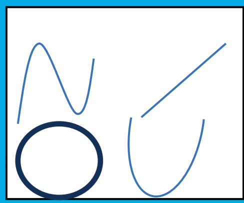
STRAND 4 GRAPHS

LEARNING OUTCOMES

Able to draw Graphs of the following functions :

- **Linear**
- **Quadratic**
- **Cubic**
- **Circle**
- **exponential**



Linear Functions

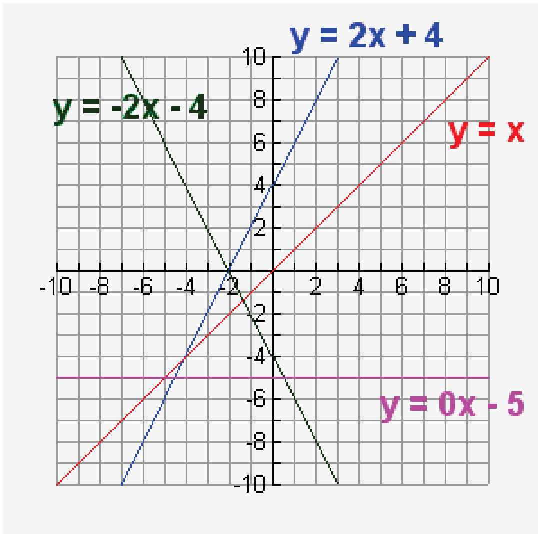
Linear functions are straight lines (power of x and y = 1).

Form: ***y = mx + c***

where **m** is the *gradient* and **c** gives the *y-intercept*

Some Observations:

1. If the slope, *m*, is positive, the line slants uphill. As the slope gets larger, the uphill slant of the line gets steeper. As the slope gets extremely large (a very big number), the line becomes nearly vertical. If the line is vertical, the slope is undefined (because it has no horizontal change).



2. As the slope gets smaller (closer to zero), the line loses steepness and starts to flatten. If the slope is zero, the line is horizontal (flat)

3. If the slope is negative the line slants downhill. As the slope decreases (remember -2 is > -3), the downhill slant of the line gets steeper.

4. Domain: $x \in R$
   Range : $y \in R$

X-Intercepts and Y-Intercepts

The **x-intercept** of a line is the point at which the line crosses the x-axis. (i.e. where the y value equals **0**)

x-intercept = (x, 0)

The **y-intercept** of a line is the point at which the line crosses the y-axis. (i.e. where the x value equals 0)

y-intercept = (0, y)

Example:

Find the x and y intercepts of the equation $3x + 4y = 12$

To find the *x*-intercept, set $y = 0$ and solve for x.

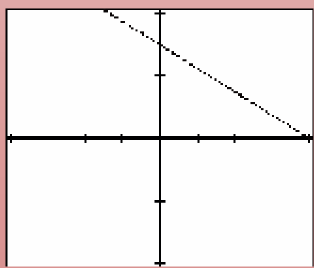
$$3x + 4(0) = 12$$
$$3x + 0 = 12$$
$$3x = 12$$
$$x = 12/3$$
$$x = 4$$

To find the *y*-intercept, set $x = 0$ and solve for y.

$$3(0) + 4y = 12$$
$$0 + 4y = 12$$
$$4y = 12$$
$$y = 12/4$$
$$y = 3$$

Therefore, the *x*-intercept is (4, 0) and the *y*-intercept is (0, 3).

The graph of the line looks like this:



Solving Equations Simultaneously

On occasions you will come across two or more unknown quantities, and two or more equations relating them. These are called simultaneous equations and when asked to solve them you must find values of the unknowns which satisfy all the given equations at the same time.

Some methods of solving equations simultaneously are the **graphical method, elimination and substitution methods.**

Graphical Method

Example . Solve simultaneously for *x* and *y*:

$$x + y = 10$$
$$x - y = 2$$

This means that we must find values of *x* and *y* that will solve both equations. We must find two numbers *whose sum is 10 and whose difference is 2*.

The two numbers, obviously, are 6 and 4:

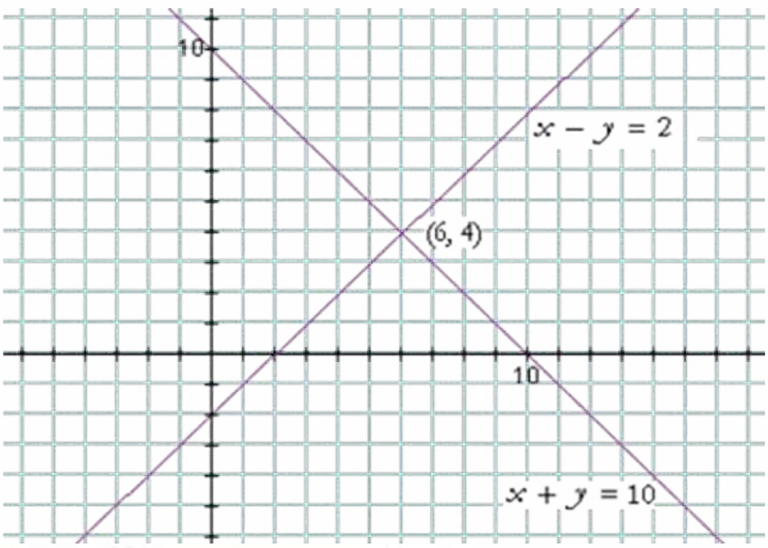
$$6 + 4 = 10$$
$$6 - 4 = 2$$

Let us represent the solution as the ordered pair (6, 4).

Now, these two equations

$$x + y = 10$$
$$x - y = 2$$

are linear equations . Hence, the graph of each one is a straight line. Here are the two graphs:



The solution to the simultaneous equations is their **point of intersection**. Why? Because that coordinate pair solves both equations. That point is the one and only point on both lines.

Solve this system of equations graphically.

$$4x - 6y = 12$$

$$2x + 2y = 6$$

Solve graphically:

To solve a system of equations graphically, graph both equations and see where they intersect. The intersection point is the solution.

First, solve each equation for "*y* = ". (make ***y*** the subject of the formula)

4*x* - 6*y* = 12

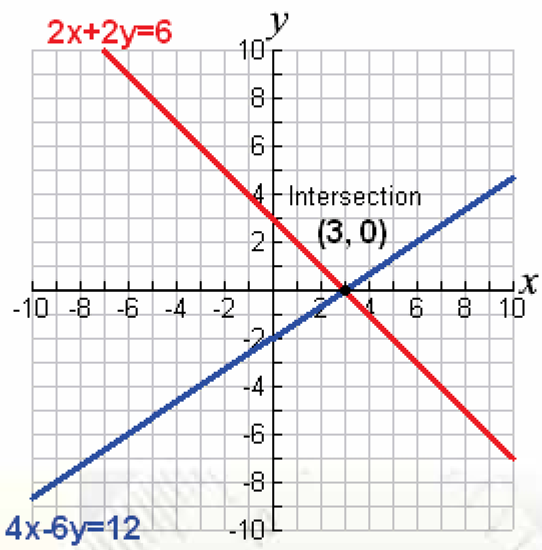
$$4x = 6y + 12$$
$$4x - 12 = 6y$$
$$6y = 4x - 12$$
$$y = \frac{4x}{6} - \frac{12}{6}$$
$$y = \frac{2}{3}x - 2$$

slope = 2/3
***y*-intercept = -2**

2*x* + 2*y* = 6

$$2x + 2y = 6$$
$$2y = -2x + 6$$
$$y = \frac{-2x}{2} + \frac{6}{2}$$
$$y = -x + 3$$

slope = -1
***y*-intercept = 3**



Graph the lines.

The slope intercept method of graphing was used in this example.

The point of intersection of the two lines, (3,0), is the solution to the system of equations.

This means that (3,0), when substituted into either equation, will make them both true. See the check.

Elimination method

Perhaps the simplest way is elimination. This is a process which involves removing or eliminating one of the unknowns to leave a single equation which involves the other unknown.

In the 'elimination' method for solving simultaneous equations, two equations are simplified by adding them or subtracting them. This eliminates one of the variables so that the other variable can be found. To add two equations, add the left hand expressions and right hand expressions separately. Similarly, to subtract two equations, subtract the left hand expressions from each other, and subtract the right hand expressions from each other. The following examples will make this clear.

Example 1: *Consider these equations:*

$$2x - 5y = 1$$
$$3x + 5y = 14$$

The first equation contains a '-5*y*' term, while the second equation contains a '+5*y*' term. These two terms will cancel if added together, so we will add the equations to eliminate '*y*'.

To add the equations, add the left side expressions and the right side expressions separately.

2x - 5y	=		1
3x + 5y	=	+	14
(2x – 5y) + (3x + 5y)	=		1 + 14

Simplifying, -5*y* and +5*y* cancel out, so we have:

$$5x = 15$$

Therefore '*x*' is 3.

By substituting 3 for 'x' into either of the two original equations we can find 'y'.

$$2x - 5y = -1$$
$$2(3) - 5y = -1$$
$$6 - 5y = -1$$
$$-5y = -7$$
$$y = \frac{7}{5}$$

Example 2. Solve simultaneously for *x* and *y*.

$$\begin{aligned} 2x + y &= 4 \\ x - y &= -1 \end{aligned}$$

Solution. In this case, the solution is not obvious. Here is a general strategy for solving simultaneous equations:

When one pair of coefficients are negatives of one another, add the equations vertically, and that unknown will cancel. We will then have one equation in one unknown, which we can solve.

Upon adding those equations, the *y*'s cancel:

$$\begin{array}{ccccc} 2x & + & Y & = & 4 \\ x & - & Y & = & -1 \\ \hline 3x & & & = & 3 \end{array}$$

$$X = \frac{3}{3}$$

$$X = 1.$$

To solve for *y*, the other unknown:

Substitute the value of *x* in one of the original equations.

Upon substituting x = 1 in the top equation:

$$2x + y = 4$$

$$2(1) + y = 4$$

$$2 + y = 4 \quad y = 2$$

If we report the solution as an ordered pair, then the solution is (1, 2). Those are the coordinates of the point of intersection of the two lines.

Example 3:

The elimination method will only work if you can eliminate one of the variables by adding or subtracting the equations as in example 1 above. But for many simultaneous equations, this is not the case.

For example, consider these equations:

$$2x + 3y = 4$$

$$x - 2y = -5$$

Adding or subtracting these equations will not cancel out the 'x' or 'y' terms.

Before using the elimination method you may have to multiply every term of one or both of the equations by some number so that equal terms can be eliminated. We could eliminate '*x*' for this example if the second equation had a '2*x*' term instead of an '*x*' term. By multiplying every term in the second equation by 2, the '*x*' term will become '2*x*', like this:

$$X \times 2 - 2y \times 2 = -5 \times 2$$

giving

$$2x - 4y = -10$$

Now the '*x*' term in each equation is the same, and the equations can be subtracted to eliminate '*x*':

$$\begin{array}{rcr} 2x + 3y & = & 4 \\ -\; 2x - 4y & = & -\; -10 \\ \hline (2x + 3y) - (2x - 4y) & = & 4 - -10 \end{array}$$

Removing the brackets and simplifying, the '2x' terms cancel out, so we have:

$$7y = 14$$

So

$$y = 2$$

The other variable, '*x*', can now be found by substituting 2 for '*y*' into either of the original equations.

$$\mathbf{x - 2y = -5}$$
$$\mathbf{x - 2(2) = -5}$$
$$\mathbf{x - 4 = 5}$$
$$\mathbf{x = 9}$$

Substitution Method

Here is the method of substitution:

- Solve one of the equations for one unknown in terms of the other.
- Then, substitute that in the other equation.
- That will yield one equation in one unknown, which we can then solve.

Consider example 2 from elimination: $2x + y = 4$ ---------→ ①

$x - y = -1$ ---------→ ②

Let us **solve** equation ① for *y*:

① $y = 4 - 2x$

And now, **substitute** this for *y* in equation ② :

② $x - (4 - 2x) = -1$

This equation has only the unknown *x*:

$$x - 4 + 2x = -1$$

$$3x = -1 + 4$$

$$3x = 3$$

$$X = 1$$

To find *y*, substitute $x = 1$ in line 1):

$$y = 4 - 2 \cdot 1$$

$$y = 2.$$

EXERCISE 4.1

1. Use the method of substitution to solve each other of the pair of simultaneous equations:

(a) $x + y = 15$, $x - y = 3$

(b) $x + y = 0$, $x - y = 2$

(c) $2x - y = 3$, $4x + y = 3$

(d) $2x - 9y = 9$, $5x + 2y = 27$

(e) $x + 4y = -4$, $3y - 5x = -1$

(f) $2x - 3y = 2$, $x + 2y = 8$

(g) $x + y = 7$, $2x - 3y = 9$

(h) $11y + 15x = -23$, $7y - 2x = 20$

(i) $5x - 6y = 2$, $6x - 5y = 9$

2. Solve each other pair of equation given below using elimination method:

(a) $x + 2y = -4$, $3x - 5y = -1$

(b) $4x + 9y = 5$, $-5x + 3y = 8$

(c) $9x - 6y = 12$, $4x + 6y = 14$

(d) $2y - \frac{3}{x} = 12$, $5y + \frac{7}{x} = 1$

(e) $\frac{3}{x} + \frac{2}{y} = \frac{9}{xy}$, $\frac{9}{x} + \frac{4}{y} = \frac{21}{xy}$

(f) $\frac{4}{y} + \frac{3}{x} = 8$, $\frac{6}{y} + \frac{5}{x} = 13$

(g) $5x + \frac{4}{y} = 7$, $4x + \frac{3}{y} = 5$

(h) $x + y = 3$, $-3x + 2y = 1$

(i) $-3x + 2y = 5$, $4x + 5y = 2$

3. Solve the following simultaneous equations:

a.	$3a + 4b = 43$	$-2a + 3b = 11$
b.	$4x - 3y = 23$	$3x + 4y = 11$
c.	$5x + (4/y) = 7$	$4x + (3/y) = 5$
d.	$4/(p - 3) + 6/(q - 4) = 5$	$5/(p - 3) - 3/(q - 4) = 1$
e.	$(l/6) - (m/15) = 4$	$(l/3) - (m/12) = 19/4$
f.	$3x + 2y = 8$	$4x + y = 9$
g.	$x - y = -1$	$2y + 3x = 12$
h.	$(3y/2) - (5x/3) = -2$	$(y/3) + (x/3) = 13/6$
i.	$x - y = 3$	$(x/3) + (y/2) = 6$
j.	$(2x/3) + (y/2) = -1$	$(-x/3) + y = 3$
k.	$5x + 8y = 9$	$2x + 3y = 4$
l.	$3 - 2(3a - 4b) = -59$	$(a - 3)/4 - (b - 4)/5 = 2^1/_{10}$

4. If $2x + 5y = 21$ and $x + y = 3$, then what is the value of x?

5. If $7x + 2y = 3$ and $x - 3y = 30$, what is the value of y?

6. If $2x + 5y = -1$ and $3x - 2y = 27$, then what is the value of x

7. If $4x - 3y = -18$ and $6x + 7y = -4$, then what is the value of y?

8. $2x - 3y = 24$ and $3x + 4y = 2$, what is the value of $x - y$?

9. $a + b = 5$ and $3a + 2b = 20$, then what is $3a + b$ equal to?

10. If $3p - 2q = 4$ and $7p - 3q = 1$, then what is $4p - 5q$ equal to?

11. Sahid has 20 balls of four colors: yellow, green, blue, and black. 17 of them are not green, 5 are black, and 12 are not yellow. How many blue balls does Sahid have?

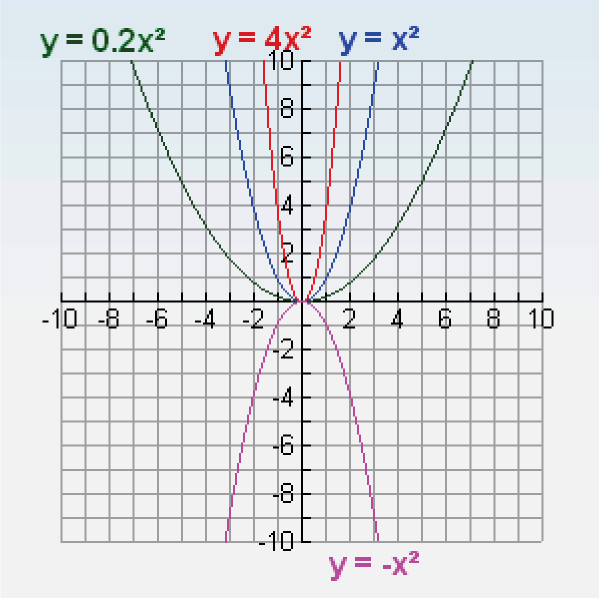
12. Ansaar bought 20 shirts at a total cost of R 980. If the large shirts cost R 50 and the small shirts cost R 40. How many of each size did he buy?

13. The diagonal of a rectangle is 25 cm more than its width. The length of the rectangle is 17 cm more than its width. What are the dimensions of the rectangle?

14. The sum of 27 and 12 is equal to 73 more than an unknown number. Find the number.

15. The sum of two consecutive odd numbers is 20 and their difference is 2. Find the two numbers.

Quadratic Functions



Quadratic functions are parabolas. Form $y = ax^2$.

Some Observations:

1. If the coefficient of x^2 gets larger, the parabola becomes thinner (narrower), closer to its line of symmetry.

2. If the coefficient of x^2 gets smaller, the parabola becomes thicker (wider), further from its line of symmetry.

3. If the coefficient of x^2 is negative, the parabola opens downward

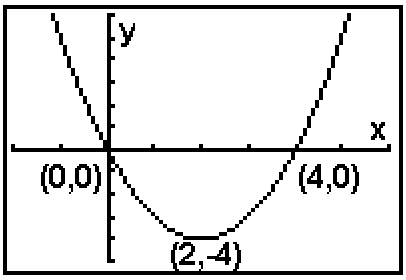
Quadratic in the form $y = x^2 - 4x$

(Remember: [-1, 5] means from x= -1 to x = 5 inclusive.)

x	$x^2 - 4x$	y
-1	$(-1)^2 - 4(-1)$	5
0	$(0)^2 - 4(0)$	0
1	$(1)^2 - 4(1)$	-3
2	$(2)^2 - 4(2)$	-4
3	$(3)^2 - 4(3)$	-3
4	$(4)^2 - 4(4)$	0
5	$(5)^2 - 4(5)$	5

In this example, the interval used for preparing the chart was given in the question. Consequently, the turning point of the parabola fell within the interval. If the question had NOT told us the interval, how would we have known which values to place in the chart to ensure that we would see the turning point of the parabola? To guarantee that the points you choose for your chart will show the turning point, start by determining the axis of symmetry of the parabola.

Plot the points generated in the chart. Draw a smooth curve through the points.



The points where the graph crosses the x-axis are called the roots of
$0 = x^2 - 4x$.
This parabola crosses the x-axis at (0, 0) and (4, 0).

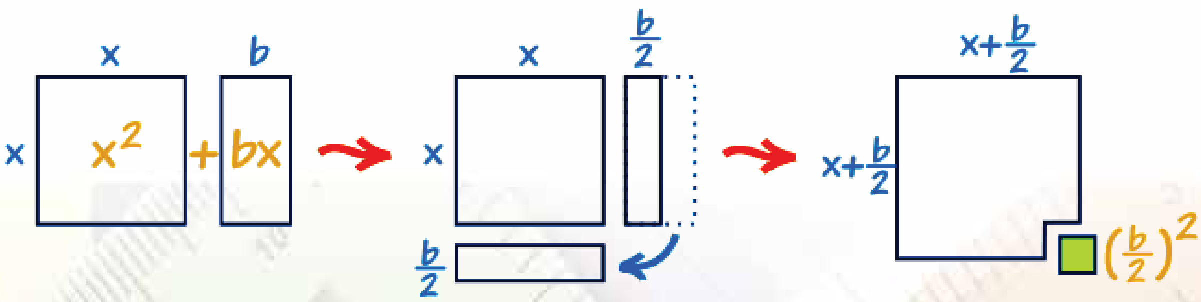
Completing the Square

"**Completing the Square**" is where we ..

take a Quadratic Equation like this: $ax^2 + bx + c = 0$ → and turn it into this: $a(x+d)^2 + e = 0$

Say we have a simple expression like $x^2 + bx$. Having x twice in the same expression can make life hard. What can we do?

Well, with a little inspiration from Geometry we can convert it, like this:



As you can see $x^2 + bx$ can be rearranged ***nearly*** into a square ...

... and we can **complete the square** with $(b/2)^2$

In Algebra, it looks like this:

$$x^2 + bx + (b/2)^2 = (x+b/2)^2$$

"Complete the Square"

So, by adding $(b/2)^2$ we can complete the square.

And $(x+b/2)^2$ has x only **once**, which is easier to use.

Keeping the Balance

Now ... we can't just ***add*** $(b/2)^2$ without also ***subtracting*** it too! Otherwise the whole value changes.

So let's see how to do it properly with an example:

Start with: $x^2 + 6x + 7$

("b" is 6 in this case)

Complete the Square:

$$x^2 + 6x + \underbrace{\quad}_{+\left(\frac{6}{2}\right)^2} + 7 \underbrace{\quad}_{-\left(\frac{6}{2}\right)^2}$$

Also **subtract** the new term

Simplify it and we are done.

$$\underbrace{x^2 + 6x + \left(\frac{6}{2}\right)^2}_{\left(x+\frac{6}{2}\right)^2} + \underbrace{7 - \left(\frac{6}{2}\right)^2}_{7-9} = (x+3)^2 - 2$$

The result:

$$x^2 + 6x + 7 = (x+3)^2 - 2$$

A Shortcut Approach

Expanding $(x + b)^2$ gets us $x^2 + 2bx + b^2$, and we have turned the question into this:

$$x^2 + 6x + 7$$

$$(x + b)^2 + c \rightarrow x^2 + 2bx + b^2 + c$$

Now we can "force" an answer:

- We know that 6x must end up as 2bx, so **b must be 3**
- Next we see that 7 must become $b^2 + c = 9 + c$, so **c must be −2**

And we get the same result $(x+3)^2 - 2$ as above!

A Simple Approach

$ax^2 + bx + c = 0 \Longrightarrow a(x+d)^2 + e = 0$

$$d = \frac{b}{2a} \qquad e = c - \frac{b^2}{4a}$$ **and**

The vertex form of a quadratic function is given by

$$f(x) = a(x - h)^2 + k$$

where (***h***, ***k***) is the vertex of the parabola.

When written in "vertex form":

- ***(h, k)*** is the vertex of the parabola, and ***x* = *h*** is the axis of symmetry.
- the ***h*** represents a horizontal shift (how far left, or right, the graph has shifted from $x = 0$).
- the ***k*** represents a vertical shift (how far up, or down, the graph has shifted from $y = 0$).
- notice that the ***h*** value is subtracted in this form, and that the ***k*** value is added. If the equation is $y = 2(x - 1)^2 + 5$, the value of ***h*** is 1, and ***k*** is 5. If the equation is $y = 3(x + 4)^2 - 6$, the value of ***h*** is -4, and ***k*** is -6.

Sketch the graph of $y = 3(x - 2)^2 - 4$

1. Start with the function in vertex form*:*

$y = 3(x - 2)^2 - 4$

2. Pull out the values for ***h*** and ***k***.

If necessary, rewrite the function so you can clearly see the *h* and *k* values.

(h, k) is the vertex of the parabola.

Plot the vertex

$y = 3(x - 2)^2 + (-4)$

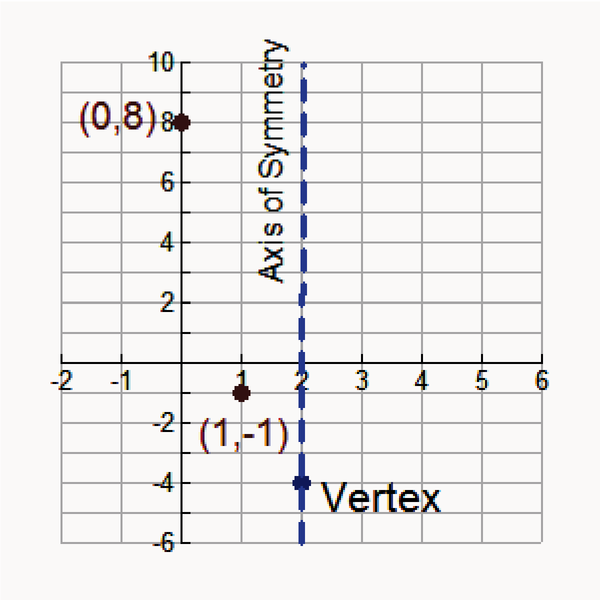
$\boldsymbol{h} = 2; \quad \boldsymbol{k} = -4$

Vertex: (2, -4)

3. The line $x = h$ is the axis of symmetry.

Draw the axis of symmetry.

$x = 2$ is the axis of symmetry



4. Find two or three points on one side of the axis of symmetry, by substituting your chosen *x*-values into the equation.

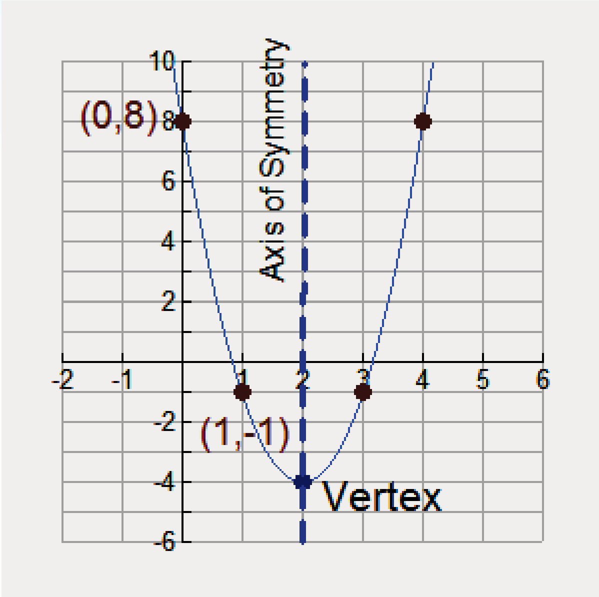
For this problem, we chose (to the left of the axis of symmetry):

$x = 1; \quad y = 3(1 - 2)^2 - 4 = -1$

$x = 0; \quad y = 3(0 - 2)^2 - 4 = 8$

Plot (1, -1) and (0,8).

5. Plot the mirror images of these points across the axis of symmetry, or plot new points on the right side. Draw the parabola.
   Remember, when drawing the parabola to avoid "connecting the dots" with straight line segments. A parabola is curved, not straight, as its slope is not constant.



Graphing Cubic Functions

A cubic function has the form

$$f(x) = a x^3 + b x^2 + c x + d \text{ – expanded form}$$

$$f(x) = (x – a)(x – b)(x – c) \text{ – Factorised form}$$

where **a, b, c** and **d** are real numbers and **a** is **not equal** to 0.

The domain of this function is the set of all real numbers. The range of f is the set of all real numbers. The **y intercept** of the graph of f is given by f(0) = d. The **x intercept(s)** are found by solving the equation

$$a x^3 + b x^2 + c x + d = 0$$

The left hand side behaviour of the graph of the cubic function is as follows:

- If the leading coefficient is positive, as x increases the graph of f is up and as x decreases indefinitely the graph of f is down.
- If the leading coefficient is negative, as x increases the graph of f is down and as x decreases indefinitely the graph of f is up.

Example 1: The basic cubic function given by

$$\boldsymbol{f}(x) = x^3$$

a. Find the x and y intercepts of the graph of ***f***.
b. Find the domain and range of ***f***.
c. Sketch the graph of ***f***.

Solution to Example 1

a - The y intercept is given by
(0 , f(0)) = (0 , 0)

- The x coordinates of the x intercepts are the solutions to
$x^3 = 0$
- The x intercept are at the points (0 , 0).

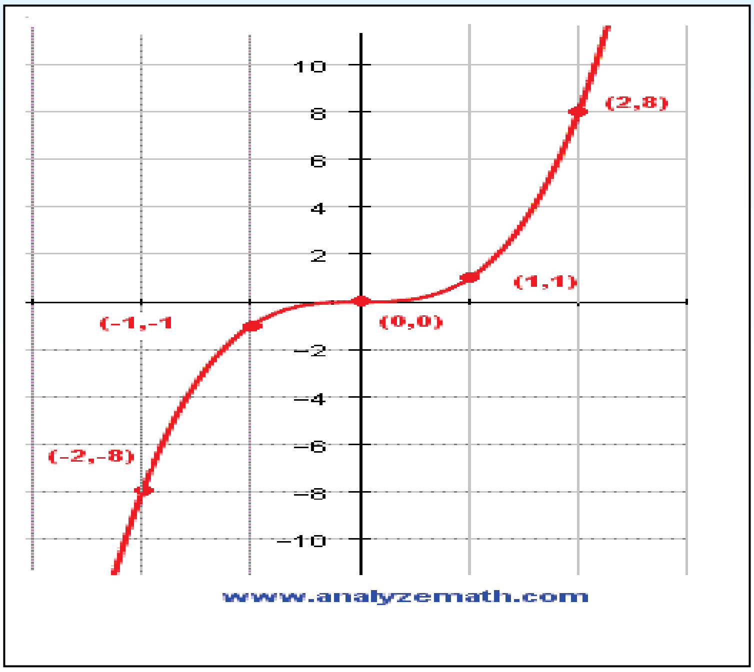
b - The domain of ***f*** (x) is the set of all real numbers.

- Since the leading coefficient (of x^3) is positive, the graph of f is up on the right and down on the left and hence the range of f is the set of all real numbers.

c - Make a table of values and graph.

x	-2	-1	0	1	2
$f(x) = x^3$	-8	-1	0	1	8

- Also since ***f***(-x) = **-** ***f***(x), function f is odd and its graph is symmetric with respect to the origin (0,0).



Example 2: f is a cubic function given by

$$f(x) = -(x-2)^3$$

a. Find y intercepts of the graph of f.

b. Find all zeros of f and their multiplicity.

c. Find the domain and range of f.

d. Use the y intercept, x intercepts and other properties of the graph of to sketch the graph of f.

Solution to Example 2

a - The y intercept is given by

$(0, f(0)) = (0, 8)$

b - The zeros of f are solutions to

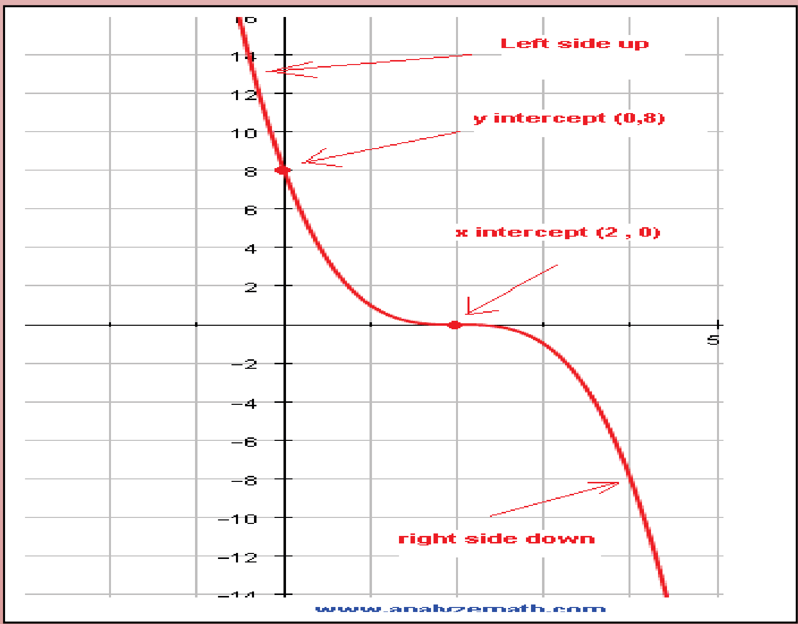
$-(x-2)^3 = 0$

- Function f has one zero at x = 2 of multiplicity 3 and therefore the graph of f cuts the x axis at x = 2.
- c - The domain of f (x) is the set of all real numbers.

- After expansion of f(x), we can see that the leading coefficient (of x^3) is negative, the graph of f is down on the right and up on the left and hence the range of f is the set of all real numbers.

d - Properties and graph.

- At x = 2, the graph cuts the x axis. The y intercept is a point on the graph of f. Also the graph of $f(x) = -(x - 2)^3$ is that of $f(x) = x^3$ shifted 2 units to the right.



- The term (x - 2) and reflected on the x axis because of the negative sign in $f(x) = -(x- 2)^3$. Adding to all these properties the left and right hand behaviour of the graph of ***f***, we have the following graph.

Example 3: f is a cubic function given by

$$f(x) = (x+2)(x-1)(x+1)$$

a. Factor f(x).
b. Find all zeros of *f* and their multiplicity.
c. Find the domain and range of *f*.
d. Use the y intercept, x intercepts and other properties of the graph of to sketch the graph of *f*.

Solution to Example 3

a - $f(x) = x^3 + 2x^2 - x - 2 = x^2(x + 2) - (x + 2) = (x + 2)(x^2 - 1)$

b - The zeros of *f* are solutions to

$(x + 2)(x^2 - 1) = 0$

- Function *f* has zeros at x = - 2, at x = 1 and x = - 1. Therefore the graph of f cuts the x axis at all these x intercepts.

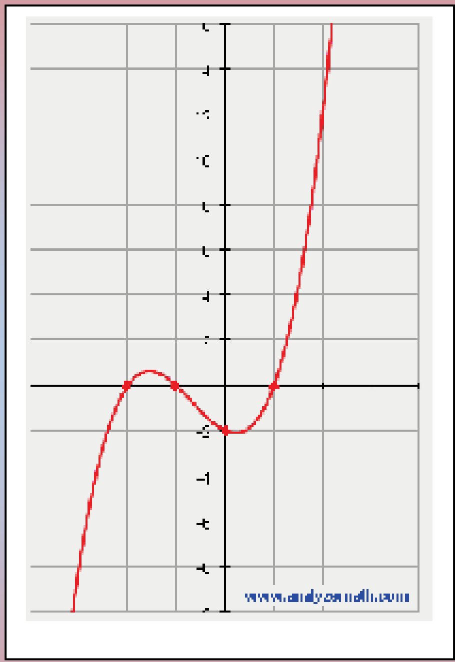
c - The domain of f (x) is the set of all real numbers.

- The leading coefficient f(x) is positive, the graph

  of *f* is down on the left and up on the right and hence

- The range of *f* is the set of all real numbers.

d - Properties and graph.



- The y intercept of the graph of *f* is at (0 , - 2). The graph cuts the x axis at x = -2, -1 and 1. Adding to all these properties the left and right hand behaviour of the graph of *f*, we have the graph shown.

Example 4: *f* is a cubic function given by

$$f(x) = -(x - 2)(x + 1)^2$$

a. Find all zeros of *f* and their multiplicity.
b. Find the domain and range of *f*.
c. Use the y intercept, x intercepts and other properties of the graph to sketch the graph of *f*.

Solution to Example 4

a. The zeros of *f* are solutions to

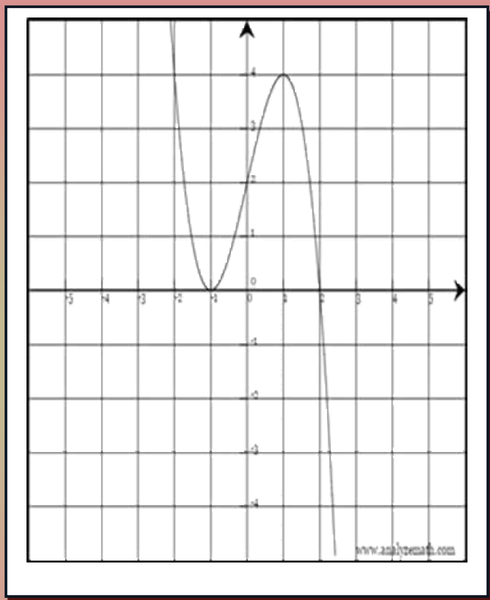
$-(x - 2)(x + 1)^2 = 0$

- Function f has zeros at x = 2 and x = - 1 with multiplicity 2. Therefore the graph cuts the x axis at x = 2 and is tangent to the x axis at x = - 1 because the multiplicity of this zero is even.

b . The domain of f (x) is the set of all real numbers.

- The leading coefficient f(x) is negative, the graph of f is up on the left and down on the right and hence the range of f is the set of all real numbers.

c . Properties and graph.



The y intercept of the graph of f is at (0 , 2). The graph cuts the x axis at x = 2 and is tangent to it at x = - 1. Adding to all these properties the left and right hand behaviour of the graph of f, we have the following graph.

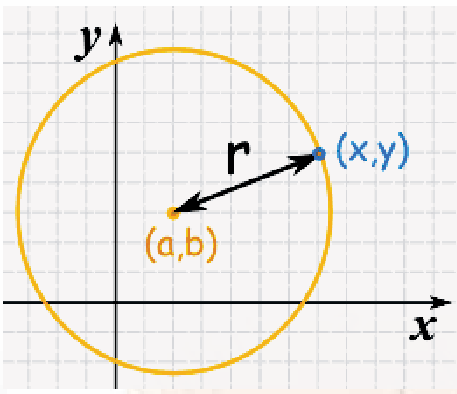
Circle

The definition of a circle is

the set of all points on a plane that are a fixed distance from a **centre**.

Let us put that center at **(a,b)**.

So the circle is **all the points (x, y)** that are **"r"** away from the centre **(a, b)**.



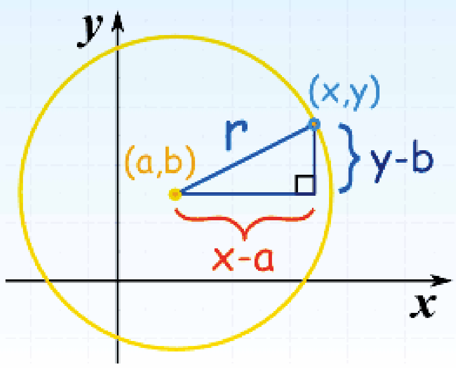
Now we can work out **exactly** where all those points are:

We make a right-angled triangle (as shown),

and then use Pythagoras ($a^2 + b^2 = c^2$):

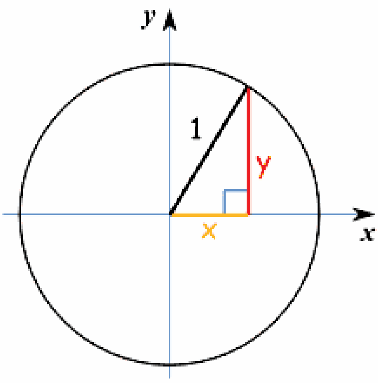
And that is the **"Standard Form"** for the equation of a circle!

$$(x - a)^2 + (y - b)^2 = r^2$$



Unit Circle

If we place the circle center at (0,0) and set the radius to 1 we get:



$$(x - a)^2 + (y - b)^2 = r^2$$

$$(x - 0)^2 + (y - 0)^2 = 1^2$$

$$x^2 + y^2 = 1$$

which is the equation of the Unit Circle.

How to Plot a Circle by Hand

1. Plot the centre **(a, b)**

2. Plot 4 points "radius" away from the centre in the up, down, left and right direction

3. Sketch it in!

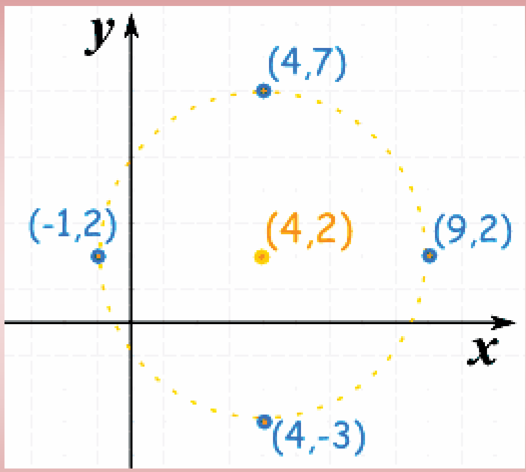
Example: Plot $(x - 4)^2 + (y - 2)^2 = 25$

The formula for a circle is $(x - a)^2 + (y - b)^2 = r^2$

So the centre is at (4, 2)

And $\mathbf{r^2}$ is **25**, so the radius is $\sqrt{25} = 5$

So we can plot:



The Centre: (4, 2)

Up: (4, 2+5) = (4, 7)

Down: (4, 2-5) = (4,-3)

Left: (4-5, 2) = (-1, 2)

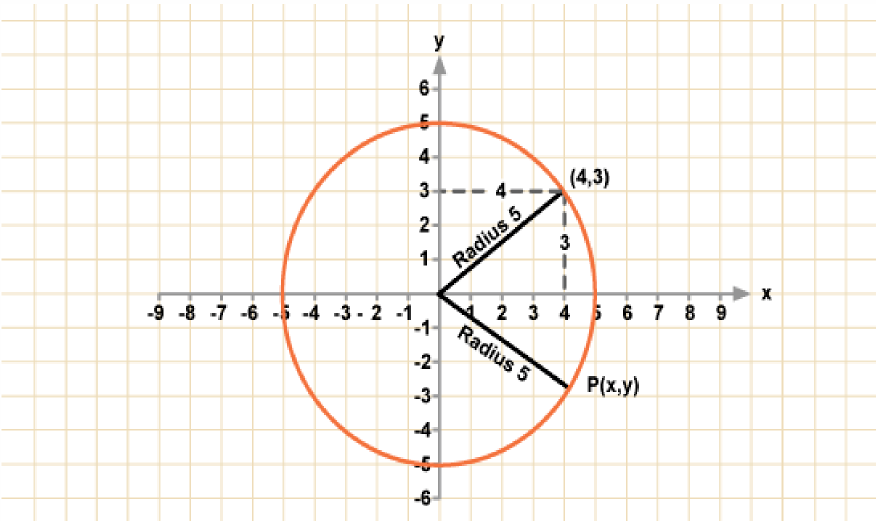
Right: (4+5, 2) = (9, 2)

Now, just sketch in the circle the best that you can!

For the circle below, r = 5, x = 4 and y = 3

So the equation of the circle is $x^2 + y^2 = 5^2$

Point (4, 3) is on the circle because $4^2 + 3^2 = 5^2$



Domain : $-5 \le x \le 5, x \in R$

Range: $-5 \le x \le 5, y \in R$

P(x,y) is on the circle $x^2 + y^2 = 5^2$

General Form

But you may see a circle equation and **not know it**!

Because it may not be in the neat "Standard Form" above.

As an example, let us put some values to **a**, **b** and **r** and then expand it

Start with:	$(x-a)^2 + (y-b)^2 = r^2$
Set (for example) a=1, b=2, r=3:	$(x-1)^2 + (y-2)^2 = 3^2$
Expand:	$x^2 - 2x + 1 + y^2 - 4y + 4 = 9$
Gather like terms:	$x^2 + y^2 - 2x - 4y + 1 + 4 - 9 = 0$

And we end up with this:

$$x^2 + y^2 - 2x - 4y - 4 = 0$$

It is a circle equation, but "in disguise"!

*So when you see something like that think " ... that **might** be a circle!"*

In fact we can write it in **"General Form"** by putting constants instead of the numbers:

$$\mathbf{x^2 + y^2 + Ax + By + C = 0}$$

Going from General Form to Standard Form

Imagine we have an equation in **General Form** (like the example above):

$$x^2 + y^2 - 2x - 4y - 4 = 0$$

How can we get it into **Standard Form** like $(x-a)^2 + (y-b)^2 = r^2$?

The answer is to Complete the Square... for x **and** for y:

Start with:	$x^2 + y^2 - 2x - 4y - 4 = 0$
Put **x**s and **y**s together on left:	$(x^2 - 2x) + (y^2 - 4y) = 4$

Now to complete the square we take half of the middle number, square it and add it.

(Also add it to the right hand side so the equation stays in balance!)

And do it for x and y.

Do it for "x": $(x^2 - 2x + (-1)^2) + (y^2 - 4y) = 4 + (-1)^2$

And for "y": $(x^2 - 2x + (-1)^2) + (y^2 - 4y + (-2)^2) = 4 + (-1)^2 + (-2)^2$

Simplify: $(x^2 - 2x + 1) + (y^2 - 4y + 4) = 9$

Finally: $(x - 1)^2 + (y - 2)^2 = 3^2$

And we have it in Standard Form

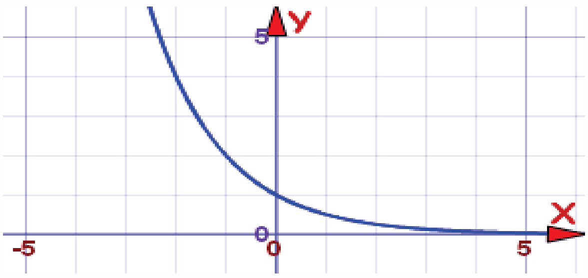
Exponential Function

This is the Exponential Function: $f(x) = a^x$ **a** is any value greater than 0.

Properties depend on value of "a"

- When **a = 1**, the graph is a *horizontal line* at **y=1**
- Apart from that there are two cases to look at:

a between 0 and 1

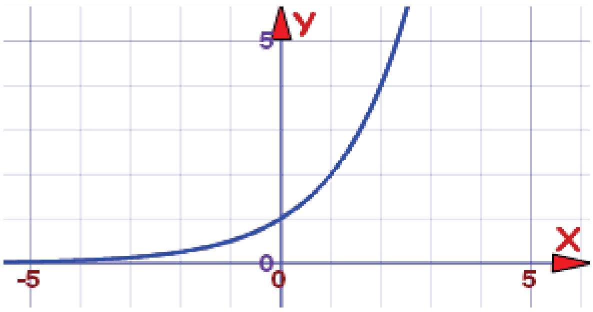


Example: **$f(x) = (0.5)^x$**

For **a** between 0 and 1

- As **x** increases, **f(x)** heads to 0
- As **x** decreases, **f(x)** heads to infinity
- Domain : x ∈ R
- Range : y > 0, y ∈ R
- It has a Horizontal Asymptote along the x-axis (y=0).

a above 1



Example: **$f(x) = (2)^x$**

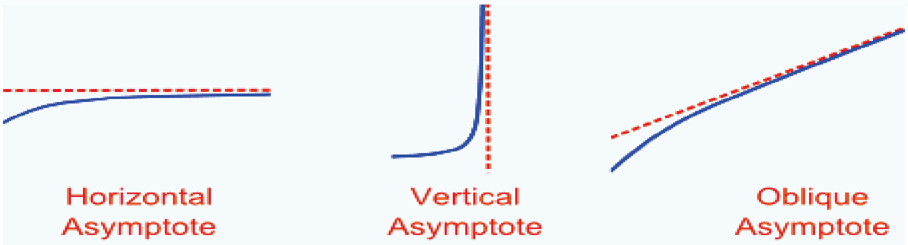
For **a** above 1:

As **x** increases, **f(x)** heads to infinity
As **x** decreases, **f(x)** heads to 0
Domain : x ∈ R
Range : y > 0, y ∈ R
It has a Horizontal Asymptote along the x-axis (y=0).

Note: (Only) Range will change with the equation of the exponential function

Asymptote

A **line** that a curve approaches, as it heads towards infinity:



It can be in a negative direction, the curve can approach from any side (such as from above or below for a horizontal asymptote)

The important point is that:

The **distance** between the curve and the asymptote **tends to zero** as they head to infinity (or −infinity)

EXERCISE 4.2

1. Sketch the graph of $y = 4x^2$ 1nd $y = 6x^2$ on the same pair of axis .
2. Sketch the graph of $y = 5x^2$ and $y = -5x^2$ on the same pair of axis.
3. Sketch the graph of the following quaratic equation showing the intercepts.

   [i] $y = x^2 - 9$ [ii] $y = x^2 - 3x$ [iii] $y = 4x - x^2$ [iv] $y = 2x - x^2$
4. Sketch the grph of the following giving the coordinates of the vertex and the equation of the axis of symmetry.

   [i] $y = (x - 2)^2 + 3$ [ii] $y = (x + 1)^2 + 3$ [iii] $y = 2(x - 3)^2 - 3$

   [iv] $y + 1 = (x - 1)^2$ [v] $y - 4 = 3(x + 1)^2$ [vi] $y = 2 - 2(x - 4)^2$

5. What is the coordinates of the vertex of the parabola $f(x) = 2x^2 + 8x - 12$?

6. Change the following equations in the form $f(x) = a(x - h)^2 + k$, using perfect square and sketch its graph showing the intercepts and vertex.

[i] $y = x^2 - 2x + 3$ [ii] $y = x^2 + 6x - 2$ [iii] $y = x^2 - 3x - 3$

[iv] $y = 2x^2 - 4x + 3$ [v] $y = 3x^2 - 2x - 1$ [vi] $y = 3x^2 - 5x + 3$

7. Sketch the graph of $y = 3x^3$ and $y = -3x^3$ on the same pair of axis

8. Sketch the graph of the following cubic functions:

[a] $y = (x-1)(x+2)(x-3)$ [b] $y = (x+3)(x+2)(x-1)$

[c] $y = -(x-3)(x+2)(x+3)$ [d] $y = y = x(x+2)(x-3)$

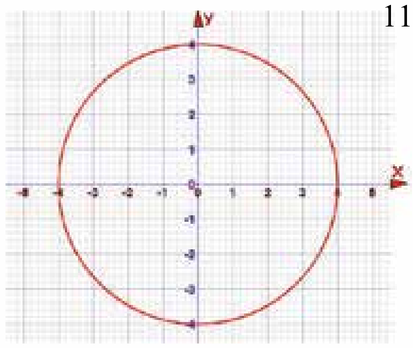
[e] $y = (x-1)^2(x-3)$ (f) $y = (x-1)^2(x+2)$

[g] $y = (2-x)^2(x-3)$ [h] $y = (1-x^2)(2-x)$

9. Sketch the graph of the following exponential functions

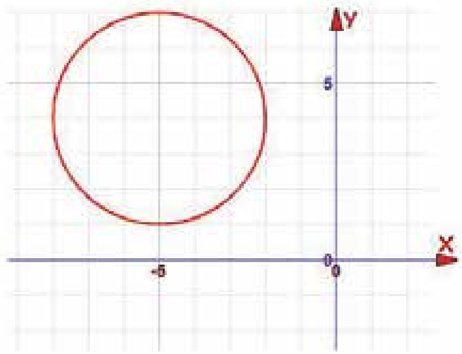
[i] $y = 3^x$ [ii] $y = 5^x$ [iii] $y = (5/8)^x$ [iv] $y = ½^x$

10. Write down the equation of the circle in standard Form



11.

11. Write down the equation of the circle in standard Form



12 . Plot the following circles

[a] $x^2 + y^2 = 16$ [b] $2x^2 + 2y^2 = 100$

[c] $(x-2)^2 + (y-5)^2 = 81$ [d] $(x+1)^2 + (y-2)^2 = 125$

[e] $(x+2)^2 + (y+4)^2 = 64$ [f] $(x+1)^2 + (y-2)^2 = 125$

[g] $x^2 + y^2 - 2x - 4y - 4 = 0$

13 . What is the centre of the circle $(x - 5)^2 + (y + 3)^2 = 49$?

14. What is the radius of the circle $(x + 2)^2 + (y - 4)^2 = 36$?

15. What is the centre of the circle $x^2 + y^2 + 8x - 12y + 27 = 0$?

16. What is the radius of the circle $x^2 + y^2 - 6x + 14y - 23 = 0$?

17. The equation of a circle in Standard Form is $(x + 11)^2 + (y - 9)^2 = 16$.

What is the equation of the circle in General Form?

18. The equation of a circle in Standard Form is $(x - 2.5)^2 + (y + 3.5)^2 = 10$.

What is the equation of the circle in General Form?

19. The General Form of the equation of a circle is $x^2 + y^2 + x + 3y - 6.5 = 0$.

Write the equation in Standard Form.