STRAND 3 → NUMBERS

HISTORY OF INDICES

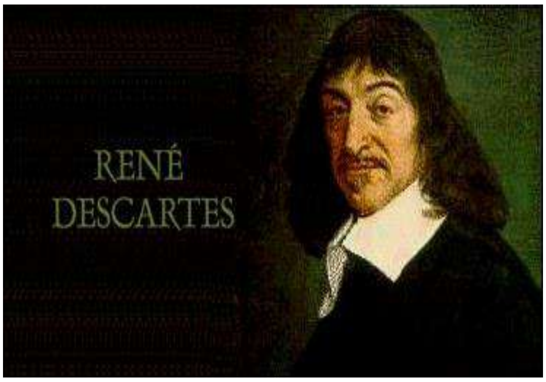
*The word **exponere** (exponent) originated from Latin, **expo**, meaning **out of**, and **ponere**, meaning **place.** While the word **exponent** came to mean different things, the first recorded modern use of exponent in mathematics was in a book called "Arithemetica Integra," written in 1544 by English author and mathematician Michael Stifel. But he worked only with a base of two, so the exponent 3 would mean the number of 2s you would need to multiply to get 8. It would look like this $2^3=8$. The way Stifel would say it is kind of backwards when compared to the way we think about it today. He would say "3 is the 'setting out' of 8." Today, we would refer the equation simply as 2 cubed. He was working exclusively with a base or factor of 2 and translating from Latin a little more literally than we do today.*

The idea of squaring or cubing goes all the way back to Babylonian times, part of Mesopotamia now Iraq. The earliest mention of Babylon was found on a tablet dating to the 23rd Century BC. And they were using the concept of exponents even then, although their numbering system used symbols to denote mathematical formulas.

What the Earliest Exponents Looked Like

The word "raised" is usually omitted, and very often "power" as well, so 3^5 is typically pronounced "three to the fifth" or "three to the five". The exponentiation b^n can be read as b raised to the n-th power, or b raised to the power of n, or b raised by the exponent of n, or most briefly as b to the n.

The modern notation for exponentiation was introduced by René Descartes in his Géométrie (Geometry) of 1637



Source: http://en.wikipedia.org/wiki/Exponentiation

Source: http://www.ehow.com/about_5134780_history-exponents.html

3.1 Expressing Numbers in Indices Form

LEARNING OUTCOMES

Students should be able to:

- To introduce indices
- To write numbers in the base index form and vice versa.

Activity:

A long time ago in the Duavata Kingdom lived a beautiful and hardworking girl. She was working for a King who was the strict ruler of Duavata Kingdom. The King was really appreciative of the work being done by the girl that he summoned her to his office to offer a reward. Below is the conversation that took place between the King and the girl.

King: “Girl, I am going to give you anything you want from my Kingdom. Please tell me what is it you really want me to give you”.

Girl: “My Lord, I come from a very poor family. My requests are as follows: Today- 1 gold coin, 1st day of work tomorrow-2 gold coins, 2nd day-4 gold coins, 3rd day-8 gold coins, 4th day-16 gold coins and to continue for 30days”.

King: “Is that all? You must be joking! You mean to tell me that that is enough for me to reward you for all the work that you have been doing for me. Fine, may your request be granted. Here is the 1st gold. My advisors will do the calculations and you will come and collect your rewards on a daily basis as you have requested”.

The King called his advisors who then made all the calculations. After receiving the breakdown for each day until the 30th day, the King fainted.

Source: https://www.google.com/search?q=google&ie=utf-8&oe=utf 8#q=images+of+a+cartoon+king+and+his+money

Questions:

1. How many gold coins would the girl receive on the 5th day of work?
2. Why did the King faint? [Clue: Do all the calculations for 30 days?

Solutions:

1. 1st day: no work done= 1 gold coin[$2^0 = 1$], 1st day: after work = 2 gold coins[$2^1 = 2$], 2nd day: after work= 4 gold coins[$2^2 = 4$], 3rd day: after work= 8 coins[$2^3 = 8$], 4th day: after work= 16 gold coins [$2^4 = 16$], 5th day: after work= 32 coins [$2^5 = 32$]

2. The King fainted because of the amount of gold coins that he has to give after the 30days beginning from the day of their conversation i.e. 1,073,741,823 gold coins

The girl used the concept of base index to formulate her request!!!!!! A very smart girl.

Index

Base

$$x^4 = \overbrace{x \times x \times x \times x}^{\text{expanded form}}$$

x^4 Is called the power of x

Exponent: another name for index

Example 3. 1

Write the following in base-index form

a. $c \times c \times c \times c$ b. $4 \times 4 \times 4 + 5 \times 5$

Solution:

a. $c \times c \times c \times c = c^4$ [4 factors so the index is 4]

b. $4 \times 4 \times 4 + 5 \times 5 = 4^3 + 5^2$

Example 3.2

Write in expanded form:

a. $2h^5$ b. $3g^5 + 4f^2$

Solution:

a. $2h^5 = 2 \times h \times h \times h \times h \times h$

b. $3g^5 + 4f^2 = 3 \times g \times g \times g \times g \times g + 4 \times f \times f$

Exercise 3.1

1. Write in base-index form

a. $q \times q \times q \times q \times q \times q$ b. $6 \times y \times y \times y$ c. $2 \times a \times a \times a + 3 \times z \times z$

d. $2 \times 2 \times 2 \times 2$ e. $-5 \times -5 \times -5$ f. $3 \times 3 \times 3 + 4 \times 4 \times 4 \times 4$

g. $p \times p \times q \times p \times q \times p$ h. $-3 \times g \times h \times g \times h \times 4 \times g \times h \times g$

2. Write in expanded form

a. x^6 b. $2d^4$ c. $3n^3 + 4a^7$

d. $3e^5 - 6e^5$ e. $9r^2 - 2r^3 + 3r^2 + 4r^3$

LEARNING OUTCOMES

Students should be able to:

- To establish rules of indices using numerals and pronumerals.
- To apply law of indices

As with all mathematical formulae, to simplify numerals and pronumerals with indices, certain laws need to be followed.

3.2 Index Rules/ Laws

When multiplying numbers or variables having the same base, we add the index.

1. Index Law: 1

$$x^2 \times x^3$$
$$= x^{2+3}$$
$$= x^5$$

Example:

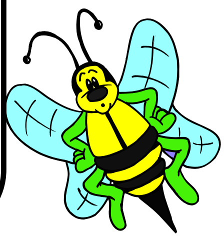
$$2^4 \times 2^3$$
$$= 2^{4+3}$$
$$= 2^7$$

2. Index Law: 2

$$x^5 \div x^3$$
$$= x^{5-3}$$
$$= x^2$$

Example:

$$2^5 \div 2^2$$
$$= 2^{5-2}$$
$$= 2^3$$



When dividing numbers or variables having the same base, we subtract the indices.

3. Index Law: 3

$$(x^2)^4$$

$$= x^{2\times4}$$

$$= x^8$$

Example:

$$(2^3)^4$$

$$= 2^{3\times4}$$

$$= 2^{12}$$

Where the index form is raised to another power, the indices are multiplied.

4. Index Law: 4

$$x^0$$

$$= 1$$

Example:

$$2^0$$

$$= 1$$

Any number or variable raised to the power of zero is equal to 1.

5. Index Law: 5

$$x^{-2}$$

$$= \frac{1}{x^2}$$

Example:

$$2^{-3}$$

$$= \frac{1}{2^3}$$

$$= \frac{1}{2\times2\times2}$$

$$= \frac{1}{8}$$

A negative index will give a fraction when simplified.

Exercise 3.2

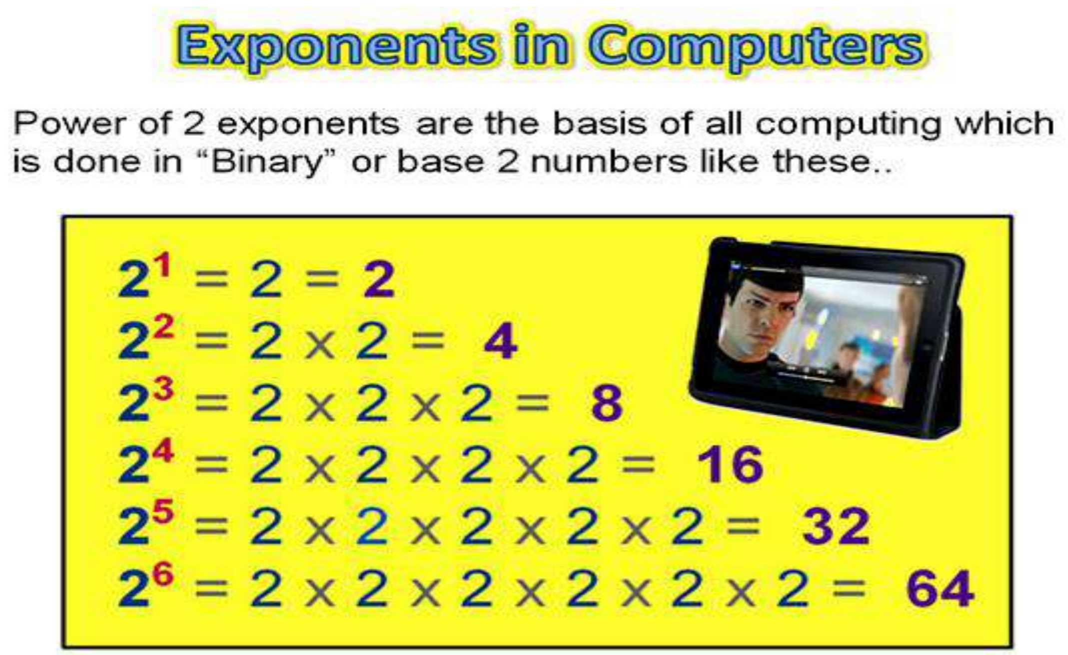
Simplify. Your answer should contain only positive exponents.

1. $a^2 \times a$	2. $w^3 \times w^4$
3. $k^6 \times k^3$	4. $p^2 \times p^3 \times p$
5. $r^6 \div r^5$	6. $t^7 \div t^7$
7. $\frac{q^5}{q^2}$	8. y^0
9. $(g^5)^2$	10. $(2^3)^2$
11. 2^{-3}	12. 3^{-2}

Indices in real life situations

Most people who use Exponents are Economists, Bankers, Financial Advisors, Insurance Risk Assessors, Biologists, Engineers, Computer Programmers, Chemists, Physicists, Geographers, Sound Engineers, Statisticians, Mathematicians, Geologists and many other professions

Exponents are fundamental, especially in Base 2 and Base 16, as well as in Physics and Electronics formulas involved in Computing.



Source: http://passyworldofmathematics.com/exponents-in-the-real-world/

There has been an Exponential increase in the speed and power of computers over recent years, and by around 2030 computing power is predicted to match that of the human brain.

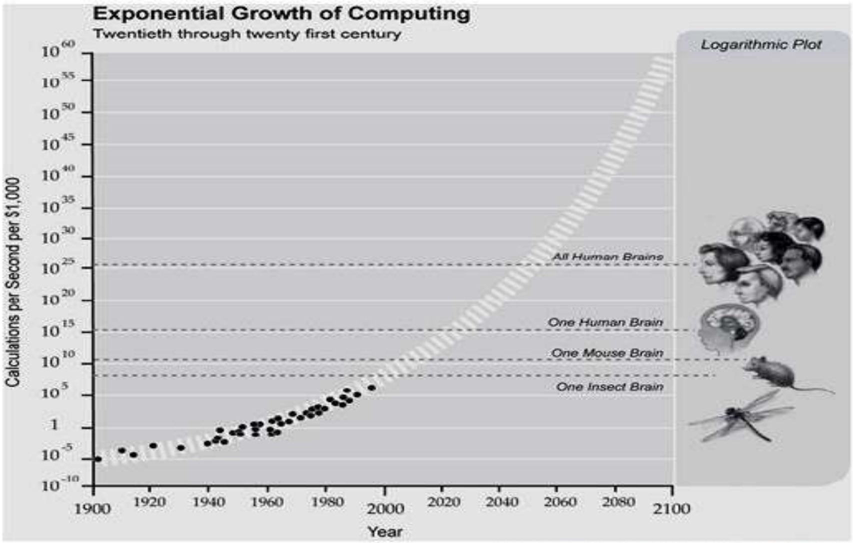
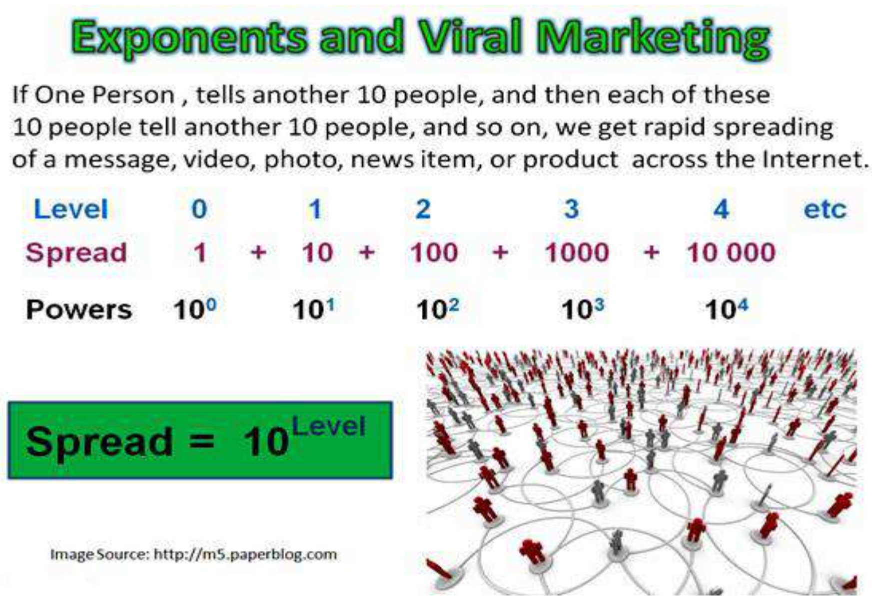
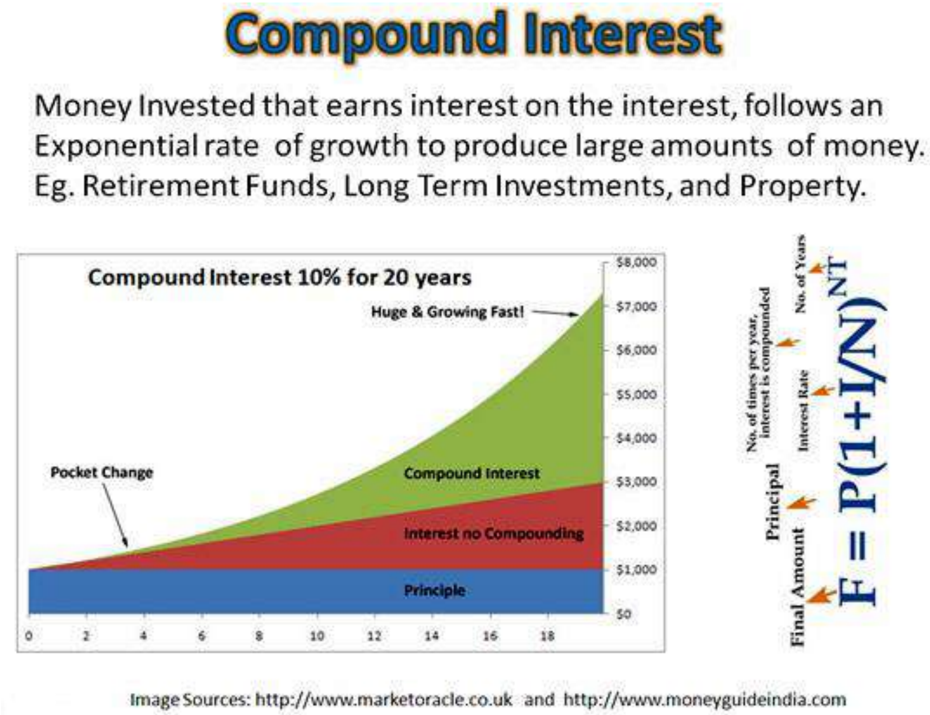


Image Source: http://www.singularity.com

Exponents are critically important in modern Internet based Sales and Marketing,

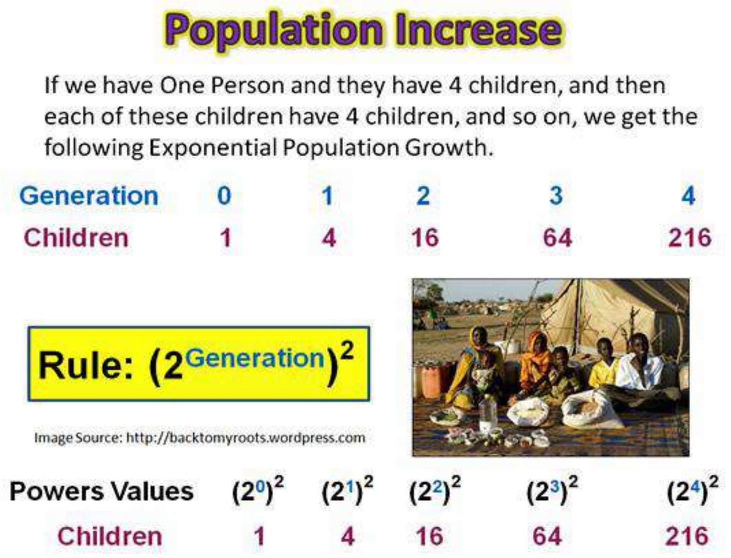


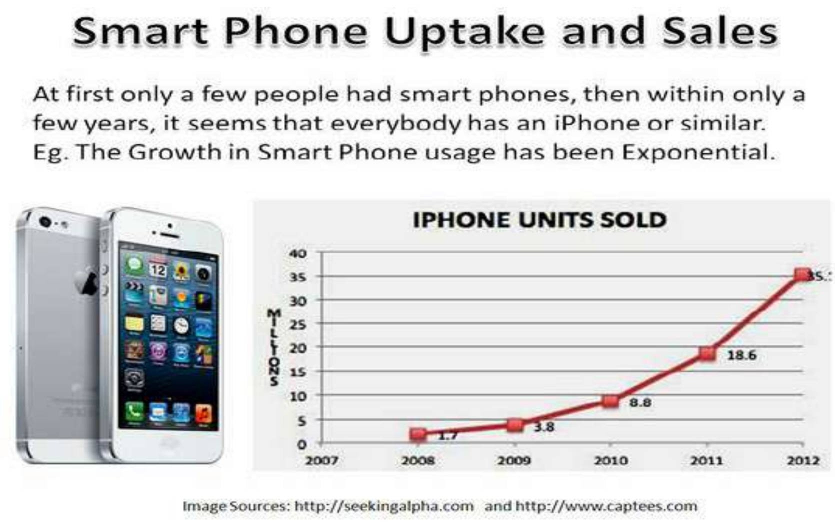
Exponents are important in Investing and Finance.



Compound Interest also works against people with a Credit Card debt they do not pay off, because the debt grows faster and faster each billing period and can quickly become out of control.

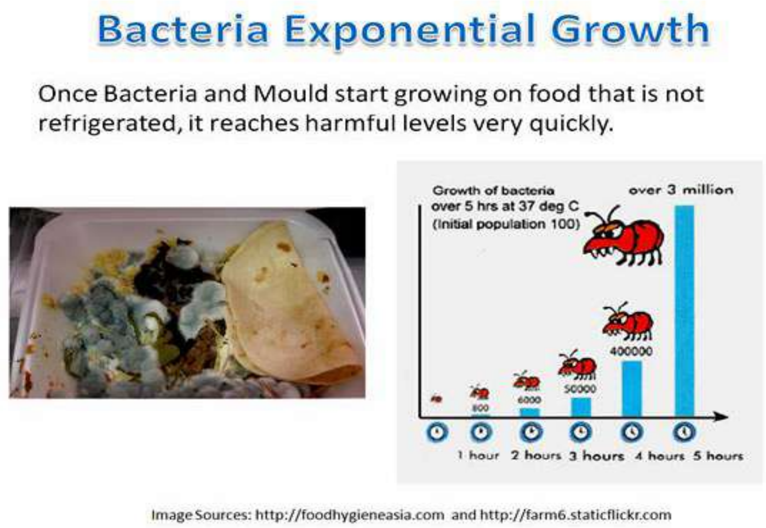
Exponents are the basis of “Demographics” (Population Growth)





Consumer Credit Debt has increased over recent years to record high levels.

Exponents are also part of Food Technology and Microbiology.



Virus Illness, (as well as many email and computer viruses), can spread at ever increasing rates causing major widespread infected areas.

This happens the same way that Viral Marketing branches out in ever increasingly wide branches of more and more people passing something onto more and more other people.

Spread of Viruses and Disease

Viruses such as Flus, and HIV AIDS can spread at Exponential Rates causing Epidemics affecting millions of people.

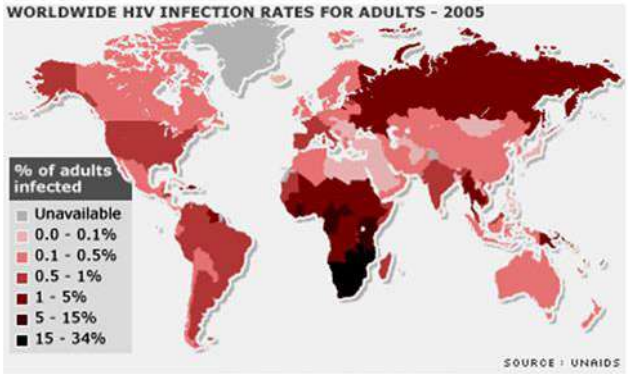


Image Source: http://news.bbc.co.uk

In explosions we get an uncontrolled massively increasing output of energy and force within a very short time period.

Picture this as a very steep exponential graph, compared to a burning match giving out energy in a fairly flat straight line graph.

Exponential Energy Release

In an uncontrolled Nuclear Explosion, Energy increases at an Exponential Rate as per Einstein's Equation: $E = mc^2$.

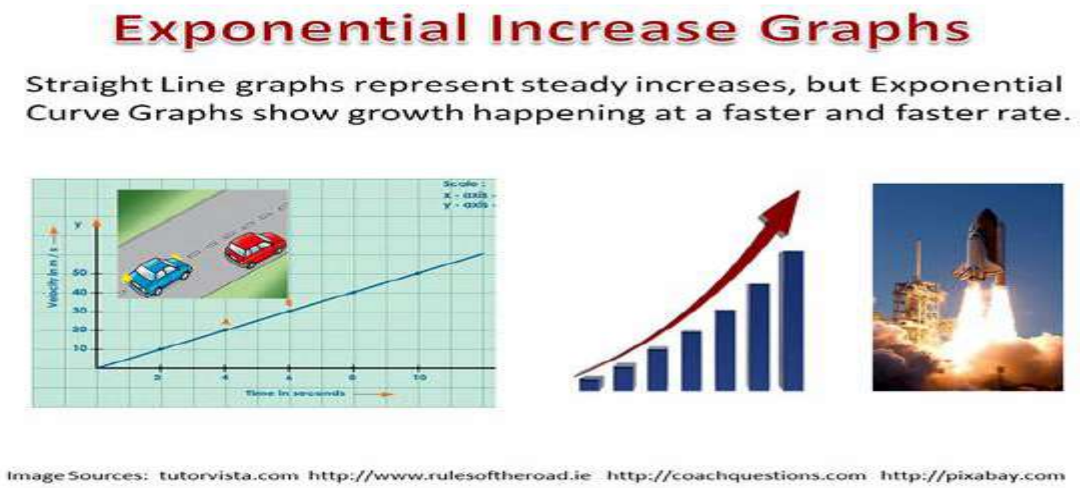
Image Source: http://www.zeusbox.com

Exponential Growth

The situations we have been considering so far involve "Exponential Growth". The equations for graphs of these situations contain exponents, and this results in the graph starting off slow, but then increasing very rapidly.

Eg. Think of Square Numbers and how they quickly get bigger and bigger:

1 4 9 16 25 36 49 64 81 100 121 132 etc . It only takes us nine square numbers to reach 100. Exponential Growth situations when graphed look like the diagram below.

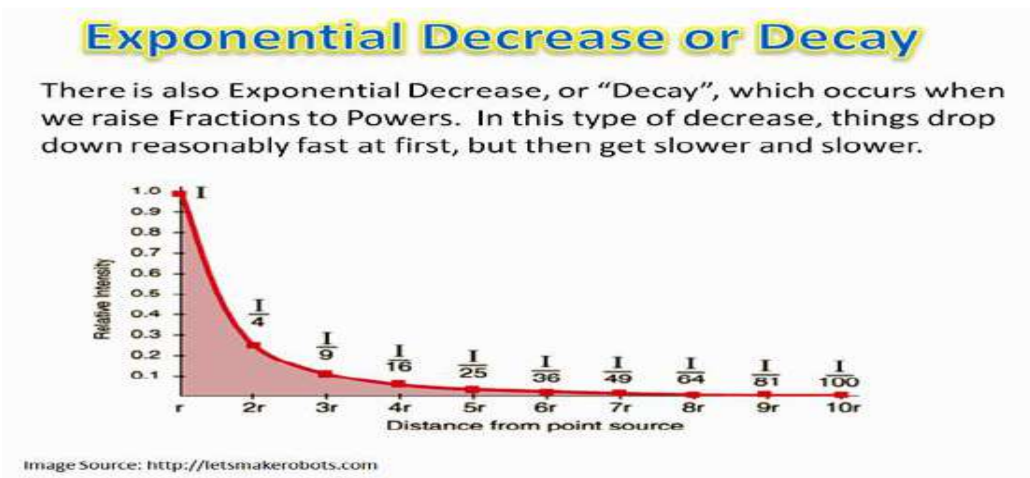


The opposite of "Exponential Growth", is when we apply exponents to fractions which results in "Exponential Decay".

Exponential Decay

Using negative power values results in fractions, and when these fractions have exponents applied to them we get "Decay".

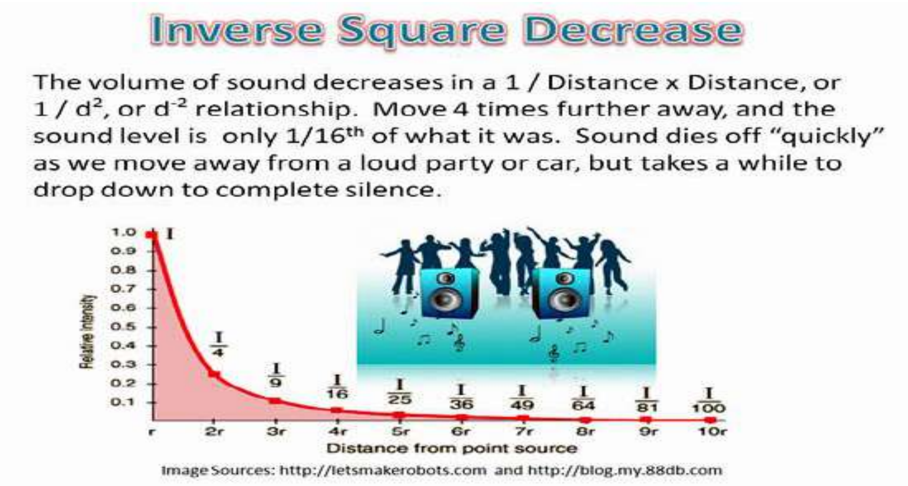
In a "Decay" process the amount involved drops off fairly quickly at the start, but then the drop off becomes slower and slower. A typical Exponential Decay graph looks like this:

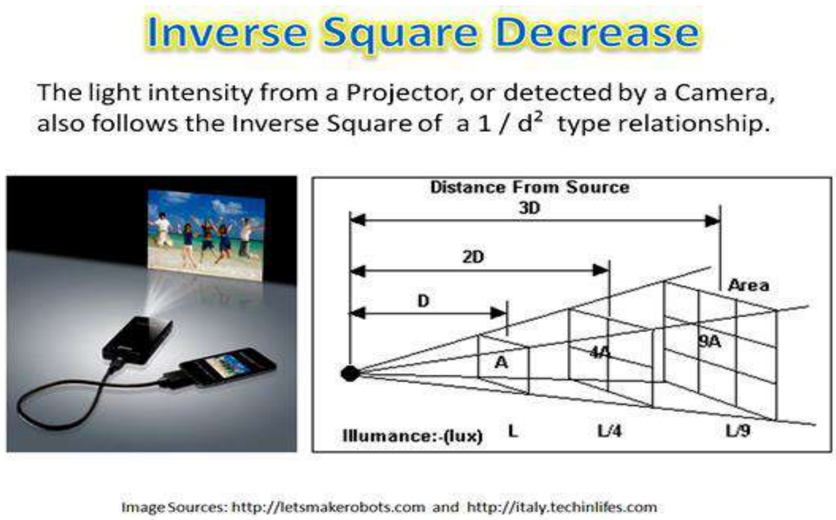


A fun way to make an Exponential decay graph is to take a pack of M&M's or Skittles and keep pouring them out of a cup, but each time removing any candies which land with the letter side showing.

Exponential Decay – Real Life Examples

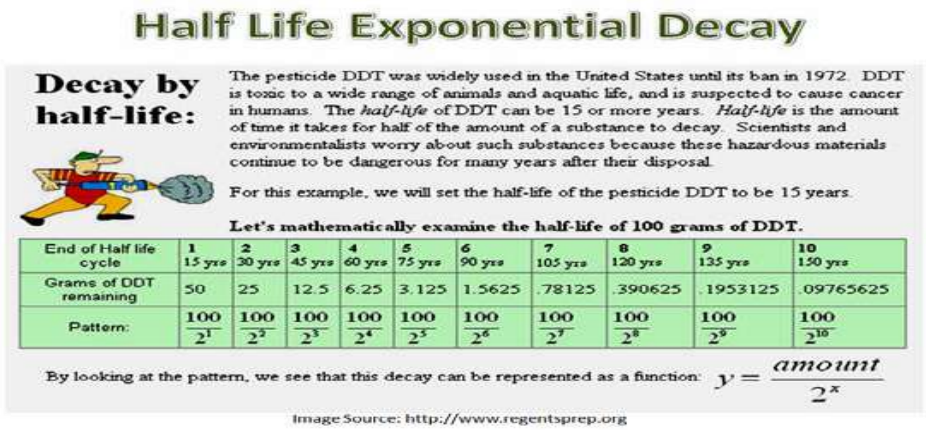
Some examples of Exponential Decay in the real world are the following.





Exponential Decay and Half Life

Many harmful materials, especially radioactive waste, take a very long time to break down to safe levels in the environment. This is because these materials undergo exponential decay, and even a small amount of the material still remaining can be harmful.



Half Life Exponential Decay

Decay by half-life:

The pesticide DDT was widely used in the United States until its ban in 1972. DDT is toxic to a wide range of animals and aquatic life, and is suspected to cause cancer in humans. The *half-life* of DDT can be 15 or more years. *Half-life* is the amount of time it takes for half of the amount of a substance to decay. Scientists and environmentalists worry about such substances because these hazardous materials continue to be dangerous for many years after their disposal.

For this example, we will set the half-life of the pesticide DDT to be 15 years.

Let's mathematically examine the half-life of 100 grams of DDT.

End of Half life cycle	1 15 yrs	2 30 yrs	3 45 yrs	4 60 yrs	5 75 yrs	6 90 yrs	7 105 yrs	8 120 yrs	9 135 yrs	10 150 yrs
Grams of DDT remaining	50	25	12.5	6.25	3.125	1.5625	.78125	.390625	.1953125	.09765625
Pattern:	$\frac{100}{2^1}$	$\frac{100}{2^2}$	$\frac{100}{2^3}$	$\frac{100}{2^4}$	$\frac{100}{2^5}$	$\frac{100}{2^6}$	$\frac{100}{2^7}$	$\frac{100}{2^8}$	$\frac{100}{2^9}$	$\frac{100}{2^{10}}$

By looking at the pattern, we see that this decay can be represented as a function: $y = \frac{amount}{2^x}$

Image Source: http://www.regentsprep.org

Exponential Scales

The Richter Scale is used to measure how powerful earthquakes are.

The actual energy from each quake is a power of 10, but on the scale we simply take the index value of 1, 2, 3, 4, etc rather than the full exponent quantity.

This means that a Richter Scale 6 earthquake is actually 10 times stronger than a Richter Scale 5 quake. (Eg. 1000000 vs 100000).

Likewise, a Richter Scale 7 earthquake is actually 100 times stronger than a Richter Scale 5 quake. (Eg. 10000000 vs 100000).

Image Source: http://lh3.googleusercontent.com

The pH Scale for measuring the Acidity of materials is also created by taking the Power Values from measured powers of 10 acid concentration values.

Exercise 3.3

1. Evaluate the following using the index laws. Leave your answers in base-index form.

a. $2^3 \times 2$ b. $3^6 \div 3^4$ c. $\left(-3^2\right)^3$

d. $6^5 \times 6^{-5}$ e. $4^{-2} \div 4^4$ d. $\left(5^{-2}\right)^1$

2. Simplify the following

a. $6x^3 \times 2x^2$ b. $18b^3 \div 3b$ c. $3a^2b \times 5ab^5$

d. $\dfrac{12x^4y^5z^8}{4x^2y^3z^6}$ e. $\dfrac{18n^3j^4}{5h^2l^6} \times \dfrac{10hl^8}{6n^2j}$ f. $\dfrac{6f^5g^3}{3r^2s} \div \dfrac{2gf^3}{12r^5s^5}$

g. $\dfrac{\left(2x^3y^2\right)^3}{12x^3y} \times \dfrac{4x^2y^3}{3xy}$

3. Write the following in base-index form using the number in brackets as the base.

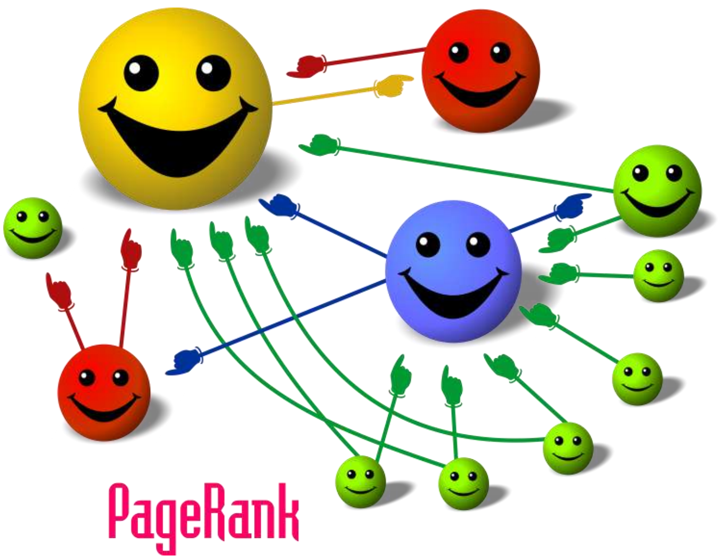
a. 64 (4) b. 125 (5) c. 2401 (7)

d. 1/8 (2) e. 1/81 (3) f. 1/6 (6)

4. Evaluate the following

a. $4^2 \times 2^3$ b. $\left(3^6\right)^3 \div 3^5$ c. $\left(5^4\right)^2 \times 4^2 \div 5^3$

d. $2x^0 + \left(5xy^2\right)^0$ e. $-7x^0 - (100y)^0$



GLOSSARY		
6	Compound interest	When the interest rate is applied to the original principal and any accumulated interest
1	Exponent	A number placed above and to the right of another number to show that it has been raised to a power
5	Exponential Decay	It occurs when a population decreases at a consistent rate over time. For exponential decay, the total value decreases but the proportion that leaves remains constant over time
2	Exponential function	Use to model a relationship in which a constant change in the independent variable gives the same proportional change (i.e. percentage increase or decrease) in the dependent variable.
4	Exponential Growth	It is a growth that increases at a consistent rate, and it is a common occurrence in everyday life
3	Half Life	The amount of time it takes for the amount of the substance to diminish by half
9	Index (exponent, power)	The index of a number says how many times to use the number in a multiplication
7	Numeral	A symbol used to represent a number
8	Pro - numeral	A letter that is used to represent a number (or numeral) in a problem