 and so $\sqrt{10}$ will be closer to 3 than to 4.
   3,1 or 3,2 are suitable estimates.

   b) $\sqrt{82}$
   Solution:
   Since $9^2 = 81$ and $10^2 = 100$, $\sqrt{82}$ must lie between 9 and 10. But we note that 82 is closer to 81 than to 100 and so $\sqrt{82}$ will be closer to 9 than to 10.
   9,1 is a suitable estimate.

   c) $\sqrt{15}$
   Solution:
   Since $3^2 = 9$ and $4^2 = 16$, $\sqrt{15}$ must lie between 3 and 4. But we note that 15 is closer to 16 than to 9 and so $\sqrt{15}$ will be closer to 4 than to 3.
   3,9 is a suitable estimate.

   d) $\sqrt{90}$

Solution:
Since $9^2 = 81$ and $10^2 = 100$, $\sqrt{90}$ must lie between 9 and 10. But we note that 90 is about halfway between 81 and 100, so $\sqrt{90}$ will be halfway between 3 and 4.
3,5 is a suitable estimate.

3. Consider the following list of numbers:

   $\frac{27}{7}$; $\sqrt{19}$; 2π ; 0,45 ; $0,\overline{45}$; $-\sqrt{\frac{9}{4}}$; 6 ; $-\sqrt{8}$; $\sqrt{51}$

   Without using a calculator, rank all the numbers in ascending order.
   Solution:
   Remember that negative numbers are smaller than positive numbers. It may also be helpful to write the fractions as decimals to help you estimate the number. For the surds you can estimate between which two numbers the surd lies and use that to help you rank these numbers.

   - $\frac{27}{7} \approx 3{,}857$
   - $\sqrt{19}$ lies between 4 and 5
   - $2\pi \approx 6{,}28$
   - $-\sqrt{\frac{9}{4}} = -\frac{3}{2} = -1{,}5$
   - $-\sqrt{8}$ lies between -2 and -3
   - $\sqrt{51}$ lies between 7 and 8

   Also note that $0{,}45 < 0{,}\overline{45}$.

   Therefore we get the following order: $-\sqrt{8}$; $-\sqrt{\frac{9}{4}}$; 0,45 ; $0,\overline{45}$; $\frac{27}{7}$; $\sqrt{19}$; 6 ; 2π ; $\sqrt{51}$

For more exercises, visit www.everythingmaths.co.za and click on 'Practise Maths'.

1a. 2DDW 1b. 2DDX 1c. 2DDY 1d. 2DDZ 1e. 2DF2 1f. 2DF3
1g. 2DF4 1h. 2DF5 1i. 2DF6 1j. 2DF7 2a. 2DF8 2b. 2DF9
2c. 2DFB 2d. 2DFC 3. 2DFD

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1.6 Products

Multiplying a monomial and a binomial

Multiplying two binomials

Multiplying a binomial and a trinomial

Exercise 1 – 4:

1. Expand the following products:

   a) $2y(y + 4)$
   Solution:

   $$2y(y + 4) = 2y^2 + 8y$$

   b) $(y + 5)(y + 2)$
   Solution:

$$(y+5)(y+2) = y^2 + 2y + 5y + 10$$
$$= y^2 + 7y + 10$$

c) $(2-t)(1-2t)$
Solution:

$$(2-t)(1-2t) = 2 - 4t - t + 2t^2$$
$$= 2t^2 - 5t + 2$$

d) $(x-4)(x+4)$
Solution:

$$(x-4)(x+4) = x^2 + 4x - 4x - 16$$
$$= x^2 - 16$$

e) $-(4-x)(x+4)$
Solution:

$$-(4-x)(x+4) = -(4x + 16 - x^2 - 4x)$$
$$= -(16 - x^2)$$
$$= -16 + x^2$$
$$= x^2 - 16$$

f) $-(a+b)(b-a)$
Solution:

$$-(a+b)(b-a) = (a+b)(a-b)$$
$$= a^2 + ba - ba - 16$$
$$= a^2 - b^2$$

g) $(2p+9)(3p+1)$
Solution:

$$(2p+9)(3p+1) = 6p^2 + 2p + 27p + 9$$
$$= 6p^2 + 29p + 9$$

h) $(3k-2)(k+6)$
Solution:

$$(3k-2)(k+6) = 3k^2 + 18k - 2k - 12$$
$$= 3k^2 + 16k - 12$$

i) $(s+6)^2$
Solution:

$$(s+6)^2 = (s+6)(s+6)$$
$$= s^2 + 6s + 6s + 36$$
$$= s^2 + 12s + 36$$

j) $-(7-x)(7+x)$
Solution:

$$\begin{aligned}-(7-x)(7+x) &= -(49+7x-7x-x^2)\\ &= -(49-x^2)\\ &= x^2-49\end{aligned}$$

k) $(3x-1)(3x+1)$
Solution:

$$\begin{aligned}(3x-1)(3x+1) &= 9x^2+3x-3x-1\\ &= 9x^2-1\end{aligned}$$

l) $(7k+2)(3-2k)$
Solution:

$$\begin{aligned}(7k+2)(3-2k) &= 21k-14k^2+6-4k\\ &= -14k^2+17k+6\end{aligned}$$

m) $(1-4x)^2$
Solution:

$$\begin{aligned}(1-4x)^2 &= (1-4x)(1-4x)\\ &= 1-4x-4x+16x^2\\ &= 16x^2-8x+1\end{aligned}$$

n) $(-3-y)(5-y)$
Solution:

$$\begin{aligned}(-3-y)(5-y) &= -15+3y-5y+y^2\\ &= y^2-2y-15\end{aligned}$$

o) $(8-x)(8+x)$
Solution:

$$\begin{aligned}(8-x)(8+x) &= 64+8x-8x-x^2\\ &= -x^2+64\end{aligned}$$

p) $(9+x)^2$
Solution:

$$\begin{aligned}(9+x)^2 &= (9+x)(9+x)\\ &81+9x+9x+x^2\\ &= x^2+18x+81\end{aligned}$$

q) $(-7y+11)(-12y+3)$
Solution:

$$\begin{aligned}(-7y+11)(-12y+3) &= 84y^7-21y-132y+33\\ &= 84y^2-153y+33\end{aligned}$$

r) $(g-5)^2$
Solution:

$$\begin{aligned}(g-5)^2 &= (g-5)(g-5)\\ &= g^2-5g-5g+25\\ &= g^2-10g+25\end{aligned}$$

s) $(d+9)^2$
Solution:

$$\begin{aligned}(d+9)^2 &= (d+9)(d+9)\\ &= d^2+9d+9d+81\\ &= d^2+18d+81\end{aligned}$$

t) $(6d+7)(6d-7)$
Solution:

$$\begin{aligned}(6d+7)(6d-7) &= 36d^2-42d+42d-49\\ &= 36d^2-49\end{aligned}$$

u) $(5z+1)(5z-1)$
Solution:

$$\begin{aligned}(5z+1)(5z-1) &= 25z^2-5z+5z-1\\ &= 25z^2-1\end{aligned}$$

v) $(1-3h)(1+3h)$
Solution:

$$\begin{aligned}(1-3h)(1+3h) &= 1+3h-3h-9h^2\\ &= 1-9h^2\end{aligned}$$

w) $(2p+3)(2p+2)$
Solution:

$$\begin{aligned}(2p+3)(2p+2) &= 4p^2+4p+6p+6\\ &= 4p^2+10p+6\end{aligned}$$

x) $(8a+4)(a+7)$
Solution:

$$\begin{aligned}(8a+4)(a+7) &= 8a^2+56a+4a+28\\ &= 8a^2+60a+28\end{aligned}$$

y) $(5r+4)(2r+4)$
Solution:

$$\begin{aligned}(5r+4)(2r+4) &= 10r^2+20r+8r+16\\ &= 10r^2+28r+16\end{aligned}$$

z) $(w+1)(w-1)$
Solution:

$$\begin{aligned}(w+1)(w-1) &= w^2+w-w-1\\ &= w^2-1\end{aligned}$$

2. Expand the following products:

a) $(g+11)(g-11)$
Solution:

$$\begin{aligned}(g+11)(g-11) &= g^2+11g-11g-121\\ &= g^2-121\end{aligned}$$

b) $(4b-2)(2b-4)$
Solution:

$$\begin{aligned}(4b-2)(2b-4) &= 8b^2-16b-4b+8\\ &= 8b^2-20b+8\end{aligned}$$

c) $(4b-3)(2b-1)$
Solution:

$$\begin{aligned}(4b-3)(2b-1) &= 8b^2-4b-6b+3\\ &= 8b^2-10b+3\end{aligned}$$

d) $(6x-4)(3x+6)$
Solution:

$$\begin{aligned}(6x-4)(3x+6) &= 18x^2+36x-12x-24\\ &= 18x^2+24x-24\end{aligned}$$

e) $(3w-2)(2w+7)$
Solution:

$$\begin{aligned}(3w-2)(2w+7) &= 6w^2+21w-4w-14\\ &= 6w^2+17w-14\end{aligned}$$

f) $(2t-3)^2$
Solution:

$$\begin{aligned}(2t-3)^2 &= (2t-3)(2t-3)\\ &= 4t^2-6t-6t+9\\ &= 4t^2-12t+9\end{aligned}$$

g) $(5p-8)^2$
Solution:

$$\begin{aligned}(5p-8)^2 &= (5p-8)(5p-8)\\ &= 25p^2-40p-40p+64\\ &= 25p^2-80p+64\end{aligned}$$

h) $(4y+5)^2$
Solution:

$$\begin{aligned}(4y+5)^2 &= (4y+5)(4y+5)\\ &= 16y^2+20y+20y+25\\ &= 16y^2+40y+25\end{aligned}$$

i) $(2y^6 + 3y^5)(-5y - 12)$
Solution:

$$\begin{aligned}(2y^6 + 3y^5)(-5y - 12) &= -10y^7 - 24y^6 - 15y^6 - 36y^5 \\ &= -10y^7 - 39y^6 - 36y^5\end{aligned}$$

j) $9(8y^2 - 2y + 3)$
Solution:

$$9(8y^2 - 2y + 3) = 72y^2 - 18y + 27$$

k) $(-2y^2 - 4y + 11)(5y - 12)$
Solution:

$$\begin{aligned}(-2y^2 - 4y + 11)(5y - 12) &= -10y^3 - 20y^2 + 55y + 24y^2 + 48y - 132 \\ &= -10y^3 + 4y^2 + 103y - 132\end{aligned}$$

l) $(7y^2 - 6y - 8)(-2y + 2)$
Solution:

$$\begin{aligned}(7y^2 - 6y - 8)(-2y + 2) &= -14y^3 + 12y^2 + 16y + 14y^2 - 12y - 16 \\ &= -14y^3 + 26y^2 + 4y - 16\end{aligned}$$

m) $(10y + 3)(-2y^2 - 11y + 2)$
Solution:

$$\begin{aligned}(10y + 3)(-2y^2 - 11y + 2) &= -20y^3 - 110y^2 + 20y - 6y^2 - 33y + 6 \\ &= -20y^3 - 116y^2 - 13y + 6\end{aligned}$$

n) $(-12y - 3)(2y^2 - 11y + 3)$
Solution:

$$\begin{aligned}(-12y - 3)(2y^2 - 11y + 3) &= -24y^3 + 132y^2 - 36y - 6y^2 + 33y - 9 \\ &= -24y^3 + 126y^2 - 3y - 9\end{aligned}$$

o) $(-10)(2y^2 + 8y + 3)$
Solution:

$$(-10)(2y^2 + 8y + 3) = -20y^2 - 80y - 30$$

p) $(7y + 3)(7y^2 + 3y + 10)$
Solution:

$$\begin{aligned}(7y + 3)(7y^2 + 3y + 10) &= 49y^3 + 21y^2 + 70y + 21y^2 + 9y + 30 \\ &= 49y^3 + 42y^2 + 79y + 30\end{aligned}$$

q) $(a + 2b)(a^2 + b^2 + 2ab)$
Solution:

$$\begin{aligned}(a + 2b)(a^2 + b^2 + 2ab) &= a^3 + ab^2 + 2a^2b + 2a^2b + 2b^3 + 4ab^2 \\ &= a^3 + 4a^2b + 5ab^2 + 2b^3\end{aligned}$$

r) $(x+y)(x^2-xy+y^2)$
Solution:

$$\begin{aligned}(x+y)(x^2-xy+y^2) &= x^3-x^2y+xy^2+x^2y-xy^2+y^3\\ &= x^3+y^3\end{aligned}$$

s) $3m(9m^2+2)+5m^2(5m+6)$
Solution:

$$\begin{aligned}3m(9m^2+2)+5m^2(5m+6) &= 27m^3+6m+25m^3+30m^2\\ &= 52m^3+6m+30m^2\end{aligned}$$

t) $4x^2(10x^3+4)+4x^3(2x^2+6)$
Solution:

$$\begin{aligned}4x^2(10x^3+4)+4x^3(2x^2+6) &= 40x^5+16x^2+8x^5+24x^3\\ &= 48x^5+16x^2+24x^3\end{aligned}$$

u) $3k^3(k^2+3)+2k^2(6k^3+7)$
Solution:

$$\begin{aligned}3k^3(k^2+3)+2k^2(6k^3+7) &= 3k^5+9k^3+12k^5+14k^2\\ &= 15k^5+9k^3+14k^2\end{aligned}$$

v) $(3x+2)(3x-2)(9x^2-4)$
Solution:

$$\begin{aligned}(3x+2)(3x-2)(9x^2-4) &= (9x^2-4)(9x^2-4)\\ &= 81x^4-36x-36x+16\\ &= 81x^4-72x+16\end{aligned}$$

w) $(-6y^4+11y^2+3y)(y+4)(y-4)$
Solution:

$$\begin{aligned}(-6y^4+11y^2+3y)(y+4)(y-4) &= (-6y^4+11y^2+3y)(y^2-16)\\ &= -6y^6+96y^4+11y^4-176y^2+3y^3-48y\\ &= -6y^6+107y^4+3y^3-176y^2-48y\end{aligned}$$

x) $(x+2)(x-3)(x^2+2x-3)$
Solution:

$$\begin{aligned}(x+2)(x-3)(x^2+2x-3) &= (x^2-x-6)(x^2+2x-3)\\ &= x^4+2x^3-3x^2-x^3-2x^2+3x-6x^2-12x+18\\ &= x^4+x^3-11x^2-9x+18\end{aligned}$$

y) $(a+2)^2-(2a-4)^2$
Solution:

$$\begin{aligned}(a+2)^2-(2a-4)^2 &= a^2+4a+4-(4a^2-16a+16)\\ &= a^2+4a+4-4a^2+16a-16\\ &= -3a^2+20a-12\end{aligned}$$

3. Expand the following products:

a) $(2x+3)^2-(x-2)^2$

Solution:

$$\begin{aligned}(2x+3)^2-(x-2)^2 &= 4x^2+12x+9-(x^2-4x+4)\\ &= 4x^2+12x+9-x^2+4x-4\\ &= 3x^2+16x+5\end{aligned}$$

b) $(2a^2-a-1)(a^2+3a+2)$

Solution:

$$\begin{aligned}(2a^2-a-1)(a^2+3a+2) &= 2a^4+6a^3+4a^2-a^3-3a^2-2a-a^2-3a-2\\ &= 2a^4+5a^3-5a-2\end{aligned}$$

c) $(y^2+4y-1)(1-4y-y^2)$

Solution:

$$\begin{aligned}(y^2+4y-1)(1-4y-y^2) &= y^2-4y^3-y^4+4y-16y^2-4y^3-1+4y+y^2\\ &= -y^4-8y^3-14y^2+8y-1\end{aligned}$$

d) $2(x-2y)(x^2+xy+y^2)$

Solution:

$$\begin{aligned}2(x-2y)(x^2+xy+y^2) &= 2(x^3+x^2y+xy^2-2x^2y-2xy^2-y^3)\\ &= 2(x^3-x^2y-xy^2-y^3)\\ &= 2x^3-2x^2y-2xy^2-2y^3\end{aligned}$$

e) $3(a-3b)(a^2+3ab-b^2)$

Solution:

$$\begin{aligned}3(a-3b)(a^2+3ab-b^2) &= 3(a^3+3a^2b-ab^2-3a^2b-9ab^2+3b^3)\\ &= 3(a^3-10ab^2+3b^3)\\ &= 3a^3-30ab^2+9b^3\end{aligned}$$

f) $(2a-b)(2a+b)(2a^2-3ab+b^2)$

Solution:

$$\begin{aligned}(2a-b)(2a+b)(2a^2-3ab+b^2) &= (4a^2-b^2)(2a^2-3ab+b^2)\\ &= 8a^4-12a^3b+4a^2b^2-2a^2b^2+3ab^3-b^4\\ &= 8a^4-12a^3b+2a^2b^2+3ab^3-b^4\end{aligned}$$

g) $2(3x+y)(3x-y)-(3x-y)^2$

Solution:

$$\begin{aligned}2(3x+y)(3x-y)-(3x-y)^2 &= 2(9x^2-y^2)-9x^2+6xy-y^2\\ &= 18x^2-2y^2-9x^2+6xy-y^2\\ &= 9x^2+6xy-3y^2\end{aligned}$$

h) $(x+y)(x-3y)+(2x-y)^2$

Solution:

$$\begin{aligned}(x+y)(x-3y)+(2x-y)^2 &= x^2-3xy+xy-3y^2+4x^2-4xy+y^2\\ &= 5x^2-6xy-2y^2\end{aligned}$$

i) $\left(\frac{x}{3}-\frac{3}{x}\right)\left(\frac{x}{4}+\frac{4}{x}\right)$

Solution:

$$\begin{aligned}\left(\frac{x}{3}-\frac{3}{x}\right)\left(\frac{x}{4}+\frac{4}{x}\right) &= \frac{x^2}{12}+\frac{4}{3}-\frac{3}{4}+\frac{12}{x^2}\\ &= \frac{x^2}{12}+\frac{16}{12}-\frac{9}{12}+\frac{12}{x^2}\\ &= \frac{x^2}{12}+\frac{7}{12}+\frac{3}{x^2}\end{aligned}$$

j) $\left(x-\frac{2}{x}\right)\left(\frac{x}{3}+\frac{4}{x}\right)$

Solution:

$$\begin{aligned}\left(x-\frac{2}{x}\right)\left(\frac{x}{3}+\frac{4}{x}\right) &= \frac{x^2}{3}+4-\frac{2}{3}-\frac{8}{x^2}\\ &= \frac{x^2}{3}+\frac{12}{3}-\frac{2}{3}-\frac{8}{x^2}\\ &= \frac{x^2}{3}+\frac{10}{3}-\frac{8}{x^2}\end{aligned}$$

k) $\frac{1}{2}(10x-12y)+\frac{1}{3}(15x-18y)$

Solution:

$$\begin{aligned}\frac{1}{2}(10x-12y)+\frac{1}{3}(15x-18y) &= 5x-6y+5x-6y\\ &= 10x-12y\end{aligned}$$

l) $\frac{1}{2}a(4a+6b)+\frac{1}{4}(8a+12b)$

Solution:

$$\frac{1}{2}a(4a+6b)+\frac{1}{4}(8a+12b) = 2a^2+3ab+2a+3b$$

4. What is the value of b, in $(x+b)(x-1)=x^2+3x-4$

Solution:

$$(x+b)(x-1) = x^2-x+bx-b$$

From the constant term we see that $b=4$. We can check the x term: $-x+4x=3x$.

5. What is the value of g, in $(x-2)(x+g)=x^2-6x+8$

Solution:

$$(x-2)(x+g) = x^2+gx-2x-2g$$

From the constant term we see that $-2g=8$, therefore $g=-4$. We can check the x term: $-4x-2x=-6x$.

6. In $(x-4)(x+k)=x^2+bx+c$:

a) For which of these values of k will b be positive?
$-3;-1;0;3;5$

Solution:

$$(x-4)(x+k) = x^2+kx-4x-4k$$

The x term is $kx-4x$ so for b to be positive $k>4$. Therefore $k=5$.

b) For which of these values of k will c be positive?
$-3;-1;0;3;5$

Solution:

$$(x-4)(x+k) = x^2 + kx - 4x - 4k$$

The constant term is $-4k$ so for c to be positive $k < 0$. Therefore $k = -3$ or $k = -1$.

c) For what real values of k will c be positive?

Solution:

From the previous question we see that $k < 0$ will make c positive.

d) For what real values of k will b be positive?

Solution:

From earlier we see that $k > 4$ will make b positive.

7. Answer the following:

a) Expand $\left(x + \frac{4}{x}\right)^2$.

Solution:

$$\begin{aligned}\left(x + \frac{4}{x}\right)^2 &= \left(x + \frac{4}{x}\right)\left(x + \frac{4}{x}\right) \\ &= x^2 + 8 + \frac{16}{x^2}\end{aligned}$$

b) Given that $\left(x + \frac{4}{x}\right)^2 = 14$, determine the value of $x^2 + \frac{16}{x^2}$ without solving for x.

Solution:

$$\left(x + \frac{4}{x}\right)^2 = x^2 + 8 + \frac{16}{x^2}$$

Now we note that the above expression can also be written as $x^2 + \frac{16}{x^2} + 8$. Since $\left(x + \frac{4}{x}\right)^2 = 14$ we get:

$$\begin{aligned}14 &= x^2 + 8 + \frac{16}{x^2} \\ 14 - 8 &= x^2 + \frac{16}{x^2} \\ 6 &= x^2 + \frac{16}{x^2}\end{aligned}$$

8. Answer the following:

a) Expand: $\left(a + \frac{1}{a}\right)^2$

Solution:

$$\left(a + \frac{1}{a}\right)^2 = a^2 + 2 + \frac{1}{a^2}$$

b) Given that $\left(a + \frac{1}{a}\right) = 3$, determine the value of $\left(a + \frac{1}{a}\right)^2$ without solving for a.

Solution:

$$\begin{aligned}\left(a + \frac{1}{a}\right)^2 &= 3^2 \\ &= 9\end{aligned}$$

c) Given that $\left(a - \frac{1}{a}\right) = 3$, determine the value of $\left(a + \frac{1}{a}\right)^2$ without solving for a.

Solution:

We note that:

$$\left(a+\frac{1}{a}\right)^2 = a^2+2+\frac{1}{a^2}$$

$$\left(a-\frac{1}{a}\right)^2 = a^2-2+\frac{1}{a^2}$$

Next we note that if we add 4 to $\left(a-\frac{1}{a}\right)^2$ we get $\left(a+\frac{1}{a}\right)^2$. Therefore:

$$\begin{aligned}\left(a+\frac{1}{a}\right)^2 &= a^2-2+\frac{1}{a^2}+4\\ &= 3^2+4\\ &= 9+4\\ &= 13\end{aligned}$$

9. Answer the following:

   a) Expand: $\left(3y+\frac{1}{2y}\right)^2$

   Solution:

$$\left(3y+\frac{1}{2y}\right)^2 = 9y^2+3+\frac{1}{4y^2}$$

   b) Given that $3y+\frac{1}{2y}=4$, determine the value of $\left(3y+\frac{1}{2y}\right)^2$ without solving for y.

   Solution:

$$\begin{aligned}\left(3y+\frac{1}{2y}\right)^2 &= 4^2\\ &= 16\end{aligned}$$

10. Answer the following:

   a) Expand: $\left(a+\frac{1}{3a}\right)^2$

   Solution:

$$\left(a+\frac{1}{3a}\right)^2 = a^2+\frac{2}{3}+\frac{1}{9a^2}$$

   b) Expand: $\left(a+\frac{1}{3a}\right)\left(a^2-\frac{1}{3}+\frac{1}{9a^2}\right)$

   Solution:

$$\begin{aligned}\left(a+\frac{1}{3a}\right)\left(a^2-\frac{1}{3}+\frac{1}{9a^2}\right) &= a^3-\frac{1}{3}a+\frac{1}{9a}+\frac{1}{3}a-\frac{1}{9a}+\frac{1}{27a^3}\\ &= a^3+\frac{1}{27a^3}\end{aligned}$$

   c) Given that $a+\frac{1}{3a}=2$, determine the value of $a^3+\frac{1}{27a^3}$ without solving for a.

   Solution:

$$a^3 + \frac{1}{27a^3} = \left(a + \frac{1}{3a}\right)\left(a^2 - \frac{1}{3} + \frac{1}{9a^2}\right)$$
$$= 2\left(a^2 - \frac{1}{3} + \frac{1}{9a^2}\right)$$

$$a^2 - \frac{1}{3} + \frac{1}{9a^2} = \left(a + \frac{1}{3a}\right)^2 - 1$$
$$= 4 - 1$$
$$= 3$$

$$a^3 + \frac{1}{27a^3} = 2(3)$$
$$= 6$$

For more exercises, visit www.everythingmaths.co.za and click on 'Practise Maths'.

1a. 2DFG	1b. 2DFH	1c. 2DFJ	1d. 2DFK	1e. 2DFM	1f. 2DFN	1g. 2DFP	1h. 2DFQ
1i. 2DFR	1j. 2DFS	1k. 2DFT	1l. 2DFV	1m. 2DFW	1n. 2DFX	1o. 2DFY	1p. 2DFZ
1q. 2DG3	1r. 2DG4	1s. 2DG5	1t. 2DG6	1u. 2DG7	1v. 2DG8	1w. 2DG9	1x. 2DGB
1y. 2DGC	1z. 2DGD	2a. 2DGF	2b. 2DGH	2c. 2DGJ	2d. 2DGK	2e. 2DGM	2f. 2DGN
2g. 2DGP	2h. 2DGQ	2i. 2DG2	2j. 2DGR	2k. 2DGS	2l. 2DGT	2m. 2DGV	2n. 2DGW
2o. 2DGX	2p. 2DGY	2q. 2DGZ	2r. 2DH2	2s. 2DH3	2t. 2DH4	2u. 2DH5	2v. 2DH6
2w. 2DH7	2x. 2DH8	2y. 2DH9	3a. 2DHB	3b. 2DHC	3c. 2DHD	3d. 2DHF	3e. 2DHG
3f. 2DHH	3g. 2DHJ	3h. 2DHK	3i. 2DHM	3j. 2DHN	3k. 2DHP	3l. 2DHQ	4. 2DHR
5. 2DHS	6. 2DHT	7. 2DHV	8. 2DHW	9. 2DHX	10. 2DHY		

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1.7 Factorisation

Common factors

Exercise 1 – 5:

Factorise:

1. $12x + 32y$
   Solution:

$$12x + 32y = 4(3x + 8y)$$

2. $-2ab^2 - 4a^2b$
   Solution:

$$-2ab^2 - 4a^2b = -2ab(b + 2a)$$

3. $18ab - 3bc$
   Solution:

$$18ab - 3bc = 3b(6a - c)$$

4. $12kj + 18kq$
   Solution:

$$12kj + 18kq = 6k(2j + 3q)$$

5. $-12a + 24a^3$
   Solution:

$$-12a + 24a^3 = 12a(-1 + 2a^2)$$

6. $-2ab - 8a$
   Solution:

$$-2ab - 8a = -2a(b + 4)$$

7. $24kj - 16k^2j$
   Solution:

$$24kj - 16k^2j = 8kj(3 - 2k)$$

8. $-a^2b - b^2a$
   Solution:

$$-a^2b - b^2a = -ab(a + b)$$

9. $72b^2q - 18b^3q^2$
   Solution:

$$72b^2q - 18b^3q^2 = 18b^2q(4 - bq)$$

10. $125x^6 - 5y^2$
    Solution:

$$\begin{aligned} 125x^6 - 5y^2 &= 5(25x^6 - y^2) \\ &= 5(5x^3 - y)(5x^3 + y) \end{aligned}$$

11. $6x^2 + 2x + 10x^3$
    Solution:

$$6x^2 + 2x + 10x^3 = 2x(3x + 1 + 5x^2)$$

12. $2xy^2 + xy^2z + 3xy$
    Solution:

$$2xy^2 + xy^2z + 3xy = xy(2y + yz + 3)$$

13. $12k^2j + 24k^2j^2$
    Solution:

$$12k^2j + 24k^2j^2 = 12k^2j(1 + 2j)$$

14. $3a^2 + 6a - 18$
    Solution:

$$3a^2 + 6a - 18 = 3(a^2 + 2a - 6)$$

15. $7a + 4$
    Solution:

$$7a + 4$$

For more exercises, visit www.everythingmaths.co.za and click on 'Practise Maths'.

1. 2DJ2 2. 2DJ3 3. 2DJ4 4. 2DJ5 5. 2DJ6 6. 2DJ7 7. 2DJ8 8. 2DJ9
9. 2DJB 10. 2DJC 11. 2DJD 12. 2DJF 13. 2DJG 14. 2DJH 15. 2DJJ

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Exercise 1 – 6:

Factorise:

1. $4(y-3)+k(3-y)$
   Solution:

$$\begin{aligned} 4(y-3)+k(3-y) &= 4(y-3)-k(y-3) \\ &= (y-3)(4-k) \end{aligned}$$

2. $a^2(a-1)-25(a-1)$
   Solution:

$$\begin{aligned} a^2(a-1)-25(a-1) &= (a-1)(a^2-25) \\ &= (a-1)(a-5)(a+5) \end{aligned}$$

3. $bm(b+4)-6m(b+4)$
   Solution:

$$\begin{aligned} bm(b+4)-6m(b+4) &= (b+4)(bm-6m) \\ &= (b+4)(m)(b-6) \end{aligned}$$

4. $a^2(a+7)+9(a+7)$
   Solution:

$$a^2(a+7)+9(a+7) = (a+7)(a^2+9)$$

5. $3b(b-4)-7(4-b)$
   Solution:

$$\begin{aligned} 3b(b-4)-7(4-b) &= 3b(b-4)+7(b-4) \\ &= (b-4)(3b+7) \end{aligned}$$

6. $3g(z+6)+2(6+z)$
   Solution:

$$\begin{aligned} 3g(z+6)+2(6+z) &= 3g(z+6)+2(z+6) \\ &= (z+6)(3g+2) \end{aligned}$$

7. $4b(y+2)+5(2+y)$
   Solution:

$$\begin{aligned} 4b(y+2)+5(2+y) &= 4b(y+2)+5(y+2) \\ &= (y+2)(4b+5) \end{aligned}$$

8. $3d(r+5)+14(5+r)$
   Solution:

$$\begin{aligned} 3d(r+5)+14(5+r) &= 3d(r+5)+14(r+5) \\ &= (r+5)(3d+14) \end{aligned}$$

9. $(6x+y)^2-9$
Solution:

$$(6x+y)^2-9=(6x+y-3)(6x+y+3)$$

10. $4x^2-(4x-3y)^2$
Solution:

$$\begin{aligned}4x^2-(4x-3y)^2&=(2x+4x-3y)(2x-(4x-3y))\\&=(6x-3y)(3y-2x)\\&=3(2x-y)(3y-2x)\end{aligned}$$

11. $16a^2-(3b+4c)^2$
Solution:

$$\begin{aligned}16a^2-(3b+4c)^2&=(4a+3b+4c)(4a-(3b+4c))\\&=(4a+3b+4c)(4a-3b-4c)\end{aligned}$$

12. $(b-4)^2-9(b-5)^2$
Solution:

$$\begin{aligned}(b-4)^2-9(b-5)^2&=(b-4-3(b-5))(b-4+3(b-5))\\&=(-2b+11)(4b-19)\end{aligned}$$

13. $4(a-3)^2-49(4a-5)$
Solution:

$$\begin{aligned}4(a-3)^2-49(4a-5)^2&=(2(a-3)-7(4a-5))(2(a-3)+7(4a-5))\\&=(2a-6-28a+35)(2a-6+28a-35)\\&=(29-26a)(30a-41)\end{aligned}$$

14. $16k^2-4$
Solution:

$$16k^2-4=(4k-2)(4k+2)$$

15. $a^2b^2c^2-1$
Solution:

$$a^2b^2c^2-1=(abc-1)(abc+1)$$

16. $\frac{1}{9}a^2-4b^2$
Solution:

$$\frac{1}{9}a^2-4b^2=\left(\frac{1}{3}a-2b\right)\left(\frac{1}{3}a+2b\right)$$

17. $\frac{1}{2}x^2-2$
Solution:

$$\begin{aligned}\frac{1}{2}x^2-2&=2\left(\frac{1}{4}x^2-1\right)\\&=2\left(\frac{1}{2}x+1\right)\left(\frac{1}{2}x-1\right)\end{aligned}$$

18. $y^2 - 8$
Solution:
Note that $\left(\sqrt{8}\right)^2 = 8$

$$y^2 - 8 = (y - \sqrt{8})(y + \sqrt{8})$$

19. $y^2 - 13$
Solution:
Note that $\left(\sqrt{13}\right)^2 = 13$

$$y^2 - 13 = (y - \sqrt{13})(y + \sqrt{13})$$

20. $a^2(a - 2ab - 15b^2) - 9b^2(a^2 - 2ab - 15b^2)$
Solution:

$$\begin{aligned} a^2(a - 2ab - 15b^2) - 9b^2(a^2 - 2ab - 15b^2) &= (a^2 - 2ab - 15b^2)(a^2 - 9b^2) \\ &= (a - 5b)(a + 3b)(a - 3b)(a + 3b) \\ &= (a - 3b)(a - 5b)(a + 3b)^2 \end{aligned}$$

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1. 2DJM 2. 2DJN 3. 2DJP 4. 2DJQ 5. 2DJR 6. 2DJS 7. 2DJT 8. 2DJV
9. 2DJW 10. 2DJX 11. 2DJY 12. 2DJZ 13. 2DK2 14. 2DK3 15. 2DK4 16. 2DK5
17. 2DK6 18. 2DK7 19. 2DK8 20. 2DK9

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Factorising by grouping in pairs

Exercise 1 – 7:

Factorise the following:

1. $6d - 9r + 2t^5d - 3t^5r$
Solution:

$$\begin{aligned} 6d - 9r + 2t^5d - 3t^5r &= 3(2d - 3r) + t^5(2d - 3r) \\ &= (2d - 3r)(3 + t^5) \end{aligned}$$

2. $9z - 18m + b^3z - 2b^3m$
Solution:

$$\begin{aligned} 9z - 18m + b^3z - 2b^3m &= 9(z - 2m) + b^3(z - 2m) \\ &= (z - 2m)(9 + b^3) \end{aligned}$$

3. $35z - 10y + 7c^5z - 2c^5y$
Solution:

$$\begin{aligned} 35z - 10y + 7c^5z - 2c^5y &= 5(7z - 2y) + c^5(7z - 2y) \\ &= (7z - 2y)(5 + c^5) \end{aligned}$$

4. $6x + a + 2ax + 3$

Solution:

$$\begin{aligned} 6x + a + 2ax + 3 &= 6x + 3 + a + 2ax \\ &= 3(2x + 1) + a(2x + 1) \\ &= (3 + a)(2x + 1) \end{aligned}$$

5. $x^2 - 6x + 5x - 30$
Solution:

$$\begin{aligned} x^2 - 6x + 5x - 30 &= x(x - 6) + 5(x - 6) \\ &= (x + 5)(x - 6) \end{aligned}$$

6. $5x + 10y - ax - 2ay$
Solution:

$$\begin{aligned} 5x + 10y - ax - 2ay &= 5(x + 2y) - a(x + 2y) \\ &= (5 - a)(x + 2y) \end{aligned}$$

7. $a^2 - 2a - ax + 2x$
Solution:

$$\begin{aligned} a^2 - 2a - ax + 2x &= a(a - 2) - x(a - 2) \\ &= (a - x)(a - 2) \end{aligned}$$

8. $5xy - 3y + 10x - 6$
Solution:

$$\begin{aligned} 5xy - 3y + 10x - 6 &= y(5x - 3) + 2(5x - 3) \\ &= (y + 2)(5x - 3) \end{aligned}$$

9. $ab - a^2 - a + b$
Solution:

$$\begin{aligned} ab - a^2 - a + b &= -a^2 - a + ab + b \\ &= -a(a + 1) + b(a + 1) \\ &= (-a + b)(a + 1) \end{aligned}$$

10. $14m - 4n + 7jm - 2jn$
Solution:

$$\begin{aligned} 14m - 4n + 7jm - 2jn &= 2(7m - 2n) + j(7m - 2n) \\ &= (7m - 2n)(2 + j) \end{aligned}$$

11. $28r - 20x + 7gr - 5gx$
Solution:

$$\begin{aligned} 28r - 20x + 7gr - 5gx &= 4(7r - 5x) + g(7r - 5x) \\ &= (7r - 5x)(4 + g) \end{aligned}$$

12. $25d - 15m + 5yd - 3ym$
Solution:

$$\begin{aligned} 25d - 15m + 5yd - 3ym &= 5(5d - 3m) + y(5d - 3m) \\ &= (5d - 3m)(5 + y) \end{aligned}$$

13. $45q - 18z + 5cq - 2cz$
Solution:

$$\begin{aligned} 45q - 18z + 5cq - 2cz &= 9(5q - 2z) + c(5q - 2z) \\ &= (5q - 2z)(9 + c) \end{aligned}$$

14. $6j - 15v + 2yj - 5yv$
Solution:

$$\begin{aligned} 6j - 15v + 2yj - 5yv &= 3(2j - 5v) + y(2j - 5v) \\ &= (2j - 5v)(3 + y) \end{aligned}$$

15. $16a - 40k + 2za - 5zk$
Solution:

$$\begin{aligned} 16a - 40k + 2za - 5zk &= 8(2a - 5k) + z(2a - 5k) \\ &= (2a - 5k)(8 + z) \end{aligned}$$

16. $ax - bx + ay - by + 2a - 2b$
Solution:

$$\begin{aligned} ax - bx + ay - by + 2a - 2b &= x(a - b) + y(a - b) + 2(a - b) \\ &= (a - b)(x + y + 2) \end{aligned}$$

17. $3ax + bx - 3ay - by - 9a - 3b$
Solution:

$$\begin{aligned} 3ax + bx - 3ay - by - 9a - 3b &= x(3a + b) - y(3a + b) - 3(3a + b) \\ &= (3a + b)(x - y - 3) \end{aligned}$$

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1. 2DKB 2. 2DKC 3. 2DKD 4. 2DKF 5. 2DKG 6. 2DKH 7. 2DKJ
8. 2DKK 9. 2DKM 10. 2DKN 11. 2DKP 12. 2DKQ 13. 2DKR 14. 2DKS
15. 2DKT 16. 2DKV 17. 2DKW

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Factorising a quadratic trinomial

General procedure for factorising a trinomial

Exercise 1 – 8:

Factorise the following:

1. $x^2 + 8x + 15$
Solution:

$$x^2 + 8x + 15 = (x + 5)(x + 3)$$

2. $x^2 + 9x + 8$
Solution:

$$x^2 + 9x + 8 = (x + 8)(x + 1)$$

3. $x^2 + 12x + 36$
Solution:

$$\begin{aligned} x^2 + 12x + 36 &= (x + 6)(x + 6) \\ &= (x + 6)^2 \end{aligned}$$

4. $2h^2 + 5h - 3$
Solution:

$$2h^2 + 5h - 3 = (h + 3)(2h - 1)$$

5. $3x^2 + 4x + 1$
Solution:

$$3x^2 + 4x + 1 = (x + 1)(3x + 1)$$

6. $3s^2 + s - 10$
Solution:

$$3s^2 + s - 10 = (s + 2)(3s - 5)$$

7. $x^2 - 2x - 15$
Solution:

$$x^2 - 2x - 15 = (x + 3)(x - 5)$$

8. $x^2 + 2x - 3$
Solution:

$$x^2 + 2x - 3 = (x + 3)(x - 1)$$

9. $x^2 + x - 20$
Solution:

$$x^2 + x - 20 = (x + 5)(x - 4)$$

10. $x^2 - x - 20$
Solution:

$$x^2 - x - 20 = (x - 5)(x + 4)$$

11. $2x^2 - 22x + 20$
Solution:

$$\begin{aligned} 2x^2 + 22x + 20 &= 2(x^2 + 11x + 10) \\ &= 2(x + 1)(x + 10) \end{aligned}$$

12. $6a^2 + 14a + 8$
Solution:

$$\begin{aligned} 6a^2 + 14a + 8 &= 2(3a^2 + 7a + 4) \\ &= 2\,(a + 1)\,(3a + 4) \end{aligned}$$

13. $6v^2 - 27v + 27$
Solution:

$$\begin{aligned} 6v^2 - 27v + 27 &= 3(2v^2 - 9v + 9) \\ &= 3\,(2v - 3)\,(v - 3) \end{aligned}$$

14. $6g^2 - 15g - 9$
Solution:

$$\begin{aligned} 6g^2 - 15g - 9 &= 3(2g^2 - 5g - 3) \\ &= 3\,(g - 3)\,(2g + 1) \end{aligned}$$

15. $3x^2 + 19x + 6$
Solution:

$$3x^2 + 19x + 6 = (3x + 1)(x + 6)$$

16. $3x^2 + 17x - 6$
Solution:

$$3x^2 + 17x - 6 = (3x - 1)(x + 6)$$

17. $7x^2 - 6x - 1$
Solution:

$$7x^2 - 6x - 1 = (7x + 1)(x - 1)$$

18. $6x^2 - 15x - 9$
Solution:

$$\begin{aligned} 6x^2 - 15x - 9 &= 3(2x^2 - 5x - 3) \\ &= 3(2x + 1)(x - 3) \end{aligned}$$

19. $a^2 - 7ab + 12b$
Solution:

$$a^2 - 7ab + 12b^2 = (a - 4b)(a - 3b)$$

20. $3a^2 + 5ab - 12b^2$
Solution:

$$3a^2 + 5ab - 12b^2 = (3a - 4b)(a + 3b)$$

21. $98x^4 + 14x^2 - 4$
Solution:

$$\begin{aligned} 98x^4 + 14x^2 - 4 &= 2(49x^4 - 7x^2 - 2) \\ &= 2((7x + 2)(7x - 1)) \end{aligned}$$

22. $(x - 2)^2 - 7(x - 2) + 12$
Solution:

$$\begin{aligned} (x - 2)^2 - 7(x - 2) + 12 &= ((x - 2) - 4)((x - 2) - 3) \\ &= (x - 6)(x - 5) \end{aligned}$$

23. $(a - 2)^2 - 4(a - 2) - 5$
Solution:

$$\begin{aligned} (a - 2)^2 - 4(a - 2) - 5 &= ((a - 2) - 5)((a - 2) + 1) \\ &= (a - 7)(a - 1) \end{aligned}$$

24. $(y + 3)^2 - 3(y + 3) - 18$
Solution:

$$\begin{aligned} (y + 3)^2 - 3(y + 3) - 18 &= ((y + 3) - 6)((y + 3) + 3) \\ &= (y - 3)(y + 6) \end{aligned}$$

25. $3(b^2 + 5b) + 12$
Solution:

$$\begin{aligned} 3(b^2 + 5b) + 12 &= 3(b^2 + 5b) + 3(4) \\ &= 3(b^2 + 5b + 4) \\ &= 3(b + 4)(b + 1) \end{aligned}$$

26. $6(a^2 + 3a) - 168$
Solution:

$$\begin{aligned} 6(a^2 + 3a) - 168 &= 6(a^2 + 3a) - 6(28) \\ &= 6(a^2 + 3a - 28) \\ &= 6(a + 7)(a - 4) \end{aligned}$$

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1. 2DKY 2. 2DKZ 3. 2DM2 4. 2DM3 5. 2DM4 6. 2DM5 7. 2DM6 8. 2DM7
9. 2DM8 10. 2DM9 11. 2DMB 12. 2DMC 13. 2DMD 14. 2DMF 15. 2DMG 16. 2DMH
17. 2DMJ 18. 2DMK 19. 2DMM 20. 2DMN 21. 2DMP 22. 2DMQ 23. 2DMR 24. 2DMS
25. 2DMT 26. 2DMV

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Sum and difference of two cubes

Exercise 1 – 9:

Factorise:

1. $w^3 - 8$
Solution:

$$w^3 - 8 = (w - 2)(w^2 + 2w + 4)$$

2. $g^3 + 64$
Solution:

$$g^3 + 64 = (g + 4)(g^2 - 4g + 16)$$

3. $h^3 + 1$
Solution:

$$h^3 + 1 = (h + 1)(h^2 - h + 1)$$

4. $x^3 + 8$
Solution:

$$\begin{aligned} x^3 + 8 &= (x + 2)[(x)^2 - (x)(2) + (2)^2] \\ &= (x + 2)(x^2 - 2x + 4) \end{aligned}$$

5. $27 - m^3$
Solution:

$$\begin{aligned} 27 - m^3 &= (3-m)[(3)^2 + (3)(m) + (m)^2] \\ &= (3-m)(9 + 3m + m^2) \end{aligned}$$

6. $2x^3 - 2y^3$
Solution:

$$\begin{aligned} 2x^3 - 2y^3 &= 2(x^3 - y^3) \\ &= 2(x-y)[(x)^2 + (x)(y) + y^2] \\ &= 2(x-y)(x^2 + xy + y^2) \end{aligned}$$

7. $3k^3 + 81q^3$
Solution:

$$\begin{aligned} 3k^3 + 81q^3 &= 3(k^3 + 27q^3) \\ &= 3(k+3q)[(k)^2 - (k)(3q) + (3q)^2] \\ &= 3(k+3q)(k^2 - 3kq + 9q^2) \end{aligned}$$

8. $64t^3 - 1$
Solution:

$$\begin{aligned} 64t^3 - 1 &= (4t-1)[(4t)^2 + (4t)(1) + (1)^2] \\ &= (4t-1)(16t^2 + 4t + 1) \end{aligned}$$

9. $64x^2 - 1$
Solution:

$$64x^2 - 1 = (8x-1)(8x+1)$$

10. $125x^3 + 1$
Solution:

$$\begin{aligned} 125x^3 + 1 &= (5x+1)[(5x)^2 - (5x)(1) + (1)^2] \\ &= (5x+1)(25x^2 - 5x + 1) \end{aligned}$$

11. $25x^3 + 1$
Solution:
Note that $\left(\sqrt[3]{25}\right)^3 = 25$.

$$\begin{aligned} 25x^3 + 1 &= (\sqrt[3]{25}x + 1)[(\sqrt[3]{25}x)^2 - (\sqrt[3]{25}x)(1) + (1)^2] \\ &= (\sqrt[3]{25}x + 1)((\sqrt[3]{25})^2 x^2 - \sqrt[3]{25}x + 1) \end{aligned}$$

12. $z - 125z^4$
Solution:

$$\begin{aligned} z - 125z^4 &= (z)(1 - 125z^3) \\ &= (z)(1-5z)[(1)^2 + (1)(5z) + (5z)^2] \\ &= (z)(1-5z)(1 + 5z + 25z^2) \end{aligned}$$

13. $8m^6 + n^9$
Solution:

$$\begin{aligned} 8m^6 + n^9 &= (2m^2)^3 + (n^3)^3 \\ &= (2m^2 + n^3)[(2m^2)^2 - (2m^2)(n^3) + (n^3)^2] \\ &= (2m^2 + n^3)(4m^4 - 2m^2n^3 + n^6) \end{aligned}$$

14. $216n^3 - k^3$
Solution:

$$216n^3 - k^3 = (6n - k)(36n^2 + 6nk + k^2)$$

15. $125s^3 + d^3$
Solution:

$$125s^3 + d^3 = (5s + d)(25s^2 - 5sd + d^2)$$

16. $8k^3 + r^3$
Solution:

$$8k^3 + r^3 = (2k + r)(4k^2 - 2kr + r^2)$$

17. $8j^3k^3l^3 - b^3$
Solution:

$$8j^3k^3l^3 - b^3 = (2jkl - b)(4j^2k^2l^2 + 2jklabc + b^2)$$

18. $27x^3y^3 + w^3$
Solution:

$$27x^3y^3 + w^3 = (3xy + w)(9x^2y^2 - 3xyw + w^2)$$

19. $128m^3 + 2f^3$
Solution:

$$\begin{aligned} 128m^3 + 2f^3 &= 2(64m^3 + f^3) \\ &= 2(4m + f)(16m^2 - 4mf + f^2) \end{aligned}$$

20. $p^{15} - \frac{1}{8}y^{12}$
Solution:

$$\begin{aligned} p^{15} - \frac{1}{8}y^{12} &= (p^5)^3 - (\frac{1}{2}y^4)^3 \\ &= \left(p^5 - \frac{1}{2}y^4\right)\left[\left(p^5\right)^2 + \left(p^5\right)\left(\frac{1}{2}y^4\right) + \left(\frac{1}{2}y^4\right)\right] \\ &= \left(p^5 - \frac{1}{2}y^4\right)\left(p^{10} + \frac{1}{2}p^5y^4 + \frac{1}{4}y^8\right) \end{aligned}$$

21. $\frac{27}{t^3} - s^3$
Solution:

$$\frac{27}{t^3} - s^3 = (\frac{3}{t} - s)(\frac{9}{t^2} + \frac{3s}{t} + s^2)$$

22. $\frac{1}{64q^3} - h^3$
Solution:

$$\frac{1}{64q^3} - h^3 = (\frac{1}{4q} - h)(\frac{1}{16q^2} + \frac{h}{4q} + h^2)$$

23. $72g^3 + \frac{1}{3}v^3$
Solution:

$$\begin{aligned} 72g^3 + \frac{1}{3}v^3 &= \frac{1}{3}(216g^3 + v^3) \\ &= \frac{1}{3}(6g + v)(36g^2 - 6gv + v^2) \end{aligned}$$

24. $1 - (x - y)^3$
Solution:

$$\begin{aligned} 1 - (x - y)^3 &= (1 - (x - y))[(1)^2 - (1)(x - y) + (x - y)^2] \\ &= (1 - x + y)(1 - x + y + x^2 - 2xy + y^2) \end{aligned}$$

25. $h^4(8g^6 + h^3) - (8g^6 + h^3)$
Solution:

$$\begin{aligned} h^4(8g^6 + h^3) - (8g^6 + h^3) &= (h^4 - 1)(8g^6 + h^3) \\ &= (h^2 - 1)(h^2 + 1)(2g^2 + h)(4g^4 - 2g^2h + h^2) \\ &= (h - 1)(h + 1)(h^2 + 1)(2g^2 + h)(4g^4 - 2g^2h + h^2) \end{aligned}$$

26. $x(125w^3 - h^3) + y(125w^3 - h^3)$
Solution:

$$\begin{aligned} x(125w^3 - h^3) + y(125w^3 - h^3) &= (x + y)(125w^3 - h^3) \\ &= (x + y)(5w - h)(25w^2 + 5wh + h^2) \end{aligned}$$

27. $x^2(27p^3 + w^3) - 5x(27p^3 + w^3) - 6(27p^3 + w^3)$
Solution:

$$\begin{aligned} x^2(27p^3 + w^3) - 5x(27p^3 + w^3) - 6(27p^3 + w^3) &= (x^2 - 5x - 6)(27p^3 + w^3) \\ &= (x - 6)(x + 1)(3p + w)(9p^2 - 3pw + w^2) \end{aligned}$$

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1. 2DMW	2. 2DMX	3. 2DMY	4. 2DMZ	5. 2DN2	6. 2DN3	7. 2DN4	8. 2DN5
9. 2DN6	10. 2DN7	11. 2DN8	12. 2DN9	13. 2DNB	14. 2DNC	15. 2DND	16. 2DNF
17. 2DNG	18. 2DNH	19. 2DNJ	20. 2DNK	21. 2DNM	22. 2DNN	23. 2DNP	24. 2DNQ
25. 2DNR	26. 2DNS	27. 2DNT					

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1.8 Simplification of fractions

Exercise 1 – 10:

1. Simplify (assume all denominators are non-zero):
   a) $\frac{3a}{15}$
   Solution:

$$\frac{3a}{15} = \frac{a}{5}$$

b) $\dfrac{2a+10}{4}$

Solution:

$$\begin{aligned}\frac{2a+10}{4} &= \frac{2(a+5)}{4}\\ &= \frac{a+5}{2}\end{aligned}$$

c) $\dfrac{5a+20}{a+4}$

Solution:

$$\begin{aligned}\frac{5a+20}{a+4} &= \frac{5(a+4)}{a+4}\\ &= 5\end{aligned}$$

d) $\dfrac{a^2-4a}{a-4}$

Solution:

$$\begin{aligned}\frac{a^2-4a}{a-4} &= \frac{a(a-4)}{a-4}\\ &= a\end{aligned}$$

e) $\dfrac{3a^2-9a}{2a-6}$

Solution:

$$\begin{aligned}\frac{3a^2-9a}{2a-6} &= \frac{3a(a-3)}{2(a-3)}\\ &= \frac{3a}{2}\end{aligned}$$

f) $\dfrac{9a+27}{9a+18}$

Solution:

$$\begin{aligned}\frac{9a+27}{9a+18} &= \frac{9(a+3)}{9(a+2)}\\ &= \frac{a+3}{a+2}\end{aligned}$$

Note restriction: $a \neq -2$.

g) $\dfrac{6ab+2a}{2b}$

Solution:

$$\begin{aligned}\frac{6ab+2a}{2b} &= \frac{2a(3b+1)}{2b}\\ &= \frac{a(3b+1)}{b}\end{aligned}$$

Note restriction: $b \neq 0$.

h) $\dfrac{16x^2y - 8xy}{12x - 6}$

Solution:

$$\begin{aligned}\frac{16x^2y - 8xy}{12x - 6} &= \frac{8xy(2x - 1)}{6(2x - 1)} \\ &= \frac{8xy}{6} \\ &= \frac{4xy}{3}\end{aligned}$$

i) $\dfrac{4xyp - 8xp}{12xy}$

Solution:

$$\begin{aligned}\frac{4xyp - 8xp}{12xy} &= \frac{4xp(y - 2)}{12xy} \\ &= \frac{p(y - 2)}{3y}\end{aligned}$$

Note restriction: $y \neq 0$.

j) $\dfrac{9x^2 - 16}{6x - 8}$

Solution:

$$\begin{aligned}\frac{9x^2 - 16}{6x - 8} &= \frac{(3x - 4)(3x + 4)}{2(3x - 4)} \\ &= \frac{3x + 4}{2}\end{aligned}$$

k) $\dfrac{b^2 - 81a^2}{18a - 2b}$

Solution:

$$\begin{aligned}\frac{b^2 - 81a^2}{18a - 2b} &= \frac{(b - 9)(b + 9)}{2(9 - b)} \\ &= -\frac{b + 9}{2}\end{aligned}$$

l) $\dfrac{t^2 - s^2}{s^2 - 2st + t^2}$

Solution:

$$\begin{aligned}\frac{t^2 - s^2}{s^2 - 2st + t^2} &= \frac{(t - s)(t + s)}{(s - t)^2} \\ &= \frac{-(s - t)(t + s)}{(s - t)^2} \\ &= \frac{-(t + s)}{s - t}\end{aligned}$$

Note restriction: $s \neq t$

m) $\dfrac{x^2 - 2x - 15}{5x - 25}$

Solution:

$$\begin{aligned}\frac{x^2 - 2x - 15}{5x - 25} &= \frac{(x - 5)(x + 3)}{5(x - 5)} \\ &= \frac{x + 3}{5}\end{aligned}$$

n) $\dfrac{x^2+2x-15}{x^2+8x+15}$

Solution:

$$\begin{aligned}\frac{x^2+2x-15}{x^2+8x+15} &= \frac{(x+5)(x-3)}{(x+3)(x+5)}\\ &= \frac{x-3}{x+3}\end{aligned}$$

Note restriction: $x \neq -3$.

o) $\dfrac{x^2-x-6}{x^3-27}$

Solution:

$$\begin{aligned}\frac{x^2-x-6}{x^3-27} &= \frac{(x-3)(x+2)}{(x-3)(x^2+3x+9)}\\ &= \frac{x+2}{x^2+3x+9}\end{aligned}$$

p) $\dfrac{a^2+6a-16}{a^3-8}$

Solution:

$$\begin{aligned}\frac{a^2+6a-16}{a^3-8} &= \frac{(a+8)(a-2)}{(a-2)(a^2+2a+4)}\\ &= \frac{a+8}{a^2+2a+4}\end{aligned}$$

q) $\dfrac{a^2-4ab-12b^2}{a^2+4ab+4b^2}$

Solution:

$$\begin{aligned}\frac{a^2-4ab-12b^2}{a^2+4ab+4b^2} &= \frac{(a-6b)(a+2b)}{(a+2b)^2}\\ &= \frac{a-6b}{a+2b}\end{aligned}$$

Note restriction: $a \neq -2b$.

r) $\dfrac{6a^2-7a-3}{3ab+b}$

Solution:

$$\begin{aligned}\frac{6a^2-7a-3}{3ab+b} &= \frac{(2a-3)(3a+1)}{b(3a+1)}\\ &= \frac{2a-3}{b}\end{aligned}$$

Note restriction: $b \neq 0$.

s) $\dfrac{2x^2-x-1}{x^3-x}$

Solution:

$$\begin{aligned}\frac{2x^2-x-1}{x^3-x} &= \frac{(2x+1)(x-1)}{x(x-1)(x+1)}\\ &= \frac{2x+1}{x(x+1)}\end{aligned}$$

Note restrictions: $x \neq -1$ and $x \neq 0$.

t) $\dfrac{qz+qr+16z+16r}{z+r}$

Solution:

$$\begin{aligned}\frac{qz+qr+16z+16r}{(z+r)} &= \frac{q(z+r)+16(z+r)}{(z+r)}\\ &= \frac{(z+r)(q+16)}{(z+r)}\\ &= q+16\end{aligned}$$

u) $\dfrac{pz-pq+5z-5q}{z-q}$

Solution:

$$\begin{aligned}\frac{pz-pq+5z-5q}{(z-q)} &= \frac{p(z-q)+5(z-q)}{(z-q)}\\ &= \frac{(z-q)(p+5)}{(z-q)}\\ &= p+5\end{aligned}$$

v) $\dfrac{hx-hg+13x-13g}{x-g}$

Solution:

$$\begin{aligned}\frac{hx-hg+13x-13g}{(x-g)} &= \frac{h(x-g)+13(x-g)}{(x-g)}\\ &= \frac{(x-g)(h+13)}{(x-g)}\\ &= h+13\end{aligned}$$

w) $\dfrac{f^2a-fa^2}{f-a}$

Solution:

$$\begin{aligned}\frac{f^2a-fa^2}{f-a} &= \frac{af(f-a)}{(f-a)}\\ &= af\end{aligned}$$

2. Simplify (assume all denominators are non-zero):

a) $\dfrac{b^2+10b+21}{3(b^2-9)} \div \dfrac{2b^2+14b}{30b^2-90b}$

Solution:

$$\begin{aligned}\frac{b^2+10b+21}{3(b^2-9)} \div \frac{2b^2+14b}{30b^2-90b} &= \frac{b^2+10b+21}{3(b^2-9)} \times \frac{30b^2-90b}{2b^2+14b}\\ &= \frac{(b+7)(b+3)}{3(b-3)(b+3)} \times \frac{30b(b-3)}{2b(b+7)}\\ &= \frac{1}{3} \times \frac{30}{2}\\ &= 5\end{aligned}$$

b) $\dfrac{x^2+17x+70}{5(x^2-100)} \div \dfrac{3x^2+21x}{45x^2-450x}$

Solution:

$$\begin{aligned}\frac{x^2+17x+70}{5(x^2-100)} \div \frac{3x^2+21x}{45x^2-450x} &= \frac{x^2+17x+70}{5(x^2-100)} \times \frac{45x^2-450x}{3x^2+21x} \\ &= \frac{(x+7)(x+10)}{5(x-10)(x+10)} \times \frac{45x(x-10)}{3x(x+7)} \\ &= \frac{1}{5} \times \frac{45}{3} \\ &= 3\end{aligned}$$

c) $\dfrac{z^2+17z+66}{3(z^2-121)} \div \dfrac{2z^2+12z}{24z^2-264z}$

Solution:

$$\begin{aligned}\frac{z^2+17z+66}{3(z^2-121)} \div \frac{2z^2+12z}{24z^2-264z} &= \frac{z^2+17z+66}{3(z^2-121)} \times \frac{24z^2-264z}{2z^2+12z} \\ &= \frac{(z+6)(z+11)}{3(z-11)(z+11)} \times \frac{24z(z-11)}{2z(z+6)} \\ &= \frac{1}{3} \times \frac{24}{2} \\ &= 4\end{aligned}$$

d) $\dfrac{3a+9}{14} \div \dfrac{7a+21}{a+3}$

Solution:

$$\begin{aligned}\frac{3a+9}{14} \div \frac{7a+21}{a+3} &= \frac{3(a+3)}{14} \div \frac{7(a+3)}{a+3} \\ &= \frac{3(a+3)}{14} \div 7 \\ &= \frac{3(a+3)}{14} \times \frac{1}{7} \\ &= \frac{3(a+3)}{98}\end{aligned}$$

e) $\dfrac{a^2-5a}{2a+10} \times \dfrac{4a}{3a+15}$

Solution:

$$\begin{aligned}\frac{a^2-5a}{2a+10} \times \frac{4a}{3a+15} &= \frac{a(a-5)}{2(a+5)} \times \frac{4a}{3(a+5)} \\ &= \frac{[a(a-5)][4a]}{[2(a+5)][3(a+5)]} \\ &= \frac{4a^2(a-5)}{6(a+5)^2}\end{aligned}$$

Note restriction: $a \neq -5$.

f) $\dfrac{3xp+4p}{8p} \div \dfrac{12p^2}{3x+4}$

Solution:

$$\begin{aligned}\frac{3xp+4p}{8p} \div \frac{12p^2}{3x+4} &= \frac{p(3x+4)}{8p} \div \frac{12p^2}{3x+4} \\ &= \frac{3x+4}{8} \times \frac{3x+4}{12p^2} \\ &= \frac{[3x+4][3x+4]}{[8][12p^2]} \\ &= \frac{(3x+4)^2}{96p^2}\end{aligned}$$

Note restriction: $p \neq 0$.

g) $\dfrac{24a-8}{12} \div \dfrac{9a-3}{6}$

Solution:

$$\begin{aligned}\frac{24a-8}{12} \div \frac{9a-3}{6} &= \frac{8(3a-1)}{12} \div \frac{3(a-1)}{6} \\ &= \frac{2(3a-1)}{3} \times \frac{2}{a-1} \\ &= \frac{[2(3x-1)][2]}{[3][a-1]} \\ &= \frac{4(3a-1)}{3(a-1)}\end{aligned}$$

Note restriction: $a \neq 1$.

h) $\dfrac{a^2+2a}{5} \div \dfrac{2a+4}{20}$

Solution:

$$\begin{aligned}\frac{a^2+2a}{5} \div \frac{2a+4}{20} &= \frac{a(a+2)}{5} \div \frac{2(a+2)}{20} \\ &= \frac{a(a+2)}{5} \times \frac{10}{a+2} \\ &= \frac{[a(a+2)][10]}{[5][a+2]} \\ &= \frac{10a}{5} \\ &= 2a\end{aligned}$$

i) $\dfrac{p^2+pq}{7p} \times \dfrac{21q}{8p+8q}$

Solution:

$$\begin{aligned}\frac{p^2+pq}{7p} \times \frac{21q}{8p+8q} &= \frac{p(p+q)}{7p} \times \frac{21q}{8(p+q)} \\ &= \frac{[p(p+q)][21q]}{[7p][8(p+q)]} \\ &= \frac{21pq}{56p} \\ &= \frac{3q}{8}\end{aligned}$$

j) $\dfrac{5ab-15b}{4a-12} \div \dfrac{6b^2}{a+b}$

Solution:

$$\begin{aligned}\frac{5ab-15b}{4a-12} \div \frac{6b^2}{a+b} &= \frac{5b(a-3)}{4(a-3)} \div \frac{6b^2}{a+b} \\ &= \frac{5b}{4} \times \frac{a+b}{6b^2} \\ &= \frac{[5b][a+b]}{[4][6b^2]} \\ &= \frac{30b^3}{4(a+b)}\end{aligned}$$

Note restriction: $a \neq -b$.

k) $\dfrac{16-x^2}{x^2-x-12} \times \dfrac{x+3}{x+4}$

Solution:

$$\begin{aligned}\frac{16-x^2}{x^2-x-12} \times \frac{x+3}{x+4} &= \frac{(4-x)(4+x)}{(x-4)(x+3)} \times \frac{x+3}{x+4} \\ &= -1\end{aligned}$$

l) $\dfrac{a^3+b^3}{a^3} \times \dfrac{5a+5b}{a^2+2ab+b^2}$

Solution:

$$\begin{aligned}\frac{a^3+b^3}{a^3} \times \frac{5a+5b}{a^2+2ab+b^2} &= \frac{(a+b)(a^2-ab+b^2)}{a^3} \times \frac{5(a+b)}{(a+b)^2} \\ &= \frac{a^2-ab+b^2}{a^3} \times 5 \\ &= \frac{5(a^2-ab+b^2)}{a^3}\end{aligned}$$

Note restrictions: $a \neq \pm 0$.

m) $\dfrac{a-4}{a+5a+4} \times \dfrac{a^2+2a+1}{a^2-3a-4}$

Solution:

$$\begin{aligned}\frac{a-4}{a+5a+4} \times \frac{a^2+2a+1}{a^2-3a-4} &= \frac{a-4}{(a+4)(a+1)} \times \frac{(a+1)^2}{(a-4)(a+1)} \\ &= \frac{1}{a+4}\end{aligned}$$

Note restrictions: $a \neq -4$.

n) $\dfrac{3x+2}{x^2-6x+8} \times \dfrac{x-2}{3x^2+8x+4}$

Solution:

$$\begin{aligned}\frac{3x+2}{x^2-6x+8} \times \frac{x-2}{3x^2+8x+4} &= \frac{3x+2}{(x-4)(x-2)} \times \frac{x-2}{(3x+2)(x+2)} \\ &= \frac{1}{(x-4)(x+2)}\end{aligned}$$

Note restrictions: $x \neq 4$ and $x \neq -2$.

o) $\dfrac{a^2-2a+8}{a^2+6a+8} \times \dfrac{a^2+a-12}{3} - \dfrac{3}{2}$

Solution:

$$\begin{aligned}\frac{a^2-2a+8}{a^2+6a+8} \times \frac{a^2+a-12}{3} - \frac{3}{2} &= \frac{(a-4)(a+2)}{(a+2)(a+4)} \times \frac{(a+4)(a-3)}{3} - \frac{3}{2} \\ &= \frac{(a-4)(a-3)}{3} - \frac{3}{2} \\ &= \frac{2(a-4)(a-3)-9}{6} \\ &= \frac{2(a^2-7a+12)-9}{6} \\ &= \frac{2a^2-14a+15}{6}\end{aligned}$$

p) $\dfrac{4x^2-1}{3x^2+10x+3} \div \dfrac{6x^2+5x+1}{4x^2+7x-3} \times \dfrac{9x^2+6x+1}{8x^2-6x+1}$

Solution:

$$\begin{aligned}&\frac{4x^2-1}{3x^2+10x+3} \div \frac{6x^2+5x+1}{4x^2+7x-3} \times \frac{9x^2+6x+1}{8x^2-6x+1}\\ &= \frac{(2x-1)(2x+1)}{(x+3)(3x+1)} \times \frac{(x+3)(4x-1)}{(2x+1)(3x+1)} \times \frac{(3x+1)^2}{(2x-1)(4x-1)}\\ &= 1\end{aligned}$$

q) $\frac{x+4}{3} - \frac{x-2}{2}$

Solution:

$$\begin{aligned}\frac{x+4}{3} - \frac{x-2}{2} &= \frac{2(x+4)-3(x-2)}{6}\\ &= \frac{2x+8-3x+6}{6}\\ &= \frac{14-x}{6}\end{aligned}$$

r) $\frac{p^3+q^3}{p^2} \times \frac{3p-3q}{p^2-q^2}$

Solution:

$$\begin{aligned}\frac{p^3+q^3}{p^2} \times \frac{3p-3q}{p^2-q^2} &= \frac{(p+q)(p^2-pq+q^2)}{p^2} \times \frac{3(p-q)}{(p-q)(p+q)}\\ &= \frac{(p+q)(p^2-pq+q^2)}{p^2} \times \frac{3}{p+q}\\ &= \frac{3(p^2-pq+q^2)}{p^2}\end{aligned}$$

Note restriction: $p \neq 0$.

3. Simplify (assume all denominators are non-zero):

a) $\frac{x-3}{3} - \frac{x+5}{4}$

Solution:

$$\begin{aligned}\frac{x-3}{3} - \frac{x+5}{4} &= \frac{4(x-3)-3(x+5)}{12}\\ &= \frac{4x-12-3x-15}{12}\\ &= \frac{x-27}{12}\end{aligned}$$

b) $\frac{2x-4}{9} - \frac{x-3}{4} + 1$

Solution:

$$\begin{aligned}\frac{2x-4}{9} - \frac{x-3}{4} + 1 &= \frac{4(2x-4)-9(x-3)+36}{36}\\ &= \frac{8x-16-9x+27+36}{36}\\ &= \frac{47-x}{36}\end{aligned}$$

c) $1 + \frac{3x-4}{4} - \frac{x+2}{3}$

Solution:

$$1+\frac{3x-4}{4}-\frac{x+2}{3}=\frac{12+3(3x-4)-4(x+2)}{12}$$
$$=\frac{12+9x-12-4x-8}{12}$$
$$=\frac{5x-8}{12}$$

d) $\frac{11}{a+11}+\frac{8}{a-8}$

Solution:

$$\frac{11}{a+11}+\frac{8}{a-8}=\frac{11(a-8)+8(a+11)}{(a+11)(a-8)}$$
$$=\frac{11a-88+8a+88}{(a+11)(a-8)}$$
$$=\frac{19a}{(a+11)(a-8)}$$

Note restrictions: $a\neq -11$ and $a\neq 8$.

e) $\frac{12}{x-12}-\frac{6}{x-6}$

Solution:

$$\frac{12}{x-12}-\frac{6}{x-6}=\frac{12(x-6)-6(x-12)}{(x-12)(x-6)}$$
$$=\frac{12x-72-6x+72}{(x-12)(x-6)}$$
$$=\frac{6x}{(x-12)(x-6)}$$

Note restriction: $x\neq 12$ and $x\neq 6$.

f) $\frac{12}{r+12}+\frac{8}{r-8}$

Solution:

$$\frac{12}{r+12}+\frac{8}{r-8}=\frac{12(r-8)+8(r+12)}{(r+12)(r-8)}$$
$$=\frac{12r-96+8r+96}{(r+12)(r-8)}$$
$$=\frac{20r}{(r+12)(r-8)}$$

Note restriction: $r\neq -12$ and $r\neq 8$.

g) $\frac{2}{xy}+\frac{4}{xz}+\frac{3}{yz}$

Solution:

$$\frac{2}{xy}+\frac{4}{xz}+\frac{3}{yz}=\frac{2z}{xyz}+\frac{4y}{xyz}+\frac{3x}{xyz}$$
$$=\frac{2z+4y+3x}{xyz}$$

Note restrictions: $x\neq 0$; $y\neq 0$ and $z\neq 0$.

h) $\frac{5}{t-2}-\frac{1}{t-3}$

Solution:

$$\begin{aligned}\frac{5}{t-2}-\frac{1}{t-3} &= \frac{(5)(t-3)}{(t-3)(t-2)}-\frac{1(t-2)}{(t-2)(t-3)}\\ &= \frac{5(t-3)-(t-3)}{(t-2)(t-3)}\\ &= \frac{5t-15-t+3}{(t-2)(t-3)}\\ &= \frac{4t-12}{(t-2)(t-3)}\end{aligned}$$

Note restrictions: $t \neq 2$ and $t \neq 3$.

i) $\dfrac{k+2}{k^2+2}-\dfrac{1}{k+2}$

Solution:

$$\begin{aligned}\frac{k+2}{k^2+2}-\frac{1}{k+2} &= \frac{(k+2)(k+2)}{(k^2+2)(k+2)}-\frac{1(k^2+2)}{(k^2+2)(k+2)}\\ &= \frac{(k+2)^2-(k^2+2)}{(k^2+2)(k+2)}\\ &= \frac{k^2+4k+4-k^2-2}{(k^2+2)(k+2)}\\ &= \frac{4k+2}{(k^2+2)(k+2)}\\ &= \frac{2(k+2)}{(k^2+2)(k+2)}\end{aligned}$$

Note restrictions: $k \neq -2$ and $k^2 \neq \pm\sqrt{2}$.

j) $\dfrac{t+2}{3q}+\dfrac{t+1}{2q}$

Solution:

$$\begin{aligned}\frac{t+2}{3q}+\frac{t+1}{2q} &= \frac{(t+2)(2q)}{(3q)(2q)}+\frac{(t+1)(3q)}{(3q)(2q)}\\ &= \frac{(2tq+4q)+(3tq+3q)}{6q^2}\\ &= \frac{q(5t+7)}{6q^2}\\ &= \frac{5t+7}{6q}\end{aligned}$$

Note restriction: $q \neq 0$.

k) $\dfrac{3}{p^2-4}+\dfrac{2}{(p-2)^2}$

Solution:

$$\begin{aligned}\frac{3}{p^2-4}+\frac{2}{(p-2)^2} &= \frac{3(p-2)^2}{(p^2-4)(p-2)^2}+\frac{2(p^2-4)}{(p^2-4)(p-2)^2}\\ &= \frac{3(p-2)(p-2)+2(p-2)(p+2)}{(p+2)(p-2)^3}\\ &= \frac{[p-2][3(p-2)+2(p+2)]}{(p+2)(p-2)^3}\\ &= \frac{3p-6+2p+4}{(p+2)(p-2)^2}\\ &= \frac{5p-2}{(p+2)(p-2)^2}\end{aligned}$$

Note restriction: $p \neq \pm 2$.

l) $\dfrac{x}{x+y} + \dfrac{x^2}{y^2-x^2}$

Solution:

$$\begin{aligned}\frac{x}{x+y} + \frac{x^2}{y^2-x^2} &= \frac{x}{x+y} + \frac{x^2}{(x+y)(x-y)} \\ &= \frac{x(x-y)+x^2}{(x+y)(x-y)} \\ &= \frac{x^2-xy+x^2}{(x+y)(x-y)} \\ &= \frac{2x^2-xy}{(x+y)(x-y)}\end{aligned}$$

Note restriction: $x \neq \pm y$.

m) $\dfrac{1}{m+n} + \dfrac{3mn}{m^3+n^3}$

Solution:

$$\begin{aligned}\frac{1}{m+n} + \frac{3mn}{m^3+n^3} &= \frac{1}{m+n} + \frac{3mn}{(m+n)(m^2-mn+n^2)} \\ &= \frac{1(m^2-mn+n^2)+3mn}{(m+n)(m^2-mn+n^2)} \\ &= \frac{m^2+2mn+n^2}{(m+n)(m^2-mn+n^2)} \\ &= \frac{m+n}{m^2-mn+n^2}\end{aligned}$$

n) $\dfrac{h}{h^3-f^3} - \dfrac{1}{h^2+hf+f^2}$

Solution:

$$\begin{aligned}\frac{h}{h^3-f^3} - \frac{1}{h^2+hf+f^2} &= \frac{h}{(h-f)(h^2+hf+f^2)} - \frac{1}{h^2+hf+f^2} \\ &= \frac{h-h+f}{(h+f)(h^2+hf+f^2)} \\ &= \frac{f}{(h+f)(h^2+hf+f^2)}\end{aligned}$$

o) $\dfrac{x^2-1}{3} \times \dfrac{1}{x-1} - \dfrac{1}{2}$

Solution:

$$\begin{aligned}\frac{x^2-1}{3} \times \frac{1}{x-1} - \frac{1}{2} &= \frac{(x^2-1)(1)}{(3)(x-1)} - \frac{1}{2} \\ &= \frac{x^2-1}{3x-3} - \frac{1}{2} \\ &= \frac{(x^2-1)(2)}{2(3x-3)} - \frac{3x-3}{2(3x-3)} \\ &= \frac{2x^2-2-3x+3}{6x-6} \\ &= \frac{(x-1)(2x-1)}{6(x-1)} \\ &= \frac{2x-1}{6}\end{aligned}$$

p) $\dfrac{x^2-2x+1}{(x-1)^3} - \dfrac{x^2+x+1}{x^3-1}$

Solution:

$$\begin{aligned}\frac{x^2-2x+1}{(x-1)^3}-\frac{x^2+x+1}{x^3-1} &= \frac{(x-1)^2}{(x-1)^3}-\frac{x^2+x+1}{x^3-1}\\ &= \frac{1}{(x-1)}-\frac{x^2+x+1}{(x-1)(x^2+x+1)}\\ &= \frac{1}{(x-1)}-\frac{1}{(x-1)}\\ &= 0\end{aligned}$$

q) $\dfrac{1}{(x-1)^2}-\dfrac{2x}{x^3-1}$

Solution:

$$\begin{aligned}\frac{1}{(x-1)^2}-\frac{2x}{x^3-1} &= \frac{1}{(x-1)^2}-\frac{2x}{(x-1)(x^2+x+1)}\\ &= \frac{x^2+x+1-2x(x-1)}{(x-1)^2(x^2+x+1)}\\ &= \frac{x^2+x+1-2x^2+2x}{(x-1)^2(x^2+x+1)}\\ &= \frac{-x^2+3x+1}{(x-1)^2(x^2+x+1)}\end{aligned}$$

r) $\dfrac{t^2+2t-8}{t^2+t-6}+\dfrac{1}{t^2-9}+\dfrac{t+1}{t-3}$

Solution:

$$\begin{aligned}\frac{t^2+2t-8}{t^2+t-6}+\frac{1}{t^2-9}+\frac{t+1}{t-3} &= \frac{(t+4)(t-2)}{(t+3)(t-2)}+\frac{1}{(t-3)(t+3)}+\frac{t+1}{t-3}\\ &= \frac{t+4}{t+3}+\frac{1}{(t-3)(t+3)}+\frac{t+1}{t-3}\\ &= \frac{(t-3)(t+4)+1+(t+1)(t+3)}{(t-3)(t+3)}\\ &= \frac{t^2+t-12+1+t^2+4t+3}{(t-3)(t+3)}\\ &= \frac{2t^2+5t-8}{(t-3)(t+3)}\\ &= \frac{2t^2+5t-8}{t^2-9}\end{aligned}$$

Note restriction: $t \neq \pm 3$.

s) $\dfrac{x^2-3x+9}{x^3+27}+\dfrac{x-2}{x^2+4x+3}-\dfrac{1}{x-2}$

Solution:

$$\begin{aligned}\frac{x^2-3x+9}{x^3+27}+\frac{x-2}{x^2+4x+3}-\frac{1}{x-2} &= \frac{x^2-3x+9}{(x+3)(x^2-3x+9)}+\frac{x-2}{(x+3)(x+1)}-\frac{1}{x-2}\\ &= \frac{(x+1)(x-2)+(x-2)^2-(x+3)(x+1)}{(x+3)(x+1)(x-2)}\\ &= \frac{x^2-x-2+x^2-4x+4-x^2-4x-3}{(x+3)(x+1)(x-2)}\\ &= \frac{x^2-9x-1}{(x+3)(x+1)(x-2)}\end{aligned}$$

Note restrictions: $x \neq -3$; $x \neq -1$ and $x \neq 2$.

t) $\frac{1}{a^2-4ab+4b^2}+\frac{a^2+2ab+b^2}{a^3-8b^3}-\frac{1}{a^2-4b^2}$

Solution:

$$\begin{aligned}
&\frac{1}{a^2-4ab+4b^2}+\frac{a^2+2ab+b^2}{a^3-8b^3}-\frac{1}{a^2-4b^2}\\
&=\frac{1}{(a-2b)(a-2b)}+\frac{a^2+2ab+4b^2}{(a-2b)(a^2+2ab+4b^2)}-\frac{1}{(a-2b)(a+2b)}\\
&=\frac{(a+2b)+(a-2b)(a+2b)-(a-2b)}{(a-2b)^2(a+2b)}\\
&=\frac{a+2b+a^2-4b^2-a+2b}{(a-2b)^2(a+2b)}\\
&=\frac{a^2+4b-4b^2}{(a-2b)^2(a+2b)}
\end{aligned}$$

Note restriction: $a \neq \pm 2b$.

4. What are the restrictions in the following:

a) $\frac{1}{x-2}$

Solution:

We need to find the value of x that will make the denominator equal to 0. Therefore:

$$\begin{aligned}
x-2 &\neq 0\\
x &\neq 2
\end{aligned}$$

b) $\frac{3x-9}{4x+4}$

Solution:

First simplify the fraction:

$$\frac{3x-9}{4x+4}=\frac{3(x-1)}{4(x+1)}$$

Now we can determine the restriction:

$$\begin{aligned}
4(x+1) &\neq 0\\
x+1 &\neq 0\\
x &\neq -1
\end{aligned}$$

c) $\frac{3}{x}-\frac{1}{x^2-1}$

Solution:

First simplify the fraction:

$$\frac{3}{x}-\frac{1}{x^2-1}=\frac{3}{x}-\frac{1}{(x-1)(x+1)}$$

Now we can determine the restrictions. There are three restrictions in this case:

$$\begin{aligned}
x &\neq 0\\
x-1 &\neq 0\\
x+1 &\neq 0
\end{aligned}$$

Therefore: $x \neq 0$ and $x \neq \pm 1$

For more exercises, visit www.everythingmaths.co.za and click on 'Practise Maths'.

1a. 2DNW	1b. 2DNX	1c. 2DNY	1d. 2DNZ	1e. 2DP2	1f. 2DP3	1g. 2DP4	1h. 2DP5
1i. 2DP6	1j. 2DP7	1k. 2DP8	1l. 2DP9	1m. 2DPB	1n. 2DPC	1o. 2DPD	1p. 2DPF
1q. 2DPG	1r. 2DPH	1s. 2DPJ	1t. 2DPK	1u. 2DPM	1v. 2DPN	1w. 2DPP	2a. 2DPQ
2b. 2DPR	2c. 2DPS	2d. 2DPT	2e. 2DPV	2f. 2DPW	2g. 2DPX	2h. 2DPY	2i. 2DPZ
2j. 2DQ2	2k. 2DQ3	2l. 2DQ4	2m. 2DQ5	2n. 2DQ6	2o. 2DQ7	2p. 2DQ8	2q. 2DQ9
2r. 2DQB	3a. 2DQC	3b. 2DQD	3c. 2DQF	3d. 2DQG	3e. 2DQH	3f. 2DQJ	3g. 2DQK
3h. 2DQM	3i. 2DQN	3j. 2DQP	3k. 2DQQ	3l. 2DQR	3m. 2DQS	3n. 2DQT	3o. 2DQV
3p. 2DQW	3q. 2DQX	3r. 2DQY	3s. 2DQZ	3t. 2DR2	4a. 2DR3	4b. 2DR4	4c. 2DR5

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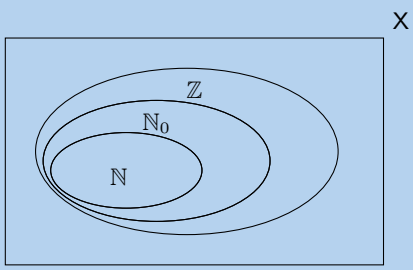
1.9 Chapter summary

End of chapter Exercise 1 – 11:

1. The figure here shows the Venn diagram for the special sets $\mathbb{N}, \mathbb{N}_0$ and $\mathbb{Z}$.



   a) Where does the number 2,13 belong in the diagram?
   Solution:
   2,13 is in its simplest form, therefore it is not in $\mathbb{N}$, $\mathbb{N}_0$ or $\mathbb{Z}$). It is in the space between the rectangle and $\mathbb{Z}$

   b) In the following list, there are two false statements and one true statement. Which of the statements is **true**?
   - Every natural number is an integer.
   - Every whole number is a natural number.
   - There are fractions in the integers.

   Solution:
   Consider each statement:
   - Integers are natural numbers and negative natural numbers. Therefore this statement is true.
   - 0 is not a natural number, therefore this statement is false.
   - Integers are natural numbers and negative natural numbers, no fractions. Therefore this is false.

   The only true statement is (i).

2. State whether the following numbers are real, non-real or undefined.

   a) $-\sqrt{-5}$
   Solution:
   This is the square root of a negative number and so is non-real.

   b) $\frac{\sqrt{8}}{0}$
   Solution:
   We are dividing by 0 and so this is undefined.

   c) $-\sqrt{15}$
   Solution:
   This is the square root of a positive number and so is real.

d) $-\sqrt{7}$

Solution:

This is the square root of a positive number and so is real.

e) $\sqrt{-1}$

Solution:

This is the square root of a negative number and so is non-real.

f) $\sqrt{2}$

Solution:

This is the square root of a positive number and so is real.

3. State whether each of the following numbers are rational or irrational.

a) $\sqrt[3]{4}$

Solution:

Irrational. It cannot be simplified to a fraction of integers.

b) 45π

Solution:

Irrational. It cannot be simplified to a fraction of integers

c) $\sqrt{9}$

Solution:

$\sqrt{9} = 3$

Rational. Can be simplified to an integer

d) $\sqrt[3]{8}$

Solution:

$\sqrt[3]{8} = 2$

Rational. Can be simplified to an integers.

4. If a is an integer, b is an integer and c is irrational, which of the following are rational numbers?

a) $\dfrac{-b}{a}$

Solution:

We have a fraction of integers and so this is rational.

b) $c \div c$

Solution:

When we divide a number by itself we get 1 and so this is rational.

c) $\dfrac{a}{c}$

Solution:

We are dividing an integer by an irrational number and so this is irrational. However if $a = 0$ then the fraction is equal to 0 and the number is rational.

d) $\dfrac{1}{c}$

Solution:

We are dividing an integer by an irrational number and so this is irrational.

5. Consider the following list of numbers:

$$\sqrt[3]{26}\ ;\ \frac{3}{2}\ ;\ \sqrt{-24}\ ;\ \sqrt{39}\ ;\ 7{,}\dot{1}\dot{1}\ ;\ \pi^2\ ;\ \frac{\pi}{2}\ ;\ 7{,}12\ ;\ -\sqrt{24}\ ;\ \frac{\sqrt{2}}{0}\ ;\ 3\pi\ ;\ \sqrt{78}\ ;\ 9\ ;\ \pi$$

a) Which of the numbers are non-real numbers?

Solution:

Only $\sqrt{-24}$ is non-real as it is the square root of a negative number.

b) Without using a calculator, rank all the real numbers in ascending order.

Solution:

We exclude $\sqrt{-24}$ from the list as it is non-real. We also exclude $\frac{\sqrt{2}}{0}$ as it is undefined. Then we note that:

- $\sqrt[3]{26}$ lies between 2 and 8
- $\frac{3}{2} = 1{,}5$
- $\sqrt{39}$ lies between 6 and 7

- $\pi^2 \approx 9{,}8696$
- $\frac{\pi}{2} \approx 1{,}5708$
- $-\sqrt{24}$ lies between -4 and -5
- $3\pi \approx 9{,}4248$
- $\sqrt{78}$ lies between 8 and 9
- $\pi \approx 3{,}1416$

Therefore the ordering is: $-\sqrt{24}\ ;\ \frac{3}{2}\ ;\ \frac{\pi}{2}\ ;\ \sqrt[3]{26}\ ;\ \pi\ ;\ \sqrt{39}\ ;\ 7{,}\dot{1}\dot{1}\ ;\ 7{,}12\ ;\ \sqrt{78}\ ;\ 9\ ;\ 3\pi\ ;\ \pi^2$

c) Which of the numbers are irrational numbers?

Solution:

Any number that cannot be written as a fraction of integers is irrational. Therefore $-\sqrt{24}\ ;\ \frac{\pi}{2}\ ;\ \sqrt[3]{26}\ ;\ \pi\ ;\ \sqrt{39}\ ;\ \sqrt{78}\ ;\ 3\pi\ ;\ \pi^2$ are all irrational.

d) Which of the numbers are rational numbers?

Solution:

All numbers that can be written as a fraction of integers are rational numbers. Therefore $\frac{3}{2}\ ;\ 7{,}\dot{1}\dot{1}\ ;\ 7{,}12\ ;\ 9$ are all rational numbers.

e) Which of the numbers are integers?

Solution:

Only 9 is an integer.

f) Which of the numbers are undefined?

Solution:

Any fraction that has a denominator of 0 is undefined, therefore only $\frac{\sqrt{2}}{0}$ is undefined.

6. Write each decimal as a simple fraction.

a) 0,12

Solution:

$$\begin{aligned} 0{,}12 &= \frac{1}{10} + \frac{2}{100} \\ &= \frac{12}{100} \\ &= \frac{3}{25} \end{aligned}$$

b) 0,006

Solution:

$$\begin{aligned} 0{,}006 &= \frac{6}{1000} \\ &= \frac{3}{500} \end{aligned}$$

c) $4{,}\overline{14}$

Solution:

$$\begin{aligned} x &= 4{,}141414\ldots \\ 100x &= 414{,}141414\ldots \\ 100x - x &= (414{,}141414\ldots) - (4{,}141414\ldots) \\ 99x &= 410 \\ \therefore x &= \frac{410}{99} \end{aligned}$$

d) 1,59

Solution:

$$\begin{aligned} 1{,}59 &= 1 + \frac{5}{10} + \frac{9}{100} \\ &= 1\frac{59}{100} \end{aligned}$$

e) $12{,}27\dot{7}$
Solution:

$$\begin{aligned} x &= 12{,}27\dot{7} \\ 10x &= 122{,}\dot{7} \\ 100x &= 1227{,}\dot{7} \\ \therefore 100x - 10x &= 90x = 1105 \\ \therefore x &= \frac{1105}{90} \\ &= \frac{221}{18} \end{aligned}$$

f) $0{,}8\dot{2}$
Solution:

$$\begin{aligned} 0{,}8\dot{2} &= 0{,}82222{,}.. \\ x &= 0{,}8222... \\ 10x &= 8{,}222... \\ 100x &= 82{,}222... \\ 100x - 10x &= 82{,}222 - 8{,}222... \\ 90x &= 74{,}000 \\ 90x &= 74 \\ \therefore x &= \frac{37}{45} \end{aligned}$$

g) $7{,}\overline{36}$
Solution:

$$\begin{aligned} x &= 7{,}363636... \\ 100x &= 736{,}363636... \\ 100x - x &= (736{,}363636...) - (7{,}363636...) \\ 99x &= 729 \\ \therefore x &= \frac{81}{11} \end{aligned}$$

7. Show that the decimal $3{,}21\dot{1}\dot{8}$ is a rational number.
Solution:

$$\begin{aligned} x &= 3{,}21\dot{1}\dot{8} \\ 1000x &= 32\ 118{,}\overline{18} \\ \therefore 10\ 000x - x &= 9999x = 32\ 115 \\ \therefore x &= \frac{32\ 115}{9999} \end{aligned}$$

This is a rational number because both the numerator and denominator are integers.

8. Write the following fractions as decimal numbers:

a) $\frac{1}{18}$
Solution:

$$
\begin{aligned}
18|\overline{1{,}0000} &= 0 \text{ remainder } 0\\
18|\overline{1{,}^{1}0000} &= 0 \text{ remainder } 0\\
18|\overline{1{,}^{10}0^{1}000} &= 5 \text{ remainder } 10\\
18|\overline{1{,}^{10}0^{1}0^{1}00} &= 5 \text{ remainder } 10\\
\frac{1}{18} &= 0{,}05555\ldots\\
&= 0{,}0\dot{5}
\end{aligned}
$$

b) $1\frac{1}{2}$
Solution:

$$
\begin{aligned}
1\frac{1}{2} &= \frac{3}{2}\\
2|\overline{3{,}0000} &= 1 \text{ remainder } 1\\
2|\overline{3{,}^{1}0000} &= 5 \text{ remainder } 0\\
&= 1{,}5
\end{aligned}
$$

9. Express $0{,}\overline{78}$ as a fraction $\frac{a}{b}$ where $a, b \in \mathbb{Z}$ (show all working).
Solution:

$$
\begin{aligned}
x &= 0{,}\overline{78}\\
100x &= 78{,}\overline{78}\\
\therefore 100x - x &= 99\\
\therefore x &= \frac{78}{99}
\end{aligned}
$$

10. For each of the following numbers:
    - write the next three digits;
    - state whether the number is rational or irrational.

    a) 1,11235...
    Solution:
    - The number does not terminate (this is shown by the . . .). There is also no indication of a repeating pattern of digits since there is not dot or bar over any of the numbers. The next three digits could be any numbers.
    - Irrational, there is no repeating pattern.

    b) $1{,}\dot{1}$
    Solution:
    - Since there is a dot over the 1 we know that the 1 repeats. The next three digits are: 111
    - Rational, there is a repeating pattern of digits.

11. Write the following rational numbers to 2 decimal places.

    a) $\frac{1}{2}$
    Solution:
    To write to two decimal places we must convert to decimal: $\frac{1}{2} = 0{,}50$.

    b) 1
    Solution:
    To write to two decimal places just add a comma and two 0's: 1,00.

    c) $0{,}11111\overline{1}$
    Solution:
    We mark where the cut off point is, determine if it has to be rounded up or not and then write the answer. In this case there is a 1 after the cut off point so we do not round up. The final answer is: $0{,}11111\overline{1} \approx 0{,}11$.

    d) $0{,}99999\overline{1}$
    Solution:
    We mark where the cut off point is, determine if it has to be rounded up or not and then write the answer. In this case there is a 9 after the cut off point so we round up. The final answer is: $0{,}99999\overline{1} \approx 1{,}00$.

12. Round off the following irrational numbers to 3 decimal places.

    a) 3,141592654...
    Solution:
    3,142 (round up as there is a 5 after the cut off point).

    b) 1,618033989...
    Solution:
    1,618 (no rounding as there is a 0 after the cut off point).

    c) 1,41421356...
    Solution:
    1,414 (no rounding as there is a 2 after the cut off point).

    d) 2,71828182845904523536...
    Solution:
    2,718 (no rounding as there is a 2 after the cut off point).

13. Round off the number 1523,00195593 to 4 decimal places.
    Solution:
    $1523{,}00195593 \approx 1523{,}0020$

14. Round off the number 1982,94028996 to 6 decimal places.
    Solution:
    $1982{,}94028996 \approx 1982{,}940290$

15. Round off the number 101,52378984 to 4 decimal places.
    Solution:
    $101{,}52378984 \approx 101{,}5238$

16. Use your calculator and write the following irrational numbers to 3 decimal places.

    a) $\sqrt{2}$
    Solution:
    $\sqrt{2} \approx 1{,}414213562... \approx 1{,}414$

    b) $\sqrt{3}$
    Solution:
    $\sqrt{3} \approx 1{,}732050808... \approx 1{,}732$

    c) $\sqrt{5}$
    Solution:
    $\sqrt{5} \approx 2{,}236067977... \approx 2{,}236$

    d) $\sqrt{6}$
    Solution:
    $\sqrt{6} \approx 2{,}449489743... \approx 2{,}449$

17. Use your calculator (where necessary) and write the following numbers to 5 decimal places. State whether the numbers are irrational or rational.

    a) $\sqrt{8}$
    Solution:
    $\sqrt{8} \approx 2{,}828427125... \approx 2{,}82843$
    Irrational number.

    b) $\sqrt{768}$
    Solution:
    $\sqrt{768} \approx 27{,}71281292... \approx 27{,}71281$
    Irrational number.

    c) $\sqrt{0{,}49}$
    Solution:
    $\sqrt{0{,}49} = 0{,}70000$
    Rational number.

    d) $\sqrt{0{,}0016}$
    Solution:
    $\sqrt{0{,}0016} = 0{,}04000$
    Rational number.

    e) $\sqrt{0{,}25}$

Solution:
$\sqrt{0{,}25} = 0{,}50000$
Rational number.

f) $\sqrt{36}$
Solution:
$\sqrt{36} = 6{,}00000$
Rational number.

g) $\sqrt{1960}$
Solution:
$\sqrt{1960} \approx 44{,}27188724... \approx 44{,}27189$
Irrational number.

h) $\sqrt{0{,}0036}$
Solution:
$\sqrt{0{,}0036} = 0{,}06000$
Rational number.

i) $-8\sqrt{0{,}04}$
Solution:
$-8\sqrt{0{,}04} = -8(0{,}20000) = -1{,}60000$
Rational number.

j) $5\sqrt{80}$
Solution:
$5\sqrt{80} \approx 5(8{,}94427191...) \approx 44{,}72136$
Irrational number.

18. Round off:

a) $\frac{\sqrt{2}}{2}$ to the nearest 2 decimal places.
Solution:

$$\begin{aligned}\frac{\sqrt{2}}{2} &\approx 0{,}7071... \\ &\approx 0{,}71\end{aligned}$$

b) $\sqrt{14}$ to the nearest 3 decimal places.
Solution:

$$\begin{aligned}\sqrt{14} &\approx 3{,}741657... \\ &\approx 3{,}742\end{aligned}$$

19. Write the following irrational numbers to 3 decimal places and then write each one as a rational number to get an approximation of the irrational number.

a) 3,141592654...
Solution:

$$\begin{aligned}3{,}141592654... &\approx 3{,}142 \\ &\approx 3\frac{142}{1000} \\ &\approx \frac{1571}{500}\end{aligned}$$

b) 1,618033989...
Solution:

$$\begin{aligned}1{,}618033989... &\approx 1{,}618 \\ &\approx 1\frac{618}{1000} \\ &\approx \frac{809}{500}\end{aligned}$$

c) 1,41421356...
Solution:

$$\begin{aligned} 1{,}41421356... &\approx 1{,}414 \\ &\approx 1\frac{414}{1000} \\ &\approx \frac{707}{500} \end{aligned}$$

d) 2,71828182845904523536...
Solution:

$$\begin{aligned} 2{,}71828182845904523536... &\approx 2{,}718 \\ &\approx 2\frac{718}{1000} \\ &\approx \frac{1359}{500} \end{aligned}$$

20. Determine between which two consecutive integers the following irrational numbers lie, without using a calculator.

a) $\sqrt{5}$
Solution:
2 and 3 ($2^2 = 4$ and $3^2 = 9$)

b) $\sqrt{10}$
Solution:
3 and 4 ($3^2 = 9$ and $4^2 = 16$)

c) $\sqrt{20}$
Solution:
4 and 5 ($4^2 = 16$ and $5^2 = 25$)

d) $\sqrt{30}$
Solution:
5 and 6 ($5^2 = 25$ and $6^2 = 36$)

e) $\sqrt[3]{5}$
Solution:
1 and 2 ($1^3 = 1$ and $2^3 = 8$)

f) $\sqrt[3]{10}$
Solution:
2 and 3 ($2^3 = 8$ and $3^3 = 27$)

g) $\sqrt[3]{20}$
Solution:
2 and 3 ($2^3 = 8$ and $3^3 = 27$)

h) $\sqrt[3]{30}$
Solution:
3 and 4 ($3^3 = 27$ and $4^3 = 64$)

i) $\sqrt{90}$
Solution:
9 and 10 ($9^2 = 81$ and $10^2 = 100$)

j) $\sqrt{72}$
Solution:
8 and 9 ($8^2 = 64$ and $9^2 = 81$)

k) $\sqrt[3]{58}$
Solution:
3 and 4 ($3^3 = 27$ and $4^3 = 64$)

l) $\sqrt[3]{118}$
Solution:
4 and 5 ($4^3 = 64$ and $5^3 = 125$)

21. Estimate the following surds to the nearest 1 decimal place, without using a calculator.

a) $\sqrt{14}$
Solution:
$\sqrt{14}$ lies between 3 and 4. Since $3^2 = 9$ and $4^2 = 16$ it lies closer to 4 than to 3.
Therefore 3,7 or 3,8 are suitable estimates.

b) $\sqrt{110}$
Solution:
$\sqrt{110}$ lies between 10 and 11. Since $10^2 = 100$ and $11^2 = 121$ it lies almost exactly between 10 and 11.
Therefore 10,5 is a suitable estimate.

c) $\sqrt{48}$
Solution:
$\sqrt{48}$ lies between 6 and 7. Since $6^2 = 36$ and $7^2 = 49$ it lies closer to 7 than to 6.
Therefore 6,9 is a suitable estimate.

d) $\sqrt{57}$
Solution:
$\sqrt{57}$ lies between 7 and 8. Since $7^2 = 49$ and $8^2 = 64$ it lies almost exactly between the two numbers.
Therefore 4,5 or 4,6 are suitable estimates.

22. Expand the following products:

a) $(a + 5)^2$
Solution:

$$\begin{aligned}(a+5)^2 &= (a+5)(a+5)\\ &= a^2 + 5a + 5a + 25\\ &= a^2 + 10a + 25\end{aligned}$$

b) $(n + 12)^2$
Solution:

$$\begin{aligned}(n+12)^2 &= (n+12)(n+12)\\ &= n^2 + 12n + 12n + 144\\ &= n^2 + 24n + 144\end{aligned}$$

c) $(d - 4)^2$
Solution:

$$\begin{aligned}(d-4)^2 &= (d-4)(d-4)\\ &= d^2 - 4d - 4d + 16\\ &= d^2 - 8d + 16\end{aligned}$$

d) $(7w + 2)(7w - 2)$
Solution:

$$\begin{aligned}(7w+2)(7w-2) &= 49w^2 - 14w + 14w - 4\\ &= 49w^2 - 4\end{aligned}$$

e) $(12q + 1)(12q - 1)$
Solution:

$$\begin{aligned}(12q+1)(12q-1) &= 144q^2 - 12q + 12q - 1\\ &= 144q^2 - 1\end{aligned}$$

f) $-(-x-2)(x+2)$
Solution:

$$\begin{aligned}-(-x-2)(x+2) &= (x+2)(x+2)\\ &= x^2+2x+2x+4\\ &= x^2+4x+4\end{aligned}$$

g) $(5k-4)(5k+4)$
Solution:

$$\begin{aligned}(5k-4)(5k+4) &= 25k^2+20k-20k-16\\ &= 25k^2-16\end{aligned}$$

h) $(5f+4)(2f+2)$
Solution:

$$\begin{aligned}(5f+4)(2f+2) &= 10f^2+10f+8f+8\\ &= 10f^2+18f+8\end{aligned}$$

i) $(3n+6)(6n+5)$
Solution:

$$\begin{aligned}(3n+6)(6n+5) &= 18n^2+15n+36n+30\\ &= 18n^2+51n+30\end{aligned}$$

j) $(2g+6)(g+6)$
Solution:

$$\begin{aligned}(2g+6)(g+6) &= 2g^2+12g+6g+36\\ &= 2g^2+18g+36\end{aligned}$$

k) $(4y+1)(4y+8)$
Solution:

$$\begin{aligned}(4y+1)(4y+8) &= 16y^2+32y+4y+8\\ &= 16y^2+36y+8\end{aligned}$$

l) $(d-3)(7d+2)$
Solution:

$$\begin{aligned}(d-3)(7d+2) &= 7d^2+2d-21d-6\\ &= 7d^2-19d-6\end{aligned}$$

m) $(6z-4)(z-2)$
Solution:

$$\begin{aligned}(6z-4)(z-2) &= 6z^2-12z-4z+8\\ &= 6z^2-16z+8\end{aligned}$$

n) $(5w-11)^2$
Solution:

$$\begin{aligned}(5w-11)^2 &= (5w-11)(5w-11)\\ &= 25w^2-55w-55w+121\\ &= 25w^2-110w+121\end{aligned}$$

o) $(5s-1)^2$
Solution:

$$\begin{aligned}(5s-1)^2 &= (5s-1)(5s-1)\\ &= 25s^2-5s-5s+1\\ &= 25s^2-10s+1\end{aligned}$$

p) $(3d-8)^2$
Solution:

$$\begin{aligned}(3d-8)^2 &= (3d-8)(3d-8)\\ &= 9d^2-24d-24d+64\\ &= 9d^2-48d+64\end{aligned}$$

q) $5f^2(3f+5)+7f(3f^2+7)$
Solution:

$$\begin{aligned}5f^2(3f+5)+7f(3f^2+7) &= 15f^3+25f^2+21f^3+49f\\ &= 36f^3+25f^2+49f\end{aligned}$$

r) $8d(4d^3+2)+6d^2(7d^2+4)$
Solution:

$$\begin{aligned}8d(4d^3+2)+6d^2(7d^2+4) &= 32d^4+16d+42d^4+24d^2\\ &= 74d^4+16d+24d^2\end{aligned}$$

s) $5x^2(2x+2)+7x(7x^2+7)$
Solution:

$$\begin{aligned}5x^2(2x+2)+7x(7x^2+7) &= 10x^3+10x^2+49x^3+49x\\ &= 59x^3+10x^2+49x\end{aligned}$$

23. Expand the following:

a) $(y^4+3y^2+y)(y+1)(y-2)$
Solution:

$$\begin{aligned}(y^4+3y^2+y)(y+1)(y-2) &= (y^4+3y^2+y)(y^2-y-2)\\ &= y^6-y^5-2y^4+3y^4-3y^3-6y^2+y^3-y^2-2y\\ &= y^6-y^5+y^4-2y^3-7y^2-2y\end{aligned}$$

b) $(x+1)^2-(x-1)^2$
Solution:

$$\begin{aligned}(x+1)^2-(x-1)^2 &= x^2+2x+1-(x^2-2x+1)\\ &= x^2+2x+1-x^2+2x-1\\ &= 4x\end{aligned}$$

c) $(x^2+2x+1)(x^2-2x+1)$
Solution:

$$\begin{aligned}(x^2+2x+1)(x^2-2x+1) &= x^4-2x^3+x^2+2x^3-4x^2+2x+x^2-2x+1\\ &= x^4-2x^2+1\end{aligned}$$

d) $(4a-3b)(16a^2+12ab+9b^2)$
Solution:

$$\begin{aligned}(4a-3b)(16a^2+12ab+9b^2) &= 64a^3+48a^2b+36ab^2-48a^2b-36ab^2-27b^3\\ &= 64a^3-27b^3\end{aligned}$$

e) $2(x+3y)(x^2-xy-y^2)$
Solution:

$$\begin{aligned}2(x+3y)(x^2-xy-y^2) &= 2(x^3-x^2y-xy^2+3x^2y-3xy^2-3y^3)\\ &= 2x^3+4x^2y-8xy^2-6y^3\end{aligned}$$

f) $(3a-5b)(3a+5b)(a^2+ab-b^2)$
Solution:

$$\begin{aligned}(3a-5b)(3a+5b)(a^2+ab-b^2) &= (9a^2-25b^2)(a^2+ab-b^2)\\ &= 9a^4+9a^3-9a^2b^2-25a^2b^2+25ab^3-25b^4\\ &= 9a^4+9a^3-34a^2b^2+25ab^3-25b^4\end{aligned}$$

g) $\left(y-\frac{1}{y}\right)\left(y+\frac{1}{y}\right)$
Solution:

$$\begin{aligned}\left(y-\frac{1}{y}\right)\left(y+\frac{1}{y}\right) &= y^2+1-1+\frac{1}{y^2}\\ &= y^2-\frac{1}{y^2}\end{aligned}$$

h) $\left(\frac{a}{3}-\frac{3}{a}\right)\left(\frac{a}{3}+\frac{3}{a}\right)$
Solution:

$$\begin{aligned}\left(\frac{a}{3}-\frac{3}{a}\right)\left(\frac{a}{3}+\frac{3}{a}\right) &= \frac{a^2}{9}+1-1+\frac{3}{a^2}\\ &= \frac{a^2}{9}-\frac{3}{a^2}\end{aligned}$$

i) $\frac{1}{3}(12x-9y)+\frac{1}{6}(12x+18y)$
Solution:

$$\begin{aligned}\frac{1}{3}(12x-9y)+\frac{1}{6}(12x+18y) &= 4x-3y+2x+3y\\ &= 6x\end{aligned}$$

j) $(x+2)(x-2)-(x+2)^2$
Solution:

$$\begin{aligned}(x+2)(x-2)-(x+2)^2 &= x^2-4-(x^2+4x+4)\\ &= -4x-8\end{aligned}$$

24. What is the value of e in $(x-4)(x+e)=x^2-16$?
Solution:

$$(x-4)(x+e) = x^2+ex-4x-4e$$

From the constant term we see that $4e=16$, therefore $e=4$.

25. In $(x+2)(x+k) = x^2 + bx + c$:

a) For which of these values of k will b be positive?
$-6\ ;\ -1\ ;\ 0\ ;\ 1\ ;\ 6$
Solution:

$$\begin{aligned}(x+2)(x+k) &= x^2 + kx + 2x + 2k \\ &= x^2 + (k+2)x + 2k\end{aligned}$$

The b term is $k+2$ and so any value greater than -2 will make the b term positive.
Therefore $-1\ ;\ 0\ ;\ 1\ ;\ 6$

b) For which of these values of k will c be positive?
$-6\ ;\ -1\ ;\ 0\ ;\ 1\ ;\ 6$
Solution:
From above we see that the c term is $2k$. Therefore any positive value of k will make c positive.
Therefore $0\ ;\ 1\ ;\ 6$

c) For what values of k will c be positive?
Solution:
From above we see that the c term is $2k$. Therefore any positive value of k will make c positive.
Therefore $k > 0$

d) For what values of k will b be positive?
Solution:
From above we see that any value greater than -2 will make the b term positive.
Therefore $k > -2$.

26. Answer the following:

a) Expand: $\left(3a - \frac{1}{2a}\right)^2$
Solution:

$$\left(3a - \frac{1}{2a}\right)^2 = 9a^2 + 3 + \frac{1}{4a^2}$$

b) Expand: $\left(3a - \frac{1}{2a}\right)\left(9a^2 + \frac{3}{2} + \frac{1}{4a^2}\right)$
Solution:

$$\begin{aligned}\left(3a - \frac{1}{2a}\right)\left(9a^2 + \frac{3}{2} + \frac{1}{4a^2}\right) &= 27a^3 + \frac{9}{2}a + \frac{3}{4a} - \frac{9}{2}a - \frac{3}{4a} - \frac{1}{8a^3} \\ &= 27a^3 - \frac{1}{8a^3}\end{aligned}$$

c) Given that $3a - \frac{1}{2a} = 7$, determine the value of $27a^3 - \frac{1}{8a^3}$ without solving for a.
Solution:

$$\begin{aligned}27a^3 - \frac{1}{8a^3} &= \left(3a - \frac{1}{2a}\right)\left(9a^2 + \frac{3}{2} + \frac{1}{4a^2}\right) \\ &= 7\left(9a^2 + \frac{3}{2} + \frac{1}{4a^2}\right)\end{aligned}$$

$$\begin{aligned}9a^2 + \frac{3}{2} + \frac{1}{4a^2} &= \left(3a - \frac{1}{2a}\right)^2 + \frac{9}{2} \\ &= 7^2 + \frac{9}{2}\end{aligned}$$

$$27a^3 - \frac{1}{8a^3} = 374\frac{1}{2}$$

27. Solve by factorising:

a) $17^2 - 15^2$

Solution:

$$\begin{aligned} 17^2 - 15^2 &= (17 - 15)(17 + 15) \\ &= 2(32) \\ &= 64 \end{aligned}$$

b) $13^2 - 12^2$

Solution:

$$\begin{aligned} 13^2 - 12^2 &= (13 - 12)(13 + 12) \\ &= 25 \end{aligned}$$

c) $120045^2 - 120035^2$

Solution:

$$\begin{aligned} 120045^2 - 120035^2 &= (120045 - 120035)(120045 + 120035) \\ &= 10(240080) \\ &= 2400800 \end{aligned}$$

d) $26^2 - 24^2$

Solution:

$$\begin{aligned} 26^2 - 24^2 &= (26 - 24)(26 + 24) \\ &= 2(50) \\ &= 100 \end{aligned}$$

28. Represent the following as a product of its prime factors:

a) 143

Solution:

$$\begin{aligned} 143 &= 144 - 1 \\ &= (12 - 1)(12 + 1) \\ &= 11 \times 13 \end{aligned}$$

b) 168

Solution:

$$\begin{aligned} 168 &= 169 - 1 \\ &= (13 - 1)(13 + 1) \\ &= 12(14) \\ &= 3 \times 2^2 \times 2 \times 7 \\ &= 2^3 \times 3 \times 7 \end{aligned}$$

c) 899

Solution:

$$\begin{aligned} 899 &= 900 - 1 \\ &= (30 - 1)(30 + 1) \\ &= 29 \times 31 \end{aligned}$$

d) 99
Solution:

$$\begin{aligned} 99 &= 100 - 1 \\ &= (10-1)(10+1) \\ &= 3^2 \times 11 \end{aligned}$$

e) 1599
Solution:

$$\begin{aligned} 1599 &= 1600 - 1 \\ &= (40-1)(40+1) \\ &= 39(41) \\ &= 3 \times 13 \times 41 \end{aligned}$$

29. Factorise:

a) $a^2 - 9$
Solution:

$$a^2 - 9 = (a-3)(a+3)$$

b) $9b^2 - 81$
Solution:

$$\begin{aligned} 9b^2 - 81 &= 9(b^2 - 9) \\ &= 9(b-3)(b+3) \end{aligned}$$

c) $m^2 - \frac{1}{9}$
Solution:

$$m^2 - \frac{1}{9} = \left(m - \frac{1}{3}\right)\left(m + \frac{1}{3}\right)$$

d) $5 - 5a^2b^6$
Solution:

$$\begin{aligned} 5 - 5a^2b^6 &= 5(1 - a^2b^6) \\ &= 5(1 - ab^3)(1 + ab^3) \end{aligned}$$

e) $16ba^4 - 81b$
Solution:

$$\begin{aligned} 16ba^4 - 81b &= b(16a^4 - 81) \\ &= b(4a^2 - 9)(4a^2 + 9) \\ &= b(2a-3)(2a+3)(4a^2+9) \end{aligned}$$

f) $a^2 - 10a + 25$
Solution:

$$a^2 - 10a + 25 = (a-5)(a-5)$$

g) $16b^2 + 56b + 49$
Solution:

$$16b^2 + 56b + 49 = (4b+7)(4b+7)$$

h) $-4b^2 - 144b^8 + 48b^5$
Solution:

$$\begin{aligned} -4b^2 - 144b^8 + 48b^5 &= -4b^2(1 + 36b^6 - 12b^3) \\ &= -4b^2(6b^3 - 1)(6b^3 - 1) \\ &= -4b^2(6b^3 - 1)^2 \end{aligned}$$

i) $16 - x^4$
Solution:

$$\begin{aligned} 16 - x^4 &= (4 - x^2)(4 + x^2) \\ &= (4 + x^2)(2 + x)(2 - x) \end{aligned}$$

j) $7x^2 - 14x + 7xy - 14y$
Solution:

$$\begin{aligned} 7x^2 - 14x + 7xy - 14y &= 7(x^2 - 2x + xy - 2y) \\ &= 7(x(x - 2) + y(x - 2)) \\ &= 7(x - 2)(x + y) \end{aligned}$$

k) $y^2 - 7y - 30$
Solution:

$$y^2 - 7y - 30 = (y - 10)(y + 3)$$

l) $1 - x - x^2 + x^3$
Solution:

$$\begin{aligned} 1 - x - x^2 + x^3 &= (1 - x) - x^2(1 - x) \\ &= (1 - x)(1 - x^2) \\ &= (1 - x)(1 - x)(1 + x) \\ &= (1 - x)^2(1 + x) \end{aligned}$$

m) $-3(1 - p^2) + p + 1$
Solution:

$$\begin{aligned} -3(1 - p^2) + p + 1 &= -3(1 - p)(1 + p) + (1 + p) \\ &= (1 + p)[-3(1 - p) + 1] \\ &= (1 + p)(-2 + 3p) \end{aligned}$$

n) $x^2 - 2x + 1 - y^4$
Solution:

$$\begin{aligned} x^2 - 2x + 1 - y^4 &= x(x - 2) + (1 - y^2)(1 + y^2) \\ &= x(x - 2) + (1 + y)(1 - y)(1 + y^2) \end{aligned}$$

o) $4b(x^3 - 1) + x(1 - x^3)$
Solution:

$$\begin{aligned} 4b(x^3 - 1) + x(1 - x^3) &= (x^3 - 1)(4b - x) \\ &= (x - 1)(x^2 + x + 1)(4b - x) \end{aligned}$$

p) $3m(v-7)+19(-7+v)$
Solution:

$$\begin{aligned}3m(v-7)+19(-7+v) &= 3m(v-7)+19(v-7)\\ &= (v-7)(3m+19)\end{aligned}$$

q) $3f(z+3)+19(3+z)$
Solution:

$$\begin{aligned}3f(z+3)+19(3+z) &= 3f(z+3)+19(z+3)\\ &= (3f+19)(z+3)\end{aligned}$$

r) $3p^3-\frac{1}{9}$
Solution:

$$3p^3-\frac{1}{9} = 3(p-\frac{1}{3})(p^2+\frac{p}{3}+\frac{1}{9})$$

s) $8x^6-125y^9$
Solution:

$$8x^6-125y^9 = (2x^2-5y^3)(4x^4+10x^2y^3+25y^6)$$

t) $(2+p)^3-8(p+1)^3$
Solution:

$$\begin{aligned}(2+p)^3-8(p+1)^3 &= [(p+2)-2(p+1)][(p+2)^2+2(p+2)(p+1)+4(p+1)^2]\\ &= [p+2-2p-2][p^2+4p+4+2p^2+6p+4+4p^2+8p+4]\\ &= (-p)(12+18p+7p^2)\end{aligned}$$

u) $\frac{1}{3}a^3-a^2b+2a^2b-6ab^2+3ab^2-9b^3$
Solution:

$$\begin{aligned}\frac{1}{3}a^3-a^2b+2a^2b-6ab^2+3ab^2-9b^3 &= \frac{1}{3}a^2(a-3b)+2ab(a-3b)+3b^2(a-3b)\\ &= (\frac{1}{3}a^2+2ab+3b^2)(a-3b)\\ &= \frac{(a^2+6ab+9b^2)(a-3b)}{3}\\ &= \frac{(a+3b)^2(a-3b)}{3}\end{aligned}$$

v) $6a^2-17a+5$
Solution:

$$6a^2-17a+5 = (2a-5)(3a-1)$$

w) $s^2+2s-15$
Solution:

$$s^2+2s-15 = (s-3)(s+5)$$

x) $16v+24h+2j^5v+3j^5h$
Solution:

$$\begin{aligned}16v+24h+2j^5v+3j^5h &= 8(2v+3h)+j^5(2v+3h)\\ &= (2v+3h)(8+j^5)\end{aligned}$$

y) $18h - 45g + 2m^3h - 5m^3g$
Solution:

$$\begin{aligned} 18h - 45g + 2m^3h - 5m^3g &= 9(2h - 5g) + m^3(2h - 5g) \\ &= (2h - 5g)(9 + m^3) \end{aligned}$$

z) $63d - 18s + 7u^2d - 2u^2s$
Solution:

$$\begin{aligned} 63d - 18s + 7u^2d - 2u^2s &= 9(7d - 2s) + u^2(7d - 2s) \\ &= (7d - 2s)(9 + u^2) \end{aligned}$$

30. Factorise the following:

a) $6a^2 + 14a + 8$
Solution:

$$\begin{aligned} 6a^2 + 14a + 8 &= 2(3a^2 + 7a + 4) \\ &= 2\,(a + 1)\,(3a + 4) \end{aligned}$$

b) $6g^2 - 15g - 9$
Solution:

$$\begin{aligned} 6g^2 - 15g - 9 &= 3(2g^2 - 5g - 3) \\ &= 3\,(g - 3)\,(2g + 1) \end{aligned}$$

c) $125g^3 - r^3$
Solution:

$$125g^3 - r^3 = (5g - r)(25g^2 + 5gr + r^2)$$

d) $8r^3 + z^3$
Solution:

$$8r^3 + z^3 = (2r + z)(4r^2 - 2rz + z^2)$$

e) $14m - 4n + 7jm - 2jn$
Solution:

$$\begin{aligned} 14m - 4n + 7jm - 2jn &= 2(7m - 2n) + j(7m - 2n) \\ &= (7m - 2n)(2 + j) \end{aligned}$$

f) $25d - 15m + 5yd - 3ym$
Solution:

$$\begin{aligned} 25d - 15m + 5yd - 3ym &= 5(5d - 3m) + y(5d - 3m) \\ &= (5d - 3m)(5 + y) \end{aligned}$$

g) $g^3 - 27$
Solution:

$$g^3 - 27 = (g - 3)(g^2 + 3g + 9)$$

h) $z^3 + 125$
Solution:

$$z^3 + 125 = (z + 5)(z^2 - 5z + 25)$$

i) $b^2 - (3a - 2b)^2$
Solution:

$$\begin{aligned} b^2 - (3a - 2b)^2 &= (b - (3a - 2b))(b + 3a - 2b) \\ &= (3b - 3a)(3a - b) \\ &= 3(b - a)(3a - b) \end{aligned}$$

j) $9y^2 - (4x + 2y)^2$
Solution:

$$\begin{aligned} 9y^2 - (4x + 2y)^2 &= (3y + 4x + 2y)(3y - (4x + 2y)) \\ &= (4x + 5y)(y - 4x) \end{aligned}$$

k) $16x^6 - 3y^8$
Solution:

$$\begin{aligned} 16x^6 - 3y^8 &= 4(4x^6 - 9y^8) \\ &= 4(4x^6 - 9y^8) \\ &= 4(4x^3 - 3y^4)(4x^3 + 3y^4) \end{aligned}$$

l) $\frac{1}{6}a^2 - 24b^4$
Solution:

$$\begin{aligned} \frac{1}{6}a^2 - 24b^4 &= \frac{1}{6}\left(a^2 - 144b^4\right) \\ &= \frac{1}{6}\left(a - 12b^2\right)\left(a + 12b^2\right) \end{aligned}$$

m) $4(a - 3) - 81x^2(a - 3)$
Solution:

$$\begin{aligned} 4(a - 3) - 81x^2(a - 3) &= (a - 3)(4 - 81x^2) \\ &= (a - 3)(2 - 9x)(2 + 9x) \end{aligned}$$

n) $(2 + b)^2 - 11(2 + b) - 12$
Solution:

$$\begin{aligned} (2 + b)^2 - 11(2 + b) - 12 &= ((2 + b) + 1)((2 + b) - 12) \\ &= (b + 3)(b - 10) \end{aligned}$$

o) $2x^2 + 7xy + 5y^2$
Solution:

$$2x^2 + 7xy + 5y^2 = (2x + 5y)(x + y)$$

p) $x^2 - 2xy - 15y^2$
Solution:

$$x^2 - 2xy - 15y^2 = (x - 5y)(x + 3y)$$

q) $4x^4 + 11x^2 + 6$
Solution:

$$4x^4 + 11x^2 + 6 = (4x^2 + 3)(x^2 + 2)$$

r) $6x^4 - 38x^2 + 40$
Solution:

$$\begin{aligned} 6x^4 - 38x^2 + 40 &= 2(3x^4 - 19x^2 + 20) \\ &= 2(3x^2 - 4)(x^2 - 5) \end{aligned}$$

s) $9a^2x + 9a^2y + 27a^2 - b^2x - b^2y - 3b^2$
Solution:

$$\begin{aligned} 9a^2x + 9a^2y + 27a^2 - b^2x - b^2y - 3b^2 &= (9a^2 - b^2)(x + y + 3) \\ &= (3a + b)(3a - b)(x + y + 3) \end{aligned}$$

t) $2(2y^2 - 5y) - 24$
Solution:

$$\begin{aligned} 2(2y^2 - 5y) - 24 &= 2(2y^2 - 5y) - 2(12) \\ &= 2(2y^2 - 5y - 12) \\ &= 2(2y + 3)(y - 4) \end{aligned}$$

u) $\frac{1}{2}x^3 - \frac{9}{2}x - 2x^2 + 18$
Solution:

$$\begin{aligned} \frac{1}{2}x^3 - \frac{9}{2}x - 2x^2 + 18 &= \frac{x^3 - 9x - 4x^2 + 36}{2} \\ &= \frac{x^2(x - 4) - 9(x - 4)}{2} \\ &= \frac{(x - 4)(x^2 - 9)}{2} \\ &= \frac{(x - 4)(x - 3)(x + 3)}{2} \end{aligned}$$

v) $27r^3s^3 - 1$
Solution:

$$27r^3s^3 - 1 = (3rs - 1)(9r^2s^2 + 3rs + 1)$$

w) $\frac{1}{125h^3} + r^3$
Solution:

$$\frac{1}{125h^3} + r^3 = \left(\frac{1}{5h} + r\right)\left(\frac{1}{25h^2} - \frac{r}{5h} + r^2\right)$$

x) $j(64n^3 - b^3) + k(64n^3 - b^3)$
Solution:

$$\begin{aligned} j(64n^3 - b^3) + k(64n^3 - b^3) &= (j + k)(64n^3 - b^3) \\ &= (j + k)(4n - b)(16n^2 + 4nb + b^2) \end{aligned}$$

31. Simplify the following:

a) $(a - 2)^2 - a(a + 4)$
Solution:

$$\begin{aligned} (a - 2)^2 - a(a + 4) &= a^2 - 4a + 4 - a^2 - 4a \\ &= -8a + 4 \end{aligned}$$

b) $(5a-4b)(25a^2+20ab+16b^2)$

Solution:

$$\begin{aligned}(5a-4b)(25a^2+20ab+16b^2) &= 125a^3+100a^2b+80ab^2-100a^2b-80ab^2-64b^3\\ &= 125a^3-64b^3\end{aligned}$$

c) $(2m-3)(4m^2+9)(2m+3)$

Solution:

$$\begin{aligned}(2m-3)(4m^2+9)(2m+3) &= (4m^2-9)(4m^2+9)\\ &= 16m^4-81\end{aligned}$$

d) $(a+2b-c)(a+2b+c)$

Solution:

$$\begin{aligned}(a+2b-c)(a+2b+c) &= a^2+2ab+ac+2ab+4b^2+2bc-ac-2bc-c^2\\ &= a^2+4ab+4b^2-c^2\end{aligned}$$

e) $\dfrac{m^2+11m+18}{4(m^2-4)} \div \dfrac{3m^2+27m}{24m^2-48m}$

Solution:

$$\begin{aligned}\frac{m^2+11m+18}{4(m^2-4)} \div \frac{3m^2+27m}{24m^2-48m} &= \frac{m^2+11m+18}{4(m^2-4)} \times \frac{24m^2-48m}{3m^2+27m}\\ &= \frac{(m+9)(m+2)}{4(m-2)(m+2)} \times \frac{24m(m-2)}{3m(m+9)}\\ &= \frac{1}{4} \times \frac{24}{3}\\ &= 2\end{aligned}$$

f) $\dfrac{t^2+9t+18}{5(t^2-9)} \div \dfrac{4t^2+24t}{100t^2-300t}$

Solution:

$$\begin{aligned}\frac{t^2+9t+18}{5(t^2-9)} \div \frac{4t^2+24t}{100t^2-300t} &= \frac{t^2+9t+18}{5(t^2-9)} \times \frac{100t^2-300t}{4t^2+24t}\\ &= \frac{(t+6)(t+3)}{5(t-3)(t+3)} \times \frac{100t(t-3)}{4t(t+6)}\\ &= \frac{1}{5} \times \frac{100}{4}\\ &= 5\end{aligned}$$

g) $\dfrac{4-b^2}{3b-6}$

Solution:

$$\begin{aligned}\frac{4-b^2}{3b-6} &= \frac{(2-b)(2+b)}{3(b-2)}\\ &= -\frac{2+b}{3}\end{aligned}$$

h) $\dfrac{x^2+2x+4}{x^3-8}$

Solution:

$$\frac{x^2+2x+4}{x^3-8} = \frac{x^2+2x+4}{(x-2)(x^2+2x+4)}$$
$$= \frac{1}{x-2}$$

i) $\dfrac{x^2-5x-14}{3x+6}$

Solution:

$$\frac{x^2-5x-14}{3x+6} = \frac{(x-7)(x+2)}{3(x+2)}$$
$$= \frac{x-7}{3}$$

j) $\dfrac{d^2+23d+132}{5(d^2-121)} \div \dfrac{4d^2+48d}{100d^2-1100d}$

Solution:

$$\frac{d^2+23d+132}{5(d^2-121)} \div \frac{4d^2+48d}{100d^2-1100d} = \frac{d^2+23d+132}{5(d^2-121)} \times \frac{100d^2-1100d}{4d^2+48d}$$
$$= \frac{(d+12)(d+11)}{5(d-11)(d+11)} \times \frac{100d(d-11)}{4d(d+12)}$$
$$= \frac{1}{5} \times \frac{100}{4}$$
$$= 5$$

k) $\dfrac{a-2}{a^2+4a+3} \div \dfrac{(a-1)(a+1)}{a-1} \times \dfrac{a^2-2a-15}{a-2}$

Solution:

$$\frac{a-2}{a^2+4a+3} \div \frac{(a-1)(a+1)}{a-1} \times \frac{a^2-2a-15}{a-2}$$
$$= \frac{a-2}{(a+1)(a+3)} \div \frac{(a-1)(a+1)}{a-1} \times \frac{(a+3)(a-5)}{a-2}$$
$$= \frac{a-2}{(a+1)(a+3)} \times \frac{a-1}{(a-1)(a+1)} \times \frac{(a+3)(a-5)}{a-2}$$
$$= \frac{a-5}{(a+2)^2}$$

l) $\dfrac{a+6}{a^2+12a+11} \times \dfrac{a^2+14a+33}{a+3} \div \dfrac{a^3+216}{a+1}$

Solution:

$$\frac{a+6}{a^2+12a+11} \times \frac{a^2+14a+33}{a+3} \div \frac{a^3+216}{a+1}$$
$$= \frac{a+6}{(a+11)(a+1)} \times \frac{(a+11)(a+3)}{a+3} \times \frac{a+1}{(a+6)(a^2+6a+36)}$$
$$= \frac{1}{a^2+6a+36}$$

m) $2 \div \dfrac{a+b}{a+2b} \times \dfrac{b^2-ba-6a^2}{a^2-4b^2} \times \dfrac{a^2-b-2b^2}{3a-b}$

Solution:

$$2 \div \frac{a+b}{a+2b} \times \frac{b^2-ba-6a^2}{a^2-4b^2} \times \frac{a^2-b-2b^2}{3a-b}$$
$$= 2 \times \frac{a+2b}{a+b} \times \frac{(b-3a)(b+2a)}{(a-2b)(a+2b)} \times \frac{(a-2b)(a+b)}{3a-b}$$
$$= -2(2a+b)$$

n) $\dfrac{st+sb+31t+31b}{t+b}$

Solution:

$$\begin{aligned}\frac{st+sb+31t+31b}{(t+b)} &= \frac{s(t+b)+31(t+b)}{(t+b)}\\ &= \frac{(t+b)(s+31)}{(t+b)}\\ &= s+31\end{aligned}$$

o) $\dfrac{ny+nq+8y+8q}{y+q}$

Solution:

$$\begin{aligned}\frac{ny+nq+8y+8q}{(y+q)} &= \frac{n(y+q)+8(y+q)}{(y+q)}\\ &= \frac{(y+q)(n+8)}{(y+q)}\\ &= n+8\end{aligned}$$

p) $\dfrac{p^2-q^2}{p} \div \dfrac{p+q}{p^2-pq}$

Solution:

$$\begin{aligned}\frac{p^2-q^2}{p} \div \frac{p+q}{p^2-pq} &= \frac{(p-q)(p+q)}{p} \times \frac{p(p-q)}{p+q}\\ &= (p-q)^2\\ &= p^2-2pq+q^2\end{aligned}$$

q) $\dfrac{2}{x}+\dfrac{x}{2}-\dfrac{2x}{3}$

Solution:

$$\begin{aligned}\frac{2}{x}+\frac{x}{2}-\frac{2x}{3} &= \frac{12+3x^2-4x^2}{6x}\\ &= \frac{12-x^2}{6x}\end{aligned}$$

r) $\dfrac{1}{a+7}-\dfrac{a+7}{a^2-49}$

Solution:

$$\begin{aligned}\frac{1}{a+7}-\frac{a+7}{a^2-49} &= \frac{1}{a+7}-\frac{a+7}{(a+7)(a-7)}\\ &= \frac{-14}{(a+7)(a-7)}\end{aligned}$$

s) $\dfrac{x+2}{2x^3}+16$

Solution:

$$\begin{aligned}\frac{x+2}{2x^3}+16 &= \frac{(x+2)+16(2x^3)}{2x^3}\\ &= \frac{32x^3+x+2}{2x^3}\end{aligned}$$

t) $\dfrac{1-2a}{4a^2-1} - \dfrac{a-1}{2a^2-3a+1} - \dfrac{1}{1-a}$

Solution:

$$\begin{aligned}\frac{1-2a}{4a^2-1} - \frac{a-1}{2a^2-3a+1} - \frac{1}{1-a} &= \frac{1-2a}{(2a-1)(2a+1)} - \frac{a-1}{(2a-1)(a-1)} + \frac{1}{a-1} \\ &= -\frac{(2a-1)}{(2a-1)(2a+1)} - \frac{1}{2a-1} + \frac{1}{a-1} \\ &= \frac{4a-1}{(2a+1)(2a-1)(a-1)}\end{aligned}$$

u) $\dfrac{1}{2}x + \dfrac{x-2}{3} + 4$

Solution:

$$\begin{aligned}\frac{1}{2}x + \frac{x-2}{3} + 4 &= \frac{3x+2(x-2)+(2)(3)(4)}{6} \\ &= \frac{3x+2x-4+24}{6} \\ &= \frac{5x+20}{6}\end{aligned}$$

v) $\dfrac{1}{x^2+2x} + \dfrac{4x^2-x-3}{x^2+2x-3}$

Solution:

$$\begin{aligned}\frac{1}{x^2+2x} + \frac{4x^2-x-3}{x^2+2x-3} &= \frac{1}{x(x+2)} + \frac{(4x+3)(x-1)}{(x-1)(x+3)} \\ &= \frac{1}{x(x+2)} + \frac{4x+3}{x+3} \\ &= \frac{x+3+x(4x+3)(x+2)}{x(x+2)(x+3)} \\ &= \frac{x+3+x(4x^2+11x+6)}{x(x+2)(x+3)} \\ &= \frac{4x^3+11x^2+7x+3}{x(x+2)(x+3)}\end{aligned}$$

w) $\dfrac{b^2+6b+9}{b^2-9} + \dfrac{b^2-6b+8}{(b-2)(b+3)} + \dfrac{1}{b+3}$

Solution:

$$\begin{aligned}\frac{b^2+6b+9}{b^2-9} + \frac{b^2-6b+8}{(b-2)(b+3)} + \frac{1}{b+3} &= \frac{(b+3)^2}{(b+3)(b-3)} + \frac{(b-4)(b-2)}{(b-2)(b+3)} + \frac{1}{b+3} \\ &= \frac{b+3}{b-3} + \frac{b-4}{b+3} + \frac{1}{b+3} \\ &= \frac{(b+3)^2+(b-3)(b-4)+b-3}{(b-3)(b+3)} \\ &= \frac{b^2+6b+9+b^2-7b+12+b-3}{(b-3)(b+3)} \\ &= \frac{2b^2+18}{(b-3)(b+3)} \\ &= \frac{2(b^2+9)}{(b-3)(b+3)}\end{aligned}$$

x) $\dfrac{x^2+2x}{x^2+x+6} \times \dfrac{x^2+2x+1}{x^2+3x+2}$

Solution:

$$\frac{x^2+2x}{x^2+x+6} \times \frac{x^2+2x+1}{x^2+3x+2} = \frac{x(x+2)}{x^2+x+6} \times \frac{(x+1)(x+1)}{(x+2)(x+1)}$$
$$= \frac{x(x+1)}{x^2+x+6}$$

y) $\frac{12}{z+12} + \frac{5}{z-5}$

Solution:

$$\frac{12}{z+12} + \frac{5}{z-5} = \frac{12(z-5)+5(z+12)}{(z+12)(z-5)}$$
$$= \frac{12z-60+5z+60}{(z+12)(z-5)}$$
$$= \frac{17z}{(z+12)(z-5)}$$

z) $\frac{11}{w-11} - \frac{4}{w-4}$

Solution:

$$\frac{11}{w-11} - \frac{4}{w-4}$$
$$= \frac{11(w-4)-4(w-11)}{(w-11)(w-4)}$$
$$= \frac{11w-44-4w+44}{(w-11)(w-4)}$$
$$= \frac{7w}{(w-11)(w-4)}$$

32. Show that $(2x-1)^2-(x-3)^2$ can be simplified to $(x+2)(3x-4)$.

Solution:

$$(2x-1)^2-(x-3)^2 = (2x-1)(2x-1)-(x-3)(x-3)$$
$$= 4x^2-2x-2x+1-(x^2-3x-3x-9)$$
$$= 3x^2+2x-8$$
$$= (3x-4)(x+2)$$

33. What must be added to x^2-x+4 to make it equal to $(x+2)^2$?

Solution:

Suppose A must be added to the expression to get the desired result.

$$\therefore (x^2-x+4)+A = (x+2)^2$$
$$\therefore A = (x+2)(x+2)-(x^2-x+4)$$
$$= x^2+2x+2x+4-x^2+x-4$$
$$= 5x$$

Therefore $5x$ must be added.

34. Evaluate $\frac{x^3+1}{x^2-x+1}$ if $x = 7{,}85$ without using a calculator. Show your work.

Solution:

First simplify the expression:

$$\frac{x^3+1}{x^2-x+1} = \frac{(x+1)(x^2-x+1)}{x^2-x+1}$$
$$= x+1$$

Now substitute the value of x: $7{,}85+1 = 8{,}85$.

35. With what expression must $(a - 2b)$ be multiplied to get a product of $(a^3 - 8b^3)$?
Solution:

$$(a - 2b)(a^2 + 2ab + 4b^2) = a^3 - 8b^3$$

So, the expression is $a^2 + 2ab + 4b^2$.

36. With what expression must $27x^3 + 1$ be divided to get a quotient of $3x + 1$?
Solution:

$$27x^3 + 1 = (3x + 1)(9x^2 - 3x + 1)$$
$$\frac{(3x + 1)(9x^2 - 3x + 1)}{9x^2 - 3x + 1} = 3x + 1$$

Therefore the expression is $9x^2 - 3x + 1$.

37. What are the restrictions on the following?

a) $\dfrac{4}{3x^2 + 2x - 1}$
Solution:

$$\frac{4}{3x^2 + 2x - 1} = \frac{4}{(3x - 1)(x + 1)}$$
$$x \neq \frac{1}{3} \text{ and } x \neq -1$$

b) $\dfrac{a}{3(b - a) + ab - a^2}$
Solution:

$$\frac{a}{3(b - a) + ab - a^2} = \frac{a}{3(b - a) + a(b - a)}$$
$$= \frac{a}{(b - a)(a + 3)}$$
$$a \neq b \text{ and } a \neq -3$$

For more exercises, visit www.everythingmaths.co.za and click on 'Practise Maths'.

1. 2DR7	2a. 2DR8	2b. 2DR9	2c. 2DRB	2d. 2DRC	2e. 2DRD	2f. 2DRF	3a. 2DRG	
3b. 2DRH	3c. 2DRJ	3d. 2DRK	4. 2DRM	5. 2DRN	6a. 2DRP	6b. 2DRQ	6c. 2DRR	
6d. 2DRS	6e. 2DRT	6f. 2DRV	6g. 2DRW	7. 2DRX	8a. 2DRY	8b. 2DRZ	9. 2DS2	
10a. 2DS3	10b. 2DS4	11a. 2DS5	11b. 2DS6	11c. 2DS7	11d. 2DS8	12a. 2DS9	12b. 2DSB	
12c. 2DSC	12d. 2DSD	13. 2DSF	14. 2DSG	15. 2DSH	16a. 2DSJ	16b. 2DSK	16c. 2DSM	
16d. 2DSN	17a. 2DSP	17b. 2DSQ	17c. 2DSR	17d. 2DSS	17e. 2DST	17f. 2DSV	17g. 2DSW	
17h. 2DSX	17i. 2DSY	17j. 2DSZ	18a. 2DT2	18b. 2DT3	19a. 2DT4	19b. 2DT5	19c. 2DT6	
19d. 2DT7	20a. 2DT8	20b. 2DT9	20c. 2DTB	20d. 2DTC	20e. 2DTD	20f. 2DTF	20g. 2DTG	
20h. 2DTH	20i. 2DTJ	20j. 2DTK	20k. 2DTM	20l. 2DTN	21a. 2DTP	21b. 2DTQ	21c. 2DTR	
21d. 2DTS	22a. 2DTT	22b. 2DTW	22c. 2DTX	22d. 2DTY	22e. 2DTZ	22f. 2DV2	22g. 2DV3	
22h. 2DV4	22i. 2DV5	22j. 2DV6	22k. 2DV7	22l. 2DV8	22m. 2DV9	22n. 2DVB	22o. 2DVC	
22p. 2DVD	22q. 2DVF	22r. 2DVG	22s. 2DVH	23a. 2DVJ	23b. 2DVK	23c. 2DVM	23d. 2DVN	
23e. 2DVP	23f. 2DVQ	23g. 2DVR	23h. 2DVS	23i. 2DVT	23j. 2DVV	24. 2DVW	25. 2DVX	
26. 2DVY	27a. 2DVZ	27b. 2DW2	27c. 2DW3	27d. 2DW4	28a. 2DW5	28b. 2DW6	28c. 2DW7	
28d. 2DW8	28e. 2DW9	29a. 2DWB	29b. 2DWC	29c. 2DWD	29d. 2DWF	29e. 2DWG	29f. 2DWH	
29g. 2DWJ	29h. 2DWK	29i. 2DWM	29j. 2DWN	29k. 2DWP	29l. 2DWQ	29m. 2DWR	29n. 2DWS	
29o. 2DWT	29p. 2DWV	29q. 2DWW	29r. 2DWX	29s. 2DWY	29t. 2DWZ	29u. 2DX2	29v. 2DX3	
29w. 2DX4	29x. 2DX5	29y. 2DX6	29z. 2DX7	30a. 2DX8	30b. 2DX9	30c. 2DXB	30d. 2DXC	
30e. 2DXD	30f. 2DXF	30g. 2DXG	30h. 2DXH	30i. 2DXJ	30j. 2DXK	30k. 2DXM	30l. 2DXN	
30m. 2DXP	30n. 2DXQ	30o. 2DXR	30p. 2DXS	30q. 2DXT	30r. 2DXV	30s. 2DXW	30t. 2DXX	
30u. 2DXY	30v. 2DXZ	30w. 2DY2	30x. 2DY3	31a. 2DY4	31b. 2DY5	31c. 2DY6	31d. 2DY7	
31e. 2DY8	31f. 2DY9	31g. 2DYB	31h. 2DYC	31i. 2DYD	31j. 2DYF	31k. 2DYG	31l. 2DYH	
31m. 2DYJ	31n. 2DYK	31o. 2DYM	31p. 2DYN	31q. 2DYP	31r. 2DYQ	31s. 2DYR	31t. 2DYS	
31u. 2DYT	31v. 2DYV	31w. 2DYW	31x. 2DYX	31y. 2DYY	31z. 2DYZ	32. 2DZ2	33. 2DZ3	
34. 2DZ4	35. 2DZ5	36. 2DZ6	37a. 2DZ7	37b. 2DZ8				

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