STRAND 1

BASIC MATHEMATICS

LEARNING OUTCOMES

- Manipulate Binary operations
- Identify Binary properties
- Determine Closure of a set
- Find identity and Inverse elements
- Know properties of zero
- Use order of operation
- Identify everyday measurements like length, area , volume , mass , capacity, money and time

Sub Strand

Basic Number Theory

- Study and use properties of binary operations
- Explain and work out closure, identity and inverse element of a system
- Use calculator and to work with properties of zero.

BINARY OPERATION

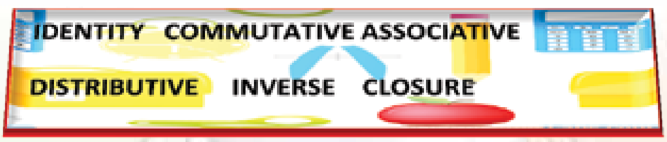
A *binary operation* on *A* is a rule that assigns to every pair of elements of a unique element of *A*.

We are used to addition and multiplication of real numbers and these are examples of binary operations.

Formally, a binary operation is a function. For example, addition on the integers could be defined as the function+ (A + B →C) with our familiar way of producing the answer. Thus +(2,3) = 5 and +(-4,11) = 7. This looks a little too weird and we are probably happier to see 2+3=5 and -4 + 11 =5. Likewise, if * is a binary operation on a set then we will write *a***b* rather than *(*a*,*b*).

The idea of a binary operation is just a way to produce an element of a set from a given pair of elements of the same set. In the case of a finite set we could list the rule in a table.

In studying binary operations on sets, we tend to be interested in those operations that have certain properties



Identity Element

Let *A* be a set on which there is a binary operation. An element *e* of this set is called an *identity, e,* if for all a ∈ A

$$\boxed{e\ .\ a = a}$$

For example **0** is an identity for the usual **addition** on the real numbers.

Given a binary operation on a set there might be no identity element. We tend to be familiar with the situation in which there is a unique identity. Also note that an identity for one operation does not have to be an identity for another operation. Think of addition and multiplication on the reals where the identities are 0 and 1 respectively.

Being an identity is a global property in the sense that it must work for **ALL** elements of the set.

Associative Property

A binary operation * is said to be *associative* if for all elements *a*, *b* and *c* satisfy :

$$\boxed{(a*b)*c = a*(b*c)}$$

For convenience let's drop the symbol for the operation and just write **(*ab*)*c* = *a*(*bc*).** The associative property then allows us to speak of *abc* without having to worry about whether we should find the answer to *ab* first and then that answer "multiplied" by *c* rather than evaluate *bc* first and then "multiply" *a* with that answer. Whichever way we process the expression we end up with the same element of the set. Note though that it does not say we can do the product in any order (i.e. *ab* and *ba* may not have the same value).

Commutative Properties

A binary operation ***** is *commutative* if

$$a * b = b * a$$

for ALL possible *a* and *b* in the set. Addition and multiplication in the reals are commutative operations whereas multiplication of matrices generally is not.

Note that the definition requires *ab* = *ba* for all pairs of elements. That some element commutes with all elements does not make the operation commutative.

For a finite set whose binary operation is given in a table, it is easy to observe whether the operation is commutative. If we write the elements along the rows in the same order as down the column then if the table is **symmetric about the diagonal** (i.e reflexive in the leading diagonal) then it is **commutative**.

Distributive Properties

Suppose there are two binary operations * and # defined on a set ***A***. We say that * *distributes* over # if for all *a*, *b* and *c* in *A*,

$$a * (b \# c) = a * b \# a * c \quad \text{and} \quad (a \# b) * c = a * b \# a * c.$$

Addition over real numbers is associative.

Other examples of distributive operations are set union and intersection.

Inverse

If identities exist for a certain binary operation then we can talk about inverses (should they exist). If e is an identity then a is an *inverse* of b if

$$a * b = b * a = e$$

Also we need to point out that an element might have more than one inverse or for that matter, no inverse. Certainly if identities do not exist then it does not make sense to talk about inverses.

Even if there is an identity then elements may not have an inverse.

So binary operation simply means any operation between **two** objects.

It is simply a rule for combining two objects of a given type, to obtain another object of that type. You first learned of binary operations in primary school. The objects you were using were numbers and the binary operations you investigated were addition, subtraction, multiplication and division. As you will discover in this text, binary operations need not be applied only to numbers.

CLOSURE OF A SET

A set is *closed* (under an operation) if and only if the operation on two elements of the set produces another element of the **same** set. If an element outside the set is produced, then the operation is *not closed*.

Example: If you multiply two *real numbers*, you will get another real number. Since this process is always true, it is said that the real numbers are "closed under the operation multiplication". There is simply no way to escape the set of real numbers when multiplying.

Closure: When you combine any two elements of the set, the result is also included in the set.

Example: If you add two even numbers (from the set of even numbers), is the sum even?

Checking:	10 + 14 = 24	Yes, 24 is even.
	6 + 10 = 16	Yes, 16 is even.
	8 + 200 = 208	Yes, 208 is even.

Since the sum (the answer) is always even, the set of even numbers is closed under the operation of addition.

Does this mean the even numbers are closed for all operations?

Let's check out this question. If you divide two even numbers (from the set of even numbers), is the quotient (the answer) even?

Checking:	20 ÷ 2 = 10	Yes, 10 is even.
	24 ÷ 2 = 12	Yes, 12 is even.
	30 ÷ 2 = 15	**NO, 15 is not even!**

When you find even ONE example that does not work, the set is not closed under that operation. The even numbers are **not closed** under division.

The elements in a binary table are displayed horizontally and vertically outside the table (in this table, the elements are 1, 2, 3, and 4). If the elements inside the table are limited to the elements 1, 2, 3, and 4, the table is **closed** under the indicated operation.

$\boxed{}$	1	2	3	4
1	4	3	2	1
2	3	1	4	2
3	2	4	1	3
4	1	2	3	4

Reading the table:

Read the first value from the left hand column and the second value from the top row. The answer is in the cell where the row and column intersect.

For example, **a * b = b, b * b = c, c * d = b, d * b = a**.

Example 2

*	a	b	c	d
a	a	b	c	d
b	b	c	d	a
c	c	d	a	b
d	d	a	b	c

What is the identity element for the operation * ?
(What single element will always return the original value?)
The identity element is **a.**

a * a = a, b * a = b, c * a = c,

d * a = d

This table shows the operation *. The operation is working on the finite set A = { a, b, c, d}. The table shows the 16 possible calculations using the elements of set A.

Checking for the Identity Element:

You will know the identity element when you see it, because all of the values in its row or column are the same as the row or column headings.

What is the inverse element for b ?

(What element, when paired with **b**, will return the identity element **a**?)

The inverse element of b is d. $b * d = a \rightarrow b^{-1} = d$

Is the operation * commutative?

(Does the property *ab* = *ba* hold for

ALL possible arrangements of values?)

Start testing values:

a * b = b * a is true since both sides equal b.

c * d = d * c is true since both sides equal b.

WOW! Having to test ALL possible arrangements could take forever! There must be an easier way.....

Testing for Commutativity Shortcut:

It is easy to check whether an operation defined by a table is commutative. Simply draw a diagonal line from upper left to lower right, and see if the table is symmetric about this line.

*	a	b	c	d
a	a	b	c	d
b	b	c	d	a
c	c	d	a	b
d	d	a	b	c

Since this table is symmetric about the diagonal (from upper left to lower right), the operation is commutative. It would only have taken one instance of lack of commutativity for this answer to have been "no".

The operation * is *commutative*.

True or False:

(a * b) * c = a * (b * c) ???

Perform the operations in the order indicated by the parentheses:

(a * b) * c = a * (b * c)

This question deals with only one case of the **associative property** for this table. Unfortunately **there is NO shortcut for checking associativity** as there is for checking commutativity when working with a table.

If asked the general question "Does this operation possess the associative property?", you would have to check ALL possible arrangements. On the other hand, if you find one instance where associativity fails, you are done and the answer is "NO".

EXERCISE 1.1

1. Which property is illustrated by the equation $ma + mb = m(a + b)$

A . Associative B. Commutative C. Distributive D. Identity

2. If A and X represent integers, $A + X = X + A$ is an example of which property?

A . Associative B. Commutative C. Distributive D. Identity

3. Operation # is defined on the set **{m, a, t, h }** as shown in the table below:

a. Is this operation commutative?
b. Name the identity element, or explain why none exists.
c. For each element having an inverse, name the element and its inverse.
d. True or false:

(m # a) # t = m # (a # t)

#	m	a	t	h
m	h	m	a	t
a	m	a	t	h
t	a	t	h	m
h	t	h	m	a

4 . Operation □ is defined on the set **{ a, b, c, d }** shown in the table below:

□	a	b	c	d
a	d	c	a	b
b	c	a	b	d
c	a	b	d	c
d	b	d	c	a

a . Is this operation commutative?

b. Name the identity element, or explain why none exists.

c. For each element having an inverse, name the element and its inverse.

d. True or false:

d □ (c □ b) = (d □ c) □ b

5. Operation □ is defined on the set **{ 1, 2, 3, 4 }** as shown in the table below

□	1	2	3	4
1	4	3	2	1
2	3	1	4	2
3	2	4	1	3
4	1	2	3	4

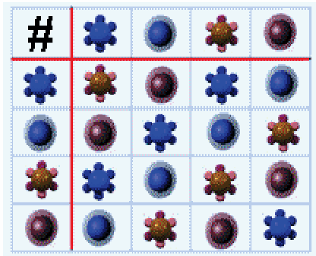
a. Is this operation commutative?

b. Name the identity element, or explain why none exists.

c. For each element having an inverse, name the element and its inverse.

d . True or false: **(1 □ 2) □ 3 = 1 □ (2 □ 3)**

6 . Operation **#** is defined on the set **{**  **}** as shown in the table below:

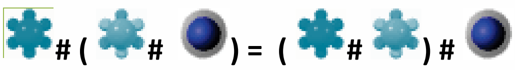


a. Is this operation commutative?

b. Name the identity element, or explain why none exists.

c . For each element having an inverse, name the element and its inverse.

d. True or false:



7. Given: a ♦ b = 2a - 3b

What is the value of - 2 ♦ 5?

8 . Given x  y = ½ b – 3a , find the value of – 2  6.

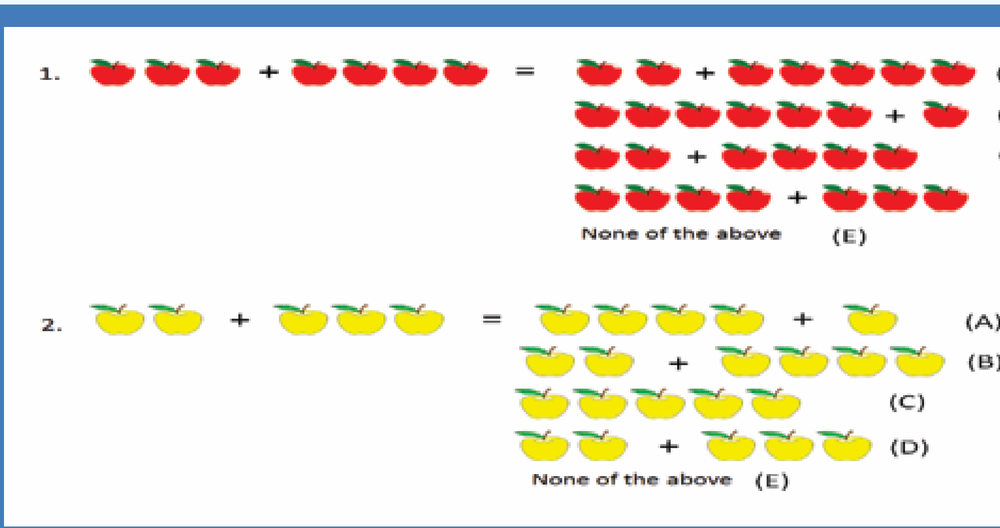
9. An operation * is defined on a set of real numbers as $a * b = 3a^2 - 5b$.

Find the value of

[i] 2 * 3 [ii] 4 * -2 [iii] -5 * -3

Test Your Knowledge

Which of the following equations follow the concept of commutative property?



ORDER OF OPERATION

When there is more than one operation involved in a mathematical problem, it must be solved by using the correct order of operations. Remember, calculators will perform operations in the order which you enter them; therefore, **you will need to enter the operations in the correct order for the calculator to give you the right answer.**

* In Mathematics, the order in which mathematical problems are solved is extremely important.

Rules

1. Calculations must be done from left to right.
2. Calculations in brackets (parenthesis) are done first. When you have more than one set of brackets, do the inner brackets first.
3. Exponents (or radicals) must be done next.
4. Multiply and divide in the order the operations occur.
5. Add and subtract in the order the operations occur.

Remember to:

- Simplify inside groupings of parentheses, brackets and braces first. Work with the innermost pair, moving outward.
- Simplify the exponents.
- Do the multiplication and division in order from left to right.
- Do the addition and subtraction in order from left to right.

Acronyms to Help you remember

How will you remember this order?
Try the following Acronyms:

* **P**lease **E**xcuse **M**y **D**ear **A**unt **S**era
*(**P**arenthesis, **E**xponents, **M**ultiply, **D**ivide, **A**dd, **S**ubtract)*

* **BEDMAS**
*(**B**rackets, **E**xponents, **D**ivide, **M**ultiply, Add, **S**ubtract)*

Examples:

$12 \div 4 + 3^2$ $12 \div 4 + 9$ $3 + 9$ 12	Rule **3**: Exponent first Rule **4**: Multiply or Divide as they appear Rule **5**: Add or Subtract as they appear
$(4^2 + 5) - 3$ $21 - 3$ 18	Rule **2**: Everything in the brackets first Rule **5**: Add or Subtract as they appear
$20 \div (12 - 2) \times 3^2 - 2$ $20 \div 10 \times 3^2 - 2$ $20 \div 10 \times 9 - 2$ $18 - 2$ 16	Rule **2**: Everything in the brackets first Rule **3**: Exponents Rule **4**: Multiply and Divide as they appear Rule **5**: Add or Subtract as they appear

Does It Make a Difference? What If I Don't Use the Order of Operations?

Mathematicians were very careful when they developed the order of operations. Without the correct order, watch what happens:

15 + 5 X 10 -- Without following the correct order, I know that 15+5 =20 multiplied by 10 gives me the answer of 200.

15 + 5 X 10 -- Following the order of operations, I know that 5X10 = 50 plus 15 = 65. ***This is the correct answer, the above is not!***

You can see that it is absolutely critical to follow the order of operations. Some of the most frequent errors students make occur when they do not follow the order of operations when solving mathematical problems. Students can often be fluent in computational work yet do not follow procedure. Use the handy acronyms to ensure that you never make this mistake again!!

PROPERTIES OF ZERO

Addition property:

The addition property says that a number does not change when adding or subtracting zero from that number

Examples: $2 + 0 = 2$ $x + 0 = x$ $(a + b) + 0 = a + b$

Additive inverse property

If you add two numbers and the sum is zero, we call the two numbers additive inverses of each other

For example: 2 is the additive inverse of -2 because $2 + -2 = 0$

-2 is also the additive inverse of 2 because $-2 + 2 = 0$

Multiplication property

The multiplication property says that zero times any number is equal to zero

Examples: $2 \times 0 = 0$ $265893748945798882 \times 0 = 0$

$x \times 0 = 0$ $(x + y + z + r) \times 0 = 0$

Power of 0

When a number is raised to the power 0, we are not actually multiplying the particular number by 0. For example, let us take 2^0. In this case we are not actually multiplying the number 2 by 0.

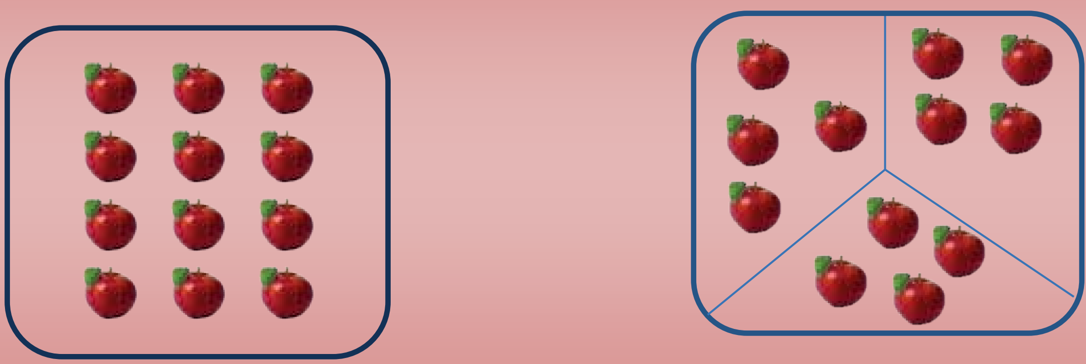
We define $2^0 = 1$, so that each power of 2 is one factor of 2 larger than the last, e.g., 1, 2,4,8,16,32... This involves the rules of exponents particularly division.

Thus, this result says that number raised to the power zero is equal to 1.

Dividing by 0

To see why we can't divide by 0, let us look at what is meant by "division": Division is splitting into equal parts or groups. It is the result of "fair sharing".

Example: there are 12 apples, and 3 friends want to share them, how do they divide the apples?



So they get 4 each: $12/3 = 4$

Now, let us try dividing the 12 apples among **zero** people, how much does each person get?

Does that question even make sense? No, of course it doesn't.

You can't share among zero people, and you can't divide by 0.

Another way of seeing it:

After dividing, can you multiply to get back again?

12 divided by 3 is 4 → 4 times 3 is 12

12 divided by 0 is ? → ? times 0 is 12

But multiplying by 0 gives 0, so that won't work.

So what is 0 ÷ 0?

0 ÷ 0 is like asking "how many 0s in 0?"

Are there no zeros in zero at all? Or perhaps there is exactly one zero in zero?

Or many zeros?

So 0 ÷ 0 is indeterminate (it could be any value).

When you try to divide by zero, things stop making sense therefore dividing by 0 is **undefined !**

I Wonder

<u>Test Your Knowledge</u>

- I know a way by which I can get a total of 120 by using five zeros 0,0,0,0,0 and any mathematical operator. Can you do this?

FORMULA MANIPULATION

A formula links one quantity to one or more other quantities. For example, **$A = \pi r^2$**

This formula can be used to determine A for any given value of r.

Changing the subject of a formula

Changing the subject of a formula is exactly the same as solving an equation. The key thing to remember is that '*whatever you do to on one side of the formula you must do to the other side*'.

To rearrange a formula you may

- add or subtract the same quantity to or from both sides
- multiply or divide both sides by the same quantity

We may know the area of a circle and need to find the radius. To do this, we rearrange the formula to make the radius the subject.

The area of a circle (A) is πr^2. So: $A = \pi r^2$

We will now rearrange the formula to make '***r***' the subject.

$A = \pi r^2$ Start by dividing both sides by π.

$\frac{A}{\pi} = r^2$ Then take the square root of both sides.

$\sqrt{\frac{A}{\pi}} = r$

Examples

1. If F = ma, what's the value of F when m = 5 and a = 3?

$$F = ma$$
$$= 5 \times 3$$
$$= 15$$

2. The formula for the volume (V) of a sphere is: $V = {}^4/_3 \pi r^3$

Rearrange the formula to make 'r' the subject $V = {}^4/_3 \pi r^3$

Multiply both sides by 3. $3V = 4\pi r^3$

Divide by 4π. $\frac{3V}{4\pi} = r^3$

Take the cube root of both sides. $\sqrt[3]{\frac{3V}{4\pi}} = r$

3. Make x the subject of the formula in y = mx + c

Subtract c from both sides $y - c = mx + c - c$

Simplify $y - c = mx$

Divide both sides by $\frac{y-c}{m} = \frac{mx}{m}$

Simplify $x = \frac{y-c}{m}$

Formulae with brackets and fractions

If there are brackets included in the formula then it is easier if you expand them first. Look out for implied brackets in fractions, (part (c) below), and include them where necessary.

Make the subject of the formula in each of the following cases.

a(x + b) = c

Expand the brackets $ax + ab = c$

Subtract ab from each side $ax = c - ab$

Divide both sides by a $x = \frac{c - ab}{a}$

EXERCISE 1.2

1 Calculate:

(a) $6 + 7 \times 2$ (b) $8 - 3 \times 2$ (c) $19 - 4 \times 3$
(d) $3 \times 6 - 9$ (e) $15 - 4 + 7 \times 2$

2. Decide whether each of the statements below is *true* or *false*.

(a) $6 \times 7 - 2 = 40$ (b) $8 \times (6 - 2) + 3 = 56$ (c) $35 - 7 \times 2 = 56$

(d) $3 + 7 \times 3 = 30$ (e) $18 - (4 + 7) = 21$

3. What is the value of this?

$$\frac{2^4 + (16 - 3 \times 4)}{(6 + 3^2) \div (7 - 4)}$$

4. What is the value of $(7 - \sqrt{9}) \times (42 - 3 + 1)$?

5. What is the value of $(33 - 9 / 3) + (4 \times 3 - 32)$?

6. What is the value of $(5^2 - 5) / (4^2 + 8 - 7 \times -5)$?

7. Make x the subject in the formulas

[i] $y = axb$

[ii] $y = w(x + t)$

[ii] $y = a(x + b)c$

[iii] $ax - y + z + b$

[iv] $ax - y = 2y$

8. In each case , make the letter at the end the subject of the formula

[i] $2s = 2ut + at^2$, (a)

[ii] $v^2 = u^2 + 2as$, (s)

[iii] $y = a^2x + b^2$, (a)

9. Make x the subject in the formulas

[i] $\frac{x}{a} = \frac{y+z}{b}$

[ii] $\frac{a(x+y)}{b} = c$

10. In each case, make the letter at the end the subject of the formula

[i] $s = ut + \frac{1}{2}at^2$, (u)

[ii] $\frac{v-u}{a} = t$, (v)

[iii] $\frac{y-x^2}{a} = 3z$, (y)

11. In each case, make the letter at the end the subject of the formula

[i] $y = \sqrt{x+3}$, (x)

[ii] $x^2 - y^2 = a^2$, (x)

[iii] $\sqrt{x^2 + y^2} = y$, (x)

[iv] $\frac{1}{m} = \frac{1}{s} + \frac{1}{t}$, (s)

12. Simplify the following

[i] $\frac{3a}{5} \cdot \frac{7ab}{3}$ [ii] $\frac{3a-1}{2b^2} \cdot \frac{3a^2b}{9a-3}$

Sub Strand Measurement

- List everyday local measurements.
- Use measurements in calculations
- Conversion in measurements
- Determine how interest rate and length of loan affect total cost of credit
- Compare 3 different savings plan
- Show advantages of early loan or credit repayment

In the metric system, each of the common kinds of measure -- length, weight, and capacity -- has *one* basic unit of measure. To measure smaller amounts, divide the basic unit into parts of ten, a hundred, or a thousand, and so on. To measure larger amounts, multiply the basic unit by ten, a hundred, or a thousand, and so on.

Brief History

The Metric system of measurement was created about two hundred years ago by a group of French scientists to simplify measurement.

Length:	**Capacity**:
1 kilometre (km) = 1000 meters (m) 1 centimetre (cm) = .01 meter (m) 1 millimetre (mm) = .001 meter (m)	1millilitre = .001 liter (l)
Weight: 1 kilogram (kg) = 1000 grams (g) 1 milligram (mg) = .001 gram (g)	*Kilo* means thousand (1000) *Hecto* means hundred (100) *Deca* means ten (10) *Deci* means one-tenth (1/10) *Centi* means one-hundredth (1/100) *Milli* means one-thousandth (1/1000)
TIME 1 hr = 60 min 1min = 60 sec	

Conversions

Sometimes you need to convert from one unit of measure to another similar unit.

Proportions will help you make conversions when working with measurements.

Create a unit **conversion ratio**, which is always equal to 1.

Example 1: Convert 4.214 kilometres to meters.

Using a conversion ratio:

$$\frac{?\,m}{4.214km} = \frac{1000m}{1km}$$

conversion ratio

Solving algebraically, the answer is 4214 m

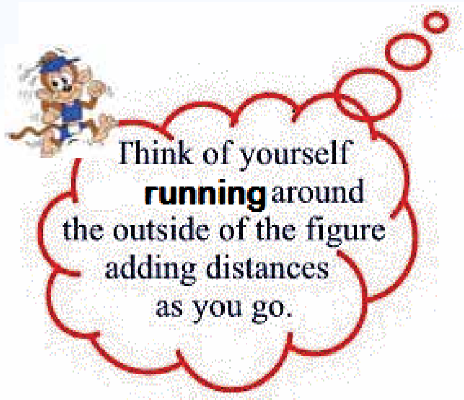
Another way to look at the problem:
When making metric conversions, arrange the prefixes from largest to smallest:

Kilo hecto deca **UNIT** deci centi milli
(meter, gram or litre)

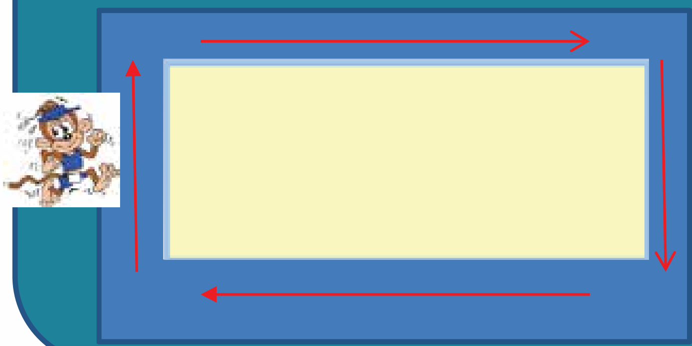
To convert 4.214 kilometres to meters, notice that in the chart above, the name metre is three places to the right of the name kilo. This implies that the decimal point in our number needs to be moved three places to the right. The answer is 4214 metres.

PERIMETER AND CIRCUMFERENCE

Perimeter is the word used to describe the distance around the outside of a figure.



To find the perimeter, add together the lengths of all of the sides of the figure.



Triangle	**3 sides**
Quadrilateral	**4 sides**
Pentagon	**5 sides**
Hexagon	**6 sides**
Heptagon or Septagon	**7 sides**
Octagon	**8 sides**
Nonagon	**9 sides**
Decagon	**10 sides**
Dodecagon	**12 sides**

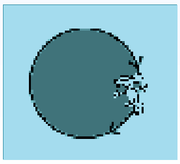
Refresh your polygon memories:

When working with perimeter, references may be made to the names of polygons. Listed at the left are some of the more common polygons whose names you should know.

Remember that "regular polygons" are polygons whose sides are all the same length and whose angles are all the same size. Not all polygons are "regular".

Circumference is the word used to describe the distance around the outside of a circle.

Like perimeter, the circumference is the distance around the outside of the figure. Unlike perimeter, in a circle there are no straight segments to measure, so a special formula is needed.



$$C = 2\pi r = \pi d$$

Use when you know the radius $C = 2\pi r$

Use when you know the diameter $C = \pi d$

Example 1

Ansaar and Muskaan are jogging around a circular track in the park.
The diameter of the track is 400m.
Find the number of kilometres they jogged if they made two complete trips around the track.

$C = d \times \pi$
$= 400 \text{ m} \times \pi$
$= 1256.6371$

Therefore for 2 rounds : 1256.6371×2
$= 2513.2741$

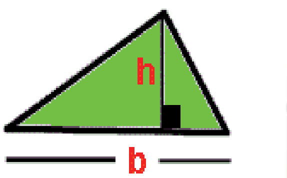
Hence $2513.2741 / 1000 = 2.51$ km

Work should be held in the calculator until the final rounding of the answer occurs.

AREA

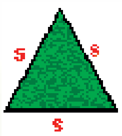
The sum of areas of all the faces of a 3D solid figure (both plane and curved) is called its total surface area.

Area (triangle)



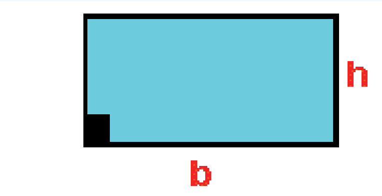
$$A = \frac{1}{2}bh$$

Area (equilateral triangle)



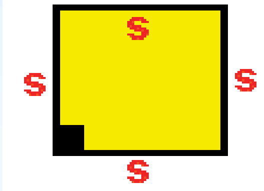
$$A = \frac{s^2\sqrt{3}}{4}$$ or $$A = \frac{1}{2}bh$$

Area (rectangle)



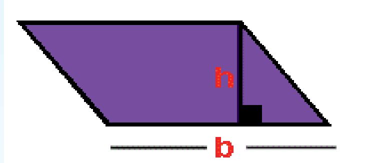
$$A = bh$$

Area (square)



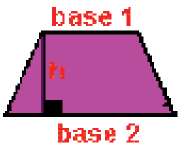
$$A = s^2 = bh$$

Area (parallelogram)



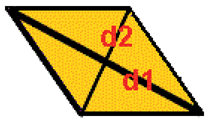
$$A = bh$$

Area (trapezium)



$$A = \frac{1}{2}h(b_1 + b_2)$$

Area (rhombus)

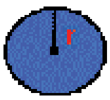


$$A = \frac{1}{2}d_1 \cdot d_2 \qquad A = bh$$

Any Sector
(fractional part of the area)

$A = \frac{n}{360}\pi r^2$ where n is the number of degrees in the central angle of the sector.	$A = \frac{C_s}{2\pi r}\pi r^2$ where C_S is the arc length of the sector.

Area (circle)

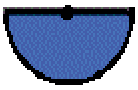


$$A = \pi r^2$$

Area of sectors of circle

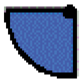
(Sectors are similar to "pizza slices" of a circle)

Semi-circle
(½ of circle = ½ of area)



$$A = \frac{1}{2}\pi r^2$$

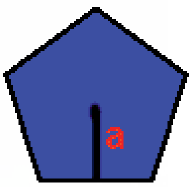
Quarter-Circle
(¼of circle = ¼ of area)

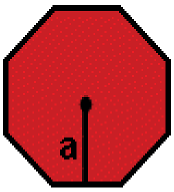


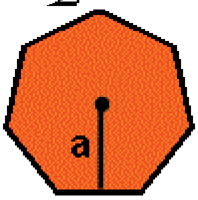
$$A = \frac{1}{4}\pi r^2$$

Area (regular polygon)

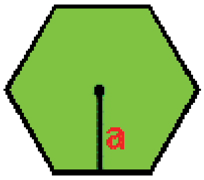
$$A = \frac{1}{2}ap$$







Regular polygons have all sides of equal length.



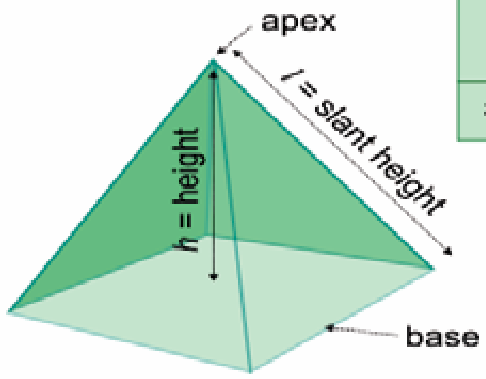
a = apoth

p = perimeter

VOLUME

The total amount of space occupied by a 3D solid figure is called its volume.

Pyramid - A three-dimensional figure whose base is a polygon of any number of sides and other faces are triangles with a common vertex, called apex.

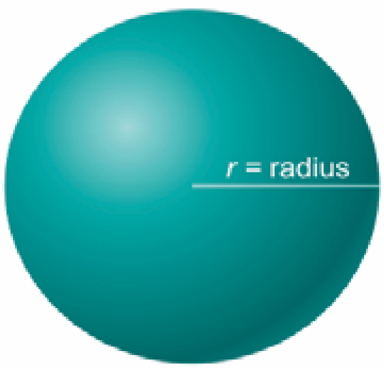


Type of Pyramid	Vertices	Edges	Surface (Faces)	
			Total	Type
	Corners of Base + 1	Sides of Base × 2	Sides of Base + 1	Type of Base, Triangular
Square	5	8	5	Square, Triangular

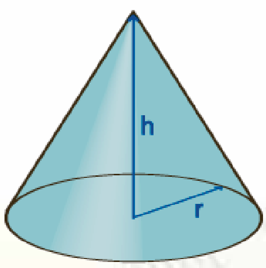
Surface Area	Area of Base + $\frac{1}{2}$ × Perimeter of Base × l
Volume	$\frac{1}{3}$ × Area of Base × h

Sphere - A sphere is a perfectly round three-dimensional geometrical object. It is the set of points which are all at the same distance from a given point in space.

Vertices	Edges	Surface (Faces)	
		Total	Type
0	0	1	Curved



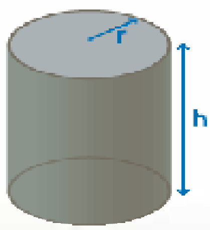
Surface Area	$4\pi r^2$
Volume	$\frac{4}{3}\pi r^3$



Cone

Surface Area = $\pi r^2 + \pi rs$

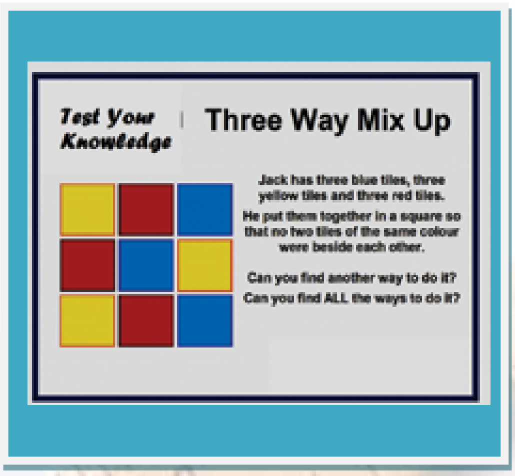
Volume = $1/3(\pi r^2 h)$



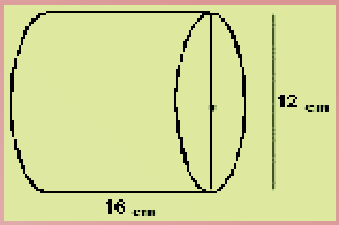
cylinder

Surface Area = $2\pi r^2 + 2\pi rh$

Volume = $\pi r^2 h$



Example 1: Consider this problem where radius is needed but not given:



a. Find the volume of this cylinder.

b. Find the surface area if this cylinder represents a can which has no lids.

NB:

1. When a formula needs a radius, **be sure that you are working with the radius** and not the diameter. In this problem 12 cm is the diameter (radius = 6 cm).

$$V = \pi r^2 h = \pi \cdot 6^2 \cdot 16 = 1809.56 \text{ cm}^3$$

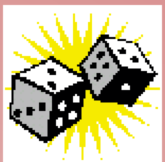
2. You may need to **amend your formula** to fit a particular situation. The surface area formula for a cylinder is

$$SA = 2\pi rh + 2\pi r^2$$

This formula includes the areas of the top and bottom (which are 2 circles). If the top and bottom are **NOT** to be considered, the formula will be

$$SA = 2\pi rh \quad = 2\pi 6 \cdot 16 = 603.19 \text{ cm}^2$$

Example 2: Consider this problem that gives "hints" on what is needed to solve the problem.



A die is a cube moulded from hard plastic. The edge of a die measures 0.62 cm. Dice are usually produced in a mould which holds 100 die at one time. To the *nearest cm³*, how much plastic is needed to fill this large mould?

When working with word problems, be sure to **read carefully to determine what the question wants you to find.** This question clearly involves volume since it states "to the nearest **cubic** cm." Also, the answer must be for 100 dice, not 1 die.

$$\textbf{Volume of one die} = lwh$$

$$= (.62)(.62)(.62) = 0.238 \text{ cm}^3$$

$$\text{For 100 dice} = (0.238)(100) = 23.8 = \mathbf{24 \text{ cm}^3}$$

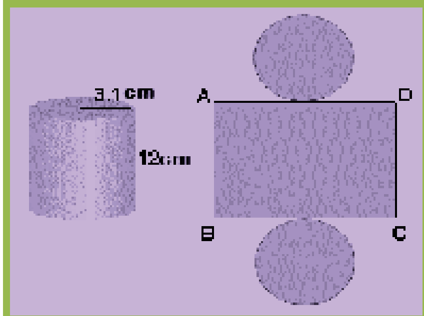
Example 3: Consider this problem with different units of measure.

A concrete truck arrives at a job site holding $15m^3$ of concrete. If the enclosure being constructed is 3m across and 2 thick, how long, to the *nearest m* will the enclosure be if constructed from the amount of concrete on the truck?

Always read carefully to determine if all of the measurements within a problem are expressed in the **same** units. This problem deals with inches, feet and cubic yards.

$$\begin{aligned} V &= lwh \\ 15 &= (l)(3)(2) \\ length &= 2.5\,m \end{aligned}$$

Example 4: Consider this problem that requires visualization.



a. Find AB

b. Find AD

c. Find the surface area of the cylinder.

When dealing with surface area it is often helpful to **imagine the figure cut apart** (called a **net**). In this example, imagine cutting off the top and bottom of the cylinder and then slicing the remaining shape and flattening it out.

a. AB = 12cm

b. AD = the length around the "edge" of the cylinder (which is a circle) = circumference of a circle

$$= 2\pi r = 2\pi(3.1)$$

$$= 19.48 \text{ cm}$$

c. Surface area

$$SA = 2\pi rh + 2\pi r^2 = 294.12\ cm^2$$

Money Management

- Compare the total cost of borrowing for any purchase of an item for two different types of credit.
- Calculate the interest rate for the loan period and its annual equivalent
- Show, by examples of calculations, how the interest rate and loan length affect the cost of credit.
- Give examples to measure the total cost of credit, including the hidden costs, not the interest rate only
- Calculate and evaluate different savings and debt options and match debt with the future value of purchases.
- Demonstrate the benefits of early repayment.

Financial Education

Financial Education teaches the management of personal finances and investment given a person's personal circumstances. Financial Education will result in a future generation of financially competent young men and women leaving schools, who are able to make informed decisions and sufficiently manage their personal finances and investments vis-à-vis their own personal circumstances, whilst contributing positively to their communities, the economy and the country.

Acquiring Financial Competence involves more than the development of mathematics skills that are traditionally included in the school curriculum, such as recognition of coins and notes, and calculations involving sums of cash. Personal Financial Education promotes social inclusion and helps break the cycle of financial exclusion. Being financially excluded means being cut off from the services and benefits of the financial services industry.

Those who are financially excluded become adept at budgeting by saving bits of money in jars or envelopes. But not only does money stored like this not accrue interest, it is also vulnerable to theft.

Financially excluded households are not able to give children the experience of managing money that others take for granted. There is evidence to suggest that such children go on to become financially excluded themselves. Certainly, "children living in poorer families learn about and experience the economic world differently from their peers in other families."5 This cycle of exclusion needs to be broken if future generations are to become financially competent, it is imperative that Financial Education begins early. This means starting Financial Education as soon as students begin their primary education, building on learning throughout the primary and secondary years of schooling. Not all students complete secondary education, so Financial Education learning at class levels 1-8 and forms 3-7 in the Fiji educational system will be fundamental to realising these goals.

Fiji Financial Education Curriculum Development Project
Senior Secondary Levels, Forms Five and Six
FinED Fiji
A Manual for Teachers
Year 11 & 12

Credit is borrowing money or buying goods and paying for them later. It is the power to obtain goods and services with an obligation on the borrower of future payment to the lender (the course of credit).

Why should borrowing money incur interest to the borrower and earn interest for the lender?

Interest is a charge that is paid by the borrower for the use of someone else's money. A lender expects to be rewarded for not having access to his money.

Table below provides different types of credit, matching the lender, the likely purpose of the credit, potential fees and interest rates to the type of credit.

Type	Lender	Purpose	Fees	Interest rates
Bank loan	Retail Bank	Small items such as a car, household items	Set up fee	Depend on bank policy 12 – 20%?
Third tier lender loan	Pay day lenders	Emergency items Funeral, temporary shortfall in income to pay expenses	Combination of Set up fees Establishment fees Insurance?	Usually 22 – 26% but can be very high if a very short term loan.
Hire purchase	Retail store	Retail goods, such as computers, whiteware, household goods	Set up fee Insurance may be compulsory	May have interest free deals, but interest rates will vary a lot
Credit card	Retail Bank	Any items	Annual fee Joint card holder fee	Standard cards very between 12 – 21%

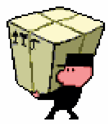
EXERCISE 1.3

1. A paper clip weighs approximately one gram, estimate how many paper clips will be in a 0.5 kilogram box of paper clips.

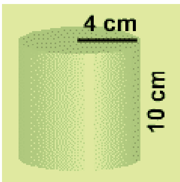
2. How many kg of ground chicken should you buy to make 120 burgers, if each burger will weigh 50grams before cooking?

3. Jone is recovering from surgery. The doctor told him to lift no more than 20 kg for the next 3 weeks .A carton of 48 powdered kitchen cleaner each weighs 500g each. Should Jone lift the carton?

4. A box needs to be covered in brown paper for mailing. If the box measures 3 m by 3 m by 2 m, what is the surface area of the box that will need to be covered?
The box is closed.



5. Find the surface area of this container assuming it has an open top



The bottom is closed.

6. The cash price of a DVD player is $280. The Hire Purchase price is $330. If Fane pays a deposit of 15% followed by 20 equal monthly payments, find how much Fane will pay per month.

7. A computer costs $1800 cash. For Muskaan's birthday her father decided to give the computer as a gift. The H.P. terms are: deposit 15%, followed by 36 payments of $64.60. Find the extra paid on H.P.

8. A television set is priced at $320 in two different shops A and B, which offer different hire purchase terms. Shop A requires 20% deposit and 12 monthly instalments of $26.60. Shop B requires 30% deposit and 12 monthly instalments of $23.50. Vani is planning to buy one. Which shop has the better deal for Vani and why?

9. Seru wished to buy a motorcycle at \$480. He chose to pay by hire purchase. The terms were 30% deposit with 12 monthly payments of \$33.80. How much was the total hire purchase price?

10. Coconut water from NIU Factory is sold in aluminium cans that measure 15cm in height and 3.5cm in diameter. How many cm^3 of coconut water are contained in a full can?

11. Ansaar works for an environmental protection agency which deals with land areas that have been contaminated by toxic waste. The contaminated soil covers an area of 1.62 hectares. He must remove the top 20 cm of soil in this area.

a. What is the area of contaminated soil in m^2?

b. What is the total volume of the contaminated soil in m^3?

c. If each truck can haul 10 m^3 of soil, how many full truckloads of contaminated soil will Ansaar be removing?

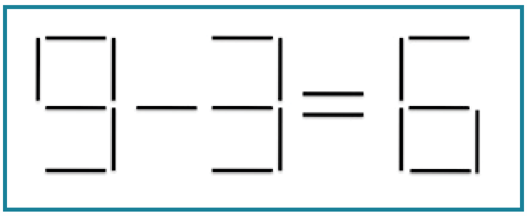
12. A pharmacist is filling medicine capsules. The capsules are cylinders with half spheres on each end. If the length of the cylinder is 12 mm and the radius is 2 mm, how many cubic mm of medication can one capsule hold?

13. What are the advantages of early repayment of hire purchases?

14 . What are some of the hidden costs in Hire Purchases?

Test Your Knowledge

Moving Matchsticks



Think, Think, Think..

Can you make the following equation correct by moving just one matchstick? There are two answers.

Oh, and you are not allowed to use the ***'not equals to'*** sign!