CHAPTER 11

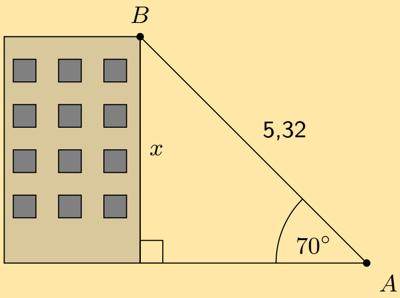
Trigonometry

- This chapter covers solving problems in two-dimensions using trigonometry.
- Emphasise the value and importance of making sketches, where appropriate.
- Prior to starting this chapter it may be appropriate to quickly revise the earlier content on trigonometry.

11.1 Two-dimensional problems

Exercise 11 – 1:

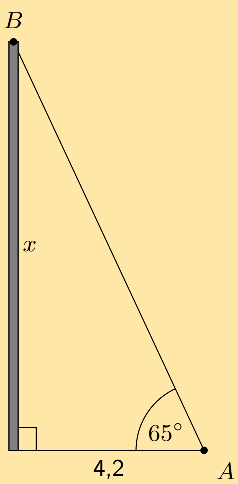
1. A person stands at point A, looking up at a bird sitting on the top of a building, point B.
   The height of the building is x meters, the line of sight distance from point A to the top of the building (point B) is 5,32 meters, and the angle of elevation to the top of the building is $70°$.
   Calculate the height of the building (x) as shown in the diagram below:

   Solution:

   $$\begin{aligned} \sin\theta &= \frac{\text{opposite}}{\text{hypotenuse}} \\ \sin 70° &= \frac{x}{5{,}32} \\ x &= 4{,}99916... \\ &\approx 5 \end{aligned}$$

   The height of the building is 5 m.
2. A person stands at point A, looking up at a bird sitting on the top of a pole (point B).
   The height of the pole is x meters, point A is 4,2 meters away from the foot of the pole, and the angle of elevation to the top of the pole is $65°$.
   Calculate the height of the pole (x), to the nearest metre.

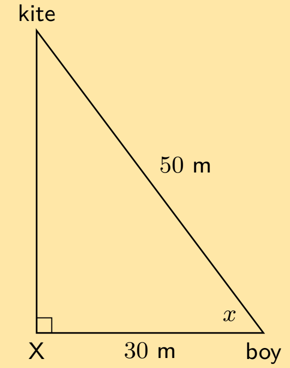
   

Solution:

$$\begin{aligned}\tan\theta &= \frac{\text{opposite}}{\text{adjacent}}\\ \tan 65^\circ &= \frac{x}{4,2}\\ x &= 9,0069...\\ &\approx 9\end{aligned}$$

The height of the pole is 9 m.

3. A boy flying a kite is standing 30 m from a point directly under the kite. If the kite's string is 50 m long, find the angle of elevation of the kite.
   Solution:
   First draw a sketch:

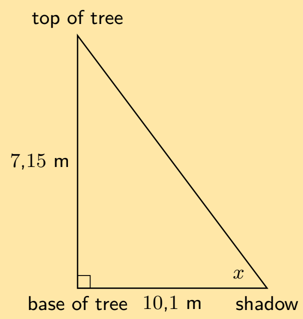


We can use the cosine ratio to find the angle of elevation (x):

$$\begin{aligned}\cos x &= \frac{30}{50}\\ x &= 53,1301...\\ &\approx 53,13^\circ\end{aligned}$$

The angle of elevation of the kite is 53,13°.

4. What is the angle of elevation of the sun when a tree 7,15 m tall casts a shadow 10,1 m long?
   Solution:
   First draw a sketch:

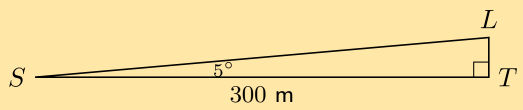


We can use the tangent ratio to find the angle of elevation (x):

$$\begin{aligned}\tan x &= \frac{7,15}{10,1}\\ x &= 35,2954...\\ &\approx 35,30^\circ\end{aligned}$$

The angle of elevation of the sun is 35,30°.

5. From a distance of 300 m, Susan looks up at the top of a lighthouse. The angle of elevation is 5°. Determine the height of the lighthouse to the nearest metre.
   Solution:
   First draw a sketch:

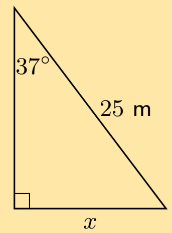


   We need to find LT. We can use the tangent ratio:

$$\begin{aligned}\tan \hat{S} &= \frac{LT}{ST} \\ LT &= 300 \tan 5° \\ &= 26{,}2465... \\ &\approx 26 \text{ m}\end{aligned}$$

   The height of the lighthouse is 26 m.

6. A ladder of length 25 m is resting against a wall, the ladder makes an angle 37° to the wall. Find the distance between the wall and the base of the ladder to the nearest metre.
   Solution:
   First draw a sketch:



   Notice that we are given the angle that the ladder makes with the wall, not the angle that the ladder makes with the ground.
   Now we can use the sine function to find x:

$$\begin{aligned}\sin 37° &= \frac{x}{25} \\ x &= 25 \sin 37° \\ &= 15{,}04537... \\ &\approx 15 \text{ m}\end{aligned}$$

   The base of the ladder is 15 m away from the wall.

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1. 2GPN 2. 2GPP 3. 2GPQ 4. 2GPR 5. 2GPS 6. 2GPT

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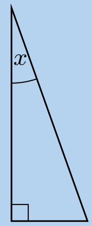
11.2 Chapter summary

End of chapter Exercise 11 – 2:

1. A ladder of length 15 m is resting against a wall, the base of the ladder is 5 m from the wall. Find the angle between the wall and the ladder.

Solution:
First draw a sketch:



Notice that we want to find the angle that the ladder makes with the wall, not the angle that the ladder makes with the ground.

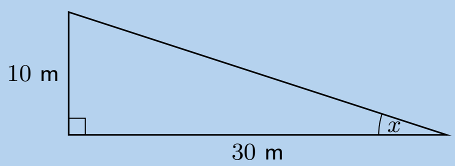
Now we use $\sin x = \frac{\text{opposite}}{\text{hypotenuse}}$:

$$\begin{aligned}
\sin x &= \frac{5}{15} \\
&= 0{,}3333... \\
x &= 19{,}4712... \\
&\approx 19{,}47°
\end{aligned}$$

The angle between the ladder and the wall is 19,47°.

2. Captain Jack is sailing towards a cliff with a height of 10 m.

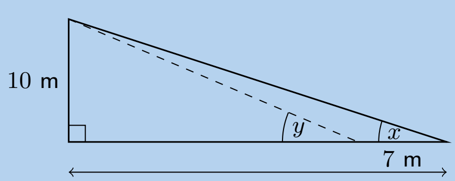
   a) The distance from the boat to the bottom of the cliff is 30 m. Calculate the angle of elevation from the boat to the top of the cliff (correct to the nearest degree).
   Solution:
   First draw a sketch:



$$\begin{aligned}
\tan x &= \frac{\text{opposite}}{\text{hypotenuse}} \\
&= \frac{10}{30} \\
x &= 18{,}4349...° \\
&= 18°
\end{aligned}$$

   The angle of elevation is 18°.

   b) If the boat sails 7 m closer to the cliff, what is the new angle of elevation from the boat to the top of the cliff?
   Solution:
   We redraw the sketch with the new information:

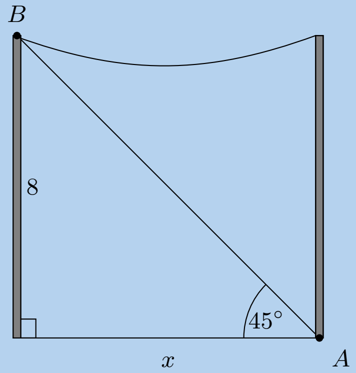


   The new distance from the boat to the cliffs is 30 m − 7 m = 23 m. The height of the cliffs has not changed.

$$\tan y = \frac{\text{opposite}}{\text{adjacent}}$$
$$= \frac{10}{23}$$
$$y = 23{,}49856...^\circ$$
$$= 23^\circ$$

The new angle of elevation is 23°.

3. Jim stands at point A at the base of a telephone pole, looking up at a bird sitting on the top of another telephone pole (point B).
The height of each of the telephone poles is 8 meters, and the angle of elevation from A to the top of B is 45°.
Calculate the distance between the telephone poles (x) as shown in the diagram below:

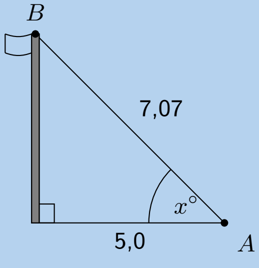


Solution:

$$\tan x = \frac{\text{opposite}}{\text{adjacent}}$$
$$\tan 45^\circ = \frac{8}{x}$$
$$x = \frac{8}{\tan 45^\circ}$$
$$= 8$$

The distance between the telephone poles is 8 m.

4. Alfred stands at point A, looking up at a flag on a pole (point B).
Point A is 5,0 meters away from the bottom of the flag pole, the line of sight distance from point A to the top of the flag pole (point B) is 7,07 meters, and the angle of elevation to the top of the flag pole is x°.
Calculate the angle of elevation to the top of the flag pole (x) as shown in the diagram below:



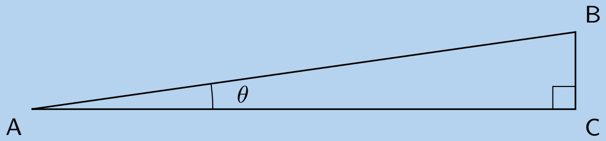
Solution:

$$\cos x = \frac{\text{adjacent}}{\text{hypotenuse}}$$
$$\cos x^\circ = \frac{5{,}0}{7{,}07}$$
$$x = 44{,}9913\ldots$$
$$\approx 44{,}99^\circ$$

The angle of elevation is $44{,}99^\circ$.

5. A rugby player is trying to kick a ball through the poles. The rugby crossbar is 3,4 m high. The ball is placed 24 m from the poles. What is the minimum angle he needs to launch the ball to get it over the bar?

Solution:

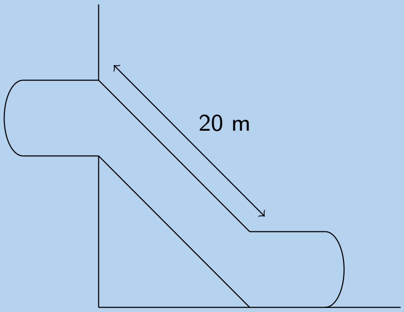


CA is the distance from the poles, 24 m; BC is the crossbar height 3,4 m. The minimum angle is the angle of elevation.

$$\tan\theta = \frac{BC}{CA}$$
$$= \frac{3{,}4}{24}$$
$$\theta = 8{,}0632\ldots$$
$$\approx 8^\circ$$

Therefore he needs to kick the ball with a minimum angle of 8°.

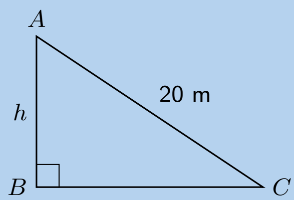
6. The escalator at a mall slopes at an angle of 30° and is 20 m long.



Through what height would a person be lifted by travelling on the escalator?

Solution:

We note that we have the following right-angled triangle:



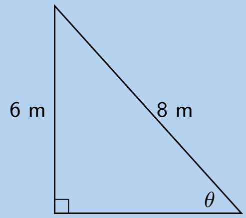
We can use the sine ratio to find the height:

$$\sin 30^\circ = \frac{h}{20}$$
$$h = 20\sin 30^\circ$$
$$= 10 \text{ m}$$

A person travelling on the escalator would be lifted through a height of 10 m.

7. A ladder is 8 metres long. It is leaning against the wall of a house and reaches 6 metres up the wall.

   a) Draw a sketch of the situation.
   Solution:



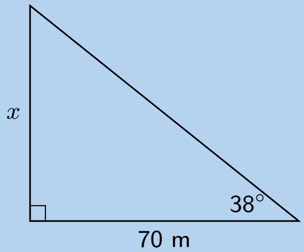
   b) Calculate the angle which the ladder makes with the flat (level) ground.
   Solution:

$$\begin{aligned}\sin\theta &= \frac{6}{8}\\ \theta &= 48{,}5903...\\ \theta &\approx 48{,}59°\end{aligned}$$

   The ladder makes an angle of 48,59° with the ground.

8. Nandi is standing on level ground 70 metres away from a tall tower. From her position, the angle of elevation of the top of the tower is 38°.

   a) Draw a sketch of the situation.
   Solution:

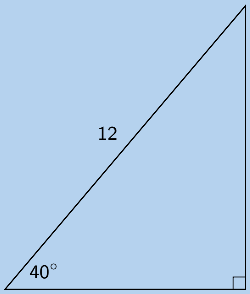


   b) What is the height of the tower?
   Solution:
   We can use the tangent ratio to find the height of the tower.

$$\begin{aligned}\tan 38 &= \frac{x}{70}\\ x &= 70\tan 38\\ &= 54{,}6899...\\ &\approx 54{,}69 \text{ m}\end{aligned}$$

   The height of the tower is 54,69 m.

9. The top of a pole is anchored by a 12 m cable which makes an angle of 40 degrees with the horizontal. What is the height of the pole?

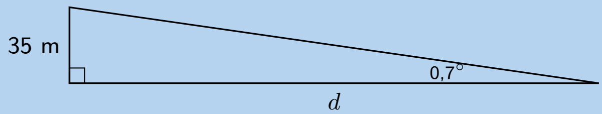


Solution:

$$\begin{aligned}\sin 40 &= \frac{h}{12}\\ h &= 12\sin 40\\ &= 7{,}713...\\ &\approx 7{,}71 \text{ m}\end{aligned}$$

The height of the pole is 7,71 m.

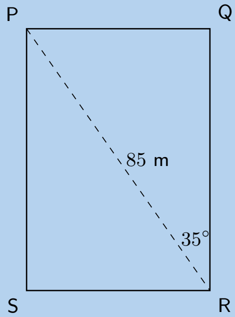
10. A ship's navigator observes a lighthouse on a cliff. According to the navigational charts the top of the lighthouse is 35 metres above sea level. She measures the angle of elevation of the top of the lighthouse to be 0,7°.
Ships have been advised to stay at least 4 km away from the shore. Is the ship safe?
Solution:
First draw a diagram:



$$\begin{aligned}\tan 0{,}7 &= \frac{35}{d}\\ d &= \frac{35}{\tan 0{,}7}\\ &= 2864{,}6464...\\ &\approx 2864{,}65 \text{ m}\\ &\approx 2{,}86 \text{ km}\end{aligned}$$

Therefore the ship is not safe.

11. Determine the perimeter of rectangle $PQRS$:



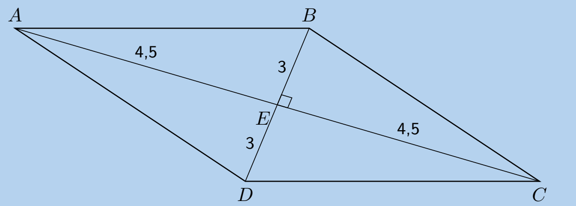
Solution:
Using trigonometric ratios we can calculate QR and PQ.

$$\begin{aligned}QR &= 85\cos 35\\ PQ &= 85\sin 35\end{aligned}$$

$$\begin{aligned}P &= 2\times(h+b)\\ &= 2(85\cos 35 + 85\sin 35)\\ &= 2(85(\cos 35+\sin 35))\\ &= 2(236{,}76 \text{ m})\\ &= 473{,}52 \text{ m}\end{aligned}$$

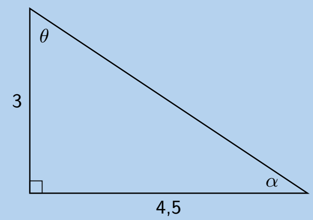
Therefore the perimeter is 473,52 m.

12. A rhombus has diagonals of lengths 6 cm and 9 cm. Calculate the sizes of its interior vertex angles.



Solution:

There are four small right-angled triangles in the rhombus: $\triangle ABE$, $\triangle BEC$, $\triangle CED$ and $\triangle DEA$. Since the diagonals bisect each other and we are given the lengths of the diagonals, we know the lengths of the sides of the triangles:



We can calculate the two angles:

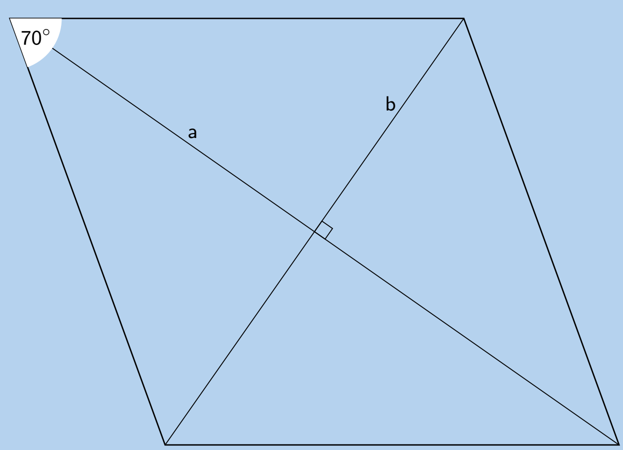
$$\begin{aligned} \tan\theta &= \frac{4{,}5}{3} \\ &= 41{,}9872... \\ \theta &\approx 53{,}31° \end{aligned}$$

$$\begin{aligned} \tan\alpha &= \frac{3}{4{,}5} \\ &= 33{,}6900... \\ \alpha &\approx 33{,}69° \end{aligned}$$

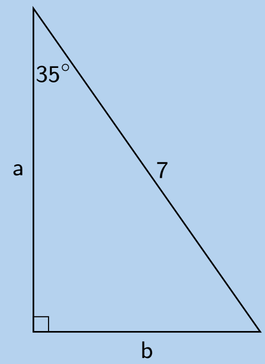
Now we note that there are two different interior angles. One of these angles is 2α and the other is 2θ.

Therefore the two angles are 106,62° and 67,38°

13. A rhombus has edge lengths of 7 cm. Its acute interior vertex angles are both 70°. Calculate the lengths of both of its diagonals.



Solution:

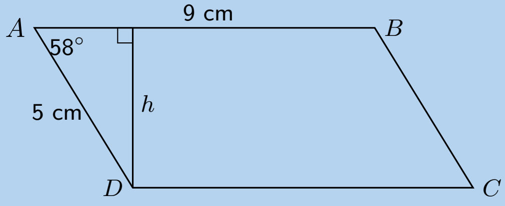


$$\begin{aligned} \cos 35° &= \frac{a}{7} \\ a &= 7\cos 35° \\ &= 5{,}734\ldots \\ \text{diagonal 1} &= 11{,}47 \text{ cm} \end{aligned}$$

$$\begin{aligned} \sin 35° &= \frac{b}{7} \\ b &= 7\sin 35° \\ &= 4{,}015\ldots \\ \text{diagonal 2} &= 8{,}03 \text{ cm} \end{aligned}$$

Therefore the two diagonals are 11,47 cm and 8,03 cm

14. A parallelogram has edge-lengths of 5 cm and 9 cm respectively, and an angle of 58° between them. Calculate the perpendicular distance between the two longer edges.



Solution:

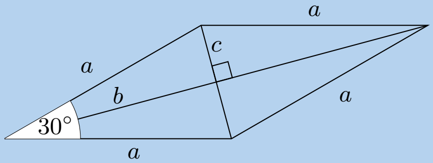
$$\begin{aligned} \sin 58° &= \frac{h}{5} \\ h &= 5\sin 58° \\ &= 4{,}24 \text{ cm} \end{aligned}$$

15. One of the angles of a rhombus with perimeter 20 cm is 30°.

    a) Find the lengths of the sides of the rhombus.

    Solution:

    First draw a sketch:



The perimeter is found by adding each side together. All the sides are equal in length, therefore the perimeter $= 4a$.

$$\begin{aligned} 20 &= 4a \\ a &= 5 \end{aligned}$$

Therefore the sides are all 5 cm in length.

b) Find the length of both diagonals.

Solution:

The diagonals of a rhombus bisect the angle, so working in one of the small triangles we can use trigonometric ratios to find b:

$$\begin{aligned} \cos 15° &= \frac{b}{5} \\ b &= 5\cos 15° \\ &= 4{,}83 \end{aligned}$$

By Pythagoras $c^2 = a^2 - b^2$:

$$\begin{aligned} c^2 &= (5)^2 - (4{,}83)^2 \\ &= 25 - 23{,}33 \\ &= 1{,}67 \\ c &= 1{,}29 \end{aligned}$$

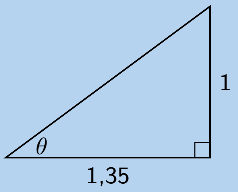
Since the diagonals bisect each other we know that the total length of each diagonal is either $2b$ or $2c$, depending on which diagonal we examine.

The one diagonal is $2(4{,}83) = 9{,}66$ cm and the other diagonal is $2(1{,}29) = 2{,}58$ cm.

16. Upright sticks and the shadows they cast can be used to judge the sun's altitude in the sky (the angle the sun makes with the horizontal) and the heights of objects.

a) An upright stick, 1 metre tall, casts a shadow which is 1,35 metres long. What is the altitude of the sun?

Solution:

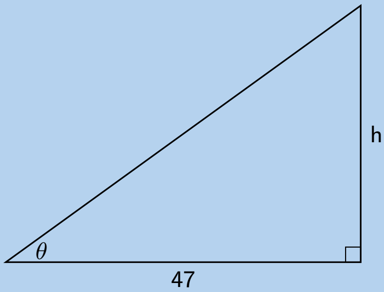


$$\begin{aligned} \tan\theta &= \frac{1}{1{,}35} \\ \theta &= 36{,}53° \end{aligned}$$

b) At the same time, the shadow of a building is found to be 47 metres long. What is the height of the building?

Solution:

We know the angle that the sun makes with the horizontal and now we can use that to find the height of the building.



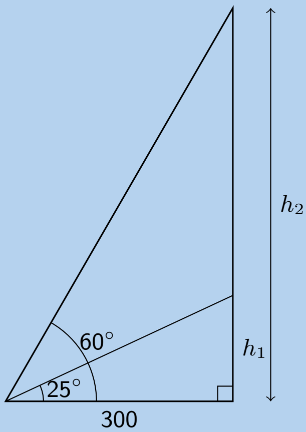
In the figure above θ is the angle that the sun makes with the horizontal.

$$\begin{aligned}\tan 36{,}53° &= \frac{h}{47}\\ h &= 47 \tan 36{,}53°\\ &= 34{,}82 \text{ m}\end{aligned}$$

17. The angle of elevation of a hot air balloon, climbing vertically, changes from 25 degrees at 11:00 am to 60 degrees at 11:02 am. The point of observation of the angle of elevation is situated 300 metres away from the take off point.

a) Draw a sketch of the situation.

Solution:



b) Calculate the increase in height between 11:00 am and 11:02 am.

Solution:

$$\begin{aligned}\tan 25° &= \frac{h_1}{300}\\ h_1 &= 300 \tan 25°\\ &= 139{,}89 \text{ m}\end{aligned}$$

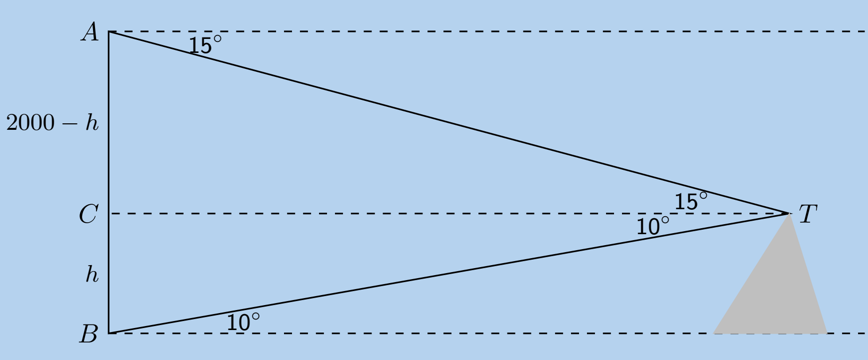
$$\begin{aligned}\tan 60° &= \frac{h_2}{300}\\ h_2 &= 300 \tan 60°\\ &= 519{,}62 \text{ m}\end{aligned}$$

The difference is:

$$519{,}62 \text{ m} - 129{,}89 \text{ m} = 379{,}73 \text{ m}$$

18. When the top, T, of a mountain is viewed from point A, 2000 m from the ground, the angle of depression (a) is equal to 15°. When it is viewed from point B on the ground, the angle of elevation (b) is equal to 10°. If the points A and B are on the same vertical line, find the height, h, of the mountain. Round your answer to one decimal place.

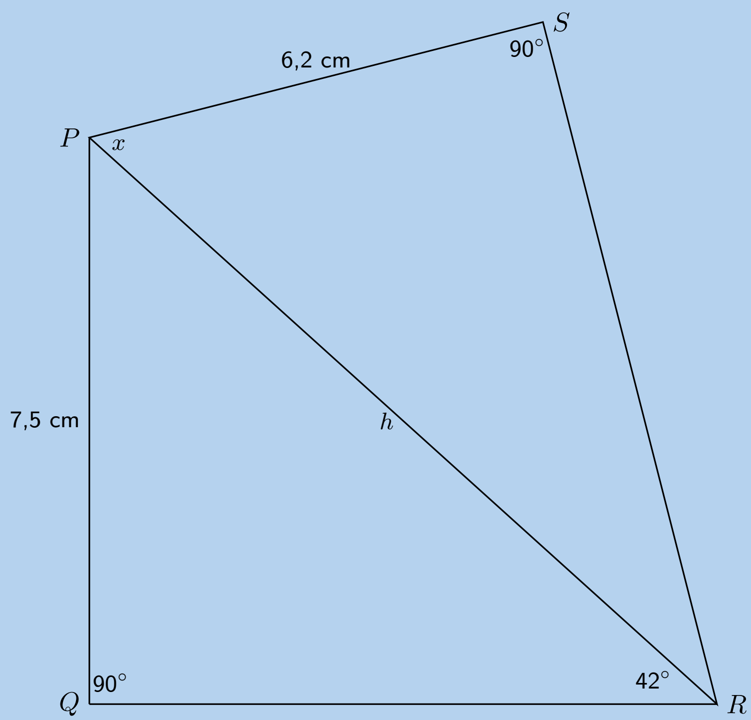
Solution:



$$\tan 10° = \frac{h}{CT}$$
$$CT = \frac{h}{\tan 10°}$$
$$\tan 15° = \frac{2000 - h}{CT}$$
$$CT = \frac{2000 - h}{\tan 15°}$$

$$\therefore \frac{h}{\tan 10°} = \frac{2000 - h}{\tan 15°}$$
$$h = 793{,}77 \text{ m}$$

19. The diagram below shows quadrilateral $PQRS$, with $PQ = 7{,}5$ cm, $PS = 6{,}2$ cm, angle $R = 42°$ and angles S and Q are right angles.



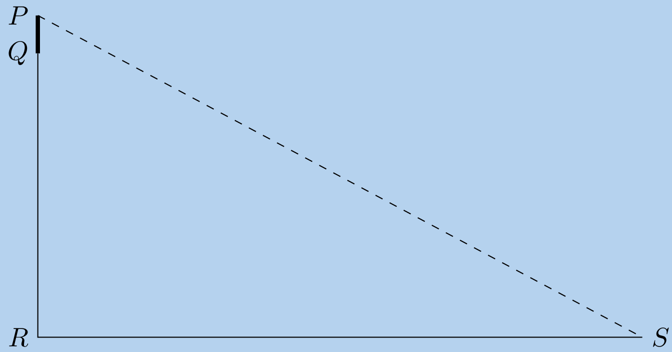
a) Find PR, correct to 2 decimal places.
Solution:

$$\frac{7{,}5}{PR} = \sin 42°$$
$$\frac{7{,}5}{\sin 42°} = PR$$
$$\therefore PR = 11{,}21 \text{ cm}$$

b) Find the size of the angle marked x, correct to one decimal place.
Solution:

$$\cos x = \frac{6{,}2}{11{,}21}$$
$$\therefore x = 56{,}4°$$

20. From a boat at sea (S), the angle of elevation of the top of a lighthouse PQ, on a cliff QR, is 27°.
The lighthouse is 10 m high and the cliff top is 75 m above sea level.
How far is the boat from the base of the cliff, to the nearest metre?
Solution:
First draw a sketch:



The distance PR is equal to the height of the lighthouse, PQ, plus the height of the cliffs, QR.

$$\frac{85}{RS} = \tan 27°$$
$$\frac{85}{\tan 27°} = RS$$
$$\therefore RS = 167 \text{ m}$$

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1. 2GPW	2. 2GPX	3. 2GPY	4. 2GPZ	5. 2GQ2	6. 2GQ3
7. 2GQ4	8. 2GQ5	9. 2GQ6	10. 2GQ7	11. 2GQ8	12. 2GQ9
13. 2GQB	14. 2GQC	15. 2GQD	16. 2GQF	17. 2GQG	18. 2GQH
19. 2GQJ	20. 2GQK				

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