MINISTRY OF EDUCATION

YEAR 12 PHYSICS

CURRICULUM DEVELOPMENT UNIT
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ACKNOWLEDGEMENT

This year 12 Physics Textbook has been prepared by the CURRICULUM DEVELOPMENT UNIT with the assistance of the following teachers.

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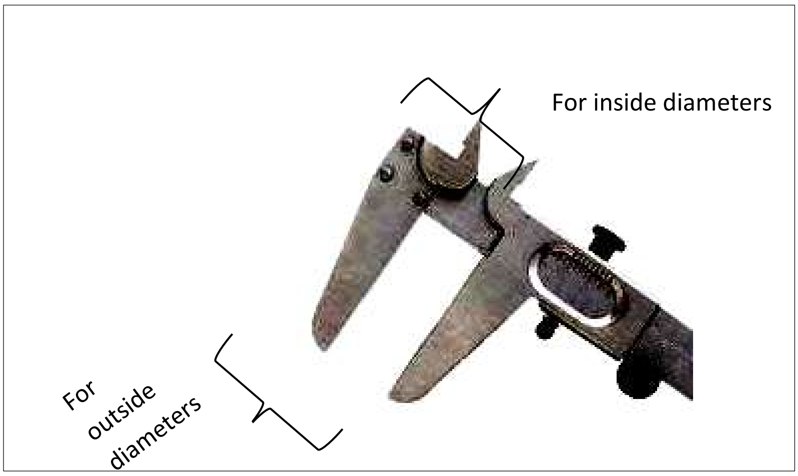
CHAPTER 1: MEASUREMENTS

1.1 MEASUREMENTS

Vernier calipers and micrometres are devices that are used in measurements. A Vernier caliper is a device which consists of a ruler and a Vernier scale attached to it. A micrometer (or a micrometer screw gauge) is a device which consists of a screw measuring system. These devices are widely used in fields such as physics, engineering, woodworking, metalworking, medicine and various other fields.

1.1.10 THE VERNIER CALIPER

A Vernier caliper is a tool used to measure small distances. Vernier calipers have two sets of jaws that allow you to measure both the inside and outside diameter of circular objects.



The problem with using a Vernier caliper to measure something is that it is not straightforward like a standard ruler. A Vernier caliper is read by using two separate scales - a **main** (fixed) scale and a **Vernier** (movable) scale. The trick to reading a Vernier caliper is learning to read the scales in conjunction with one another.

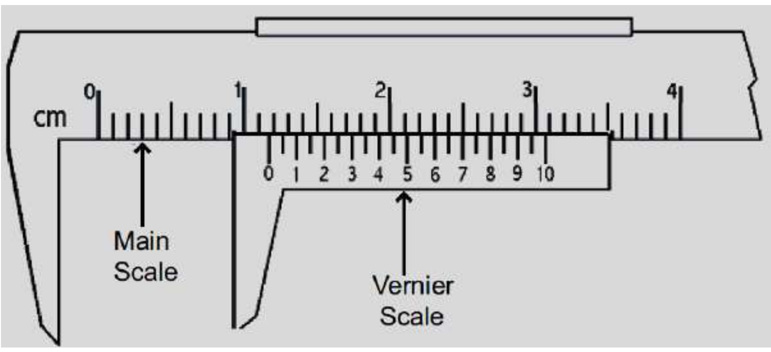


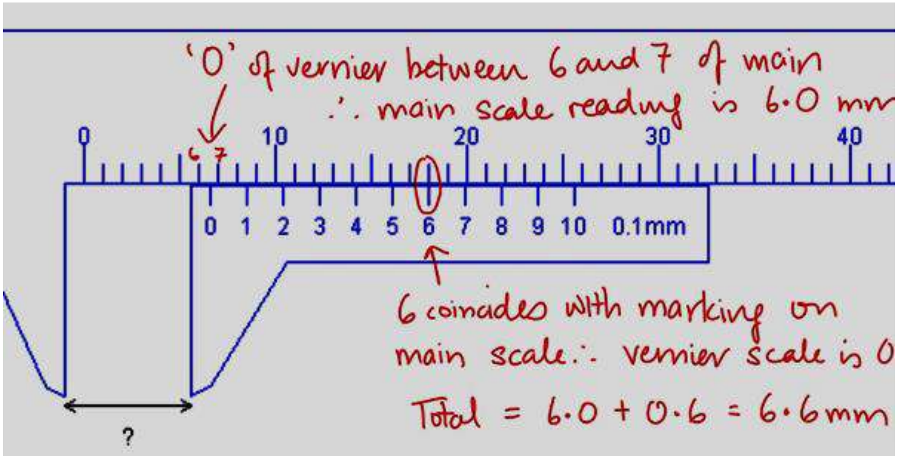
Figure 1: Vernier caliper for measuring the external size of an object

In the case of the caliper shown in Fig. 1, the smallest measurement on the main scale is 0.1 cm or 1 mm. The Vernier scale can read to 0.05 mm. So using both scales, the width can be read to the nearest 0.005 cm (or 0.05 mm).

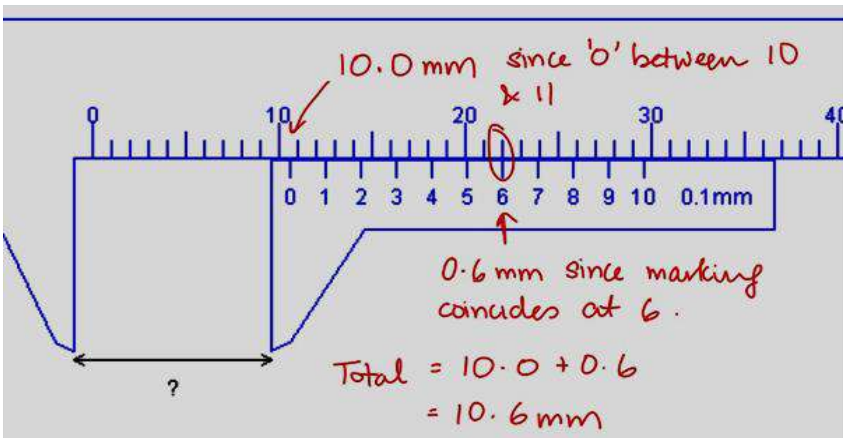
To measure the width, you read the top and bottom scale as follows:

1. Find where the 0 mark of the Vernier scale lines up on the main scale. In this case, it is between 1.1 and 1.2 cm. So, the first reading is 1.1 cm.
2. Find the mark on the Vernier scale that most closely lines up with one of the marks on the main scale. Here, 6.0 and 7.0 are very close, but 6.5 lines up best with one of the marks on the main scale. This value is the number of hundredths of centimetres (or tenths of millimetres). So, the second reading is 0.065 cm.
3. Add the two values together to get the total reading: **1.1 cm + 0.065 cm = <u>1.165 cm</u>**

Example 1



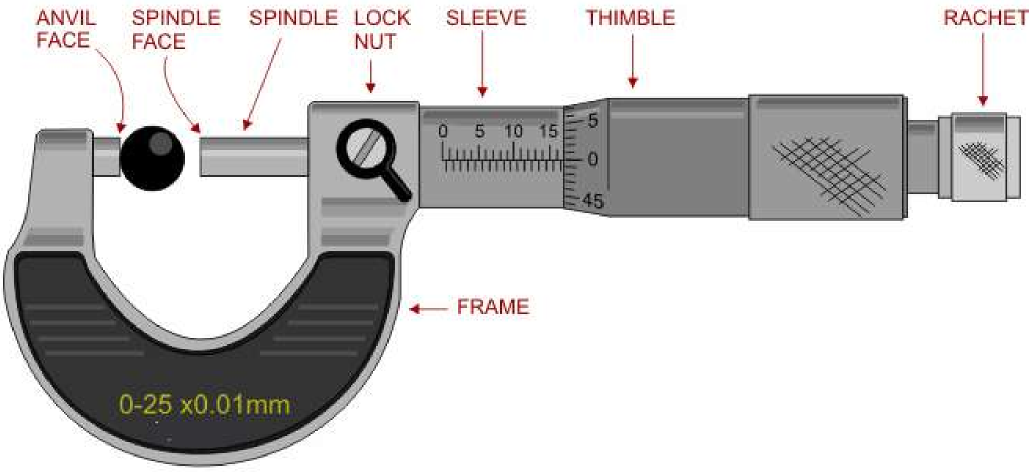
Example 2



1.1.11 THE MICROMETER

The micrometer is a precision measuring instrument, used by engineers. Each revolution of the ratchet moves the spindle face 0.5 mm towards the anvil face.

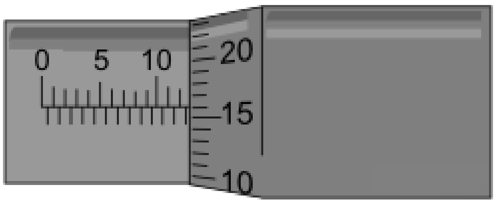
The object to be measured is placed between the anvil face and the spindle face. The ratchet is turned clockwise until the object is 'trapped' between these two surfaces and the ratchet makes a 'clicking' noise. This means that the ratchet cannot be tightened anymore and the measurement can be read.



http://www.technologystudent.com/equip1/microm1.htm

Example 1

Using the first example seen below:



SLEEVE READS FULL mm =	12.00
SLEEVE READS ½ mm =	0.50
THIMBLE READS =	0.16
TOTAL MEASUREMENT =	**12.66 mm**

1. Read the scale on the sleeve. The example clearly shows 12 mm divisions.

2. Still reading the scale on the sleeve, a further ½ mm (0.5) measurement can be seen on the bottom half of the scale. The measurement now reads 12.5 mm.

3. Finally, the thimble scale shows 16 full divisions (these are hundredths of an mm).

The final measurement is 12.5 mm + 0.16 mm = **12.66 mm.**

1.1.12 UNCERTAINTY IN MEASUREMENTS

Uncertainty refers to the fact that a measurement is only an estimation of the true value. There are three types of uncertainty:

I. Random uncertainty
II. Systematic uncertainty
III. Parallax error

Random Uncertainty:

A random uncertainty is one for which the measurement is just as likely to be larger or smaller than the true value.

Example: Students using a stop watch to measure the time for a pendulum to complete ten swings. Assuming the student has a good reaction time, the measurement may be slightly high in some trials and slightly low in others.

Sources of random errors include:

- The observer being less than perfect
- The readability of the equipment
- External effects on the observed item

Random uncertainty can be minimized by taking the average of several readings.

Systematic Uncertainty:

This type of error results from a consistent problem with a measuring device or the person using it.

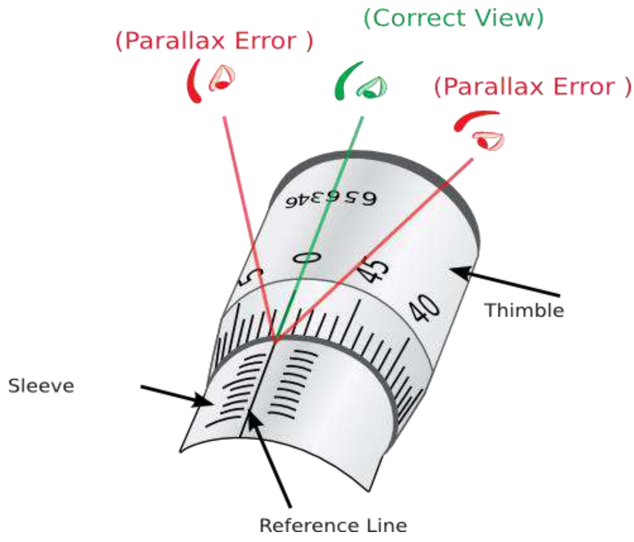
Example: Using a metre rule with loose ends, a dial instrument with needle that is not properly zeroed, or a human reaction time that is always either too late or too early.

Sources of systematic errors include:

- The observer being less than perfect in the same way every time
- An instrument with a zero offset error
- An instrument that is improperly calibrated

Parallax error:

This is the apparent shift in the object's position when the observers position changes
To overcome parallax error when reading instruments, you should view the dial and scale at a direct angle.



http://www.measuring-tools.biz/measuring-instruments/micrometer-parallax-error.html

1.1.13 Significant Figures

Some basic rules about significant figures include:

All figures other than zero are significant.

Example: 3.67 has 3 significant figures

All zeros between two non-zeros are significant.

Example: 3005 has 4 significant figures

2.3006 has 5 significant figures

Zeros placed to the right of a non-zero digit after decimal points are significant.

Example: 3.50 has 3 significant figures

Zeros used to space a decimal point are not significant. In scientific notation, powers of 10 have no significance.

Example: (i) 0.036 has 2 s.f. (ii) 5.01×10^6 has 3 s.f.

Absolute Uncertainty (Absolute Error)

The absolute uncertainty (or absolute error) is the size of the range of values in which the "true value" of the measurement probably lies. If a measurement is given as 25.4 ± 0.1 cm, the absolute uncertainty is 0.1 cm.

Percentage Uncertainty (Relative Error)

Percentage uncertainty is the ratio of the absolute uncertainty of a measurement to the best estimate. It expresses the relative size of the uncertainty of a measurement (its precision).

Percentage uncertainty = $\frac{Absolute\ Uncertainty}{best\ estimate\ of\ the\ value} \times 100\%$

For example, the measurement 35.4 $\pm$ 0.2 cm has a relative uncertainty of:

Percentage uncertainty = $\frac{0.2cm}{35.4cm} \times 100\% = 0.006 \times 100\% = 0.6\%$

1.1.14 UNCERTAINTY CALCULATIONS

Addition and Subtraction

When adding or subtracting two measurements which have uncertainties, the **absolute uncertainties** should be added together.

Example 1

A student takes two measurements which were obtained using a meter rule calibrated in millimetres and wishes to add them.

1st Measurement: 20.4 ± **0.5** mm

2nd Measurement: 32.3 ± **0.5** mm

Therefore, adding them: [20.4 ± 0.5mm] + [32.3 ± 0.5 mm]

[20.4 + 32.3] $\pm$ [**0.5 + 0.5**] mm

52.7 ± 1.0 mm

Multiplication and Division

When multiplying or dividing two measurements which have uncertainties, the **percentage uncertainties** should be added together.

1. the absolute uncertainties is converted to percentage uncertainties
2. These are then added together
3. The final step involves converting the % uncertainty back to absolute uncertainty of the final answer.
4. Rounding off the absolute uncertainty is done so that the least significant digit in the uncertainty will affect the least significant digit in the answer.

Example 2

A piece of paper is measured to be 5.63 ± 0.15 mm wide and 64.2 ± 0.7 mm long. What is the area of this piece of paper?

Area = width × length

= [5.63 ± 0.15 mm] × [64.2 ± 0.7 mm]

Width	length
% uncertainty (width) = $\frac{0.15}{5.63} \times 100 = 2.66\%$	% uncertainty (width) = $\frac{0.7}{64.2} \times 100 = 1.09\%$

Therefore, Area = [5.63 ± 2.66% mm] × [64.2 ± 1.09%]

= [5.63 × 64.2] ± [2.66% + 1.09%]

= *361.446 ± 3.75% mm²*

Converting % back to absolute : $\frac{3.75}{100} \times 361.446 =$ **13.55**

Area = *361.446 ± 13.55 mm²*

So, rounding answer to 3 significant digits: **<u>Area = 361 ± 14 mm²</u>**

1.1.15 EXERCISE

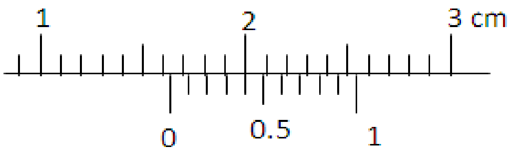
1. Given below are five measurements (in cm) of length taken during the performance of a Form 5 Physics experiment.

   5.2, 5.4, 5.5, 5.3, 5.4
   The length of its absolute uncertainty is **best** represented by
   A. 5.36 ± 0.14
   B. 5.36 ± 0.16
   C. 5.4 ± 0.1
   D. 5.4 ± 0.2

2. An area of 0.4 m^2 is the same as

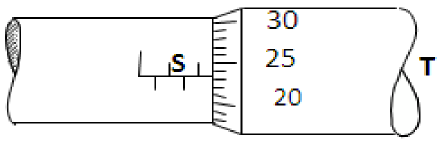
   A. 40 000 cm^2 **B.** 4 000 cm^2 **C.** 400 cm^2 **D.** 0.0004 cm^2

3. A student used a vernier caliper to measure the diameter of a wooden cylinder. The diagram shows an enlargement of the caliper scales. What reading was recorded?



A 2.40 cm **B** 1.64 cm **C** 0.62 cm **D** 0.42 cm

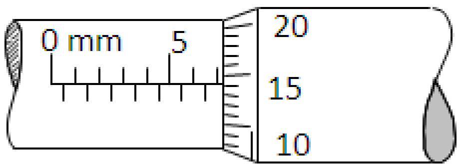
4. The diagram shows the barrel (**S**) and the rotating thimble (**T**) of a micrometre screw gauge.



The reading shown above is

A 2.20 mm **B** 2.23 mm **C** 2.25 mm **D** 2.28 mm

5. The diagram shows a micrometre. What is the reading shown?



A 5.74 mm **B** 6.l4 mm **C** 6.74 mm **D** 7.14 mm

6. A length of copper pipe, of uniform cross-section and several metres long, carries water to a tap. Which instruments are used to take measurements to calculate accurately the volume of copper in the pipe?
   A. calipers and micrometre
   B. micrometre and rule
   C. rule and tape
   D. tape and calipers

7. The diameter of a ball bearing was measured as accurately as possible using the following instruments: metre rule, vernier calipers and a micrometre.
   The table below shows the readings obtained. Complete the table to show which instrument produced each reading.

Diameter	**Instrument used**
7.12mm	
7.0 cm	
7.1mm	

Give the name of the instrument most suitable for measuring

1. the diameter of a wire, _____
2. the volume of a small stone, _
3. The diameter of a soft drinks can. ___

8. Which of the following represents the preferred accuracy in the sum

   12.4 + 11 + 63.37 + 4.2 cm ?

   A. 90.97 B. 91 C. 95.0 D. 95

9. Convert the following to relative uncertainties:

   a. 2.70 ± 0.05 cm b. 12.02 ± 0.08 cm

10. Convert the following to absolute uncertainties:

    a. 3.5 cm ± 10 % b. 16 s ± 8 %

11. Complete the following, determining the appropriate uncertainty:

    a. (2.70 ± 0.05 cm) + (12.02 ± 0.08 cm)

    b. (12.70 ±0.05 cm) − (12.02 ± 0.08 cm)

    c. (2.70 ± 0.05 cm) + (3.5 cm ± 10 %)

12. Complete the following, determining the appropriate uncertainty:

    a. (2.70 ± 0.05 cm) × (12.02 ± 0.08 cm)
    b. (12.02 ± 0.08 cm) ÷ (16 s ± 8 %)
    c. (3.5 cm ± 10 %) × (2.70 ± 0.05 cm) ÷ (16 s ± 8 %)

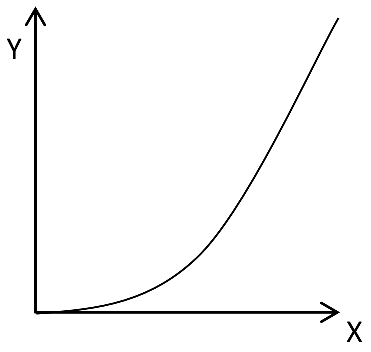
1.2 RELATIONSHIPS

One of the most important mathematical operations in physics is finding the relationship between variables. Through the study of these relationships, we can know how a change in one variable affects another variable, thus enabling us to make predictions and conclusions easily.

1.2.10 RELATIONSHIPS FROM NON LINEAR GRAPHS

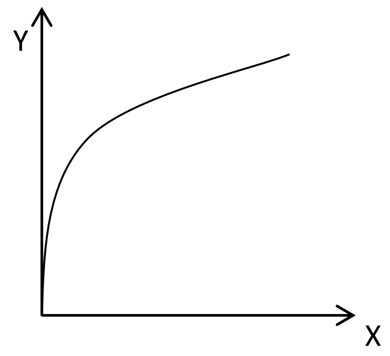
It is easy to get a relationship from experimental data which gives us a **linear** (straight line) graph – it is **Y** = m **X** + C (m is the gradient and C is the Y intercept)

There are situations where the graphical data obtained from a practical investigation is **nonlinear** – it comes out as a curve. Typically we might see curves like these:



A “power of” relationship

$Y = k X^2$



A “root of” relationship

$Y = k \sqrt{x}$

When the relationship between two variables gives a *curved graph* the variables are changed by **squaring** until the graph becomes linear. When a linear graph is obtained, the relationship is more clearly seen.

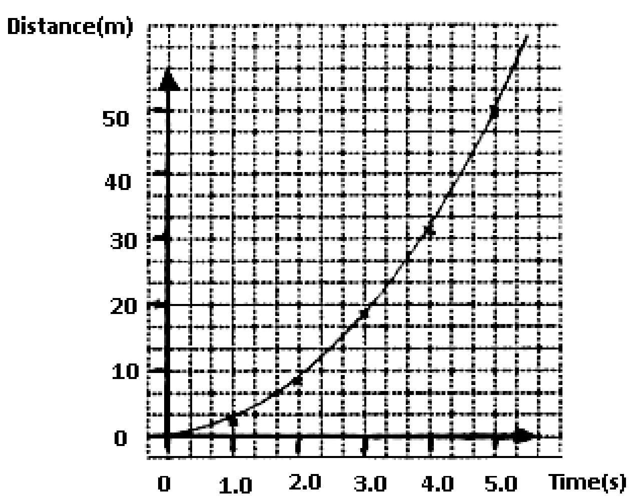
Example 1

Distance [m]	0.0	2.0	8.0	18.0	32.0	50.0
Time [s]	0.0	1.0	2.0	3.0	4.0	5.0
Time^2 [s^2]	0.0	1.0	4.0	9.0	16.0	25.0

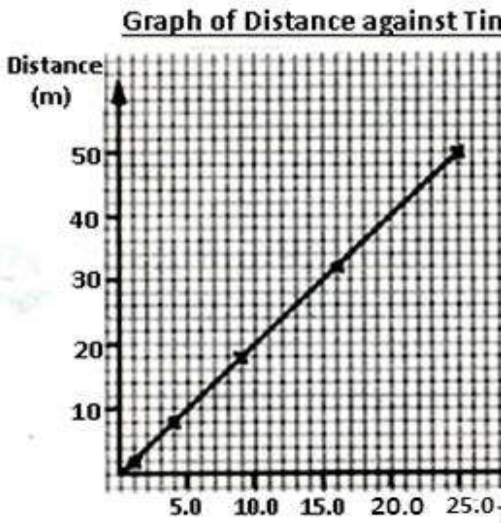
Plotting the graph of **Distance vs. Time**

This gives a curve result.

Graph of Distance against Time



It is difficult to see the relationship between the variables. Therefore, **the time measurements are squared**. A Graph of ***Distance Vs. Time²*** is plotted to give a linear or straight line graph.



The graph between the changed variables is a straight line. This means that distance is proportional to time^2.
Taking the origin (0, 0) and the co-ordinates (25 s^2, 50 m) as the two points to calculate the slope:

$$m = \frac{\Delta y}{\Delta x} = \frac{50-0}{25-0} = 2.0ms^{-2}$$

The mathematical relationship between distance and time is: [y = mx + c]

Distance = 2.0 x **t**ime^2 or

$$\boxed{\mathbf{d = 2.0t^2}}$$

1.2.11 DIRECT AND INVERSE SQUARE RELATIONSHIPS

If a ***direct square relationship*** exists between two variables, then each quantity varies with direct proportion with respect to the square of the other, i.e. if a variable increases by an amount ‘n’ then the variable that it is directly proportional to increases by an amount ‘n^2’ (n squared)

$$\boxed{A = kB^2}$$

Basically, if B "DOUBLES" then A "QUADRUPLES", due to the square. If B "triples" then A increases by a factor of NINE.

Example 1

$$E_k = \frac{1}{2}mv^2$$

In the case of kinetic energy, we see that the kinetic energy is DIRECTLY related to the “square” of the velocity, when the mass is constant.

If v doubles, then E_k increases 4 times i.e. quadruples.

If an ***inverse square relationship*** exists between two variables, then each quantity varies with inverse proportion with respect to the square of the other, i.e. if a variable increases by an amount n then the variable that it is inversely proportional to decreases by an amount n^2.

$$\boxed{A = \frac{k}{B^2}}$$

For the inverse square" if B "DOUBLES", then A DECREASES by a factor of FOUR, or it is simply ONE FOURTH its original value. If B "TRIPLES", then A is ONE NINTH its original value.

Example 2

Given the equation $F = \frac{mv^2}{r}$, what relationship exists between each of the following?

a. F and r b. F and m c. F and v

(a) $F \alpha \frac{k}{r}$.

r divides into (m x v^2) to create F. If F goes up, and (m x v^2) stays the same, then r must go down or decrease. There is an "inverse" relationship between r and F if (m x v^2) remains unchanged.

(b) $F \quad \alpha \quad km$.

If F decreases and goes down, and v^2 and r remain unchanged m must also decrease linearly.

(c) $F \quad \alpha \quad v^2$.

F increases, v must also increase as well. But, because v is squared, it will increase as the "square" to F.

Example 3

$$F = \frac{Gm_1m_2}{r^2}$$

In the case of Newton's Law of Gravitation, we see that if the force due to gravity DECREASES, the distance from Earth, r, must have INCREASED by a square factor.

What will be the value of F if:

(i) The distance, r, is doubled

$$F = \frac{Gm_1m_2}{(2r)^2} \implies F = \frac{Gm_1m_2}{4r^2} \implies F = \frac{1}{4}\frac{Gm_1m_2}{r^2}$$

Thus, <u>***F decreases by a factor of 4 i.e.***</u> $\frac{1}{4}F$

(ii) The mass m_1 and m_2 is doubled

$$F = \frac{G(2m_1)(2m_2)}{r^2} \quad \Longrightarrow \quad F = 4\frac{Gm_1m_2}{r^2}$$

Thus, ***F increases by s factor of 4 i.e. 4F***

(iii) Both the mass m_1 and the distance r, are doubled

$$F = \frac{G(2m_1)m_2}{(2r)^2} \quad \Rightarrow \quad F = \frac{2Gm_1m_2}{4r^2} \quad \Rightarrow \quad F = \frac{1}{2}\frac{Gm_1m_2}{r^2}$$

Thus, ***F decreases by a factor of 2, i.e.*** $\frac{1}{2}$ **F**

1.2.22 EXERCISE

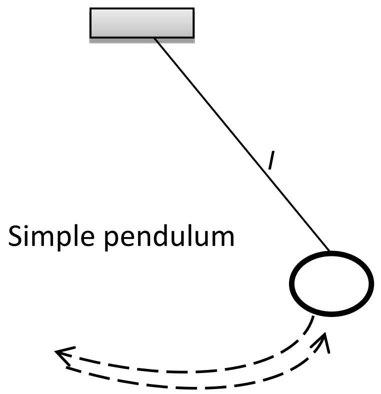
1. A toy car of mass m moving with uniform acceleration a has a velocity v at displacement s from a fixed point as follows:

s(m)	0.0	1.0	2.0	3.0	4.0
v(m/s)	0.0	2.0	2.8	3.5	4.0

(a) Sketch the graph of s against v
(b) Calculate and tabulate the values of v^2.
(c) Plot a fully labelled graph of s versus v^2.
(d) What is the slope *k* of the graph in (c)? State the units of the slope.
(e) State the expression relating s, k and v
(f) Determine graphically the acceleration of the trolley.
(g) What is the net force on the toy car of mass 1.5 kg?

2. The period T of a simple pendulum varies with its length *l* as according to the formula:

$T = 2\pi\sqrt{\frac{l}{g}}$, where g is the gravitational field strength.



In a Lab experiment to determine g, the following results were obtained for the length of the pendulum *l* and the time for 20 oscillations:

L (cm)	20	40	60	80	100	120
t(20 sec)	18.0	25.4	31.1	35.9	40.1	44.0

a) Calculate and tabulate the values for
   (i) *L* in m
   (ii) Period of the pendulum *T*
   (iii) T^2
b) Plot the graph of T^2 (vertical axis) against *l*.
c) Determine the size and units of slope of the graph in (b)
d) Determine graphically the value of g.

3. The kinetic energy of an object is given by the formula: $K.E = \frac{1}{2}mv^2$

where, K.E = kinetic energy
m = mass of the object
v = velocity of the object

a) Sketch the **graphs** that best represents the relationship between K.E and v for the object
b) The mass is now halved and the speed doubled. Calculate the new kinetic energy.

1.3 VECTORS

Many of the quantities with which we deal in Physics are vectors. Sometimes we need to add a number of vectors together. For example, calculating the resultant force acting on a car when several forces act on the car concurrently – the wind, friction, gravity and the force supplied by the engine. Sometimes we need to subtract two vectors. For example, calculating the change in velocity of a car as it goes around a bend in the road.

When the need arises to add or subtract vector quantities, this proves to be easy only when the vector quantities act along the same straight line. If the vectors act at an angle to each other we really need to draw a vector diagram to assist in solving the problem.

1.3.10 VECTOR SUBTRACTION

An important application of subtracting vectors is when you are dealing with velocity vectors. Vector subtraction is commonly used when calculating a **change** in a vector quantity. E.g., to find the **change in velocity**, Δv, from an **initial** velocity of v_i to a **final** velocity of v_f, then

$$\boxed{\Delta v = v_f - v_i}$$

CHANGE in VELOCITY = FINAL VELOCITY – INITIAL VELOCITY

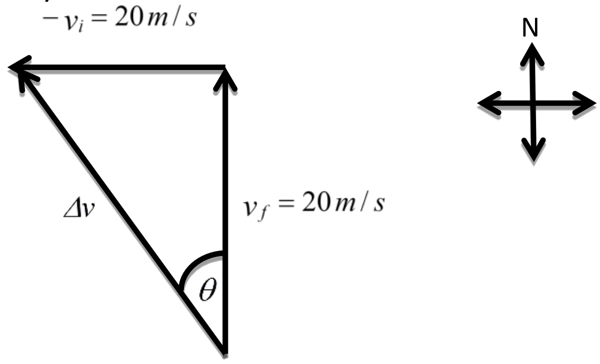
Example 1

A car is moving due East at 20 ms^{-1}. A short time later it is moving due North at 20 ms^{-1}. Calculate the ***change in velocity*** of the car.

$$\Delta v = v_f - v_i$$

This should really be written as: $\Delta v = v_f + (-v_i)$

since that is how we draw the vector diagram. We simply add the negative of the initial velocity to the final velocity



Using Pythagoras' Theorem and basic trigonometry

By **Pythagoras' Theorem**, the **magnitude** of the resultant change in velocity of the car is:

$$R = \sqrt{(20)^2 + (20)^2} \quad = \quad \underline{\mathbf{28.3\ m/s}}$$

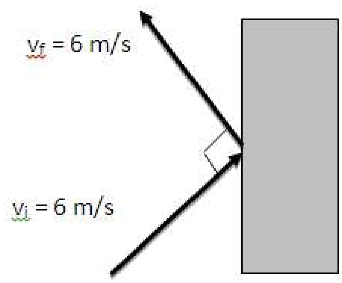
and the **direction** can be found using basic **trigonometry** as follows:

$$\theta = \tan^{-1}\left(\frac{20}{20}\right) \quad = \quad \underline{\mathbf{45^{0}}}$$

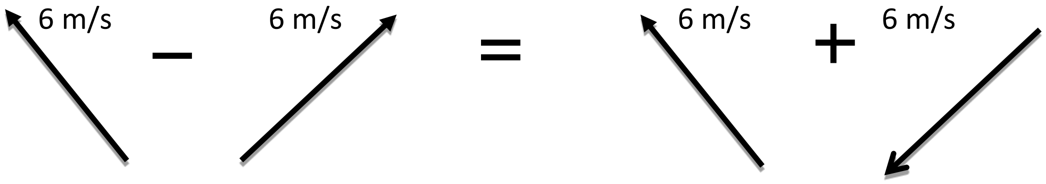
So, the ***change in velocity*** of the Car is **28.3 ms^{-1} at an angle of 45^{o} West of North (N 45^{0} W)**.

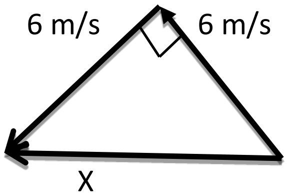
Example 2

A tennis ball thrown against a wall rebounds without loss in speed as shown. Calculate the magnitude and direction of the ball's ***change*** in velocity.



$$\Delta v = v_f - v_i \quad \Longrightarrow \quad \Delta v = v_f + (-v_i)$$





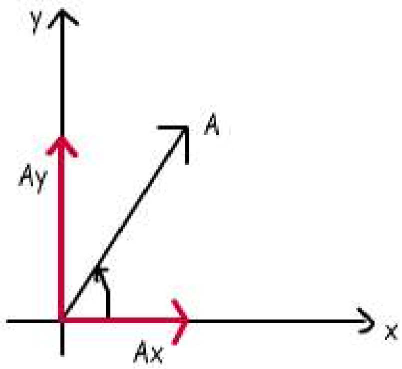
using Pythagoras theorem: $X^2 = 6^2 + 6^2 = 36 + 36$

$X^2 = 72$

$X = \sqrt{72} =$ **<u>8.48 m/s</u>**

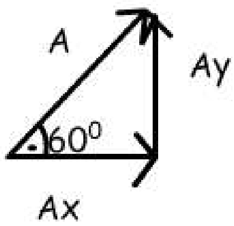
1.3.11 VECTOR COMPONENTS

Any vector can be **resolved** into a number of components and when these are added together they result in the original vector.



For example, look at vector, A given on the left, it is in northeast direction. In the figure, we see the X and Y component of this vector. In other words, addition of A_X and A_Y gives us vector A. We benefit from trigonometry at this point. There are two simple equations which you can use and find the components of any given vector.

Example 1



$\text{Sin}60^0 = \dfrac{A_y}{A}$ and, $\cos 60^0 = \dfrac{A_x}{A}$

Thus,

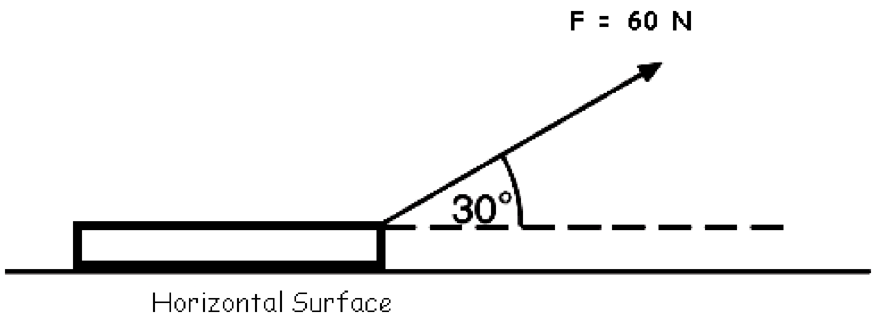
$A_x = A.\text{Cos}60^0$

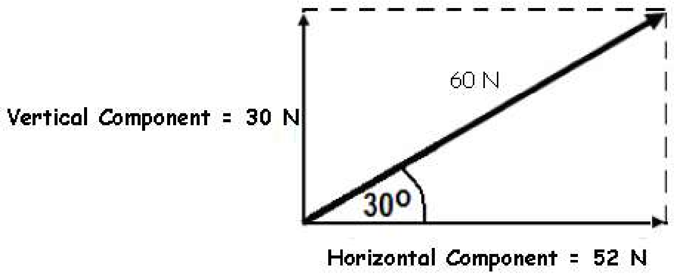
$A_y = A.\text{Sin}60^0$

All vectors can be divided into their components. Now we solve an example and see how we use this technique.

Example 2

A force of 60 Newton is applied to a rope to pull a box across a horizontal surface at a constant velocity. The rope is at an angle of 30^0 above the horizontal. Find the vertical and horizontal component of this force.





Horizontal Component of Force: Cos θ = $\frac{a}{h} \rightarrow$ Cos θ = $\frac{F_h}{h}$

Cos 30^0 = $\frac{F_h}{60} \rightarrow$ F_h = (Cos 30^0) (60) = **52 N**

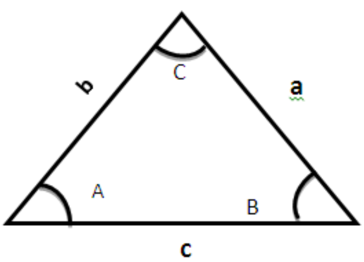
Vertical Component of Force: Sin θ = $\frac{o}{h} \rightarrow$ Sin θ = $\frac{F_v}{h}$

Sin 30^0 = $\frac{F_v}{60} \rightarrow$ F_v = (Sin 30^0) (60) = **30 N**

1.3.12 ADDING NON PERPENDICULAR VECTORS

When we add two or more vectors, the diagram we construct often results in a triangle. Therefore, it is possible to use trigonometric relationships and laws that hold for a triangle to solve for unknown sides or angles. For Non-Perpendicular Vectors, the **Sine Rule** and the **Cosine Rule** are the two basic tools one can use when analyzing triangles.

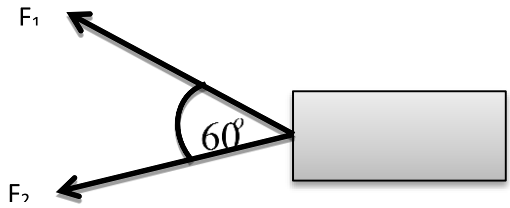
SINE RULE: $$\frac{a}{SinA} = \frac{b}{SinB} = \frac{c}{SinC}$$



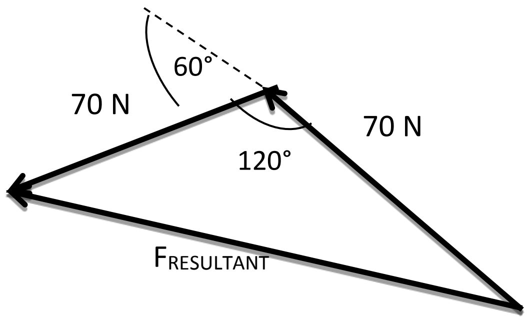
COSINE RULE: $c^2 = a^2 + b^2 - (2abCosC)$

Example 1

Two equal forces of 70 Newton act on a body at 60^0 to each other as shown on the diagram given below. Determine the **magnitude** of the resultant force acting on the body.



Therefore, drawing vector diagram showing addition of forces:



By COSINE RULE: $c^2 = a^2 + b^2 - (2abCosC)$

$F_R^2 = 70^2 + 70^2 - (2 \times 70 \times 70 \times \text{Cos } 120)$

$F_R^2 = 4900 + 4900 - - 4900$

$F_R^2 = 14,700$

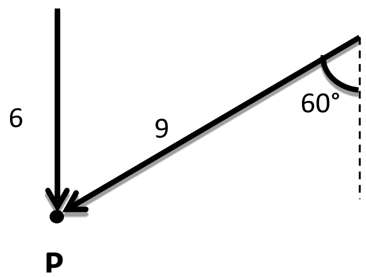
$F_R = \sqrt{14700}$

$\mathbf{F_R = \underline{121.24\ N}}$

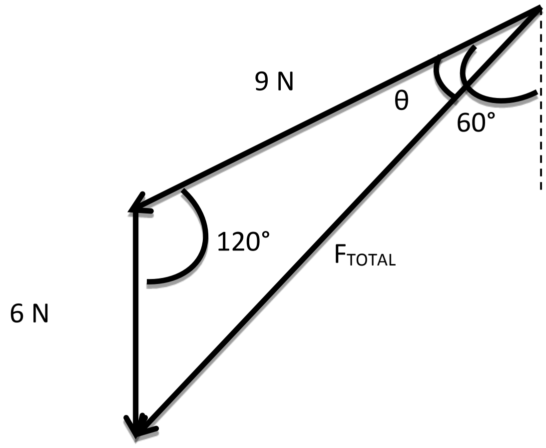
The resultant force acting on the body is 121.24 Newton.

Example 2

A force of 6.0 N south and 9.0 N S 60^0 W act on the same point **P**. Determine the total force that must be acting on that point.



Step 1: Draw vector diagram showing the addition of forces:



Step 2: Use Cosine Rule to determine the magnitude:

$$c^2 = a^2 + b^2 - (2abCosC)$$

$F_T^2 = 9^2 + 6^2 - (2 \times 9 \times 6 \times \text{Cos } 120^0)$

$F_T^2 = 81 + 36 - (-54)$

$F_T^2 = 171$

$F_T = \sqrt{171} \quad = \quad 13.1$ N

Step 3: Use Sine Rule to determine the direction:

$$\frac{SinA}{a} = \frac{SinB}{b} = \frac{SinC}{c}$$

$$\frac{Sin\theta}{6} = \frac{Sin120}{13.1} \quad \Longrightarrow \quad Sin\theta = \frac{Sin\,120}{13.1} \times 6 = \ 0.397$$

$\theta = Sin^{-1}(0.397) = 23.4^0$ (Direction with respect to reference point: 60 - 23.4 = 36.6^0)

Thus, The total force = **<u>13.1 N S 36.6⁰ W</u>**

1.3.13 RELATIVE VELOCITY IN ONE DIMENSION

Imagine you're walking along a road, heading **west at 8 km/hr**. A train track runs parallel to the road and a train is passing by, traveling at **40 km/hr west**. There is also a car driving by on the road, going **30 km/hr east**.

How fast is the train traveling relative to you?
How fast is the car traveling relative to you?
And how fast is the train traveling relative to the car?

Example 1
One way to look at it is this: in an hour, the train will be 40 km west of where you are now, but you will be 8 km west, so the train will be 32 km further west than you in an hour. Relative to you, then, the train has a velocity of 32 km/hr west. Similarly, relative to the train, you have a velocity of 32 km/hr east.

Using a subscript *Y* for you, *T* for the train, and *C* for the car, we can resolve this using vector subtraction:

i. *the velocity of the train relative to you =* $v_{Train\ rel\ You} = v_T - v_Y$

$v_{Train\ rel\ You}$ = 40 km/hr west – 8 km/hr west

$= \overleftarrow{40\ \text{km/hr}} - \overleftarrow{8\ \text{km/hr}}$

$= \overleftarrow{40\ \text{km/hr}} + \overrightarrow{8\ \text{km/hr}}$

$= -40 + 8$

$= -32 = \overleftarrow{32\ \text{km/hr}} =$ **32 km/hr west**.

ii. *the velocity of the car relative to you =* $V_{Car\ rel\ You} = V_C - V_Y$

$v_{Car\ rel\ You}$ = 30 km/hr east – 8 km/hr west

$= \overrightarrow{30\ \text{km/hr}} - \overleftarrow{8\ \text{km/hr}}$

$= \overrightarrow{30\ \text{km/hr}} + \overrightarrow{8\ \text{km/hr}}$

$= 30 + 8$

$= 38$

$\overrightarrow{38\ \text{km/hr}} =$ **38 km/hr east.**

iii. *the velocity of the train relative to car =* $v_{Train\ rel\ Car} = v_T - v_C$

$v_{Train\ rel\ Car}$ = 40 km/hr west – 30 km/hr east

$= \overleftarrow{40\ \text{km/hr}} - \overrightarrow{30\ \text{km/hr}}$

$= \overleftarrow{40\ \text{km/hr}} + \overleftarrow{30\ \text{km/hr}}$

$= -40 + -30$

$= -70 = \overleftarrow{70\ \text{km/hr}} =$ <u>**70 km/hr west**</u>**.**

1.3.14 RELATIVE VELOCITY IN TWO DIMENSIONS

1. BOAT PROBLEM

This deals with boats (or swimmers) that are in running water. There are three different velocities we deal with in these problems and each is represented by a different subscript:

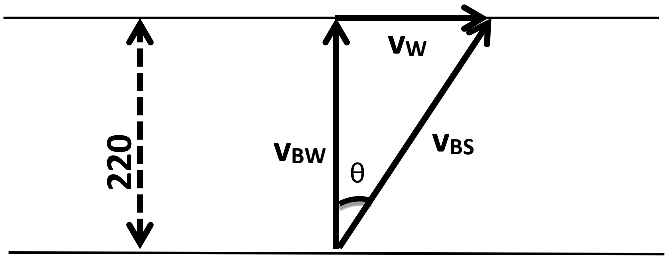
- v_{BW} – the velocity of the boat relative to the water
- v_{WS} – the velocity of the water relative to the shore
- v_{BS} – the velocity of the boat relative to the shore

Generally, problems will ask you to find the velocity of the boat relative to the shore (v_{BS}) or where the boat should aim to achieve a certain velocity. The relationship between the three velocities will change depending on which you are trying to find.

Example 1

A canoeist, capable of travelling at a speed of 5 m/s in still water, is crossing a river that is flowing with a velocity of 3 m/s [E]. The river is 220 m wide.

(a)If the canoe is aimed northward (directly across the river), what will its velocity be relative to the shore?



v_{BS} =?

V_{BW} and v_{WS} form a right angled triangle with V_{BS} as the hypotenuse. Therefore, using Pythagoras Theorem to find the velocity of the boat relative to the shore:

$$(v_{BS})^2 = (v_{BW})^2 + (v_{WS})^2$$

$$v_{BS} = \sqrt{(v_{BW})^2 + (v_{WS})^2}$$

$$v_{BS} = \sqrt{5^2 + 3^2}$$

v_{BS} = ***5.83 m/s***

Since we are dealing with vectors, we also need to find the direction of v_{BS} :

$$\text{Tan } \theta = \frac{v_{WS}}{v_{BW}}$$

$$= \tan^{-1}\left[\frac{v_{WS}}{v_{BW}}\right] = \tan^{-1}\left[\frac{3}{5}\right] = 31^0$$

So, v_{BS} = ***5.83 m/s N 31⁰ E [31⁰ E of N]***

(a) How long will it take to cross the lake?

ΔT = ???

The time it takes for the canoeist to cross the river is dependent on the **vertical component of its velocity only (v_{BW})**. The distance is must cross is in the vertical axis, the distance it moves horizontally has no effect.

$$D = v_{BW} \times T$$

$$T = \frac{D}{v_{BW}} = \frac{220m}{5m/s}$$

T = **44 s**

Therefore, ***the canoeist takes 44 s to cross the river***

(b) Where is the landing position of the canoe, relative to its original position?

Δd_x = ?

We can use the time we just calculated along with the horizontal velocity (V_{WS}) to find the horizontal displacement.

$\Delta d_x \quad = \quad v_{WS} \times T$

$\Delta d_x \quad = \quad 3 \text{ m/s} \times 44 \text{ s}$

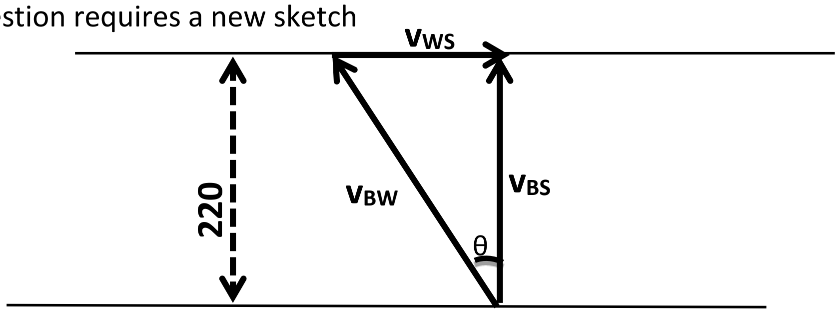
$\Delta d_x \quad = \quad$ **132 m**

Therefore, ***the canoeist will land 132 m [E] of where he left the shore.***

(c) At what angle should the canoeist aim in order to land directly across the river?

θ = ???

This question requires a new sketch



Note that the right-angled triangle has changed so that V_{BW} is now the hypotenuse and V_{BS} is now the vertical component.

$$Sin\,\theta = \frac{v_{WS}}{v_{BW}}$$

$$\boldsymbol{\theta} = Sin^{-1}\left(\frac{v_{WS}}{v_{BW}}\right) = Sin^{-1}\left(\frac{3}{4}\right) = 48.6^0$$

Therefore, *the canoeist should aim* ***48.6° W of N (N 48.6 W)*** *in order to travel straight across the river*

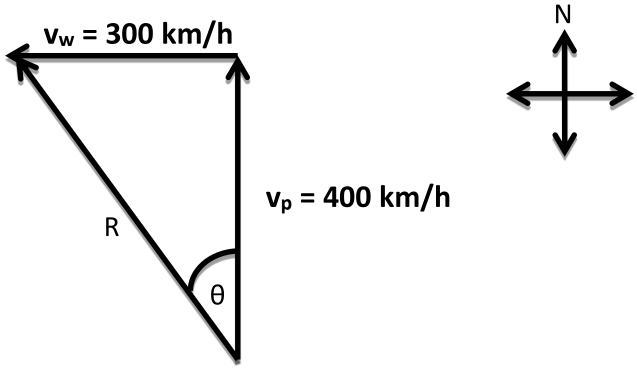
2. PLANE PROBLEM

NOTE: *For aircraft, the true airspeed (TAS) is the actual speed of the aircraft through the air (the speed of the aircraft relative to the air). The wind speed is usually measured relative to the ground. Groundspeed is the speed of the aircraft relative to the ground. The groundspeed of the aircraft is the vector sum of the true airspeed and the wind speed.*

Example 2

A pilot flies her jet with a true air speed of 400 km/h North. A crosswind from the East blows at 300 km/hr relative to the ground. Calculate the jet's resultant velocity relative to the ground.

Using Vector diagram:



By **Pythagoras' Theorem**, the **magnitude** of the resultant velocity of the jet is:

$$R = \sqrt{(300)^2 + (400)^2} = 500 \text{ km/hr}$$

and the **direction** can be found using basic **trigonometry** as follows:

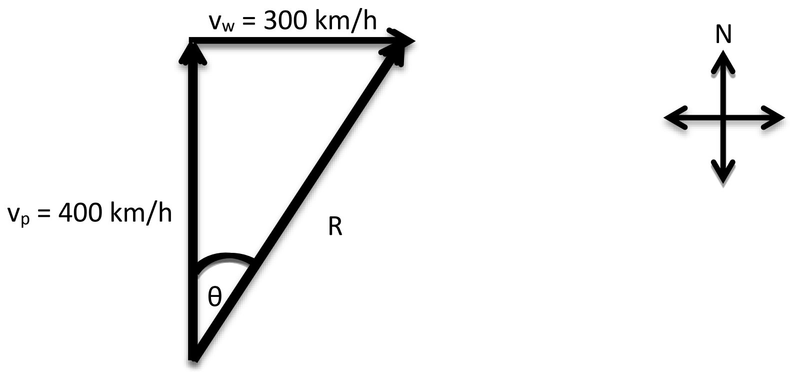
$$\theta = \tan^{-1}\left(\frac{300}{400}\right) = 36.9^0$$

So, the velocity of the jet relative to the ground is <u>500 km/hr N36.9^o W</u>.

> ***Note***: *if the angle between the two vectors being added together is other than 90^o,* ***Cosine Rule*** *and* ***Sine Rule*** *can be used to solve the problem mathematically. Note also the use of the* ***compass*** *in the diagram to establish direction.*

(a) In which direction should the pilot head and with what airspeed in order to actually fly north at 400 km/h relative to the ground?

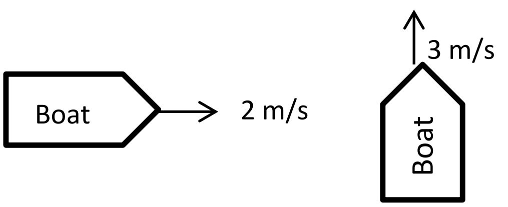
Since the pilot must fly into the wind, by vector diagram we obtain the diagram shown below:



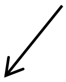
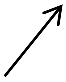
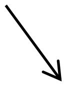
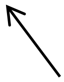
If the pilot flies **N 36.9^o E with an air speed of 500 km/h**, the wind will bring her back to a heading of due North at a speed of 400 km/h relative to the ground**.** Remember also, there is usually more than one way to give the direction. The direction the pilot should fly in this example could just as correctly be given as E53.1^{o}N or as a True Bearing of 36.9^o.

1.3.15 EXERCISE

1. A boat is observed to be travelling east at 2 m/s. It then changes direction and is observed an instant later to be travelling north at 3 m/s.



(a) The vector that **best** represents the change in velocity of the boat is

A.  B.  C.  D. 

(b) Calculate the **magnitude** of the change in velocity

2. A ball travelling at 15m/s strikes a wall at right angles and rebounds along the same path at $12ms^{-1}$. Determine the **change** in velocity.

3. Find the horizontal and vertical components:

    a) 50km at 20° east of North
    b) 70km at 30° south of East

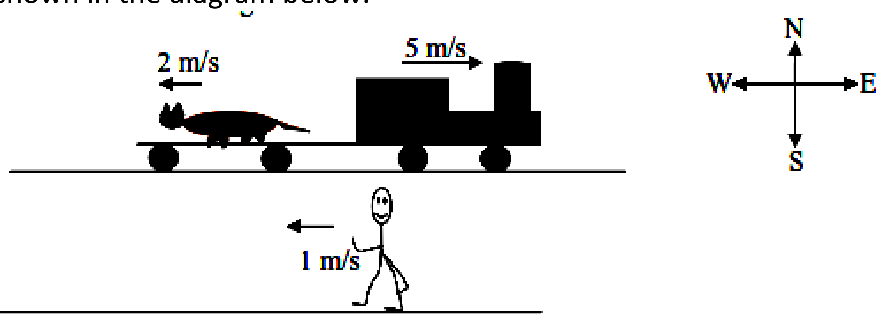
4. A lawn mower is being ***pushed*** at constant velocity through the grass. If the force applied to the handle is 80 Newton's at 30° to the ground, determine the components of the force vector.

5. Decide which trigonometric relationship should be used to add the following pairs of vectors. Then add the vectors.
    i. 6.0 km N 45^0 W and 3.0 km N 45^0 E.
    ii. 55.0 Newton S 25^0 E and 45.0 Newton N 45^0 W.
    iii. 1.3 m/s S and 2.5 m/s N 30^0 E.

6. A hiker walks due east for a distance of 25.5 km from her base camp. On the second day, she walks 41.0 km North West till she discovers the cave she wanted to see. Determine the magnitude and direction of her resultant displacement between the base camp and the cave.

7. An airplane heads north of east by 18 degrees for a distance of 67 km then heads due north for 39 km. What is the planes total displacement?

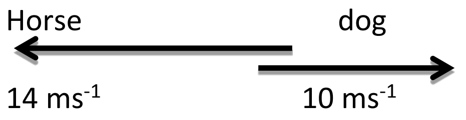
8. A cat walks on a train carriage at 2 m/s towards the west while the train moves at 5 m/s to the east as shown in the diagram below.



The velocity of the cat relative to a man walking on the ground at 1 m/s towards the west as the train passes would be **best** given as

A. 2 m/s west B. 4 m/s west C. 2 m/s east D. 4 m/s east

9. A dog runs at 10.0ms^{-1} due east at the same time that a horse runs westwards at 14.0ms^{-1}.

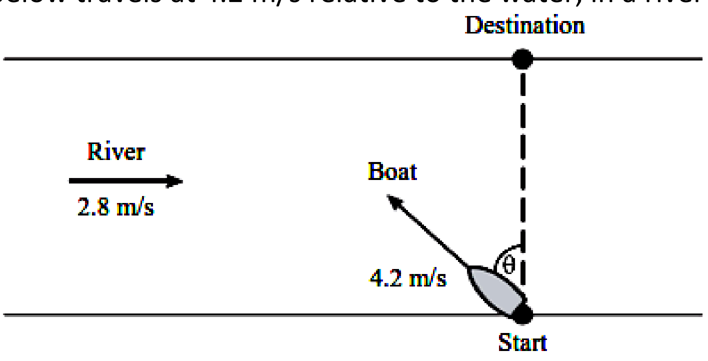


Calculate the **magnitude** and **direction** of the *velocity of the horse relative to the dog*.

10. Abdul is in a bus travelling to Nausori at 13 m/s while Jone is in another bus travelling to Suva at 8 m/s. If the buses pass each other at Nasinu, which is on the Suva - Nausori highway, calculate the **magnitude** and **direction** of *Jone's velocity relative to Abdul's*.

11. An athlete runs with a velocity of 9 m/s North against a wind which is blowing 2 m/s Southward. Calculate the **magnitude** and **direction** of the *velocity of the wind relative to the athlete*.

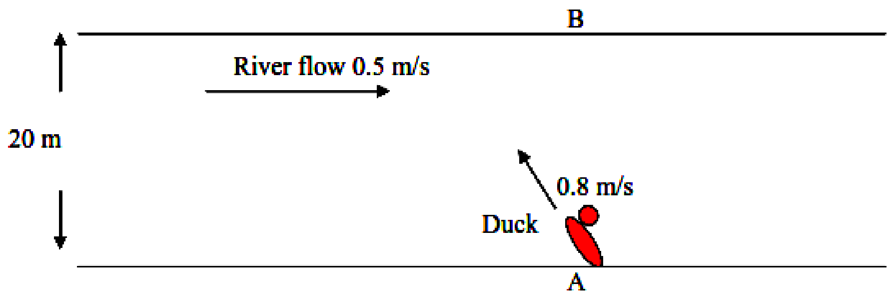
12. A boat shown below travels at 4.2 m/s relative to the water, in a river flowing at 2.8 m/s.



At what angle, θ, must the boat head to reach the destination directly across the river?

A. 56^0 B. 48^0 C. 42^0 D. 34^0

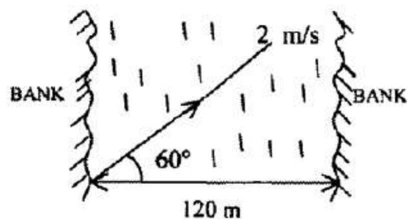
13. A duck capable of swimming at 0.8 m/s crosses a 20 m wide river from point A on one bank to point B directly opposite on the other bank. The river flows to the right at 0.5 m/s as shown below.



The duck, with a great sense of direction, points upstream in order to overcome the river flow and reach point B. The magnitude of the duck's velocity relative to the river bank at A would be

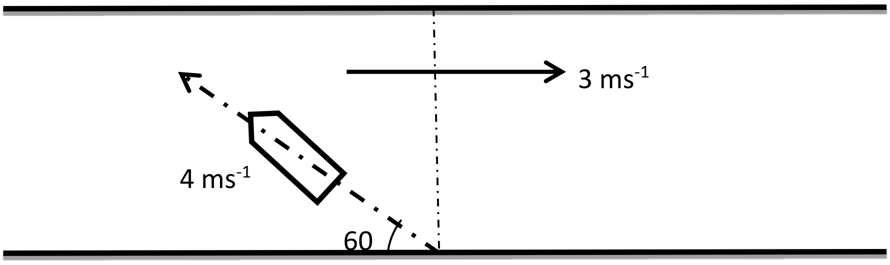
A. 0.50 m/s B. 0.62 m/s C. 0.80 m/s D. 0.94 m/s

14. A swimmer swims across a river which is 120 metres wide, at an angle *of 60°* to the horizontal as shown in the diagram given below:



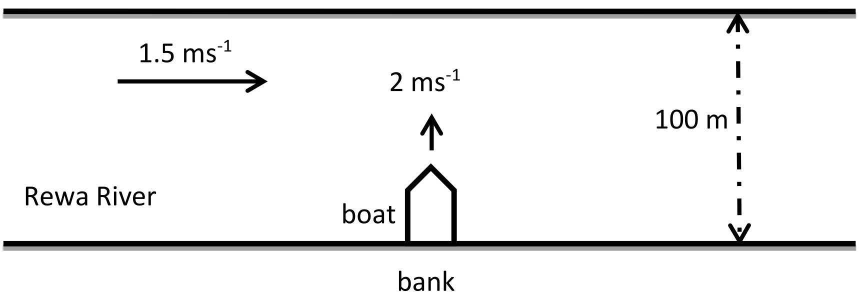
If her speed through the water is 2 m/s, how long will she take to reach the other side?

15. A boat that has a water speed of 4.0 ms^{-1} heads upstream at an angle of 60° to a river bank. The river flows downstream at 3.0 ms^{-1} and is 50 m wide.



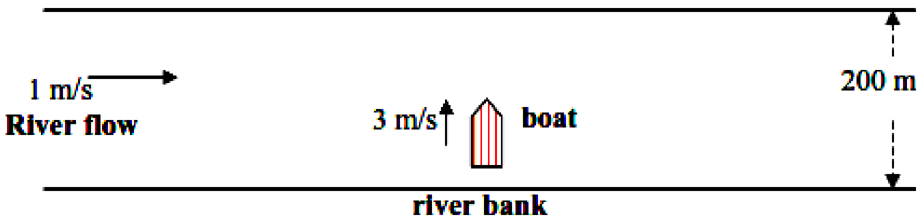
(i) Calculate the component of the boat's velocity perpendicular to the river bank.
(ii) After 5 seconds the boat will have drifted downstream a distance
(a) 5.0 m (b) 15.0 m (c) 15.5 m (d) 25.0 m

16. A man from Nausori rows a punt at a steady speed of 2ms^{-l}. He sets out at right-angles to the bank of the Rewa River which is 100 m wide at this section of the river. The river flows downstream at 1.5ms^{-l}.



(a) How long will the crossing take on :
   i. still river water ?
   ii. river water moving at 1.5ms^{-l} downstream?
(b) How far downstream does he land?
(c) What is the speed of the punt relative to the river bank?

17. A boat crosses a river at 3 m/s relative to the water. The 200 m wide river flows at 1 m/s as shown below.



What would be the velocity of the boat as observed by a stationary observer on the **river bank** from which the boat departed?

(a) How long does the boat take to cross the river?
(b) How far **downstream** is the boat taken by the river flow with reference to a point directly opposite where it started?

18. An aircraft has a constant horizontal speed of 100 m/s relative to the wind. The pilot wants to fly directly east, but there is a wind blowing from the north with a speed of 40 m/s.

(i) Draw a labelled vector diagram showing the direction in which the pilot must point the aircraft to actually fly east in the wind.
(ii) Calculate the angle at which the aircraft must fly to actually travel east.
(iii) Calculate the velocity of the plane relative to the ground.

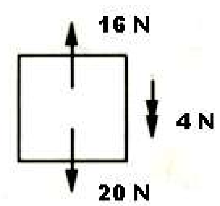
19. A ship is heading due west at a steady speed of 15 km/h. A current of 3 km/h is running due south. Calculate the velocity of the ship relative to the seabed. (Hint: The velocity of the ship plus the velocity of the current will add up to the total velocity of the ship relative to the seabed.)

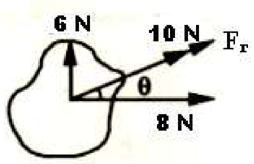
1.4 FORCES

Force (symbol F) is a vector quantity because it involves both *size* and *direction.* The SI unit for force is the newton (symbol N)

RESULTANT FORCE, F_R: When two or more forces act on the same object they can be replaced by a single force that has the same effect. This single force is called the **resultant force**. The resultant force is found by adding the forces acting, taking into account their directions.

Example





The resultant force, **F_r = 4 N downward**

By Pythagoras theorem, $F_r^2 = 6^2 + 8^2$

$F_r = \sqrt{6^2+8^2}$ = **F_r = 10 N** ↗

Newton's second law

Newton's second law states that "the relationship between the resultant force, F, acting on an object; the mass, m, of the object; and the acceleration, a, of an object is:

$$\boxed{F = ma}$$

Where F is measured in N, m in kg and a in ms^{-2}.

This shows that if you keep the mass constant and double the applied force the acceleration will double.

Example 1

A car of mass 5000 kg is accelerated by a resultant force of 600 N. Calculate the acceleration, a.

F = ma » $a = \frac{F}{m}$ » $a = \frac{600N}{5000kg}$ = **<u>0.12 ms^{-2}</u>**

Example 2

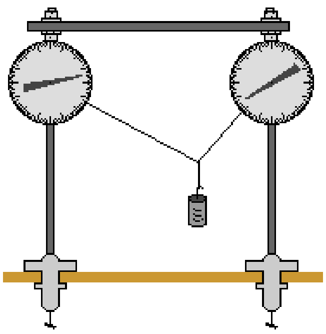
A rock is accelerated at 13 ms^{-2} when a net force of 50 N is applied to it. Find out the mass of the rock.

F = ma » $m = \frac{F}{a}$ » $m = \frac{50N}{13ms^{-2}}$ = **<u>3.85 kg</u>**

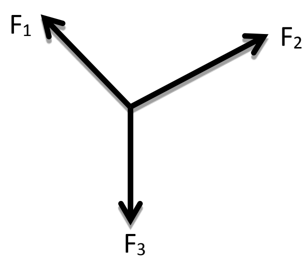
1.4.10 EQUILIBRIUM IN TWO DIMENSIONS

When two or more forces are acting on the same object at the same time and their sum is zero then the object is said to be in **equilibrium**.

Example 1



Object is at equilibrium position.
The load is static.



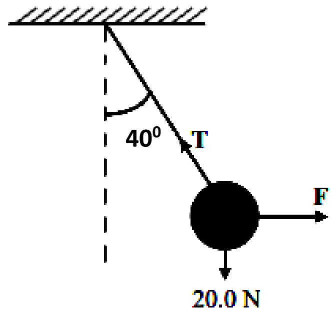
The sum of the forces F_1, F_2 and F_3 equals zero i.e. the resultant force which is the vector from the tail of the first to the head of the last is zero.

Thus, we can write the equations of equilibrium for a two-dimensional structure as

Σ F $_x$ = 0,
Σ F $_y$ = 0,
Σ M $_A$ = 0, where A is any point in the plane of the structure.

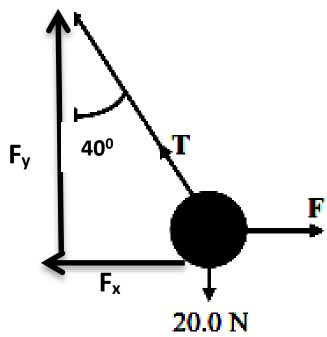
Example 2

A pendulum which weighs 20.0 N is attached to a string. It is held aside by a horizontal force **F** to make an angle of 40° to the vertical as shown in the diagram.



Calculate:
(i) The magnitude of the tension **T** in the string
(ii) The force **F**.

Sketching the free body diagram of the pendulum:



Writing equations of equilibrium:

(i) $\sum F_y = 0$ $\quad$ F_y = 20.0 N

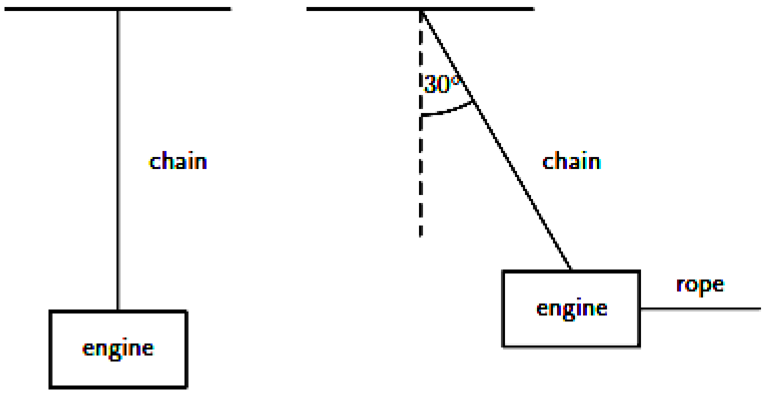
Thus Tension T: $Cos\theta = \frac{F_y}{T}$ $\quad$ $T = \frac{F_y}{Cos\theta} = \frac{20N}{Cos40^0}$ = **<u>26.1 N</u>**

(ii) $\sum F_x = 0$ $\quad$ F_x = **F**

Thus Force F: $Sin\theta = \frac{F_x}{T}$ $\quad$ $F_x = TSin\theta$ $\quad$ = 26.1 × Sin 40^0 = **<u>16.8 N</u>**

Example 3

A car engine of weight 2000 N is lifted by means of a chain and pulley system. The engine is initially suspended by the chain, hanging stationary. Then, the engine is pulled sideways by a mechanic, using a rope. The engine is held in such a position that the chain makes an angle of 30^0 with the vertical. In the questions that follow, the masses of the chain and the rope can be ignored.

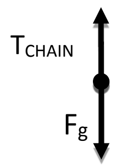


(i) Draw a free body representing the forces acting on the engine in the initial situation.
(ii) Determine the tension in the chain initially.
(iii) Draw a free body diagram representing the forces acting on the engine in the final situation.
(iv) Determine the magnitude of the applied force and the tension in the chain in the final situations.

Solution:

(i) **Initial free body diagram for the engine**:

There are only two forces acting on the engine initially: the tension in the chain, T_{chain} and the weight of the engine, F_g.



(ii) **Determine the tension in the chain**:

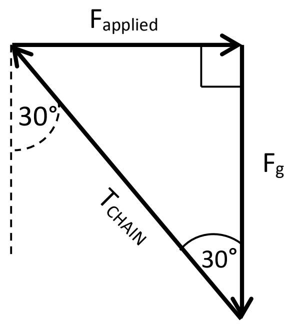
The engine is initially stationary, which means that the resultant force on the engine is zero. There are also no moments of force. Thus the tension in the chain exactly balances the weight of the engine. The tension in the chain is:

$$T_{CHAIN} = F_g$$

$$T_{CHAIN} = 2000 \text{ N}$$

(iii) **Final free body diagram for the engine**:

There are three forces acting on the engine finally: The tension in the chain, the applied force and the weight of the engine.



(iv) **Magnitude of the applied force and the tension in the chain in the final situation**:

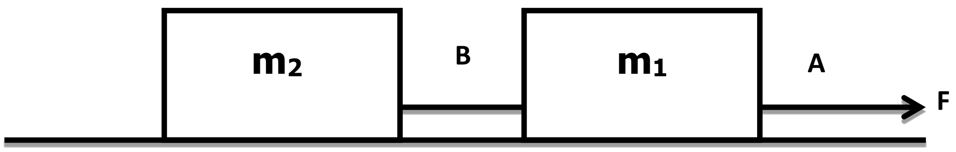
- $\tan\theta = \dfrac{O}{A}$ $\qquad$ $\tan 30^0 = \dfrac{F_{APPLIED}}{F_g}$ $\qquad$ $F_{APPLIED} = (2000)\tan 30^0$ = **1155 N**

- By Pythagoras: $\qquad$ $T_{CHAIN} = \sqrt{(2000)^2 + (1155)^2}$

  $\mathbf{T_{CHAIN} = 2310\ N}$

1.4.12 FORCES IN ONE DIMENSION

I. Masses pulled by strings



When two objects of mass m_1 and m_2 are pulled by a force **F** by means of light inextensible strings A and B we can write:

$$\mathbf{F = (m_1 + m_2)\,a}$$

Hence, $\qquad$ $\boxed{a = \dfrac{F}{(m_1 + m_2)}}$

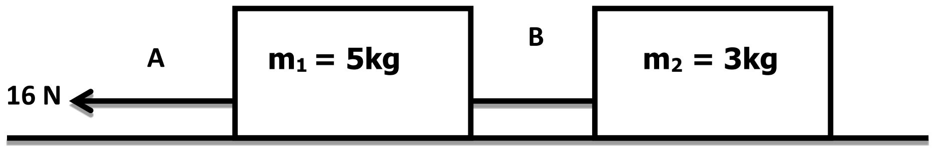
The tension in string A is *F,* and the tension in string B is the force needed to give m_2 acceleration *a,* where *a* is given by the above equation. Hence,

$$\text{Tension in the string B} = \frac{m_2 F}{(m_1 + m_2)}$$

String B pulls on both m_1 and m_2 with this force (Newton's Third Law).

Example 1

The diagram shows two masses, m_1 and m_2 connected by a string and pulled together with a force of 16N.



Calculate:

(i) The acceleration of the system
(ii) The tension in string B
(iii) The tension in string A

Soln:

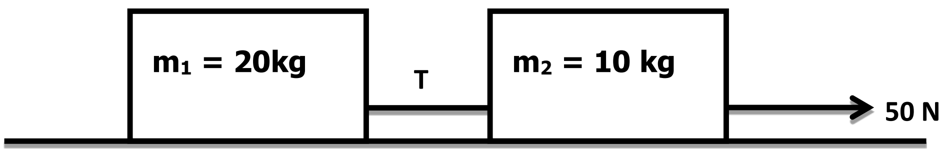
(i) $a = \dfrac{F}{m_1 + m_2} = \dfrac{16N}{(5+3)kg} =$ $\underline{\mathbf{2\ ms^{-2}}}$

(ii) Tension in string B, F = ma = (3kg) ($2ms^{-2}$) = **<u>6 N</u>**.

(iii) Tension in string A = Force F = **<u>16N</u>**.

Example 2

Two toy boxes of masses, 10 kg and 20 kg, are on a frictionless horizontal surface and are connected by a light string. A 50 N force is applies to the 10 kg box as shown below.

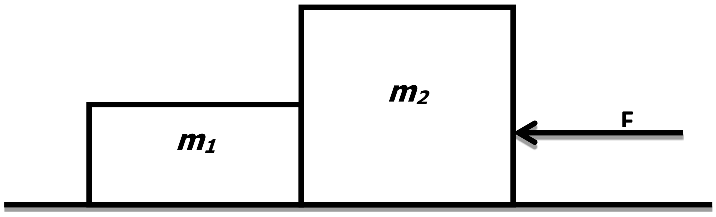


If a total frictional force of 14 N acts on the masses, calculate the acceleration of the boxes.

Soln: $F_{Net} = F_{applied} - F_{friction}$ = 50 N – 14 N = 36 N

$$\therefore a = \frac{F_{Net}}{m_1 + m_2} = \frac{36N}{(20+10)kg} = 1.2\ \underline{ms^{-2}}$$

II. Bodies in contact



If two objects in contact are accelerated by a force *F* as shown, then both will stay in contact with each other and will accelerate in the direction of the force.

$$\mathbf{F = (m_1 + m_2)\,a}$$

$$\boxed{\mathbf{a} = \frac{F}{(m_1 + m_2)}}$$

The force applied by m_2 to m_1 is given by

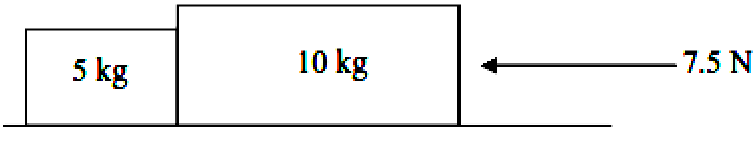
$$\frac{m_1 F}{(m_1 + m_2)}$$

This force has the same direction as the acceleration. From Newton's Third Law, this must be equal and opposite to the force applied by A to B. Hence the force applied by m_1 to m_2 is:

$\frac{m_1 F}{(m_1 + m_2)}$ but has direction opposite to that of the acceleration.

Example 3

A force of 7.5 N is applied to two objects as shown below.



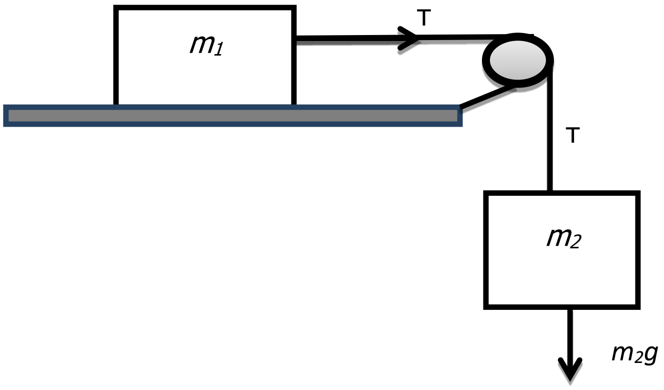
(i) Calculate the acceleration of the system.

(ii) Calculate the force applied to the 10 kg object by the 5 kg object.

Soln: - acceleration, $a = \dfrac{F}{(m_1 + m_2)}$ = $\dfrac{7.5N}{(5+10)kg}$ = **<u>0.5 ms^{-2}.</u>**

- Force applied = $\dfrac{m_1 F}{(m_1 + m_2)}$ = $\dfrac{5 \times 7.5}{(5+10)} N$ = **<u>2.5 N</u>**

III. Horizontal acceleration due to gravity



Two masses are connected by a light inextensible string over a pulley as shown above.

- The mass m_1 is accelerating horizontally across a frictionless surface, owing to the action *of* gravity on m_2.

The force acting on m_1 is the horizontal force *T,* the tension in the string.

For m_1 we can write

$$\boxed{\boldsymbol{T = m_1 a}} \qquad (1)$$

- Both masses will experience same acceleration, *a.*
- There are two forces acting on m_2, namely *T,* the tension in the string, acting upward, and the weight, m_2g, acting downward. For m_2 we can write

$$\boldsymbol{m_2 a = m_2 g - T}$$

$$\boxed{\boldsymbol{T = m_2 g - m_2 a}} \qquad (2)$$

Since *T* is the same for both m_1 and m_2, Equations 1 and 2 give us:

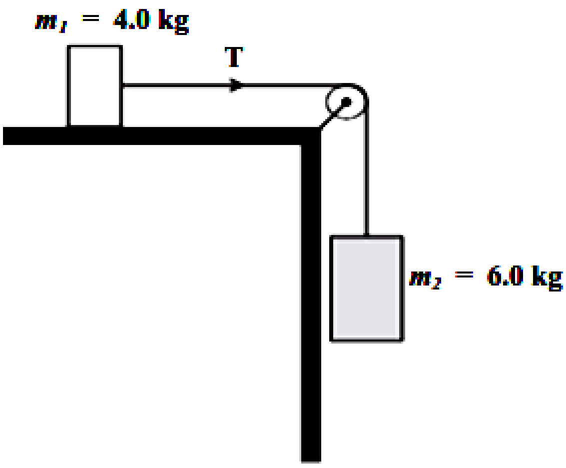
$$\boldsymbol{m_1 a = m_2 g - m_2 a}$$

$$\boxed{a = \frac{m_2 g}{(m_1 + m_2)}}$$

Example 4

Two masses are connected by a light string over a frictionless massless pulley.

Assume mass **m** is resting on a frictionless horizontal surface.

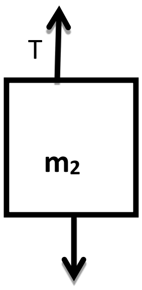


(i) Draw and label the forces acting on mass **m_2.**

(ii) What is the acceleration of mass **m_2**?

(iii) Determine the tension, **T** on mass **m_1**.

Soln: (i)



F_w = mg = (6kg)(10N/kg) = 60 N

(ii) acceleration, $a = \dfrac{m_2 g}{m_1 + m_2}$ = $\dfrac{60N}{(6+4)kg}$ = **<u>6 ms^{-2}</u>**

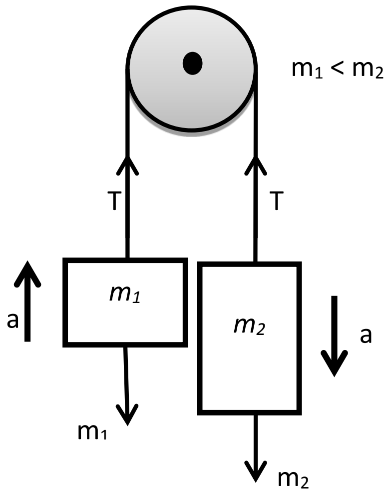
(iii) Tension, T on mass m_1 :

T = m_1a

T = (4 kg) (6 ms^{-2})

= **<u>24 N</u>**

IV. Masses over pulley



Two masses m_1 and m_2 are connected by a light inextensible string over a frictionless pulley as shown.

- Since $m_1 < m_2$, m_1 will accelerate up while m_2 will accelerate down when the system is released. Each will have acceleration *a.*
- Two forces act on m_1, namely tension *T* upward, and weight m_1g down. Since m_1 is accelerating upward, we can write:

$m_1a = T – m_1g$,

i.e. $\boxed{T = m_1g + m_1a}$ (1)

- There are two forces acting on m_2, namely tension *T* up, and weight m_2g down. Since m_2 is accelerating down, we can write:

$m_2a = m_2g - T$

i.e. $\boxed{T = m_2g - m_2a}$ (2)

From Equations 1 and 2, since T = T, we can write

$$m_1g + m_1a = m_2g – m_2a$$

Rearranging, making ‘a’ the subject:

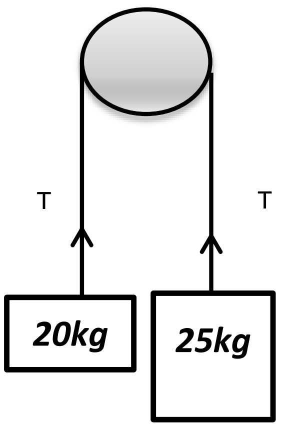
$$\boxed{a = g\frac{(m_2 - m_1)}{(m_1 + m_2)}}$$

The tension can now be found:

$$\boxed{T = m_1g + m_1a}$$

Example 5

Masses of 20 kg and 25 kg hang on opposite ends of a light string which passes over a frictionless pulley as shown below.



(i) Calculate the acceleration of the system.

$$a = g\frac{(m_2 - m_1)}{(m_1 + m_2)} \quad = \quad 10\frac{(25-20)kg}{(25+20)kg} \quad = \quad \underline{\mathbf{1.11\ ms^{-2}}}$$

(ii) What is the tension in the string?

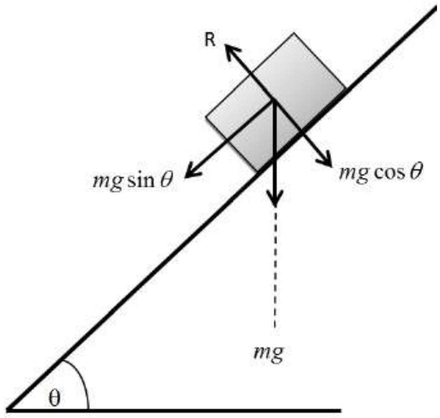
$\boldsymbol{T = m_1g + m_1a}$ =(20kg × 10 ms^{-2}) + (20kg × 1.11 ms^{-2}) = 200 N + 22.22 N

= **<u>222.22 N</u>**

1.4.13 FORCES IN TWO DIMENSION

I. Acceleration down slopes

If a mass *m* is placed on a frictionless slope inclined at an angle θ to the horizontal, the weight of the object *mg* will have a component down the slope and also at right angles to the slope.



The acceleration *a,* down the slope is due to the component of gravity down the slope.

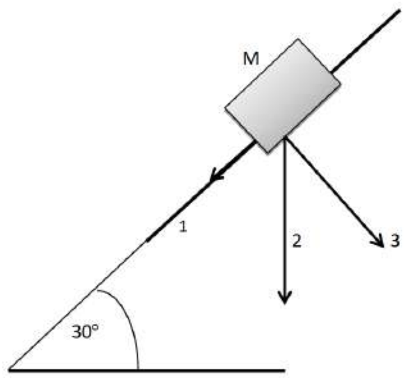
ma = mg sin θ

Thus, $\boxed{\mathbf{a = g \sin \theta}}$

The component of gravity at right angles to the surface is **mg cos θ.** From Newton's Third Law this is equal and opposite to the **normal reaction *R*** which is the force applied by the surface of the slope to the object.

Example 1

The diagram shows a mass M on an inclined plane with three arrows drawn to represent the weight and its components.



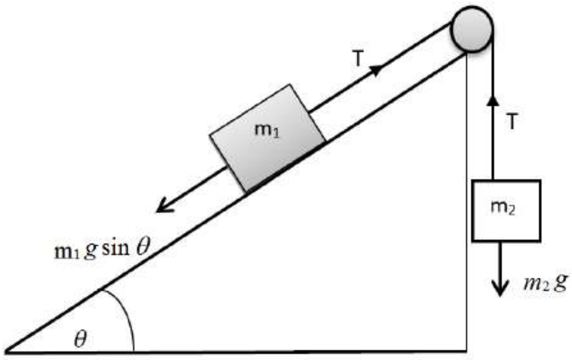
(i) Which of the arrows shows the component of the weight that determines the friction between the mass M and the inclined surface?

- Arrow 3

(ii) If M = 4kg and friction is negligible, calculate the acceleration of the mass.

- **a = gSinθ** = (10 N/kg) (Sin 30^0) = **<u>5 ms^{-2}.</u>**

II. Slopes and pulleys



The figure above shows that a mass m_1, is on a frictionless slope inclined at an angle θ to the horizontal. A light inextensible string connects m_1 to m_2 over a frictionless pulley as shown.

- There are two forces acting on m_1, i.e. $m_1 g \sin \theta$ down the slope and the *tension* in the string *T* up the slope.
- There are two forces acting on m_2, i.e. gravity ($m_2 g$) down, and *T*, the tension in the string, up.

CASE 1: If $\mathbf{m_1 g \sin \theta > m_2 g}$, m_1 will slide down the slope while m_2 will move up. Both will have the same acceleration, *a.*

- For m_1 we can write:

$$m_1 a = m_1 g \sin \theta - T$$

i.e. $\boldsymbol{T = m_1 g \sin \theta - m_1 a}$ (1)

- For m_2 we can write:

$m_2 a = T - m_2 g$

i.e. $\boldsymbol{T = m_2 a + m_1 g}$ (2)

Hence from Equations 1 and 2 we can write:

$$m_1 g \sin \theta - m_1 a = m_2 a + m_2 g$$

i.e. $$\boxed{\mathbf{a} = \frac{m_1 g \sin \theta - m_2 g}{m_1 + m_2}}$$

From this the value of T can be calculated using equation (1) or (2)

CASE 2: If $m_2g > m_1g \sin\theta$, m_1 will slide up the slope while m_2 will move downwards. Each will have acceleration a.

- For m_1 we can write:

$m_1a = T - m_1g \sin\theta$

i.e. $\mathbf{T = m_1g \sin\theta + m_1a}$ (3)

- For m_2 we can write:

$$m_2a = m_2g - T$$

i.e. $\mathbf{T = m_2g - m_2a}$ (4)

From equations 3 and 4 we can write:

$$m_1a + m_1g \sin\theta = m_2g - m_2a$$

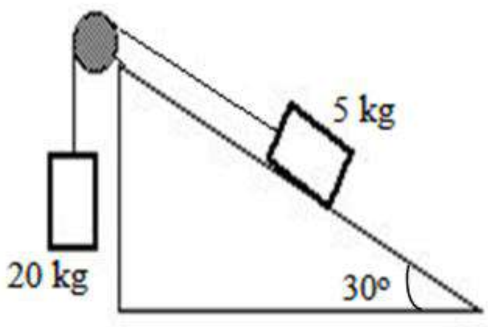
This gives the following expression for a:

$$\boxed{a = \frac{m_2g - m_1g\sin\theta}{m_1 + m_2}}$$

From this T can be calculated.

Example 2

A 5.0 kg mass is accelerated from rest at the bottom of the 4.0 m long ramp by a falling 20.0kg mass suspended over a frictionless pulley. The ramp is inclined 30^O ramp from the horizontal.



(a) Determine the acceleration of the 5.0 kg mass along the ramp.
(b) Determine the tension in the rope during the acceleration on the 5.0 kg mass along the ramp.

Soln:

- acceleration of 5 kg :

$a = \dfrac{m_2 g - m_1 g \sin\theta}{m_1 + m_2}$ = $\dfrac{(20\times 9.8) - (5\times 9.8\times \sin 30^0)}{(5+20)}$ = $\dfrac{171.5}{25}$ = **6.86 ms^{-2}.**

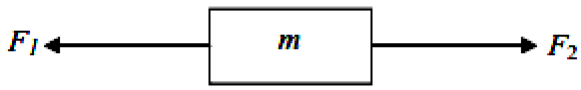
- Tension, T: T = $m_2g - m_2a$ =(20 × 9.8) – (20 × 6.86)

=**58.8 N.**

1.4.14 EXERCISE

1. Which of the following statements is Newton's third law of motion?
   A. Every force causes a reaction.
   B. To every action there is an equal and opposite reaction.
   C. The forces acting on a body are always equal and opposite.
   D. If there is no resultant force on a body then there is no acceleration.

2. The only forces acting on the object shown below are given as F_1 and F_2 with equal magnitude.

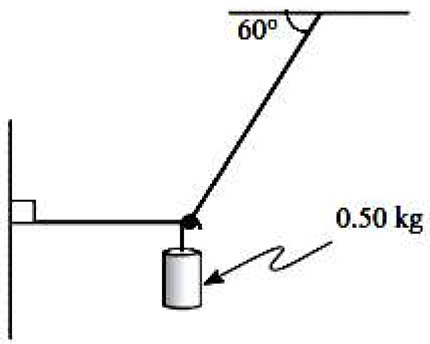


Which of the following statements is **not possible**?

   A. The object is at rest.
   B. The object is accelerating to the left
   C. The object is moving with constant velocity to the right.
   D. The object is moving with constant velocity towards the top of the page.

3. A point is acted on by two forces in equilibrium. The forces
   A. have equal magnitudes and directions.
   B. have equal magnitudes but opposite directions.
   C. act perpendicular to each other.
   D. act in the same direction.

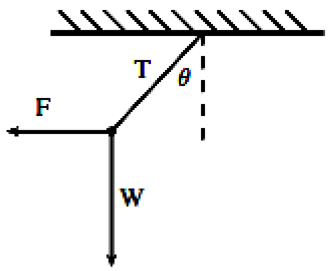
4. A 0.50 kg mass is suspended as shown in the diagram.



If the system is in a state of equilibrium, what is the tension in the horizontal string?

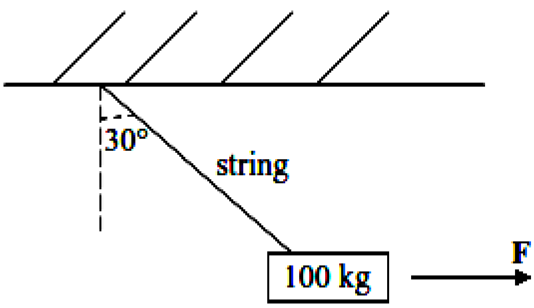
A. 2.5 N B. 2.9 N C. 4.2 N D. 4.9 N

5. The diagram shows an object of weight W, attached to a string. A horizontal force F is applied to the object so that the string makes an angle of θ with the vertical when the object is at rest. The force exerted by the string is T. Which one of the following expressions is incorrect?



A. F + T + W = 0 B. W = T Cosθ C. $Tan\theta = \frac{F}{W}$ D. W = T Sinθ

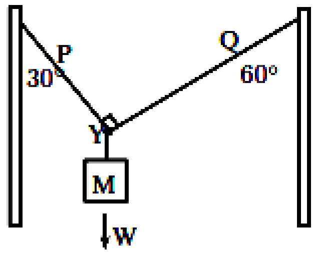
6. A 100 kg object hanging from the ceiling is pulled to the right by a force **F** as shown below.



If the system is in a state of equilibrium, calculate:

(i) the force **F**.
(ii) the tension in the string joining the mass to the ceiling.

7. A rope of negligible mass is strung between two vertical struts. A mass M of weight W hangs from the rope through a hook fixed at point Y.



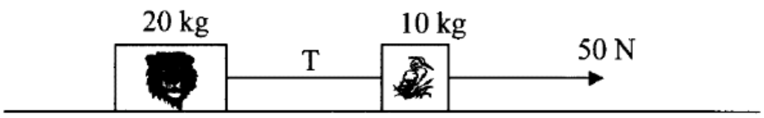
i. Draw a vector diagram, plotted head to tail, of the forces acting at point Y. Label each force and show the size of each angle.
ii. Where will the force be greatest? Part P or Q? Motivate your answer.
iii. When the force in the rope is greater than 600 N it will break. What is the maximum mass that the above set up can support?

8. The diagram given below shows a system of two masses connected by a string and acted on by a 24 N force. The surface is frictionless.

The tension in the string connecting the two masses is

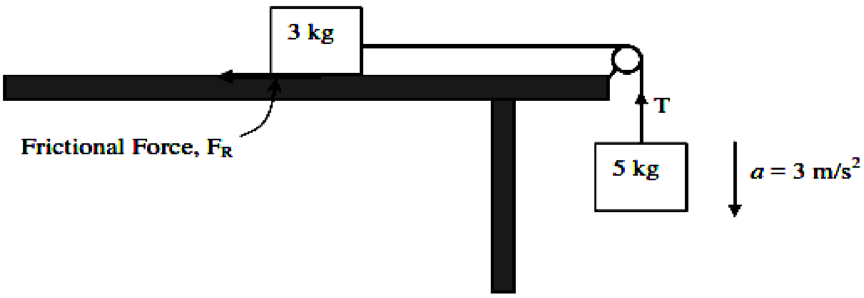
A. 24 N B. 18 N C. 8 N D. 6 N

9. Two toy boxes of masses,10 kg and 20 kg, are on a frictionless horizontal surface and are connected by a light string. A 50 N force is applied to the 10 kg box as shown below.



If a total frictional force of 14 N acts on the masses, calculate the acceleration of the boxes.

10. The diagram below shows two masses on a bench top connected by a light inextensible string.

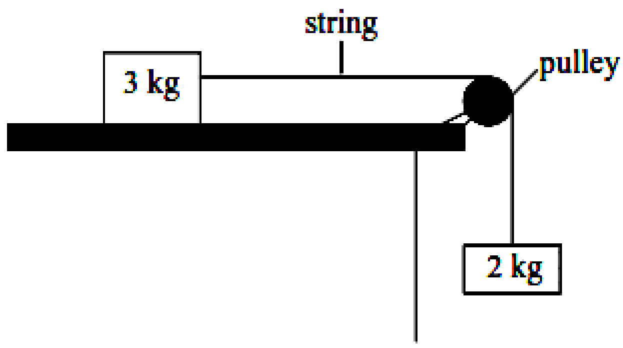


The system is accelerating at 3 m/s^2 as shown.

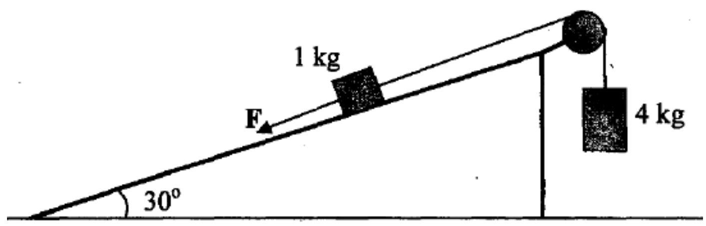
(i) Find the net force acting on the system.
(ii) Calculate the tension, T, in the string.
(iii) Find the frictional force between the 3 kg mass and the bench top.

11. The diagram below shows two masses connected by a light inextensible string

(i) Calculate the acceleration of the masses.
(ii) Determine the tension in the string.



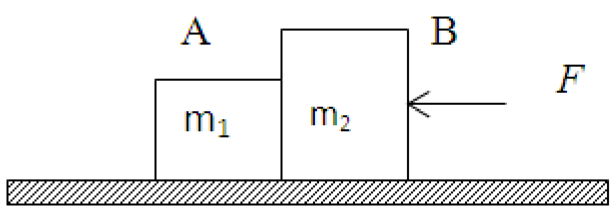
12. The diagram given below shows a 1 kg mass on a frictionless inclined plane and a 4 kg hanging mass. Both are connected by a light string over a smooth pulley.



F is the component of the weight of the 1 kg mass along the inclined plane.

(i) Find the magnitude of F.
(ii) Hence calculate the acceleration of the masses.

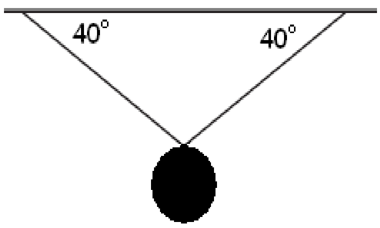
13. A force of 7.5 N is applied to two bodies to, A = 5 kg and B = 10 kg.



Calculate:

(a) the acceleration produced
(b) the force applied to B by A

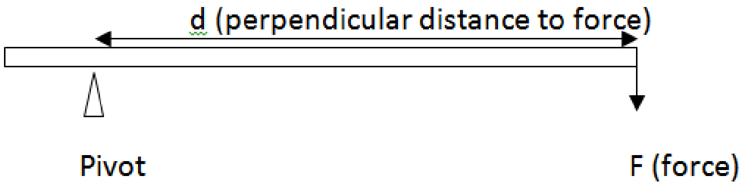
14. A black disk hangs from two wires as shown. If the tension in each cable is 230N, what is the weight of the black disk?



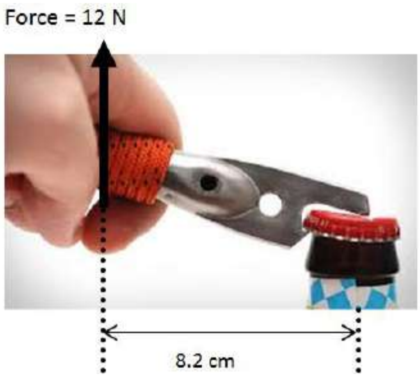
1.5 MOMENT

Torque or Moment refers to the turning effect of force and is measured by the product of the force and the perpendicular distance of the force from the turning point.

Torque or Moment = Force x perpendicular distance



Example: A force of 12 N acts at 8.2 cm from a pivot is needed to lift the cap off a bottle of soft drink

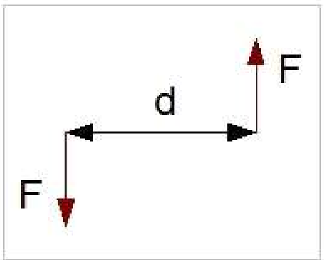


Torque applied is: Torque = Force x distance

= 12 x 0.082 (8.2 cm = 0.082 m)

= 0.98 Nm anticlockwise

Couple occurs where two equal and oppositely directed forces act at a distance apart. A couple causes rotation only and the moment of the couple about any point is Force x distance i.e. one of the force by the perpendicular distance between the two force.



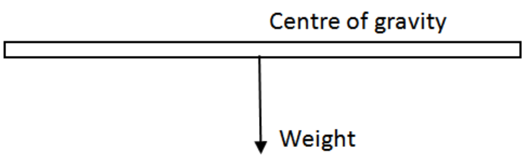
1.5.10 Equilibrium

Equilibrium occurs when an object is at rest or moving uniformly, as describe in Newton First Law. An object is describe as being in equilibrium when both the resulted force is zero and the sum of all the torque acting on the object is zero.

EQUILIBRIUM

1. The resultant force acting on the object is zero; i.e. the vector sum of the force acting is zero. **∑ F = 0,** $\boldsymbol{F_X}$ **= 0** $\boldsymbol{F_Y}$ **= 0** The acceleration is zero so the object will either be stationary or have a uniform motion.	2. The sum of all the torque acting on the object is zero. **∑ Torque = 0** As the result, the object will not twist or rotate. Clockwise moment equal Anticlockwise moment about any point on the object.

Most situations involve objects in equilibrium for example a painter organizing trestles and planks, or engineer working on bridge designs. The weight of any object always acts down from the centre of mass. It's the point at which, if an object is suspended or pivoted, the object will balance.



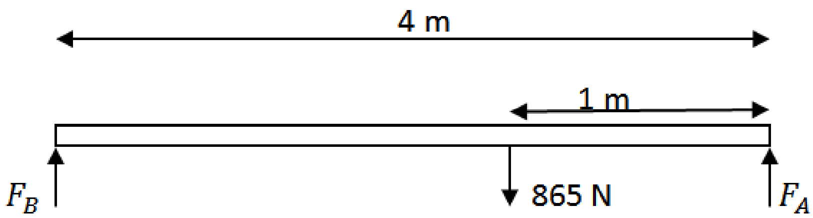
1.5.11 PAINTER ON SCAFFOLDS

Example 1:

A painter weighing 865N stands on the plank 4m long, this is supported at each end by a stepladder. If he stands 1m from one end of the plank, what force is exerted by each stepladder?

Solution:

Redraw the diagram above using force diagram.



To determine the force at A (F_A), moments are taken about pivot B. At this point, F_B produce no torque.

Anticlockwise moments = F_A x 4

= $4F_A$

Clockwise moments = 865 x 3

=2595 N

Clockwise moments = Anticlockwise moments

$2595 = 4F_A$

$F_A = \frac{2595}{4}$

F_A = 649 N

Sum of force upwards = Sum of force downwards

$F_A + F_B$ = 865 N

649 N + F_B = 865 N

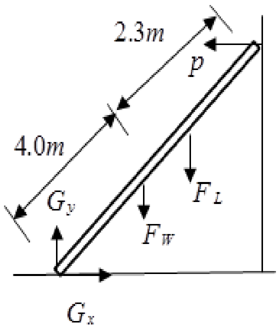
F_B = 865 N – 649 N

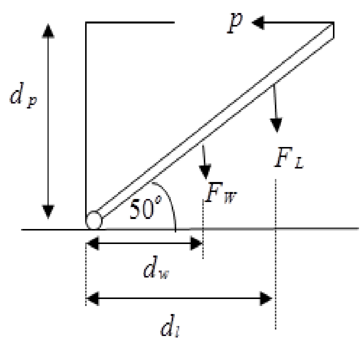
F_B = 216 N

1.5.12 LADDER

Example 2

An 8 m ladder of weight F_W = 355 N lean against a smooth vertical wall. The term smooth means that the wall can exert only a normal force directed perpendicular to the wall and cannot exert a frictional force parallel to it. A fire fighter whose weight (F_L) is 875 N, stand 6.30 m from the bottom of the ladder. Assume that the ladders weight act at the ladders centre and neglect the hose weight. Find the force that the wall and the ground exert on the ladder.





Free Body Diagram

∑ Torque = 0

∑ Torque = (F_P x d_P) + (- F_w x 6.30 Cos 50°) + (- F_L x 4 Cos 50°) (**force and perpendicular distance**)

∑ Torque = (F_p x 8 Sin 50°) - (875 x 6.30 Cos 50°) - (355 x 4 Cos 50°)

0 = 6.13 F_p - 3543 – 913

4456 = 6.13 F_P

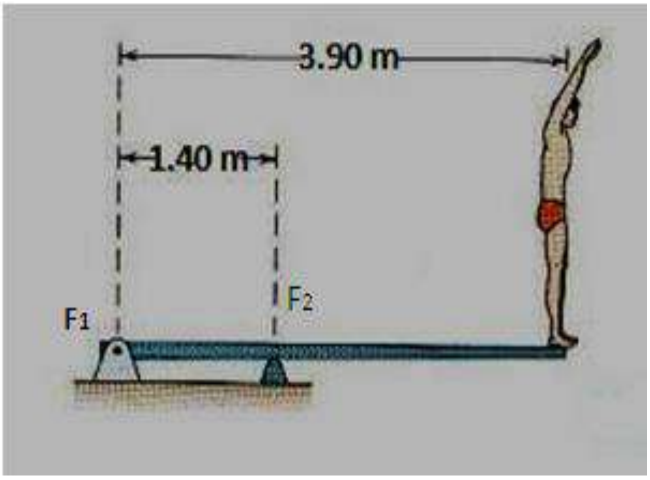
$F_P = \frac{4456}{6.13}$

F_P = 727N

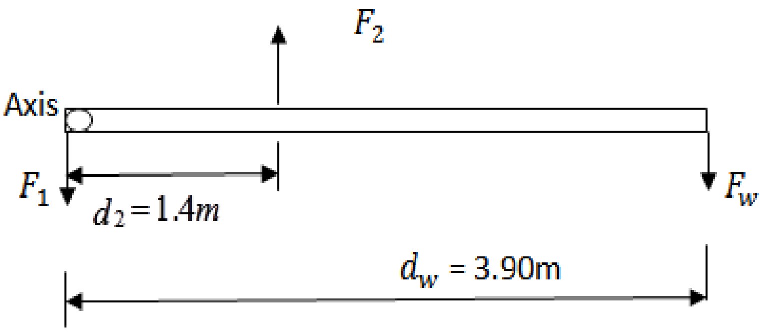
1.5.13 DIVING BOARDS

Example 3

A man whose weight is 530N is poised at the right end of the diving board, whose length is 3.90m. The board has negligible weight and is bolted down at the left end, while being supported 1.40m away from the fulcrum. Find the force (F_1) and force (F_2) that the bolt and the fulcrum, respectively exert on the ground.



Free – Body Diagram



- F_1 point downwards because the bolt will pull in that direction.
- F_2 point upwards because the board pushes downward against a fulcrum, which in reaction pushes upwards on the board.
- Board is stationary, it is in equilibrium.

$$\sum \text{Torque} = 0$$
$$(F_2 \text{ x } d_2) + (-F_w \text{ x} d_w) = 0$$
$$(F_2 \text{ x } 1.40) - (530 \text{ x } 3.90) = 0$$
$$1.40\, F_2 - 2067 = 0$$
$$1.40\, F_2 = 2067$$
$$F_2 = \frac{2067}{1.40}$$
$$F_2 = 1476 \text{ N}$$

It is in equilibrium

$$\sum F_y = 0$$

$$-F_1 + F_2 - 530\text{N} = 0$$

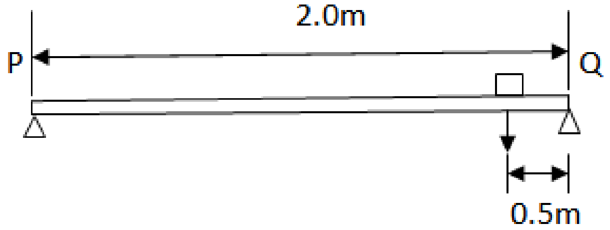
$$-F_1 + 1480 - 530 = 0$$

$$-F_1 + 950 = 0$$

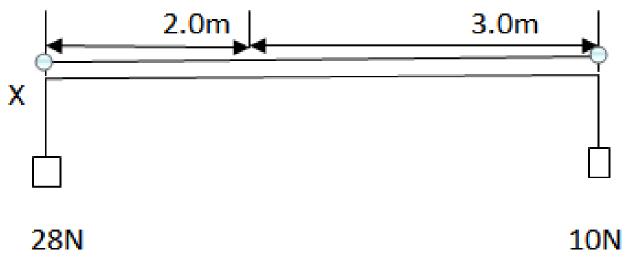
$$946\text{ N} = F_1$$

1.5.13 EXERCISE

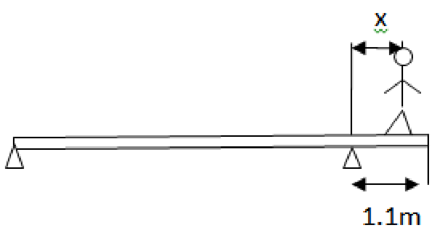
1. PQ represents a uniform beam of mass 2.0kg supported at ends P and Q. An object of mass 4.0kg is placed so that it is 0.5 m from Q. The beam is 2.0m long.
   (a) By how much does the force on Q exceed the force on P?
   (b) Calculate the total anticlockwise torque (moment) about P.



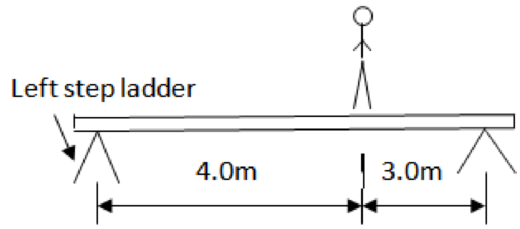
2. A uniform beam in equilibrium is suspended by a cord at X which is 2.0m from one end of the beam and 3.0m from the other end. Masses provide forces on the end of the beam of 28N and 10N. Determine the weight force of the beam.



3. A uniform plank of length 5.0m and weight 225N rest horizontally on two supports, with 1.1m of the plank hanging over the right support. Find the force exerted by the two supports.



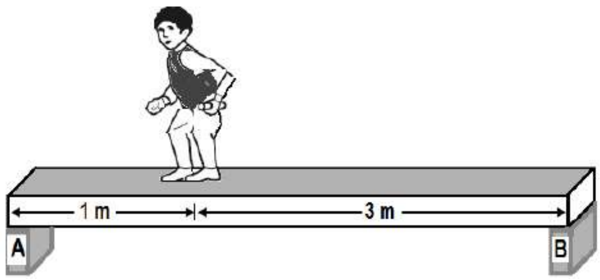
4. An 840N painter stands on a 7.0m board of negligible weight. The board is supported by two step ladders as shown below.



(a) State the principle of moments,

(b) What is the force exerted on the board by the left step ladder?

5. A 40 kg boy stands on a horizontal plank, 1 m from one end and 3m from the other. The plank is uniform and has a mass of 25kg. It is supported by a trestle at each end. Find the force each trestle exerts on the plank.



1.6 KINEMATICS

1.6.10 KINEMATIC EQUATION OF MOTION

Problems involving uniform acceleration in a straight line over a time interval can often be quickly solved using a set of formula called the **Kinematic Equation of Motion.**

1st equation: $v_f = v_i + at$

2nd equation: $d = v_i t + ½ at^2$

3rd equation: $v_f^2 = v_i^2 + 2ad$

Where:

v_f =final velocity (m/s)

v_i = initial velocity (m/s)
a = constant acceleration (m/s^2)
t = time (s)
d = distance (m)

Each equation contains four or five variables (t, d, v_i, v_f, a)

In deciding which equation to use:

1. To determine the unknown value of time (t) – we must use the 1st or 2nd equation

2. To determine the distance (d): we must use the 2nd or the 3rd equation

3. To determine the velocity (v) we must use the 1st or the 3rd equation

Depending on the variable provided so it is important to list down what is given and what do we have to find out.

Example 1

A body moving with an initial velocity of 6m/s accelerates at 2m/s^2. Find

(a) the distance gone after 3 seconds.

v_i = 6m/s
a = 2m/s^2

The unknown is the distance – use the 2nd or the 3rd equation

Since (t) is given- use 2nd equation

d = $v_i t$ + ½ at^2
d = (6) (3) + (1/2) (2) $(3)^2$
d = 18 +9
d = 27m

(b) The velocity of the body after it has gone 20m

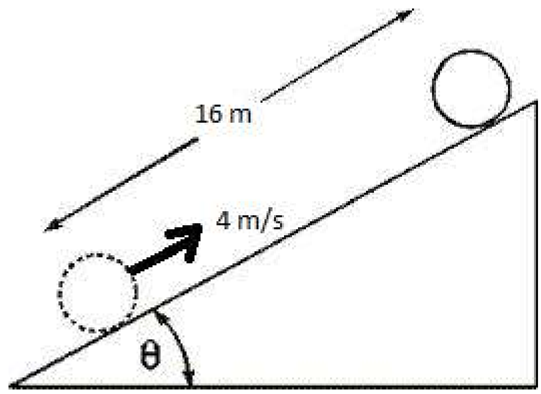
v_i = 6m/s a = 2m/s^2 d = 20m v_f

The unknown is velocity – use the 1st or the 3rd equation and since(t) is not given, use the 3rd equation

${v_f}^2$ = ${v_i}^2$ + 2 ad
${v_f}^2$ = $(6)^2$ + 2(2) (20)
${v_f}^2$ = 36 +80
${v_f}^2$ = 116
v_f =$\sqrt{116}$
v_f = 10.77m/s
≈ **10.8m/s**

Example 2

A ball initially travelling at 4.0m/s rolls up a slope and then slows uniformly to a stop 16.0m up the slope. What is the acceleration of the ball?



v_i = 4.0m/s d = 16.0m v_f =0

use the 3rd equation of motion

$v_f{}^2 = v_i{}^2$ + 2ad

$0^2 = 4^2$ + 2a (16) – (substitute the variable)

0 = 16 + 32a (re arrange to make a the subject of the formula)

-16 = 32a

∴ **a = - 0.50m/s^2**

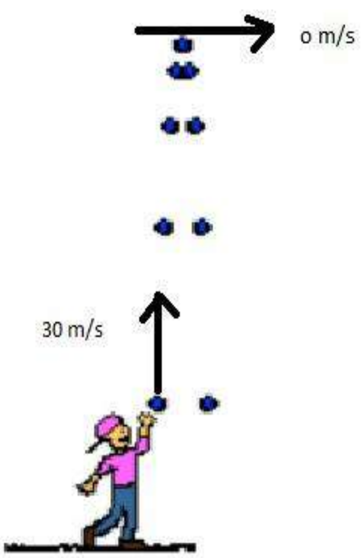
1.6.11 VERTICAL MOTION OR FREE FALL

The acceleration of bodies under the gravitational pull of the earth is 10m/s^2. Positive acceleration (a = 9.81m/s^2) and negative is moving up (a= -9.81m/s^2) if the object is moving up remember at the highest level of a body its vertical velocity is zero (v_{fy} = 0m/s)

Example 1

A body is thrown vertically upwards with an initial velocity of 30m/s

(a) Find the greatest height reached



v_I=30m/s v_f = 0 a = -9.81m/s^2 (moving up)

Use the 3rd equation of motion

$v_f{}^2 = 30^2$ + 2(-9.81) d

0 = 30^2 – 19.6d

19.6d= 900

$$\frac{19.6d}{19.6} = \frac{900}{19.6}$$

d = 46

(b) The time to get to the highest point

$a = -9.81 m/s^2$ $v_i = 30 m/s$
$v_f = 0 m/s$

Use the 1st equation of motion

$$v_f = v_i + at$$

0 = 30 + (-9.81) t

0 = 30 - 9.81t

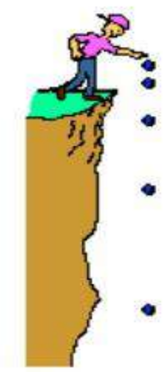
9.81t = 30

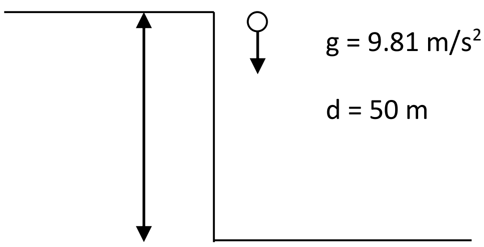
$$\frac{9.81t}{9.81} = \frac{30}{9.81}$$

t = <u>3.1 seconds</u>

Example 2

A rock (initially statutory) is dropped from a cliff 50m above the sea.





(a) How far does it fall in 2.0 seconds

v_i=0 a= 9.81m/s^2 t= 2.0s
d= 50m

Use the 2nd equation of motion

d = v_i + ½ at^2

d = (0) (2) + (1/2) (9.81) (4)

d = 19.6m

(b) How long does it take to fall 50m

a = 9.81m/s d= 50m v_i= 0m/s

Use the2nd equation of motion

d = v_i + ½ at^2

50 = (0) t + ½ (9.81) (t^2)

50 = 4.9t^2

$$\frac{50}{4.9} = \frac{4.9t^2}{4.9}$$

10.2 = t^2

$\sqrt{10.2}$ = t ∴ t= 3.2 seconds

1.6.13 EXERCISE

Use g = 9.81m/s^2 in vertical motion

1. Calculate the uniform acceleration of a sports car which:
   a) starts from rest and reaches a speed of 15.5m/s in 8 s;
   b) Changes its speed from 20m/s to 36m/s in 5 s;
   c) Starts from rest and goes a distance of 98m in 7 s;
   d) Starts from rest and travels a distance of 22m during the sixth seconds of its motion;
   e) Slows down from a speed of 67m/s and come to rest in 12 s

2. A ball roll from rest down an incline plane with a uniform acceleration of 3.6m/s^2.
   a) What is its speed after 7.2 seconds?
   b) How long will it take to reach a speed of 38m/s?
   c) How long does it take to travel a distance of 200m, and what is its speed after going this distance?
   d) How far does it travel during the third seconds of its motion?

3. A car, initially travelling at a uniform velocity, accelerates at the rate of 1m/s^2 for a period of 12 seconds. If the car travelled 190m during this 12 seconds interval, what was the velocity of the car when it started to accelerate?

4. A skydiver drops from a hovering helicopter and falls freely for 5 seconds before opening his parachute.
   a) What speed has he attained when he opens his parachute?
   b) How far did he fall on free fall?
   c) What was his average speed while falling freely?

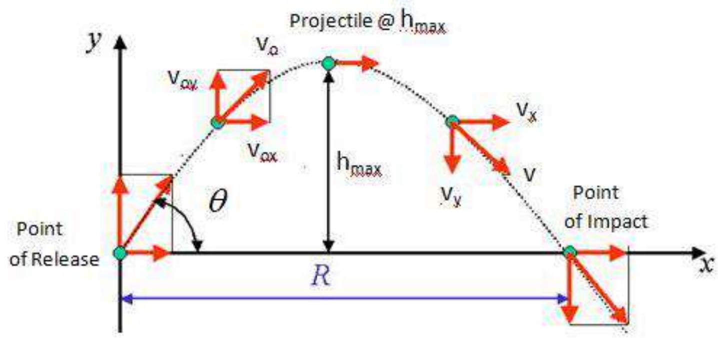
5. An object is thrown vertically upwards with an initial speed of 16m/s from the top of a bridge 25m above the water. How long does it take to reach the water?

1.7 PROJECTILE MOTION

A projectile motion is any object that moves through the air without its own source of power, only under the influence of gravity for example bullets, shot put, netball and softballs.

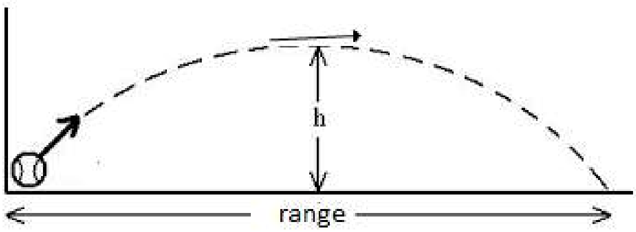
The only force acting on the projectile is its weight forces which act vertically downwards and so only the vertical component of the velocity of the projectile changes with time. Horizontal component remain constant through the motion.

1.7.10 FULL PROJECTILE



This is projectiles under the influence of gravity.

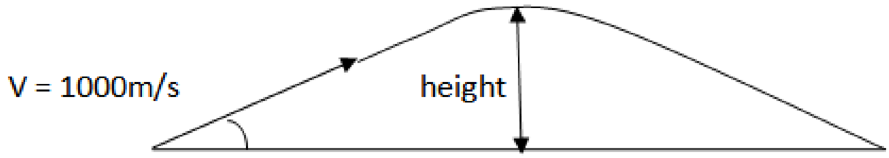
Path shows by projectile motion is called parabola.



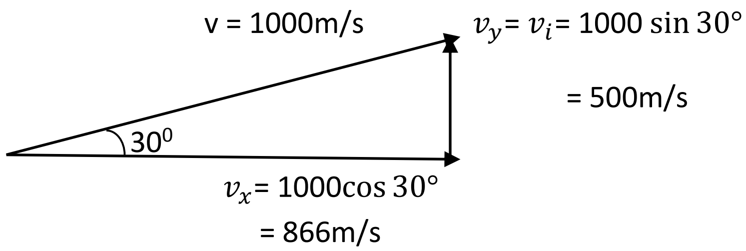
- Maximum height of flight is reached when the vertical component of the projectile velocity is zero.
- Total time of flight is twice the time taken to reach the maximum height.
- Range is the distance travelled horizontally is determined by the product of horizontal component of velocity and the total time of flight.
  Range = Velocity in horizontal component x Total time of flight
- The projectile hits the ground at the same speed as it was fired and makes the same angle to the horizontal.

Example 1

A rocket is fired at 1000m/s at an angle of 30°.



Projectile from the ground will need its components.



Calculate:

a) The greatest height reached by the rocket

v_f= 0m/s, a= -9.81m/s^2, v_i=500m/s, d=?

Use 3rd equation of motion: $v_f{}^2 = v_i{}^2 + 2ad$

$0 = (500^2) + 2(-9.81)\ d$

$0 = 250000 - 19.6d$

$19.6\ d = 250000$

$d = \frac{250000}{19.6}$

<u>d = 12755 m</u>

b) The time of flight of the rocket

time to the greatest height: a=-9.81m/s^2, v_f= 0m/s, v_i= 500m/s

use the 1st equation : $v_f = v_i + at$

o= 500 + (-9.81)t

9.81t = 500

$t = \frac{500}{9.81}$

t = 51s

Therefore time of flight = 2 x 51 = **<u>102 s</u>**

c) The Range (horizontal distance) travelled by the rocket

Range = horizontal component of velocity x total time of flight

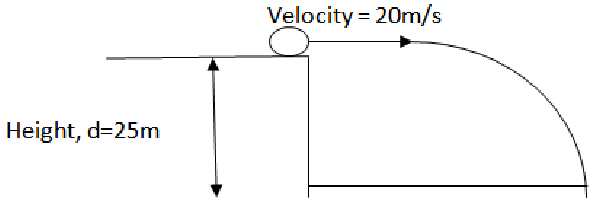
= 866m/s x 102s

= **<u>88332m</u>**

1.7.11 HALF PROJECTILE

Example 2

A 6kg projectile is launched horizontally from a height of 25m with a velocity of 20m/s as shown below.



(i) Calculate the time of flight of the projectile

(ii)

v_i= 0m/s, d = 25m, a = 9.81m/s^2

Use the 2nd equation of motion: d=v_it + ½ at^2

25 = 0 + ½ (9.81)t^2

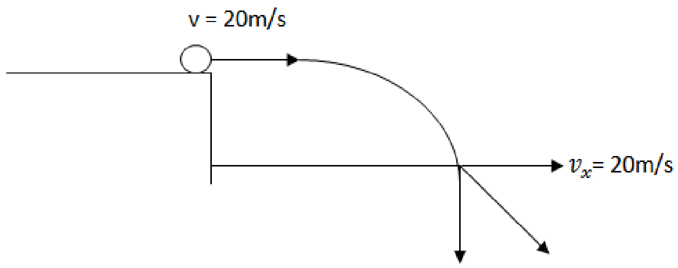
25 = 4.9t^2

$t^2 = \frac{25}{4.9}$

t = $\sqrt{5.1}$

t = 2.26 s

(iii) Calculate the velocity of projectile just before it hit the ground.



${v_y}^2 = {v_i}^2$ + 2ad

${v_y}^2$ = 0 + 2(9.8) (25)

$v_y = \sqrt{2x\ 9.8x25}$

v_y = 22.1m/s

Use Pythagoras theorem:

$v^2 = {v_x}^2 + {v_y}^2$

$v^2 = (20^2) + (22.1^2)$

v^2 = 400 + 488

v = $\sqrt{888}$

v = 29.8m/s

1.7.12 EXERCISE

1. When an airplane is at an altitude of 500m and moving horizontally at a speed of 160m/s, a small package is dropped from it.

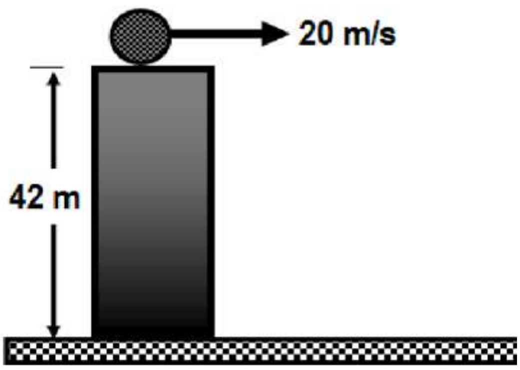
a) How long does the package take to reach the ground?
b) How far from the point over which the package is dropped does it hit the ground?
c) What is the shape of the path followed by the package?

2. A ball is thrown horizontally with a speed of 16m/s from a point 3.8m above the ground. Calculate:
   a) the time taken by the ball to reach the ground.
   b) the horizontal distance travelled in that time.
   c) its velocity when it reaches the ground.

3. A boy standing on the tray of the lorry travelling at 18km/h throw a ball vertically upwards with a speed of 10m/s and catches it again at the same level. What distance horizontally does the ball move while it is in the air?

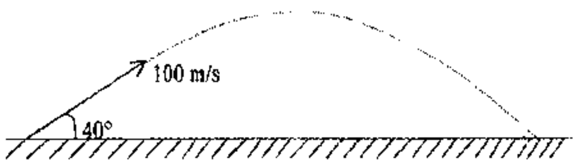
4. A cricket ball is hit with a velocity of 8m/s at an angle of 60 ° with the horizontal. Calculate:
   (a) Its horizontal and vertical displacement after 0.5s has elapsed.
   (b) The time taken to return to the level from which it was hit, and the horizontal distance travelled in this time.

5. A projectile is launched horizontally from a height of 42 meters with a velocity of 20 m/s as shown in the diagram below. Calculate;



   (a) The time of the flight.
   (b) The range of the projectile.
   (c) The velocity of the projectile just before it hits the ground.

6) A projectile is launched with a velocity of 100 m/s at an angle of 40° to the horizontal as given below:



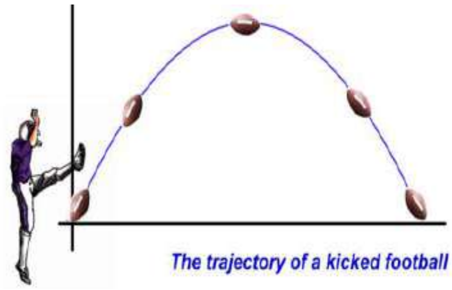
Calculate
   i. The time of the flight.
   ii. The maximum height reached.
   iii. The range (horizontal distance.

7) Osea kicks a rugby ball for conversion after scoring a stunning try. He kicks the ball at 1000 m/s at an angle of 30^o above the ground.

a) Calculate the time taken to reach H_{MAX}.

b) What is the H_{MAX} reached by the ball?

c) What horizontal distance the ball travels?



1.8 MOMENTUM.

Momentum is a useful quantity to consider when collisions between different objects occur, or when explosion break objects into pieces or push objects apart. Momentum is the product of mass and velocity and it is a vector quantity.

Momentum = Mass × Velocity.

$$\boxed{\mathbf{p = mv}}$$

Where m is the mass (kg)

v is the velocity (m/s)

P is the momentum (kg in /s)

Example 1

(a) A 35g golf ball travelling at 10m/s has momentum.

p = mv

= (0.035) × (10) (changing 35g to 0.035)

= 0.35kg m/s

b) A shopping trolley of mass 20 kg moving at 0.85 ms^{-1} to the south has momentum

p = mv

= (20) (0.85)

= **17 kgms^{-1} south**.

(c) ship of mass 40,000 tonne moving at 0.2 ms^{-1} has momentum.

p = mv

= (4 × 10^7) (0.2) (changing 40,000 ton = 4 × 10^7 kg)

= **8 × 10^6 kgms^{-1}**

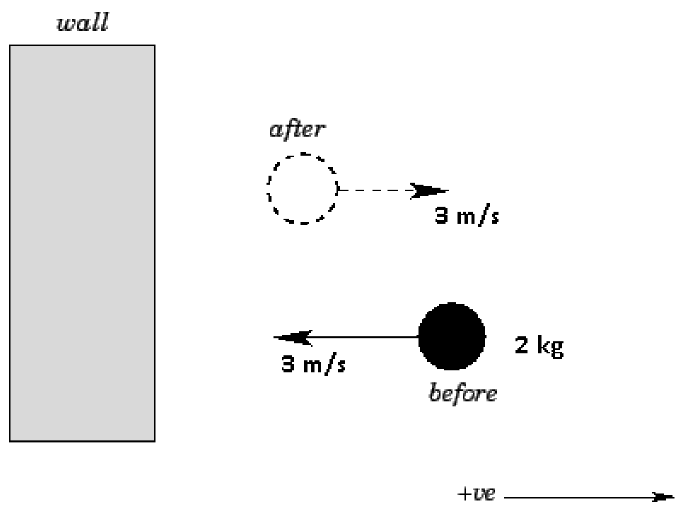
1.8.10 CHANGE IN MOMENTUM.

When force act on an object and changes its motion, the objects momentum will also change. Change in momentum (Δp) can be calculated using:

Change in momentum = Final momentum - Initial momentum

$$\boxed{\Delta p = p_f - p_i}$$

Example: A ball of 2kg was thrown against a wall at 3m/s to the left and rebounds at 3 m/s. What is the change in momentum of the body?



Initial momentum: $\rho_i = mv_i$
$= (2)(3)$
$= 6\text{kg m/s} \longleftarrow$

Final momentum: $\rho_f = mv_f$
$= (2)(3)$
$= 6\text{ kg m/s} \longrightarrow$

Method (1)

$\Delta\rho = \rho f - \rho i$

$= \underset{\longrightarrow}{6} - \underset{\longleftarrow}{6}$

$= \underset{\longrightarrow}{6} + \underset{\longrightarrow}{6}$

$\underline{= \quad 12\text{ kg m/s}}$ $\longrightarrow$

Method (2)

$\Delta\rho = \rho_f - \rho_i$

$= (2)(3) - (2)(-3)$

$= 6 + 6$

$= 12\text{ kg m/s}$

= **<u>12 kg m/s to the Right</u>**.

1.8.11 IMPULSE

Impulse is the product of the average force and the time internal over which the force is applied. This results in the acceleration of the object and changes of its momentum. Impulse is also equal to change in momentum.

Newton's 2nd Law: F = ma

$$F = m\left(\frac{v_{f-}v_i}{t}\right)$$

$$F = mv_{f-}\ mv_i \,/\, \Delta t$$

$$F \times \Delta t = \Delta p$$

Impulse = change in momentum.

(Ns) (Kgm/s)

Example: How long must a 300kg satellite, in orbit, fire its thruster rocket in order to increase its speed from 500m/s to 600m/s? The force exerted by the thruster when firing is 1500N.

Solutions: The desired change in momentum is:

$$\Delta\rho = mv_{f-}\ mv_i$$

$$\Delta\rho = m\,(v_f - v_i)$$

$$\Delta p = (300)\ (6000 - 5000)$$

$$= 300\,000 \text{ kgms}^{-1}$$

The impulse needed to cause this change in momentum is

$$F \times \Delta t = \Delta p$$

$$1500 \times \Delta t = 300000$$

$$\Delta t = \frac{300000}{1500}$$

$$\Delta t = 200 \text{ seconds}$$

The thruster rocket must be fired for **200 seconds**.

1.8.12 CONSERVATION OF MOMENTUM IN TWO DIMENSIONS

Momentum is conserved in collision and explosion. It is also conserved in all directions. If one objects moves at an angle to another object, then the angle must be taken in to account. A vector momentum diagram can be drawn for problem involving momentum in two dimensions

Momentum before collision / explosions = Momentum after collision / explosions

Masses move separately

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

Masses stick together

$$m_1u_1 + m_2u_2 = (m_1 + m_2)v_c$$

Elastic and Inelastic collisions

An **elastic collision** is one in which the total kinetic energy is the **same** before and after collision.

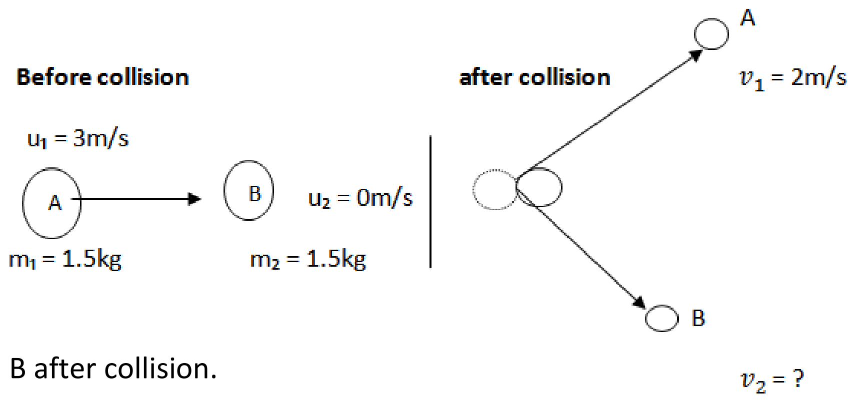
$$KE_i = KE_f$$

An **inelastic collision** is one in which the total kinetic energy before collision is **not equal** to the total kinetic energy after collision. If the object sticks together after collision, the collision is said to be **completely inelastic**.

$$KE_i \neq KE_f$$

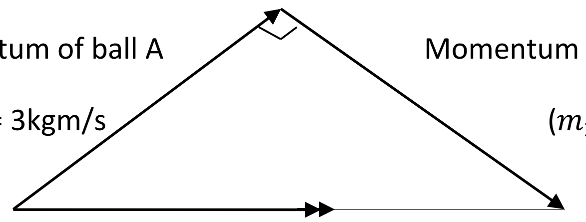
Example 1

A bowling ball, A, of mass 1.5kg and travelling to the right at 3.0m/s hits an identical ball, B , which is stationary. Ball A moves off at 2.0m/s at an angle of 90° to the direction in which B moves.



(a) Find the speed of the ball B after collision.

Total momentum before the collision = Total momentum after collision



P = $m_1 u_1 + m_2 u_2$

= (1.5 x 3) + (1.5 x 0)

= 4.5kgm/s

Magnitude of the momentum of ball B is obtained using Pythagoras theorem.

$p_B = \sqrt{4.5^2 - 3.0^2}$

p_B = 3.35kgm/s

And the speed of ball B is:

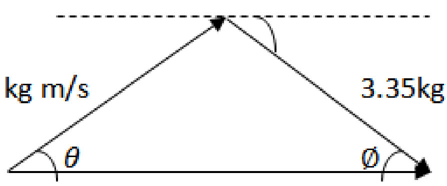
p_B = m v

3.35 = 1.5v

v $= \frac{3.35}{1.5}$

v = **2.2m/s**

(b) Find the direction of ball A after the collision.



$\cos\theta = \frac{adjacent}{hypotenuse}$

$\cos\theta = \frac{3.0}{4.5}$

$\theta = \cos^{-1}\frac{3.0}{4.5}$

= **48.2°**

(c) Find the direction of the ball B after the collision.

$\cos\emptyset = \frac{adjacent}{hypotenuse}$

$\cos\emptyset = \frac{3.35}{4.5}$

$\emptyset = \cos^{-1}\frac{3.35}{4.5}$

$\emptyset = \underline{\mathbf{42°}}$

In this example, a vector diagram of velocities would form a **similar** triangle to the momentum triangle shown. This only happens when the two object have the **same mass**.

(d) Show that collision in elastic collision.

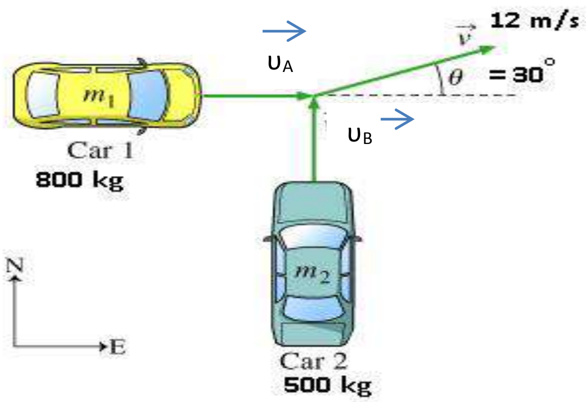
$$KE_i = \frac{1}{2}(m_1)(u1)^2 + \frac{1}{2}(m_2)(u2)^2$$
$$= \frac{1}{2}(1.5)(3)^2 + \frac{1}{2}(1.5)(0)^2$$
$$= 6.75J$$

$$KE_f = \frac{1}{2}(m_1)(v1)^2 + \frac{1}{2}(m_2)(v2)^2$$
$$= \frac{1}{2}(1.5)(2)^2 + \frac{1}{2}(1.5)(\sqrt{5})^2$$
$$= 6.75J$$

The total energy before collision is equal to total energy after the collision. Hence, it is an elastic collision

Example 2

A police car of mass 800kg travelling East, collide with another car of mass 500kg travelling North, at a road junction as shown below.



Given that the two cars stick together after collision and move off with a common velocity of 12m/s in the direction shown.

(a) Find the speed of u_A and u_B of each car before collision.

(b) Show that the collision is inelastic.

Total momentum before collision = Total momentum after collision

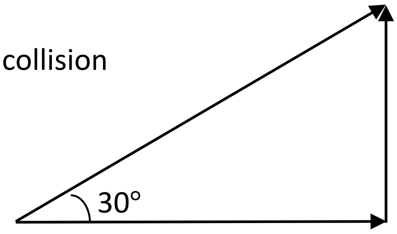
Momentum of mass B before collision

Momentum after collision

P = (m_A + m_B) v

P = (800+500)12

P = 15600kgm/s



$p_B = m_B\, u_B$

$p_B = 500\, u_B$

Momentum of mass A before collision

$p_A = m_A\, u_A$

$p_A = 800\, u_A$

(a) Velocity of mass A

$\cos\theta = \frac{adjacent}{hypotenuse}$

$\cos 30 = \frac{800u_A}{15600}$

$u_A = \frac{15600\ \cos 30}{800}$

u_A = 16.9m/s

Velocity of mass B

$\sin\theta = \frac{opposite}{hypotenuse}$

$\sin 30 = \frac{500u_B}{15600}$

$u_B = \frac{15600\ \sin 30}{500}$

u_B = 15.6m/s

(b)

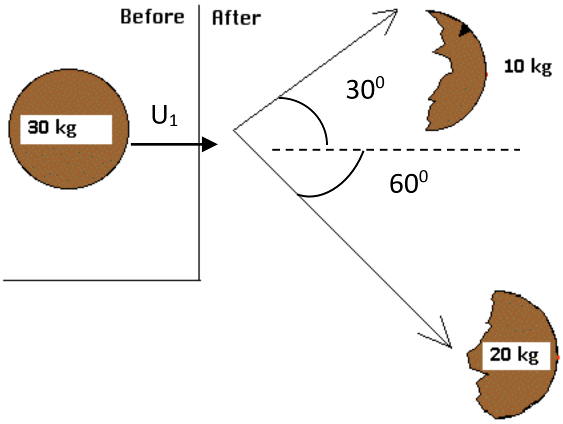
$$KE_i = \frac{1}{2}(m_1)(u_1)^2 + \frac{1}{2}(m_2)(u_2)^2$$
$$= \frac{1}{2}(800)(16.9)^2 + \frac{1}{2}(500)(15.6)^2$$
$$= 175.1kJ$$

$$KE_f = \frac{1}{2}(m_1 + m_2)v_C{}^2$$
$$= \frac{1}{2}(800 + 500)(12)^2$$
$$= 93.6kJ$$

The total energy before collision is not equal to total energy after the collision. Hence, it is an inelastic collision.

Example 3

A 30kg mass suddenly splits into two fragments. One piece (mass A) is 10kg and the other piece is 20kg. The 10kg piece travel 30° above the horizontal at 8m/s and the (mass B) 20kg piece travel 60° below the horizontal at 5m/s. Calculate the velocity of the 30kg mass before it splits.



(a) Find the momentum of the 10kg piece

P = m x v

P= 10kg x 8m/s

P = 80kgm/s and its 30° above horizontal

(b) Find the momentum of the 20kg piece

P = m x v

P = 20kg x 5m/s

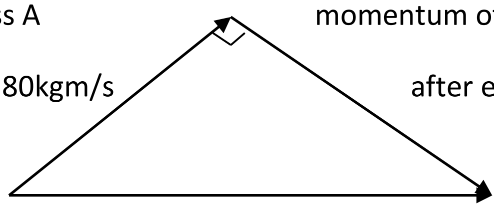
P = 100kgm/s and its 60° below the horizontal

(c) Find the velocity of the 30kg mass before it splits

Momentum before explosion = Momentum after explosion

Momentum of mass A

After explosion = 80kgm/s



momentum of mass B

after explosion = 100kgm/s

Momentum before explosion

P = $30u_1$

Use Pythagoras theorem:

$$c^2 = a^2 + b^2$$

$$(30u)^2 = 80^2 + 100^2$$

$$900u^2 = 6400 + 10000$$

$$u^2 = 16400/900$$

$$u = \sqrt{\frac{16400}{900}}$$

u = <u>4.27m/s</u>

1.8.13 EXERCISE

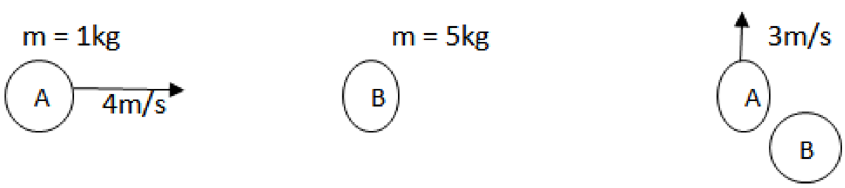
1. A truck of mass 4000kg moving at 3.5m/s collides with a stationary truck to which it becomes automatically coupled. The truck moves along together at 2m/s, find the mass of the second truck.

2. A bag of sand of mass 9kg which is suspended by a rope used as a target for a pistol bullet of mass 30g and which strikes the bag with a horizontal velocity of 301m/s. If the bullet remains embedded in the sand, determine the velocity of the bag immediately after impact.

3. A toy locomotive of mass 420g moving at 30cm/s collide with a carriage of mass 200g moving at 32cm/s in the opposite direction. If they become coupled together, what is there common speed after collision?

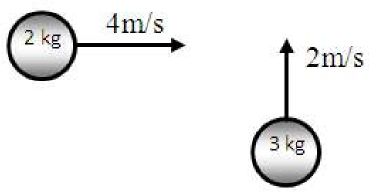
4. An explosion blows a rock in to three parts; two pieces go off at right angle to each other – a 1kg piece at 12m/s and a 2kg piece at 8m/s. The third piece flies off at 40m/s.
   (a) What is the mass of the third piece?
   (b) In what direction does the third piece fly?

5. A ball A with a mass of 1kg and moving at 4m/s strike a glancing blow on the second ball B which is initially at rest. After the collision, ball A is moving at right angle to its original direction at a speed of 3m/s as shown.



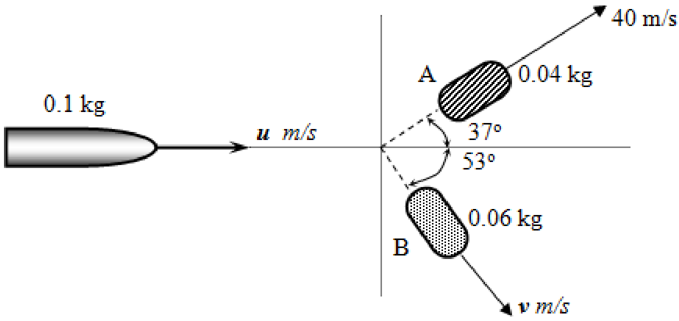
   a) What is the magnitude of the momentum of B after the collision?
   b) In what direction is B moving after the collision?
   c) If B has the mass of 5kg, what is its speed after the collision
   d) If the time of impact was 0.02s, what was the magnitude of the average force exerted on B during the collision?
   e) Was this an elastic collision? Give reason for your answer.

6. A 2kg ball traveling East at 4m/s collides with a 3kg ball traveling North at 2m/s. They stick together after collision.



a) Find the total momentum of the balls before collision.
b) What is the total momentum of the balls immediately after collision?
c) Calculate the speed of the balls immediately after collision.

7. When a firework of mass 0.1 kg reaches its highest point, it has a horizontal velocity of, μ ms^{-1}. At this point it explodes into two parts A and B as shown.



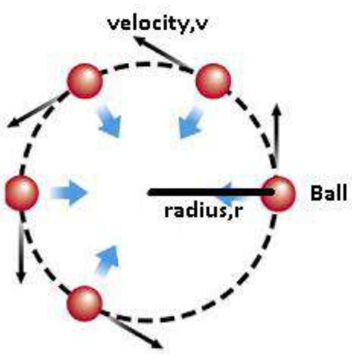
Calculate the speeds ***v*** and ***u.***

1.9 CIRCULAR MOTION

To move around once in circular motion is known as rotation. Period (T) is the time taken for one rotation or revolution.SI unit for period is the seconds. Frequency is the number of rotation made per second. The SI unit for frequency is hertz. Period and frequency are reciprocal to one another.

$$T = \frac{1}{f} \text{ and } f = \frac{1}{T}$$

A ball tied to the end of a length of string is whirled around at a constant speed in a horizontal circle.



Distance that the ball moves in one rotation is the circle circumference:

D= 2πr,   where r is the radius of the circle and π is the constant (3.14).

The speed or velocity of the object travelling in circular motion can be calculated using:

$$\boxed{v = \frac{2\pi r}{T}}$$

Where, r = radius (m)
T = period (s)
v = speed (m/s)

Example 1

The propeller blade of a single engine aircraft rotate with a frequency of 50HZ. If the blade have radius of 85cm, how fast do the tips of the propeller blades move?

Period (T) = $\frac{1}{f}$ $= \frac{1}{50}$

= 0.02 seconds.

Velocity (v) = $\frac{2\pi r}{T}$

= $\frac{2\pi\,(0.85)}{0.02}$ (85cm = 0.85m)

= **<u>267m/s</u>**.

The frequency of rotation is sometimes expressed in revolutions per minute (rpm). The frequency in rpm is 60 times greater than the frequency in hertz, because there are 60 seconds in a minute.

Example 2

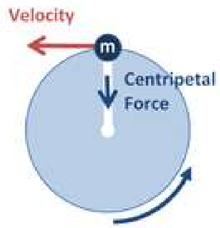
A racing car's revolution counter indicates an engine speed of 5000 rpm. Calculate this in Hertz.

5000 revolution in 60 seconds

∴ f =$\frac{5000}{60}$ = 83.3 HZ.

1.9.10 FORCE AND ACCELERATION

When a ball on a string is whirling around in a circle, various forces are acting. There is <u>tension</u> force in the string pulls the ball inward. The direction of this force is at right angles to the motion of the ball. It changes the direction of the ball but because the force is always at right angles to the direction of ball motion, the speed of the ball does not change.



The tension in the string that provides the force on the ball is called centripetal force. It causes the ball to accelerate, by changing the ball direction but not its speed.

Centripetal Force: $F_C = \frac{mv^2}{r}$ where m = mass (kg)

V = velocity

R = radius (m)

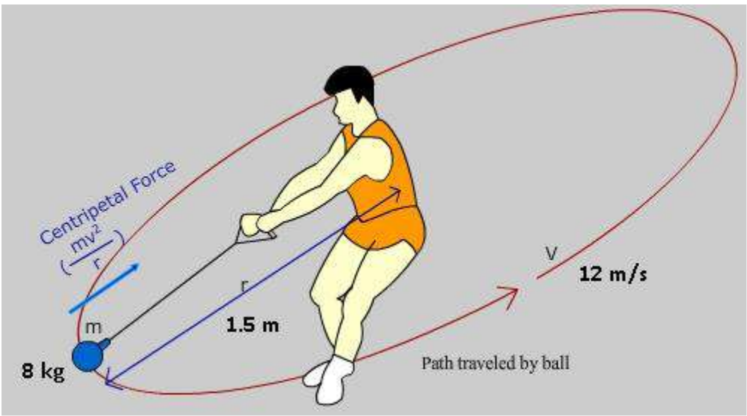
The ball continually accelerating in the direction of the centripetal force, the inwards towards the centre of the circle. Acceleration is known as centripetal acceleration.

$a_c = \frac{v^2}{r}$ where: v= velocity (m/s)

R = radius (m)

Example:

During a hammer throw, an 8.0kg steel ball is swung horizontally with a speed of 12m/s in a circle of radius 1.5m.



The force required to keep the ball moving in a circle is:

$$F = \frac{mv^2}{r}$$

$$= \frac{(8)(12)^2}{1.5}$$

= 768N inwards.

Acceleration of the steel ball is:

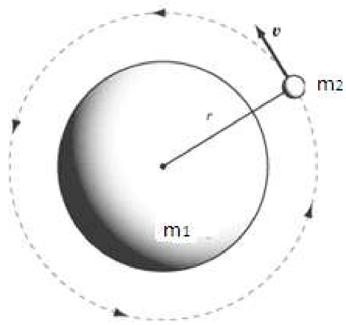
$$a_c = \frac{v^2}{r}$$

$$= \frac{(12)^2}{1.5}$$

$$= 96 \text{ m}/s^2$$

Newton’s law of gravitation

Newton's universal law of gravitational attraction states that two masses, m_1 and m_2, separated by a distance of *r*, attract each other with a force given by the following equation:



$$\boxed{F = \frac{Gm_1m_2}{r^2}}$$

$$F \alpha\, m_1 m_2$$

$$F \alpha \frac{1}{r^2}$$

Where m_1 and m_2 are masses (kg)

$r =$ Distance between masses from the center

$$G = 6.673x10^{-11}\left(N\,m^2/kg^2\right)$$

Example

1. Suppose two masses of 1 kg each are separated by 10 cm. What is the attracting force between them?

$$F = \left(6.673\times10^{-11}\right)\frac{Nm^2}{kg^2}\times\frac{(1)\,kg\times(1)\,kg}{(0.1)^2\,m^2}$$

$$= 6.673\times10^{-9}\,N$$

2. The force between two masses is $18.0N$. Determine the new force if the distance between the two masses is halved.

$$r = r/2$$

$$F = G\frac{m_1m_2}{r^2} \Rightarrow F = G\frac{m_1m_2}{\left(r/2\right)^2} \Rightarrow F = G\frac{m_1m_2}{r^2/4} \Rightarrow F = G\frac{m_1m_2}{r^2}(4)$$

❖Means the force is increased by factor 4

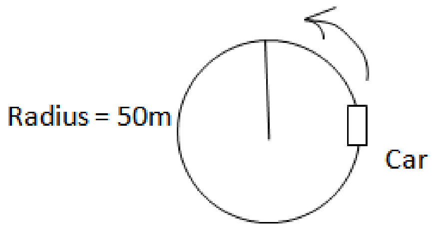
$$= 18(4) \quad = 72\text{N}$$

1.9.11 EXERCISE

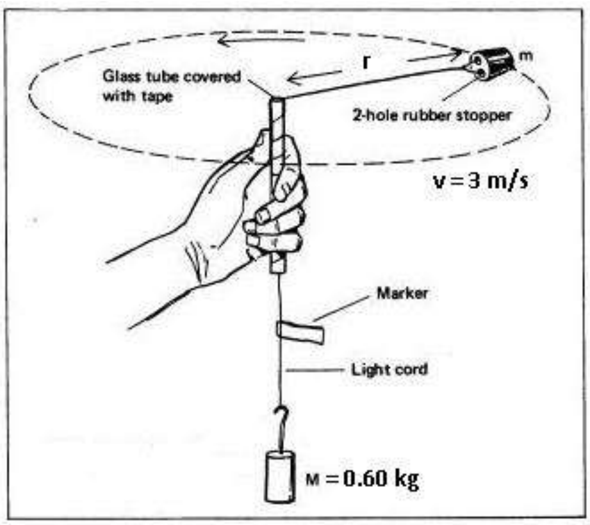
1. Calculate the acceleration of a car which travels around a circle of:
   a) 50m radius at a constant speed of 54km/hr.
   b) 18m radius at a constant speed of 12m/s.

2. A wheel of radius 20cm has a period of revolution of 0.01 seconds. Find the accelerations of a point on the rim of the wheel.

3. Calculate the speed of a car which completes a full circular lap of radius 50m in a time of 20 seconds.



4. A 0.40kg mass (m) on a frictionless horizontal table is attached to a weight of mass 0.60kg by a string passing through a smooth hole in the centre of the table. The mass (P) is moving in a circle about the hole with a uniform speed of 3m/s. Calculate the radius of the circular path.

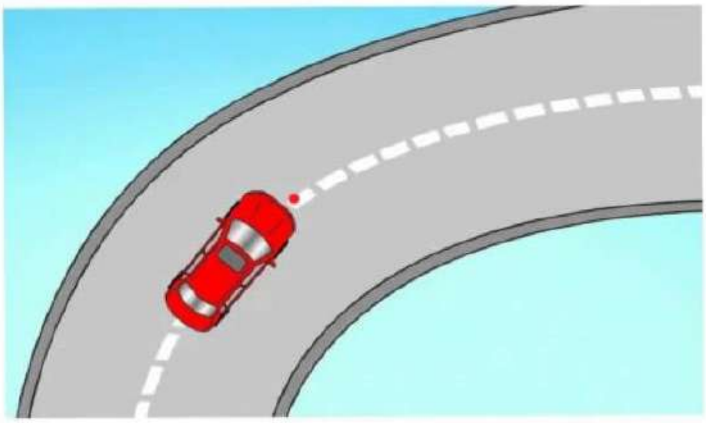


A string 5m long of diameter 2mm just supports a hanging ball without breaking.

a) If the ball is set to swinging, the string will break. Why?
b) What diameter string of the same material should be used if the ball travels 7m/s at the bottom of its swing?

5. An engine of mass $4x10^4$kg travels at a constant speed of 8.5 m/s on a level track in the form of a circular arc of radius 200m. Calculate the horizontal force exerted by the rails on the flanges of the wheel.

6. A car of mass 400kg, moves around a circular roundabout of radius 15m, with a constant speed, v. It completes half a revolution in 20 seconds.



   i. Calculate the speed of the car.
   ii. Is the car accelerating? Give a reason for your answer.
   iii. Find the force, if any, acting on the car.

7. Calculate the gravitational attraction between two spherical objects ($m_1 = 20 \times 10^3$ kg and $m_2 = 2.0 \times 10^4$ kg) separated by a distance 0f 4.0 m.

8. Find the force of attraction between the earth and the moon, using the following data:
   Mass of moon = 7.35×10^{22} kg
   Mass of earth = 5.98×10^{24} kg
   Average distance from the earth to the moon = 3.84×10^{8}m.

9. A mass of 1.5 kg moves in a circle of radius 25 cm at 2.0 Hz.
   Calculate:
   i. The velocity.
   ii. The acceleration.
   iii. The centripetal force acting on the mass.

CHAPTER 2: ENERGY

2.1 ENERGY TRANSFORMATION

2.1.10 WORK

Work is the process that transfers energy from one form to another. Amount of work done dependent on the force involve and the distance through which force act. It is defined as the product of force and distance.

Work = force × distance (force is parallel to parallel travel)

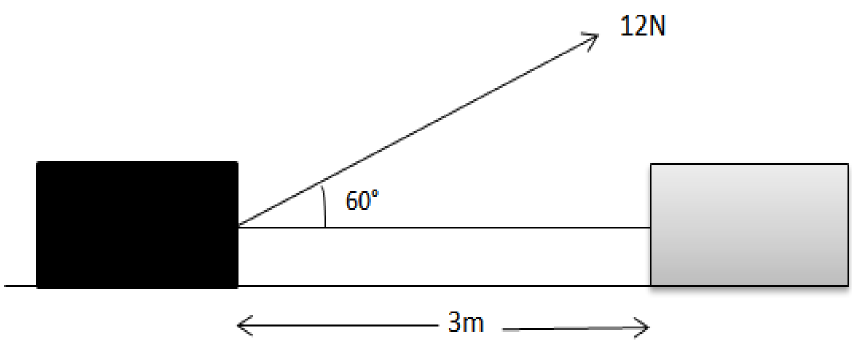
(Joules/Newton meter) (Newton) (Meter)

$$\boxed{W = F \times d}$$

Work is a scalar quantity - Although vectors are used in its calculation, the resulting work done no specification direction.

Example 1

For the diagram shown, calculate the work done in moving the body horizontal distance in 3m.



Cos θ = A/H

Cos 60 = F_x/12

F_x =12 cos 60

Work done = Force x distance.

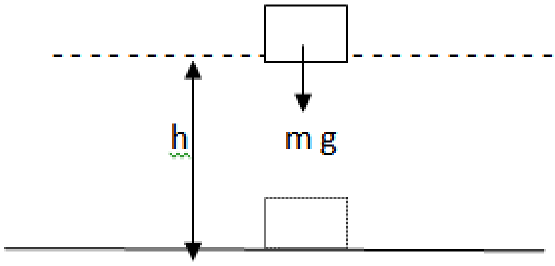
= F_x x d

= 12cos 60 x 3

= **<u>18 Joules.</u>**

2.1.11 POTENTIAL ENERGY (Joules)

Potential Energy is energy stored in lifting object of mass (m) against gravity a distance (m) from one height to a higher one. Energy has been transferred from chemical energy in a person body to gravitational potential energy (PE).



Gravitational Potential Energy = mass x gravity x height

$$\boxed{\text{P.E.} = m\,g\,h}$$

There is usually some reference level of zero potential energy. Often ground or floor level is taken to be at zero potential energy.

2.1.12 KINETIC ENERGY (Joules)

Kinetic Energy is energy a body possesses because it's moving. A body of mass (m) moving with velocity (v) has kinetic energy of

$$\text{Kinetic Energy} = \tfrac{1}{2} \text{ x mass x } velocity^2$$

$$\text{K.E.} = \tfrac{1}{2}\, m\, v^2$$

When moving object slow down and stop, their kinetic energy is transform into other forms of energy. Energy like work is a scalar quantity and the direction of motion does not matter.

Power

Power (P), is the rate at which work is done, i.e. power measures how quickly energy is transferred. Power can be calculated as a rate at which work is done.

$$\boxed{\text{Power} = \frac{Work\ done}{Time}}$$

Where P is the power (Watt), W is work done and T is time.

2.1.13 ELASTIC POTENTIAL ENERGY

Hooke's Law

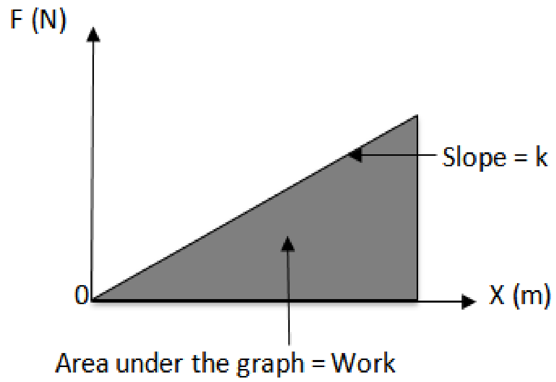
Work is done in compressing or stretching a spring. When this done, energy is stored in the spring and can be released later. This is known as elastic potential energy.

Elastic potential energy = $\frac{1}{2}$ x spring constant x $(compression\ or\ extension)^2$

$$\boxed{\text{Elastic P.E.} = \tfrac{1}{2}\,kx^2}$$

A graph of extension force against extension of spring shows a linear relationship. If F is the extending force in (N), and x is the extension in m, then

$$\boxed{F = kx}$$



Example 2

A mass of 0.5kg hung from the end of a spring extends the spring by 25cm.

a) Calculate the spring constant

F = mg
  = (0.5) (10)
  = 5N

F = k x
mg = k (0.25)   (25cm = 0.25m)
(0.5) (10) = (0.25) k
$k = \frac{5}{0.25}$
k = 20N/m

b) How much elastic potential energy is stored in the spring?

Elastic P.E. = ½ k x^2
 = ½ (20) (0.25^2)
 = 0.63 J

2.1.14 SPECIFIC HEAT

1. **Heat Energy, (H or Q):** is the total amount of energy contained by all the particles of the body.
2. **Temperature, (t):** is the degree of hotness of a body i.e. is a measure of the average kinetic energy of the particles.

Example 3

1 kg water at 50°C	10 kg water at 10°C
Higher Temperature	Lower Temperature but more heat energy because it has more particles

3. **Specific Heat Capacity, (c):** If the same quantity of heat is supplied to equal masses of different substances, different temperature changes occur.

Water 10°C	+	10 Joules	→	12°C
Metal 10°C	+	10 Joules	→	20°C

This difference in bodies is accounted for by the specific heat capacity of the body which is:

The Heat Energy required raising the temperature of 1kg of the substance by 1°C

Example 4

Specific heat capacity of water = 4200 Joules/kg°C .i.e. it requires 4200 Joules of energy to raise the temperature of 1kg of water by 1°C.

$$\mathbf{Q = m\,c\,\Delta t}$$

Where Q = Energy absorbed or liberated, m = mass of substance, c = specific heat capacity of the substance and Δt is the change in temperature

Example 5

How much energy is required to heat 100gram of water from 10°C to 15°C

m = 100g, c = 4200 J/kg°C, Δt = 15°C - 10°C

= 0.1kg = 5°C

$Q = mc\Delta t = (0.1) \times (4200) \times (5) =$ **2100 J**

2.1.15 LATENT HEAT

Latent Heat, L, of fusion or vaporization of a substance is the amount of energy to change the state of 1kg of the substance without changing its temperature.

Example 6

Latent heat of fusion of ice = 336000 Joules/kg i.e. it requires 336000 Joules of energy to melt 1kg of ice.

Remember that there is no temperature change during the changes of state, and that the energy absorbed or liberated is therefore Potential Energy.

Example 7

Find the amount of energy released when 1gram of steam at 100°C condenses and cools to 20°C.

Energy = steam $\longrightarrow$ water + water cools

At 100°C 100 °C $\longrightarrow$ 20°C

Energy = m L + mcΔt

Energy = (0.001x250000) + (0.001x4200x80)

Energy = 2250 + 336

= **<u>2586 Joules</u>**

2.1.16 CONSERVATION OF ENERGY

In all situations where work is done, energy is transferred from one form to another. Sometimes the energy is transferred into several forms of energy, such as heat, sound and light. This means that energy can be created but cannot be destroyed or lost, can only be conserved to other forms of energy.

Example 8

A body is dropped from a height of 15m.What is its velocity as it reaches the ground.

Conservation of Energy

Potential Energy = Kinetic Energy

P.E = K.E

mgh = $\frac{1}{2}mv^2$

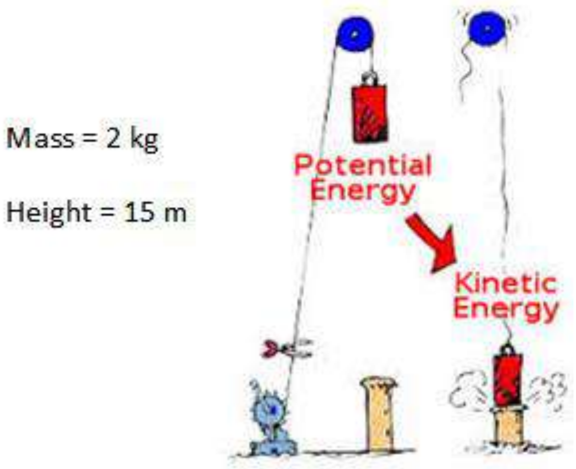
2gh = v^2

v = $\sqrt{2gh}$

v = $\sqrt{2x10x15}$

v = $\sqrt{300}$

v = <u>17.32m/s</u>

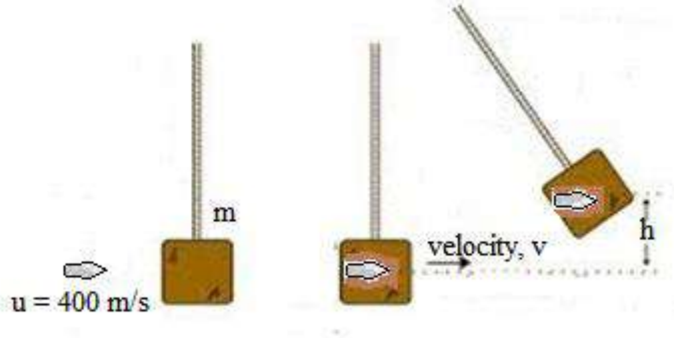


Example 9

A bullet of mass 30g is fired with a speed of 400m/s in to a sandbag. The sandbag has a mass of 10kg and is suspended by two ropes so that it can swing. What is the maximum vertical height, h, that the sandbag rises as it recoils with the bullet embedded inside?

Mass of bullet = 30g

u = 400 m/s



Solution: Speed of the sandbag and bullet immediately after the impact can be found by applying the idea that momentum is conserved when the bullet embeds itself in the sandbag.

Momentum of the bullet = Momentum of the bullet

Before collision after collision

$$m_1u_1 + m_2u_2 = (m_1 + m_2)\,v$$

$$(0.03 \times 400) + (10 \times 0) = (0.03 + 10)\,v$$

$$12 = 10.03v$$

$$v = \frac{12}{10.03}$$

v = <u>1.2m/s</u>

After the collision, the sandbag swings upward by a height (h), and the kinetic energy of the sandbag immediately after the collision was transferred to gravitational potential energy. At the maximum height of the swing, all the kinetic energy has been transferred to gravitational potential energy

Loss in kinetic energy = gain in gravitational potential energy

$$\frac{1}{2}mv^2 = mgh$$

$$\frac{1}{2} \times (10.03) \times 1.2^2 = 10.03 \times 10 \times h$$

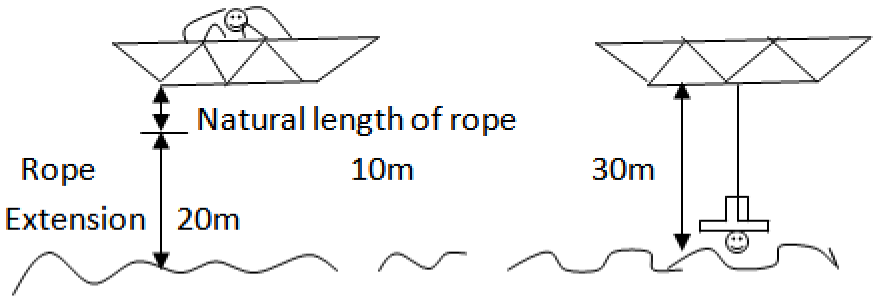
$$h = \frac{7.22}{100}$$

h = <u>0.072m</u>

I.e. the sandbag raises a vertical height of 7.2cm.

Example 10

A bunging jumper of mass 75kg jumps off a bridge over a river. The spring constant of the rubber bands that make up the bunging rope is adjusted so that the jumper head just touches the river at maximum stretch (30m) before springing back up in to the air. If the natural length of the rope is 10m, calculate the rope spring constant. Assume no energy is lost due to friction.



https://www.google.com/?gws_rd=ssl#q=bunging+jumper+images

Solution:

Conservation of energy, all the jumpers original gravitational potential energy above the river level is converted into elastic potential energy when the jumpers reached the river level.

P.E = mgh

= 75 x 10 x 30 = 22500 J

At the river level, the ropes extension from its natural length is:

x = Stretched length – natural length

x = 30 – 10

x = 20m

Potential Energy = Elastic Potential Energy

22500 J = ½ k (x^2)

22500 = ½ k (20^2)

22500 = ½ k (400)

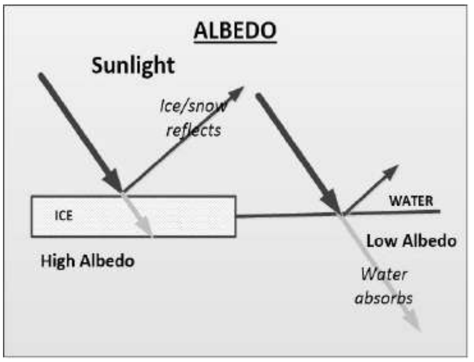
22500 = 200 k

k = $\frac{22500}{200}$

k = 112.5N/m

k = <u>110 N/m</u> (2 significant figures)

ALBEDO



When sunlight reaches the Earth's surface, some of it is absorbed and some is reflected. *The relative amount (ratio) of light that a surface reflects compared to the total sunlight that falls on it is called albedo*. In other words, Albedo is the fraction of solar energy (shortwave radiation) reflected from the Earth back into space. Surfaces that reflect a lot of the light falling on them are bright, and they have a high albedo. Surfaces that don't reflect much light are dark, and they have a low albedo.

Snow has a high albedo, and forests have a low albedo. Water is much more absorbent and less reflective. So, if there is a lot of water, more solar radiation is absorbed by the ocean than when ice dominates.

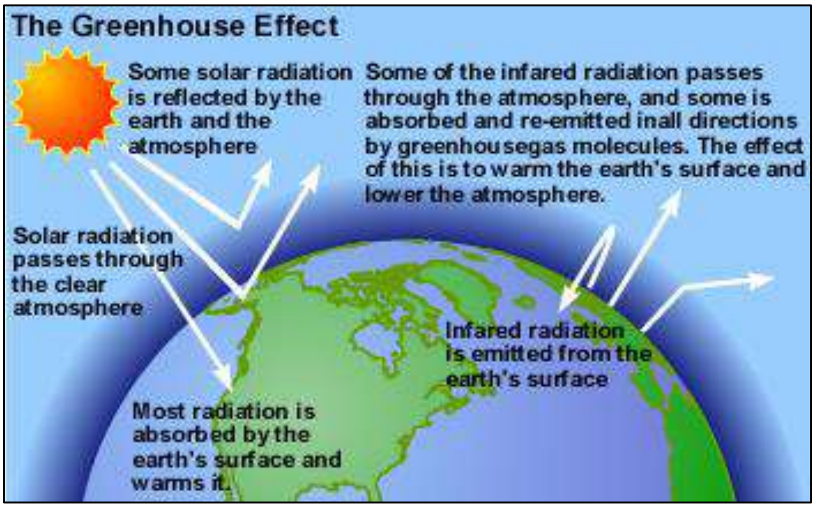
GREENHOUSE EFFECT

The greenhouse effect is the natural process by which the atmosphere traps some of the Sun's energy, warming the Earth enough to support life. Without the greenhouse effect, the earth would be much cooler than it is now and life would be difficult. However, too much greenhouse warming could raise global temperatures to a level that is significantly different than the current climate.

Joseph Fourier: The greenhouse effect is somewhat similar to the process that goes on in a real greenhouse. The original concept of the greenhouse effect dates back to 1824 with Joseph Fourier. The glass of a greenhouse allows the sun's radiation in, which warms the ground inside, which in turn, warms the air above the ground by long-wave (heat) radiation. The glass then acts like a barrier to keep the warm air inside from mixing with the cooler air outside the greenhouse.

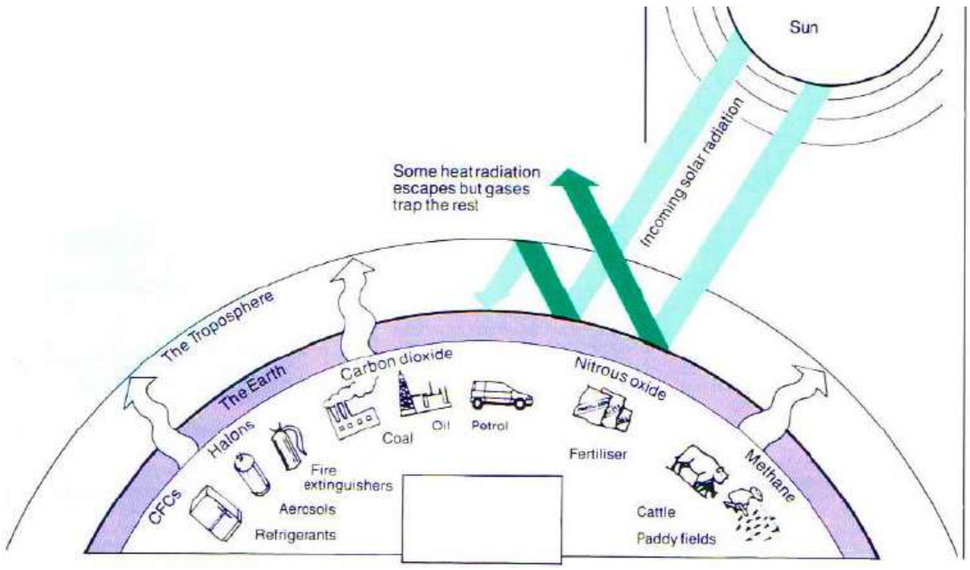
The greenhouse gases in the atmosphere allow the sun's short wavelength radiation in, and because of the chemical properties of the gases, they do not interact with sunlight. But they do absorb the long-wave radiation from the earth and emit it back into the atmosphere, different from a greenhouse which does not allow the long-wave radiation to escape through the glass. The increase in trapped energy leads to higher temperatures at the earth's surface. This has caused some people to rename the process 'the atmospheric greenhouse effect' or just 'the greenhouse effect'.

The most abundant greenhouse gases responsible for the greenhouse effect in the atmosphere are water vapour, carbon dioxide, methane, nitrous oxide, and ozone.



The greenhouse effect works like this: First, the sun's energy enters the top of the atmosphere as shortwave radiation and makes its way down to the ground without reacting with the greenhouse gases. Then the ground, clouds, and other earthly surfaces absorb this energy and release it back towards space as long-wave radiation. As the long-wave radiation goes up into the atmosphere, it is absorbed by the greenhouse gases. The greenhouse gases then emit their radiation (also long-wave), which will often keep being absorbed and emitted by various surfaces, even other greenhouse gases, until it eventually leaves the atmosphere. Since some of the re-emitted radiation goes back towards the surface of the earth, it warms up more than it would if no greenhouse gases were present.

What cause the greenhouse effect?



https://www.powercor.com.au/media/1265/fact-sheet-the-greenhouse-effect.pdf

Consequences of greenhouse-effect

The release of GHGs and their increasing concentration in the atmosphere are already having an impact on the environment, human health and the economy. These impacts are expected to become more severe, unless concerted efforts to reduce emissions are undertaken.

Consequences of

Environmental Impacts

- Overall average annual temperature is expected to increase.
- Rising sea levels and increased in coastal flooding.
- Heat waves are likely to increase in frequency.

Economic Impacts

- Agriculture, forestry and tourism affected.
- Damage to infrastructure (e.g., roads and bridges).
- Additional economic stress on health and social support systems.

Human health impacts

- Increase the risk of deaths from dehydration and heat stroke.
- Risk of water and food.
- Greater risk of respiratory and cardiovascular problems.

Possible solutions

There are various options available to us to address climate change. Our responses to climate change can be broadly split into three categories: adaptation, mitigation, and geo-engineering.

Possible solutions

Adaptation

Means responding to the negative impacts of climate change. An adaptive response to this impact would be to build sea walls or re-locate communities.

Mitigation

Refer to policies that avoid climate change in the first place. This is accomplished by reducing emissions of greenhouse gases, usually through policies.

Geo-engineering

Refers to active manipulation of the climate system. Under this approach, our society would continue adding greenhouse gases to the atmosphere, but we would intentionally change some other aspect of the climate in order to cancel the warming effects of the greenhouse gases.

Absorption Graph of the Atmosphere

Generally, the sun emits UV, Visible, and IR and the earth's atmosphere absorbs UV and IR while Visible goes through to the earth's surface.

The surface of the earth absorbs Visible and re-radiates it as IR which is absorbed by the atmosphere trapping it like greenhouses. Figures 1 and 2 will illustrate the solar spectrum and atmospheric absorption graph.

Figure 1 below shows the energy spectrum for our Sun along with the percent of energy radiated by the Sun in the ultraviolet (UV), visible. And infrared portions of electromagnetic spectrum.

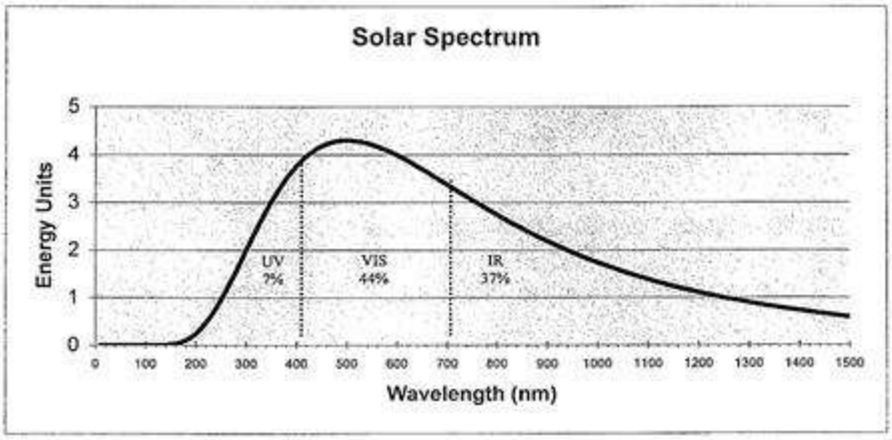


Figure 1

Source: Wikipedia and
http://faculty.otterbein.edu/utrittmann/IS2403_Activities/ActivityGreenhouse.pdf

The temperature of the surface of our planet is affected primarily by the energy we receive from the Sun that is able to reach Earth's surface. Figure 2 below shows that certain wavelengths of light are absorbed in our atmosphere before they can travel all the way to the surface. The dashed lines delimit the visible wavelength.

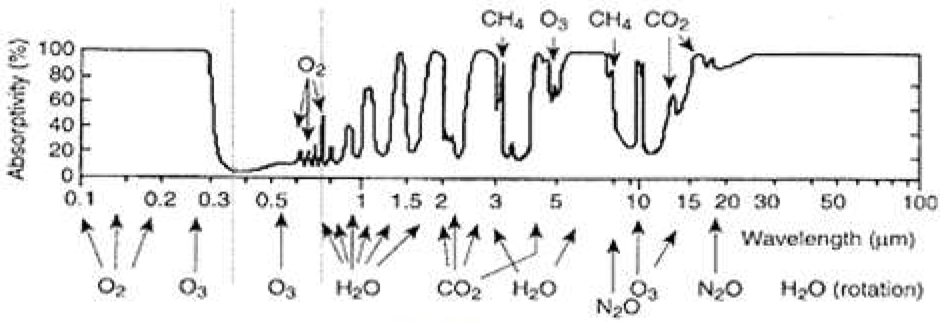


Figure 2

Source: Wikipedia and
http://faculty.otterbein.edu/utrittmann/IS2403_Activities/ActivityGreenhouse.pdf

2.1.17 EXERCISE

Use specific heat capacity of water as 4.20 x 10^3Joules/kg°C

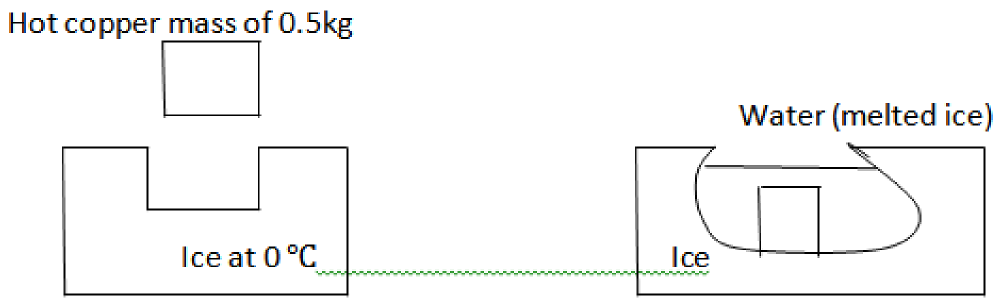
1. A lump of iron of specific heat capacity 5.03 x 10^2 J/kg°C falls from a height of 220metres. If all the energy it acquires in falling is used to heat it, find its rise in temperature.

2. If the energy possessed by a body of mass 50kg moving at a speed of 30m/s is used to heat 0.6kg of water, what would be the rise in temperature?

3. After sliding down a vertical pole a distance of 10 meters, a man of mass 70kg has a speed of 2 m/s.
   a) How much energy is converted into heat as man slide 10m down the pole?

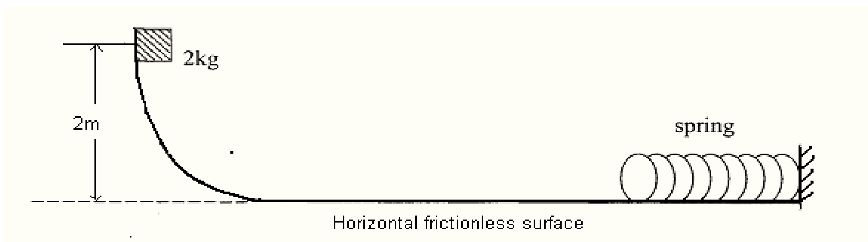
b) If this energy could be used to heat 100 gram of water, what would be the rise in temperature of water?

4. A lump of copper of mass 0.5kg is placed in an oven for some time and then transferred to a large dry block of ice at 0°C.



5. When the temperature of the copper mass has cooled to 0, it is found that 0.3kg of ice has melted. The specific heat capacity of copper is 400 J/kg.K and the heat of fusion is 320 kJ/kg. Calculate the temperature of the hot copper mass.

6. A 2 kg mass slide from a height of 2 m on a frictionless surface and compresses a spring of force constant 100 N/m on a horizontal plane.



   i. What is the speed of the mass just before it compresses the spring?
   ii. How far will the spring be compressed when the mass comes to rest?

7. A spring stretches 10cm when a force of 50N is applied to it.
   i. State **Hooke's law**.
   ii. Calculate the value of the spring constant, *k*, of the spring.
   iii. What will be the extension of the spring when a force of 200N is applied to it?
   iv. What amount of elastic potential energy is stored in the spring when stretched 10cm?

8. Discuss the possible causes and consequence of Greenhouse effect
9. Define the term ALBEDO.
10. Describe evidences of global warming and what are the solutions to this?

2.2 HEAT ENERGY

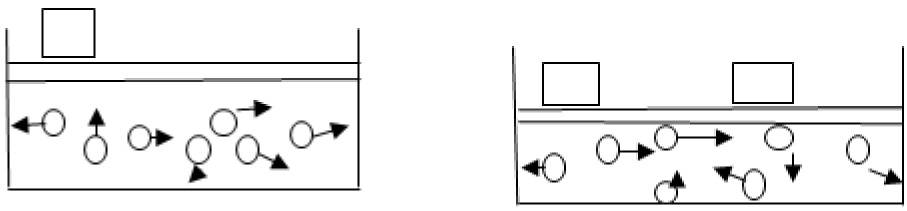
2.2.10 KINETIC THEORY OF GASES

This is a model of ideal or perfect gas based on experimental observations of real gases. Below are set of observations on behaviour of gases.

1. Gases consist of small particles in constant random motion, which are continually colliding with each other and the wall of the container. Particles of gas move in irregular path.
2. The size of the particle is negligible in comparison with the total volume of gas. It has large spaces between particles that make it easily compressible.
3. There is no force or interactions between the particles. Therefore gas has no fixed volume or shape but occupies the entire volume of its container.
4. All collisions of the particles amongst themselves and with the wall of the container are elastic collision. If they were not then energy would be lost; soon all motion would ceases and the particle would settle at the bottom of the container. This was not observed to happen. The collision of the particles with the wall of the container is responsible for the pressure set up by the gas.
5. The average Kinetic Energy of the particles is proportional to the temperature. Increases the temperature resulted in more movements of the particles.

2.2.11 BOYLE'S LAW

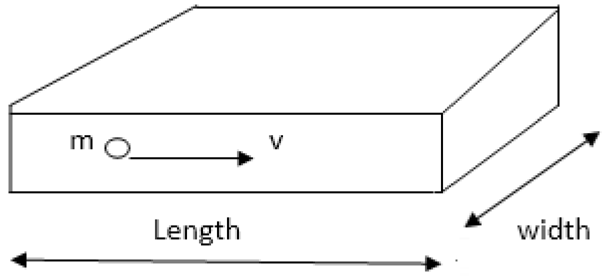
Temperature is constant; hence the average kinetic energy of the molecule is constant.



i.e., the average speed of the molecules remains constant. Pressure experienced by a gas inside cylinder depends only on the number of molecules collision per unit area of wall per second. When the volume is reduced, particles of gas collide with a wall more frequently, therefore pressure increase.

2.2.12 PRESSURE OF IDEAL GAS

Pressure is the force of particles of gas per unit. Pressure is caused by the particles bouncing off the walls. Let the container have N molecules and at any instant assume that 1/3 of these are moving in each of the three directions (length, width, and height).



Assume all molecules have speed (v). Consider particles moving length- ways, times between a particle colliding with a wall = $(distance)/velocity$ = $2l/v$
Change in momentum for each collision:

$\Delta p = p_f - p_i$
= (m) (-v) - mv
= -mv-mv
= -2mv (indicate direction)
= 2mv

∴ Force exerted by 1 molecule on the wall:

= $\frac{change\ in\ momentum}{time}$

= $\frac{2mv}{2\,l/v}$ = $\frac{mv^2}{l}$

Since we have 1/3N molecules colliding with the wall then the:
Total force = 1/3N $^{mv^2}/_{l*h*w}$

∴ Pressure on this wall = $\frac{total\ force}{area}$

P = $\frac{1}{3}$ N $^{mv^2}/_{l*h*w}$

P= $\frac{1}{3}\frac{Nmv^2}{l^3}$

∴ $p_v = \frac{1}{3}Nmv^2$

To correct for the assumption that the entire molecule had the same speed we use the average velocity ($\bar{v}$). Therefore v^2 is called the **root- mean – square speed** in short.

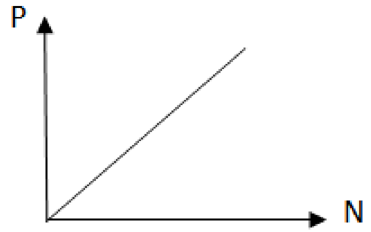
$P_V = \frac{1}{3}N\,m\bar{v}^2$ r m s speed $v_{rms} = \sqrt{v^2}$

$= \frac{2}{3}\left(\frac{1}{2}Nm\bar{v}^2\right)$ Nm = total mass of the gas

$= \frac{2}{3}\left(\frac{1}{2}M\bar{v}^2\right)$

$= \frac{2}{3}(K.E)$

- Pressure depends on number of molecules of the gas present. ($P \propto N$)



But by the gas equation we know that:

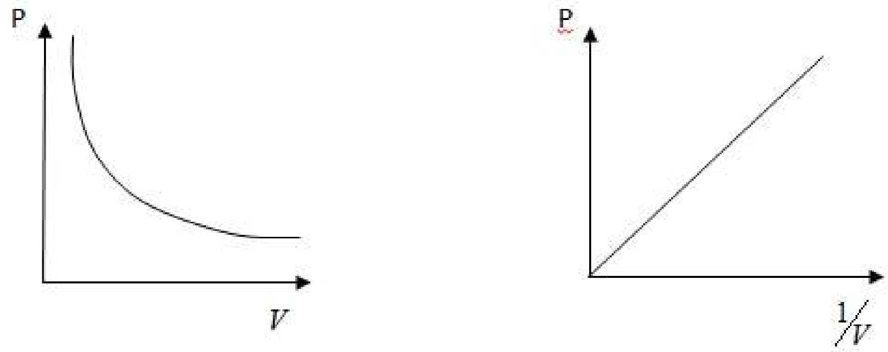
PV = constant x T

$= \frac{2}{3}$ (K.E)

$\therefore$ T$\propto K.E$

I.e. we have shown that the temperature is proportional to the average Kinetic Energy of the molecules.

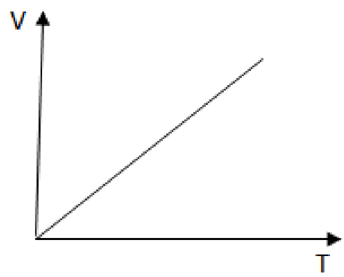
If temperature is constants then kinetic energy is also constant, and so PV= constant, which obeys Boyle's Law



2.1.13 CHARLES LAW

An increase in the temperature means a rise in the average speed.

Hence a rise in the number collision per unit area per second._To keep pressure constant we must therefore increase the volume so that the faster moving particles have greater distance, so rate of collision does not change then the pressure must increase if the temperature increases.



Volume is proportional to temperature at constant pressure ($V \propto T$ let P constant)

2.1.14 IDEAL GAS LAW

When all laws are combined together it gives gas law

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

The absolute pressure (P) of an ideal gas is proportional to the Kelvin temperature (T) and the number of moles (n) of the gas and inversely proportional to the volume (V) of the gas:

Example 1

PV = nRT where R = universal gas constant 8.31J, n= number of moles.

A certain mass of gas occupies a volume of 350cm^3 at a pressure of 76cm of mercury. What will be its volume when the pressure is reducing to 73cm?

$\frac{P1\,V1}{T1} = \frac{P2\,V2}{T2}$

$P1\,V1 = P2\,V2$

(76) (350) = (73) $V2$

$V2 = \frac{(76)(350)}{73}$

$V2$ = 364.4 cm^3

2.1.15 EXERCISE

1. 100cm^3 of air at atmospheric pressure ($10^5\ N/m^2$) is contained in a syringe. If the volume is changed at a constant temperature what is the new pressure when the volume is:
   a) 45cm^3
   b) 25cm^3

2. If the piston of a cylinder containing 360cm^3 of gas at atmospheric pressure ($10^5 N/m^2$) moves outwards so that the pressure falls to 8$\times\ 10^4 N/m^2$. Find the volume of the gas if the temperature remains constant.

3. A certain mass of oxygen has a volume of 5m^3 at 27℃. If the pressures remain constant, what will be its volume at 77℃ .

4. At the end of a trip, a driver adjusts the absolute temperature in her tires to be $2.81\times 10^5\ Pa$. When the outdoor temperature is 284K. At the end of her trip she measures the pressure to be $3.10\times\ 10^5\ Pa$. Ignoring the expansion of the tires, find the air temperature inside the tires at the end of the trip.

5. In a portable oxygen system, the oxygen (O_2) is contained in the cylinder whose volume is $0.0028 cm^3$. A full cylinder has an absolute pressure of $1.5\times 10^5 Pa$ when the temperature is 296K.
Find the mass (in kg) in the cylinder.

6. A car tire, of volume $250 cm^3$ is filled to an absolute pressure of 280kPa at 270℃. After driving some distance, the temperature of the air inside the tire rises to 57℃. Assume that the pressure inside the tire remains the same. What is the new volume of the tire?

7. Neon gas in a container was heated from 20 °C to 120 °C. Its new volume is 150ml. What was the original volume?

8. 17˚C a gas occupies a volume of 0.5 litres. The gas is then heated to a temperature of 27˚C.
   i. Calculate the new volume of the gas.
   ii. Sketch a suitable graph showing the relationship between volume and temperature (˚C) of a gas whose pressure is unchanged.

9. A certain mass of oxygen of $5m^3$ at 27℃. If the pressure remains constant, what will be its volume at 77℃.

CHAPTER 3: FLUIDS

Fluid Mechanics is the study of fluids either in motion (FLUID DYNAMICS) or at rest (FLUID STATICS). Fluids refer to substances that deforms continuously under the action of shear stress. Fluids are either gas or liquid. Solids are NOT fluids. Solids can resist a shear stress, fluids can't.

3.1 PROPERTIES OF FLUIDS

Bernoulli's principle
In 1738, the Swiss mathematician Daniel Bernoulli made a surprising discovery. It has become known as BERNOULLI'S PRINCIPLE.

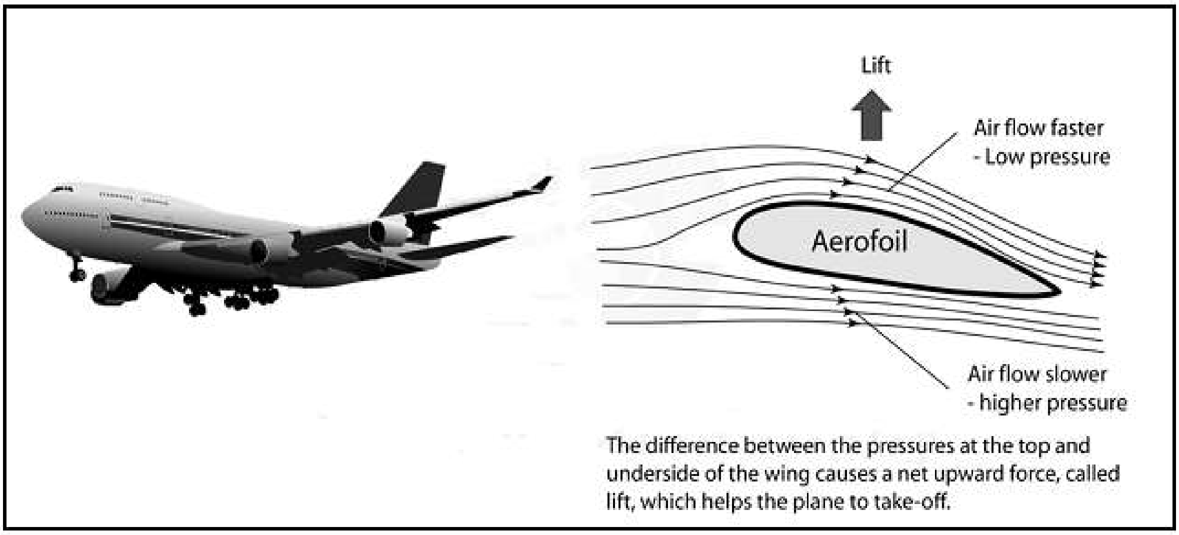
Bernoulli found that as the speed of a gas or liquid increases, its pressure drops. This means that air rushing over a surface, for example, pushes against the surface less than if the air were still.

" The Bernoulli principle states that **the pressure in a fast moving stream of air is lower than in a slower stream of air.**''
That is, fast air will produce low pressure and slow moving air will produce high pressure.

3.1.10 APPLICATIONS

I. Aeroplane wings

An aircraft wing, called an *aerofoil*, is shaped so that the air has to travel farther and so faster over the top surface than underneath



http://spmphysics.onlinetuition.com.my/2013/06/application-of-bernoullis-principle.html

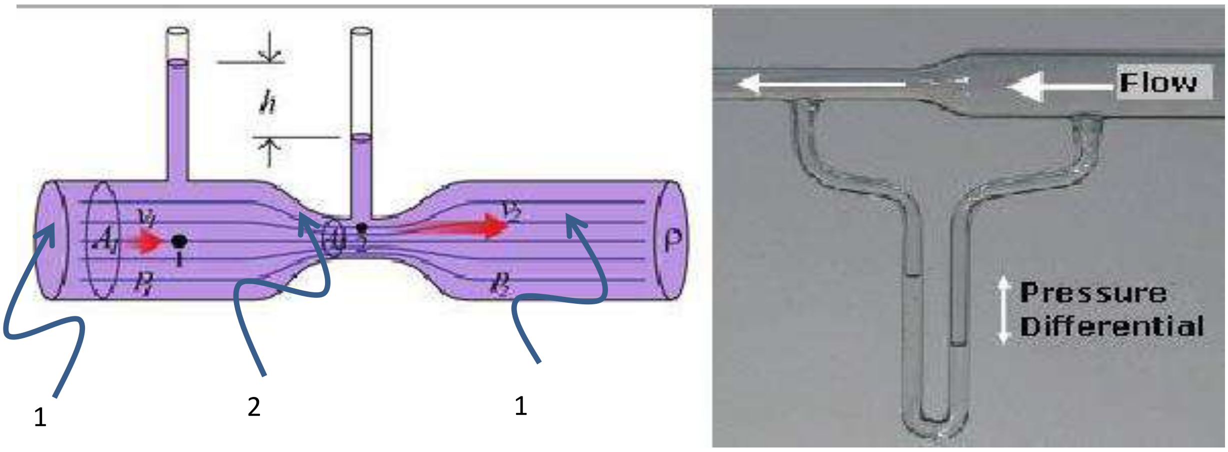
1. When a wing in the form of an aerofoil moves in air, the flow of air over the top travels faster and creates a region of low pressure. The flow of air below the wing is slower resulting in a region of higher pressure.

2. The difference between the pressures at the top and underside of the wing causes a net upward force, called **lift**, which helps the plane to take-off.

II. Venturi meter

Venturi tube, an instrument for measuring the drop in pressure that takes place as the velocity of a fluid increases. It consists of a glass tube with an inward-sloping area in the middle, and manometers, devices for measuring pressure, at three places: the entrance, the point of constriction, and the exit. The Venturi meter provided a consistent means of demonstrating Bernoulli's principle.

A flow of air through a venturi meter, showing the columns connected in a U-shape (a manometer)and partially filled with water. The meter is "read" as a differential pressure head in cm or inches of water.

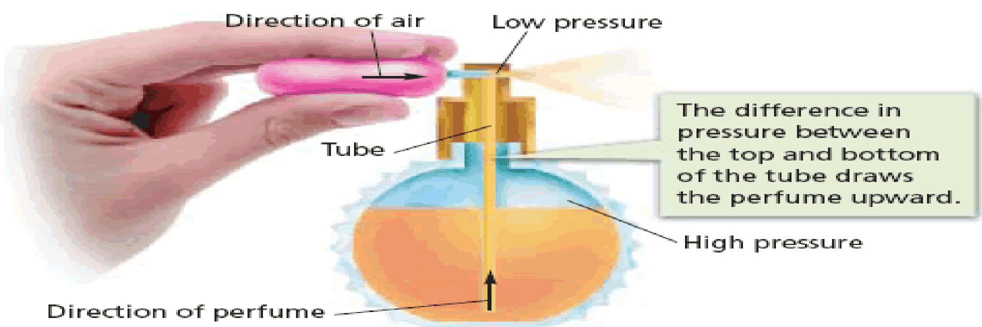


https://www.scribd.com/doc/104586852/Bernoulli-Theorem

The pressure at "1" is higher than at "2" because the fluid speed at "1" is lower than at "2".

III. Atomizer

Bernoulli’s principle can help you understand how the perfume atomizer shown in the figure below works.

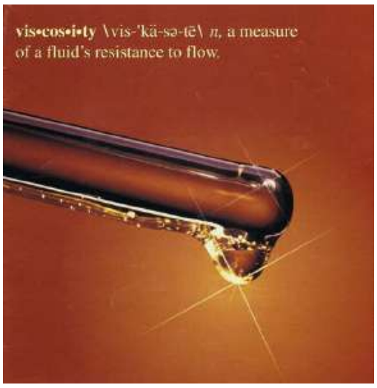


Perfume Atomizer An atomizer is an application of Bernoulli’s principle.

http://mycampus.nationalhighschool.com/doc/sc/Physical%20Science/iText/products/0-13-190327-6/ch11/ch11_s4_2.html

When you squeeze the rubber bulb, air moves quickly past the top of the tube. The moving air lowers the pressure at the top of the tube. The greater pressure in the flask pushes the liquid up into the tube. The air stream breaks the liquid into small drops, and the liquid comes out as a fine mist.

3.1.11 VISCOSITY



http://www.syntheticlubricants.ca/english/view.asp?x=968

Every type of fluid possesses differing amounts of resistances against deformation. The measure of that resistance is called viscosity. Viscosity expresses the fluid's resistance against either tensional stress, or shear stress.

In common terms, viscosity is how "thick" a fluid is. For example, we can easily move through air. It is more difficult to move through water which is 50 times higher viscosity than air. In other words, the thicker the fluid is, the higher the viscosity.

Viscosity can be viewed in two different ways. The first is a fluid's tendency to flow as is visually indicated. One can think of this as the time it takes to watch a fluid pour out of a container. The term used to describe this is **Kinematic Viscosity** and it is expressed in units indicating flow volume over a period of time.

Dynamic viscosity, also called absolute viscosity, is the more commonly used measurement. It measures the resistance of a fluid to flow — in other words, the internal friction of the fluid, or how easily it can deform under mechanical stress at a given temperature and pressure.

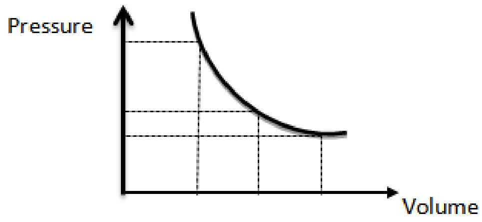
Dynamic and kinematic viscosity is expressed in different units of measurement. The International System of Units (SI) measurement units for dynamic viscosity are **pascal-seconds**. Pascal's are a measurement of pressure — in this case, the shear stress applied to the liquid — while seconds measure the time it takes to deform. Dynamic viscosity can also be measured with a unit called the **poise**, another measure relating pressure versus time. The common unit used to measure kinematic viscosity is the **stokes**, or **square centimetres per second**, although sometimes the SI unit of square meters per second is used.

3.2 STATICS FLUIDS

3.2.10 BOYLE'S LAW

The relationship between volume and pressure when temperature is held constant shows an *inverse* relationship.

The graph shows an *inverse relationship* between volume, V and pressure, P.



This is known as Boyle's Law and can be written:

$$P \alpha \frac{1}{V} \quad \text{(Temperature constant)}$$

Mathematically, when solving problems, the relationship is written:

$$\mathbf{P_1V_1 = P_2\,V_2}$$

Where P_1:is the initial pressure, V_1:is the initial volume
P_2:is the final pressure, V_2:is the final volume

Example: *An amount of air at atmospheric pressure (10^5 Pa) is contained inside a bicycle pump with its nozzle sealed. The volume of the air is 12 cm^3. What is the pressure of the air inside the pump if the handle is pulled out to a volume of 24 cm^3?*

***Solutio*n**: Initially the pressure (P_1) is 10^5 Pa and the volume, V_1 is 12 cm^3. After the handle is pulled out, the new volume, V_2, is 24 cm^3. Calculate the pressure required, P_2.

$$P_1V_1 = P_2\,V_2$$

$$10^5 \times 12 = P_2 \times 2 \quad \text{[substituting]}$$

$$P_2 = \frac{10^5 \times 12}{24} \quad \text{[rearranging]}$$

$$= \underline{\mathbf{5.0 \times 10^4\ Pa}}$$

3.2.11 GAUGE PRESSURE AND ATMOSPHERIC PRESSURE

Atmospheric pressure is the force per unit area exerted by the weight of the column of air above a measuring point. Atmospheric pressure is around 1 atmosphere at sea level but its value fluctuates. Pressure can be expressed in various units. The unit factors for converting one sets of unit to another are shown below:

1atm = 1.01325×10^5 Pa
1atm = 760 mmHg
1atm = 1.01325 Bar

Gauge pressure is the pressure measured by a gauge. This device measures the pressure relative to the atmospheric pressure i.e. it is calibrated against the atmospheric pressure.
Total (or Absolute Pressure) = $P_{GAUGE} + P_{ATMOSPHERE}$

For Pressure in a uniform fluid:
The **gauge pressure** at a particular depth is directly proportional to …

- the density of the fluid ρ,
- the acceleration due to gravity ***g***, and
- the depth ***h***.

Thus, the absolute pressure in a uniform fluid at a particular depth is given by …

$$P_{TOTAL} = P_{ATMOSPHERE} + \rho gh$$

Example 1
A diver in the ocean measures gauge pressure to be 515 kPa. What is the absolute pressure?

$P = P_A + P_G$
$P = 101 \text{ kPa} + 515 \text{ kPa}$
$P = \underline{\mathbf{616 \text{ kPa}}}$

Example 2
What is the absolute pressure at the bottom of the swimming pool whose depth is 2 m?

$P = P_A + P_G$
$P = P_A + \rho gh$
$P = 101300 \text{ Pa} + (1000 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(2 \text{ m})$
$P = \underline{\mathbf{120900 \text{ Pa} = 1.209 \times 10^5 \text{ Pa.}}}$

EXERCISE

1. The hydrogen balloons which are used to collect weather information from the atmosphere is made of plastic and never completely filled. Thus the pressure inside and outside are same. The balloon is filled with 150 litres of hydrogen, the air temperature is 27°C and the atmospheric pressure is 98 kPa. The balloon rises to a height where it radios back that the pressure is 30kPa and the temperature is -33°C.

   i. What is the Kelvin temperature equivalent to -33°C?
   ii. What is the volume of hydrogen at this height?

2. State Bernoulli's Principle.

3. Draw the flow of air over this wing. Label where there is low pressure and high pressure caused by Bernoulli's Principle.

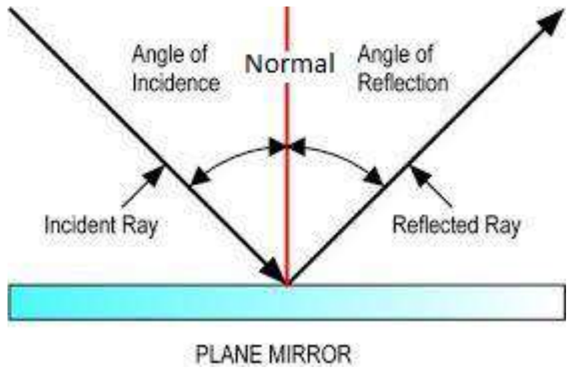
CHAPTER 4: GEOMETRICAL OPTICS AND WAVE MOTION

4.1 LIGHT

Light is an electromagnetic wave or electromagnetic radiation that travels near a speed of 3×10^8 m/s. Other forms of electromagnetic radiation is x-rays, gamma rays, short and long radio waves, light travels in a straight lines sometimes referred as straight line propagation.

4.1.10 REFLECTIONS OF LIGHT

When light reaches mirror or surfaced that is well polished it get reflected.



Laws of reflection

1. Angle of incidence = the angle of reflection.

$$\theta_1 = \theta_2 \quad \text{or} \quad \hat{i} = \hat{r}$$

2. Incident ray, reflected rays and normal all lie in the same plane.
3. Normal is the line perpendicular to the plane and to the surface at the point where incident and reflected ray meet.
4. Angle of incident and angle of reflection is always measured towards the normal.

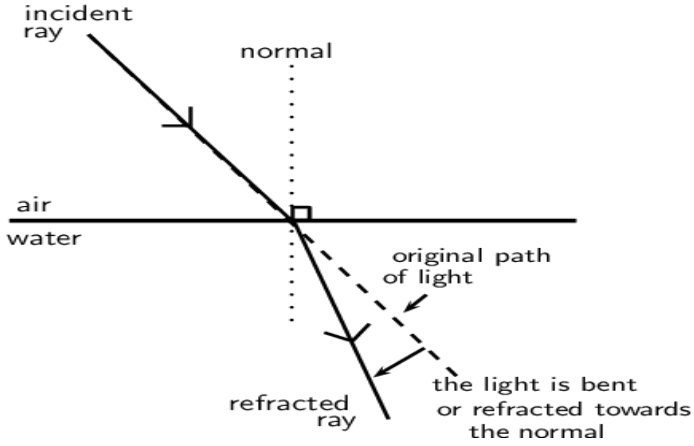
4.1.11 REFRACTION

The bending of light as it passes from one medium to another. The cause of light bending or changed direction is the change of its velocity.

Laws of Refraction:

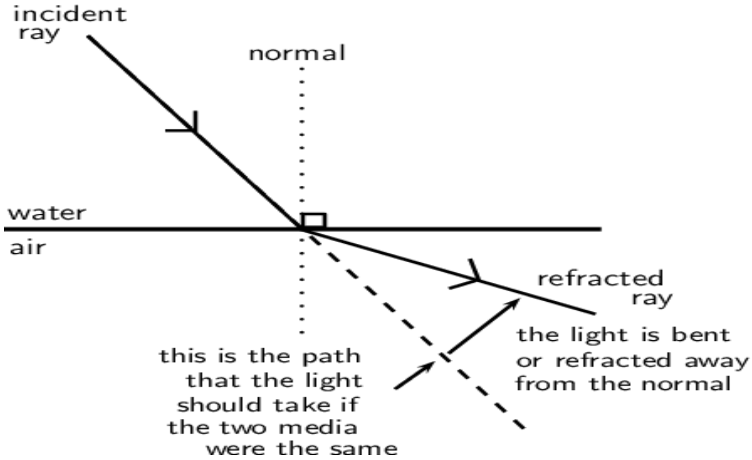
1. The incident ray, refracted ray and the planar are co-planar.
2. Angle of incident and refracted angle are measured towards the normal.

   When light travels from less dense material to more dense the refracted ray bend towards the normal

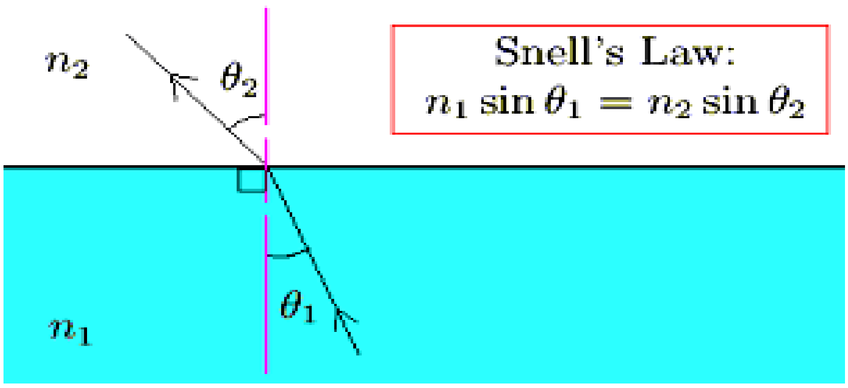


Source: http://qap.everythingmaths.co.za/science/grade-11/05-geometrical-optics/05-geometrical-optics-07.cnxmlplus

When light travels from more dense to less dense the refracted ray bend away from normal.



Snell's law determines the amount that light is bending or refraction.



Example:

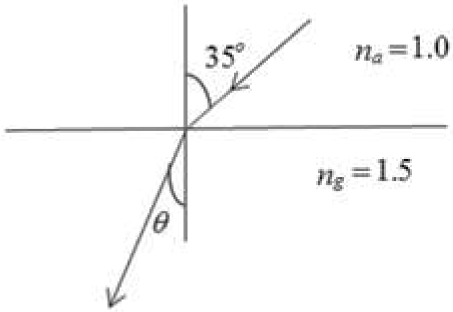
Calculate the angle of refraction for light travelling from air (n_a=1.0) into glass (ng=1.5) if the angle of incidence is 35^o.

$n_1 \sin \theta_{1=n_2} \sin \theta_2$

$1.0\sin 35 = 1.5 \sin \theta_2$

$\therefore \sin \theta_2$=0.3824

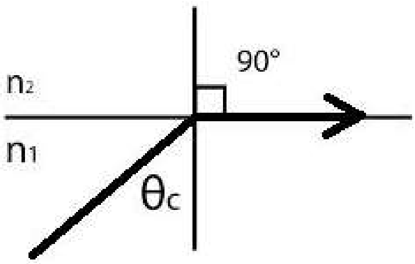
θ_2=22.5



4.1.12 CRITICAL ANGLE

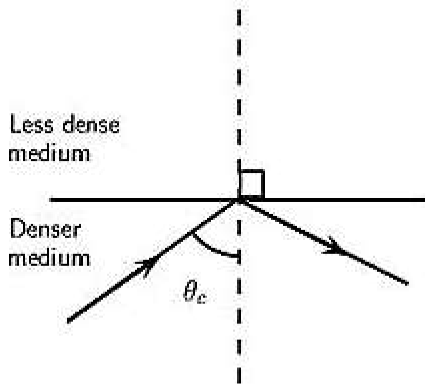
Critical angle is the angle of incidence which gives an angle of refraction of 90°. This only happens if the ray of incident is moving from more dense substance to less dense.

$$\boxed{\sin \theta_c = \frac{n_2}{n_1}}$$



4.1.13 TOTAL INTERNAL REFLECTION

Total internal reflection happens when reflection occur within the material. Angle of incidence is greater than the critical angle. It means no light is refracted into the less dense substance. Formation of mirages and apparent pools of water on bitumen roads in hot weather is caused by total internal reflection of light in the layers of hot air just above the ground.



Different substance (transparent material) has different refractive index.

Substance	n	Substance	n
diamond	2.24	Paraffin oil	1.44
Ruby	1.76	Ethanol	1.36
Flint glass	1.65	Water	1.33
Crown glass	1.52	Ice	1.31
Perspex	1.49	Air	1.00

Relative refractive index for two media is the ratio $\frac{\sin i}{\sin r}$ for light going from one substance into the other.

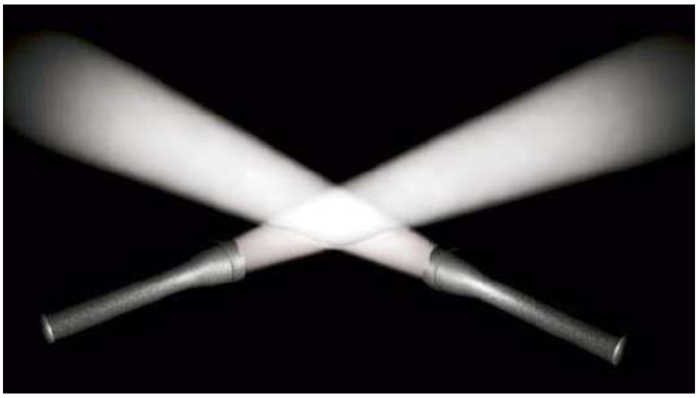
$$n_{12} = \frac{\sin i}{\sin r} = \frac{n_2}{n_1}$$

The summary formula is:

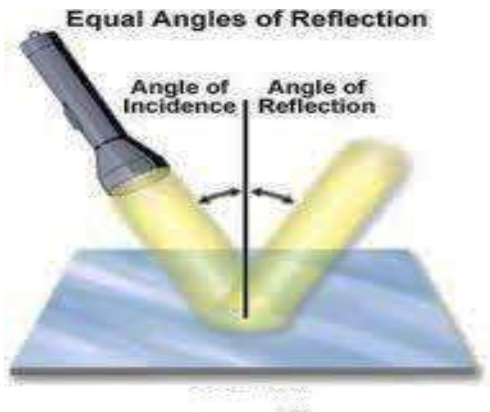
$$\boxed{n_{12} = \frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2} = \frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}}$$

4.1.14 PARTICLES MODEL OF LIGHT.

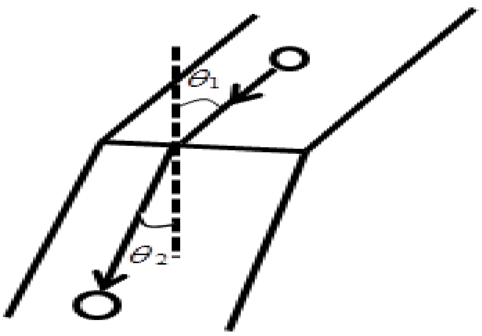
Light is a stream of particles called photons travelling at a very high speed. This can account for way of light form sharp shadows and how two beams can cross each other without apparent interaction.



Moving Particles of light also obeys law of reflection.



Particle model also explain the law of refraction .Example roll a ball down two connecting ramps of different slope.

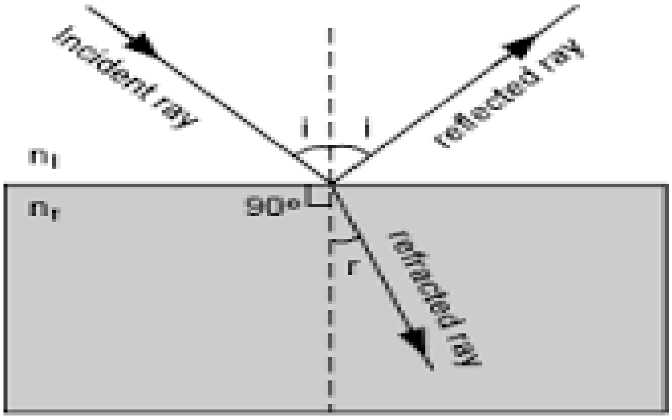


But the particle model requires the velocity to increase in the denser medium (i.e. when ray bends towards the normal).

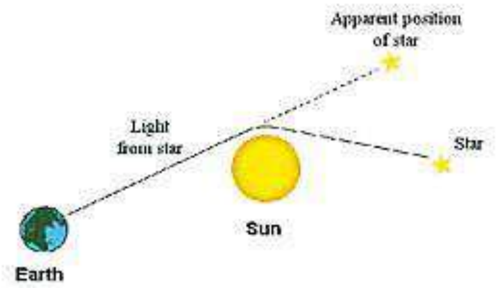
Model correctly predicts an inverse square law of intensity or illumination. It predicts that a substance which absorbs light is heated and experiment shows this to be true.

But particle model of light does not satisfactorily explain the following:

1. The existence of partial reflection and refraction at a boundary between two medium.

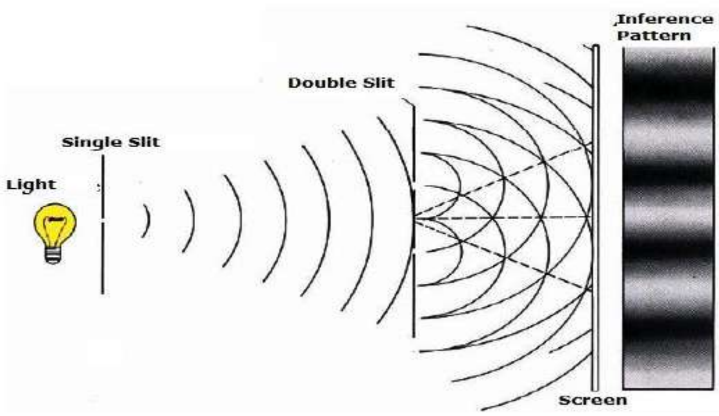


2. Diffraction- The bending of light around objects



3. The speed of light in water when measured by Foucault was found to be lower than the speed of light in air, particle theory predicting it should be greater.

4. Interference – how two beams of light incident on the screen and produce bright and dark band.



4.1.15 EXERCISE: LIGHT

1. A ray of light travels from air to glass of refractive index 1.5. If the angle of refraction is 40°, find the angle of incidence.

2. A ray of light travelling from air to water has an angle of incidence of 60° . Find the angle of refraction.

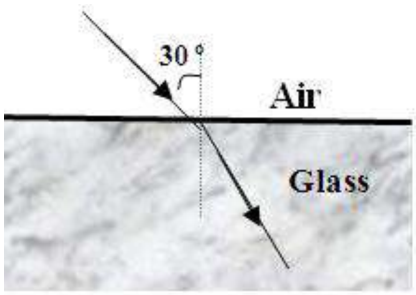
3. A ray of light travelling from toluene to air has an angle of incidence of 27° and an angle of refraction of 42°. What is the refractive index of toluene

4. Find the critical angle for the water/air surface.(n for water = 1.33)

5. The critical angle for an oil/air surface is 40°. What is the refractive index for oil?

6. Water (refractive index 1.33) is placed upon a block of glass of refractive index 1.5. What is the critical angle for light passing from the glass to the water?

7. A ray of light travels from air into glass as shown below:



8. Diamond has a refractive index of 2.4. If the speed of light in air is 3 x 10^8m/s, calculate the speed of light in diamond

4.2 WAVES

Wave motion is all happening around us. Some are mechanical waves like sand waves, small earthquakes, and vibration passing through a solid that require a material medium in which to travel.

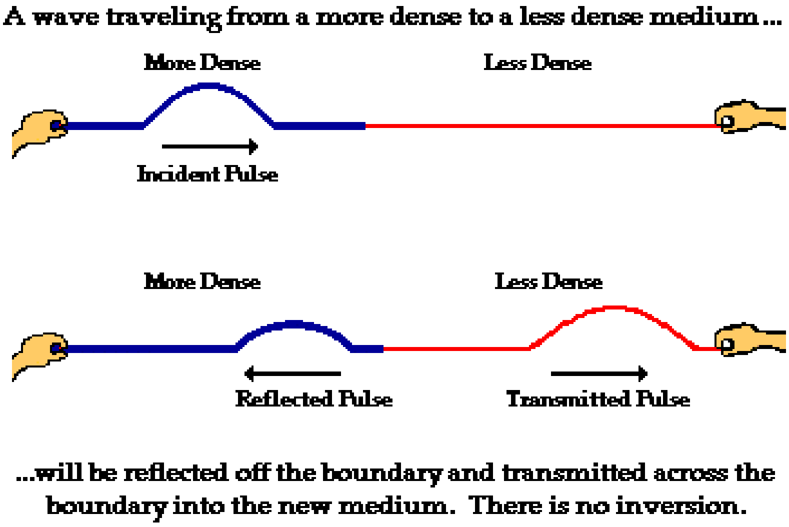
Other electromagnetic waves such as radio waves and light are produced when electrons are made to accelerate or when electron change energy level in an atom. These type of waves travel through vacuums, i.e. they do not need a medium.

4.2.10 REFLECTIONS AND TRANSMISSION OF WAVES:

When waves move from one medium to another, some of the waves are reflected. Type of reflection of waves depends on the type of medium it travels through for example; moving from a slow medium to a fast medium or the other way around.

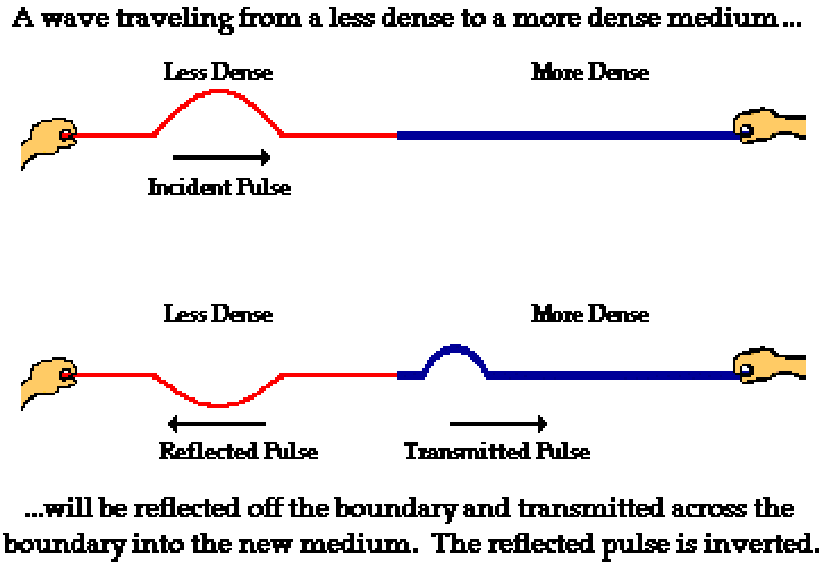
(a) Heavy to Light String:

A pulse (or wave) on a heavy string moves towards a lighter string. The pulse moves slower along the heavy string and faster along the lighter string. Small reflected pulse the same way up as the original pulse moves back along the heavy string.



(b) Light to Heavy String:

A pulse on a light string moves towards a heavy string. A small reflected pulse upside down to the original pulse moves back along the light string.

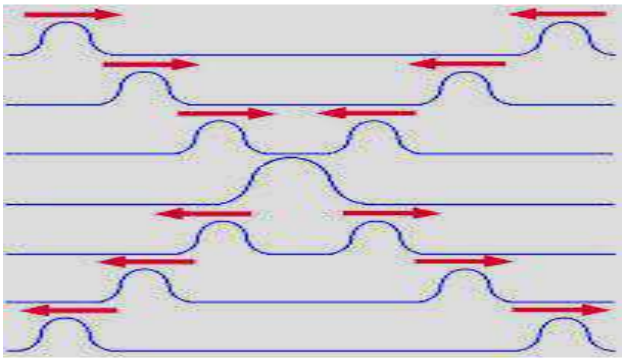


Amplitude of the reflected and transmitted waves are less, showing loss of energy. The pulse in the lighter string is further from the boundary, as they are travelling faster.

(c) Velocity of wave on a string does not depend on the **amplitude** of the waves. It depends on the tension in the string.
Similarly the velocity of light does not depend on the intensity of the light and intensity is related to amplitude in waves.

(d) **Superposition:**
Superposition is the ability of waves to add their displacements and their energy at each position with respect to time.

Constructive superposition refers to two waves of same amplitude meet then we get a resultant displacement of twice the amplitude where waves overlap.



4.2.11 INTERFERENCE

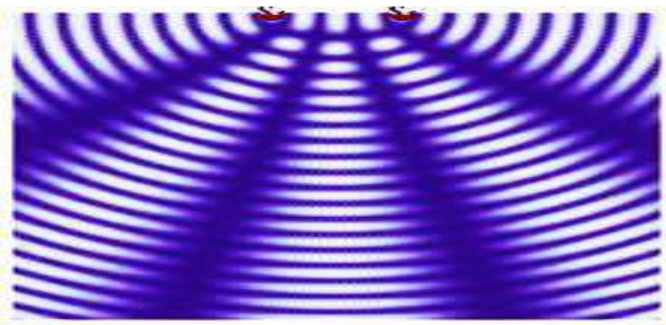
Interference pattern happens when two waves of equal amplitude and velocity moving in opposite direction overlap each other.

When crest from one wave meets a trough from another wave, this is known as destructive interference.

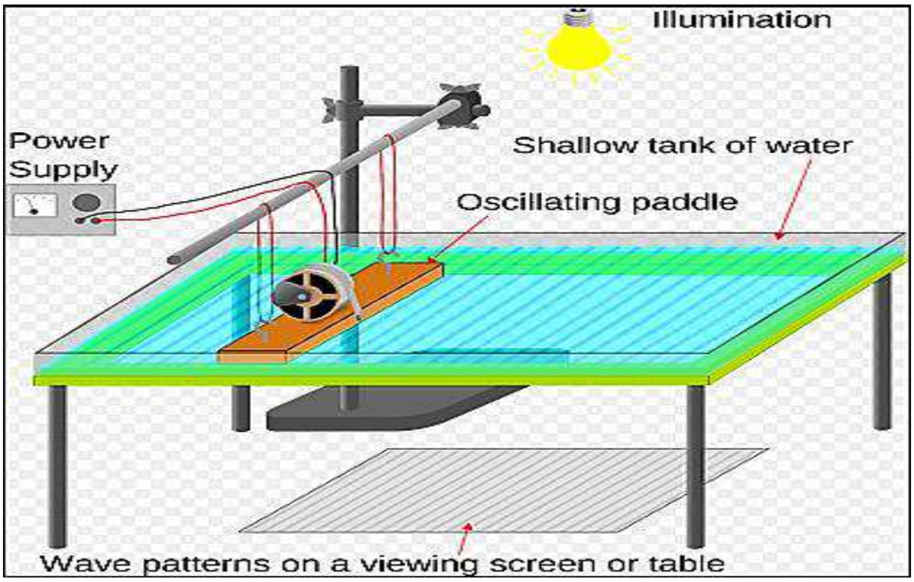
Constructive Interference:

Constructive interference happens when crest from one wave meet a crest from another wave or trough meets a trough.

As the trough and crest move away from the source, the continuous series of points appear, forming undisturbed lines of points appear, forming undisturbed lines of water. These lines are called nodal lines. Antinodes line form from when a crest meets a crest or trough meets a trough from two waves form.



In laboratory, a ripple tank is used to study waves.

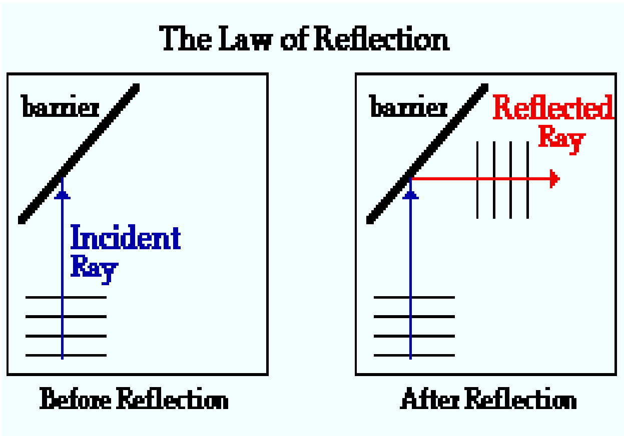


http://spmphysics.onlinetuition.com.my/2013/07/phenomena-of-waves.html

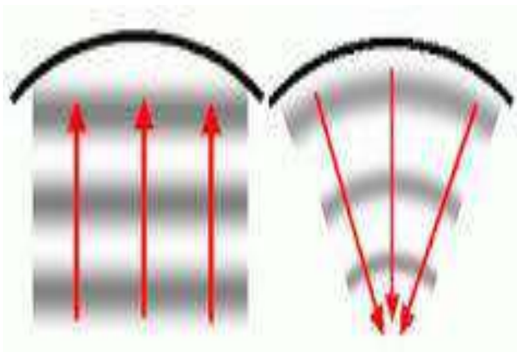
Experiments done in the ripple tank show that water waves behave like light waves.

4.2.12 REFLECTIONS

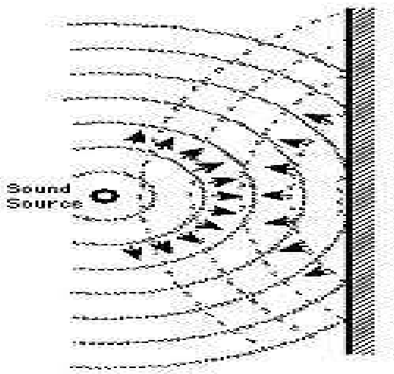
Waves are reflected off barriers, obeying the same laws of reflection as light. Note that the angle of incidence equals the angle of reflection.

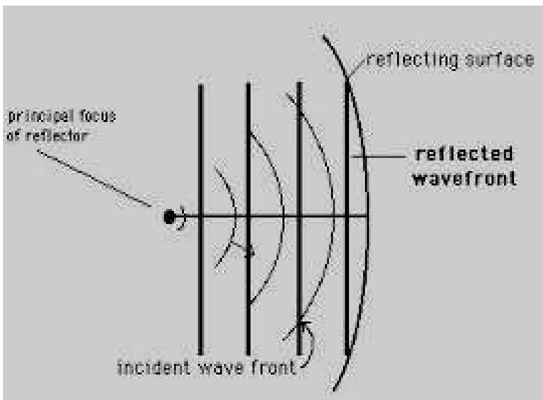


Flat waves hitting circular barrier



Circular waves hitting flat and circular barriers





4.2.13 REFRACTION

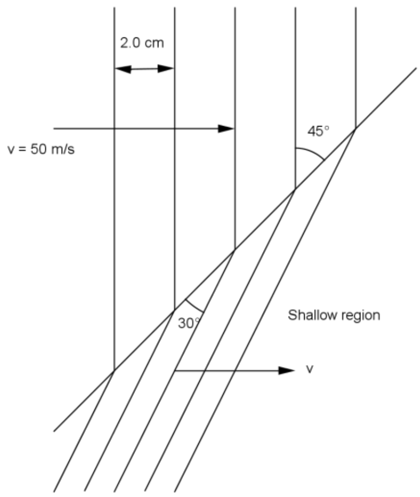
When water waves pass between a deep region and a shallow region they obey the law of refraction. In the shallow region the waves velocity decrease (wavelength decrease and since velocity = frequency x wavelength, velocity decrease since frequency remains constant)

Waves obey Snell's law.

$$\boxed{\frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2} = \frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}}$$

Example:

Water waves travelling at 5.0 cm/s and with a wavelength of 2.0cm are incident from deep water to shallow water as shown in the diagram.



A). Determine the relative refractive index.

$$n_{12} = \frac{\sin\theta_1}{\sin\theta_2}$$

$$= \frac{\sin 45}{\sin 30}$$

$$= 1.41\text{cm}$$

b). Determine the speed of the waves in the shallow water.

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2}$$

$$\frac{\sin 45}{\sin 30} = \frac{5}{v_2}$$

$$\therefore$$

$$v_2 = \frac{5.0\sin 30}{\sin 45}$$

$$v_2 = 3.54\text{cm/s}$$

c). Determine the frequency of the wave. (in water, the frequency of the wave always be constant. It will have same frequency in deep and shallow region).

Velocity = frequency x wavelength

5 = frequency x 2

Frequency = $\frac{5}{2}$

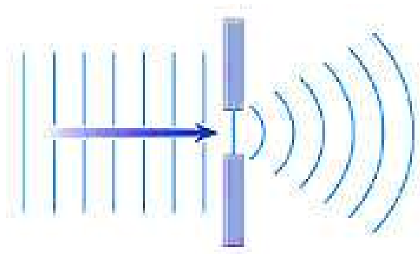
f = 2.5Hz

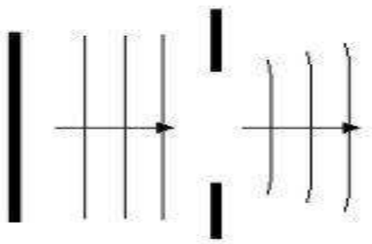
Partial Reflection and Refraction:

When waves travel from deep region to shallow region at the boundary, partial reflection and refraction happens to the incident rays.

4.2.14 DIFFRACTION:

When water waves passes through a narrow gap is equal or less than the wavelength of the incident wave. If the width of the gap is greater than the wavelength of the incident waves, the effect of the diffraction will be very small.





All the above properties are properties that light shows and so strongly suggests that light is in wave form.

1.2.15 WAVE MODEL OF LIGHT.

The particle model of light fails to explain two important facts which are known about light.

When light passes from one medium (such as air) to another (such as glass) some light if reflected and some refracted into the second medium.

When light passes from an optically less dense medium (such as air) to an optically denser medium (such as glass) the speed of light decreases.

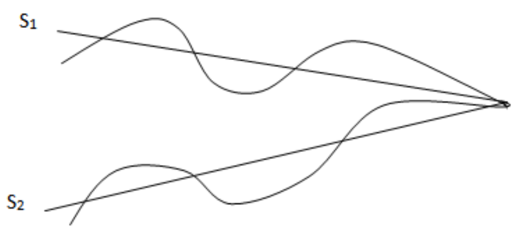
Both of the above difficulties can be explained if light is considered to be a wave motion.

The wave model of light describes light as consisting of waves with a very small wave length and travelling in straight lines forms source with a very large speed.

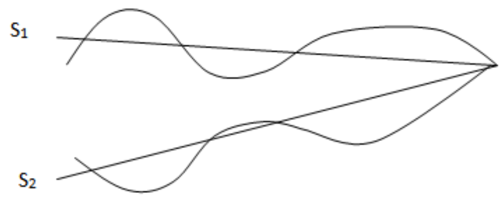
Interference of Light:

In 1810, Thomas Young demonstrated that light passing through two small holes very close together diffracts and forms an interference pattern

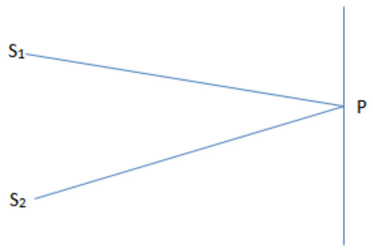
When two waves from individual slit overlap each other and arrive at a particular point at the same time (in phase) it forms Antinodes (Bright Band). It is the result of constructive interference.



When two waves arrive at a particular point not at the same time out of phase) it forms Nodes (dark band). It's the result of destructive interference.



Formation of Nodes and Antinodes is determined by path difference travelled by the two waves.



If $S_1P - S_2P = n\lambda$ where n= 0,1,2,3,……

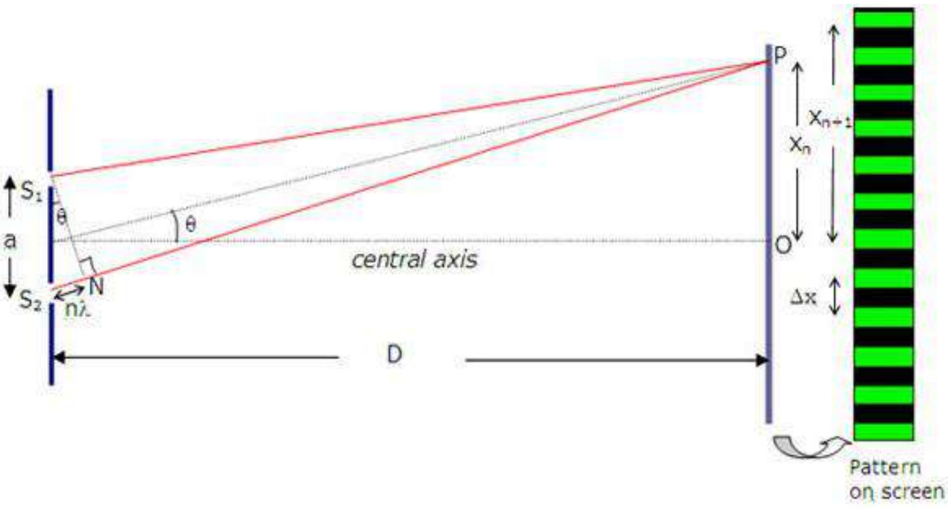
Path difference = whole number of wavelength we get Antinode or Constructive interference.

But if $S_1P - S_2P = ½ , 1½ ,……… = (n-½)\lambda$

Where n= 1, 2,3, …….

Path difference = half wavelength difference then the waves arrive out of phase and so we get Nodes or Destructive Interference.

Young's Experiment:



In Young's Interferometer the monochromatic light (light of one colour) from the source is divided into two parts using double slit arrangement.

The distance used to determine interference Light source to double slits: 20 – 100cm
Slit to screen: 1 – 5 meters
Slit width: 0.1 – 0.2 mm
distance between slits: less than 1mm.

Path difference = d $\sin\theta$ and since the angle are small then $\sin\theta \approx \tan\theta$

- We get Constructive Interference if:

$$\boxed{d \sin\theta = n\lambda = \frac{xd}{L}}$$

And Destructive Interference

$$\boxed{d \sin\theta = (n - ½)\lambda = \frac{xd}{L}}$$

Example:

Find the position of the First bright band from the central bright band if n=1 , d=1mm, L=2m and wavelength =10^{-6} m

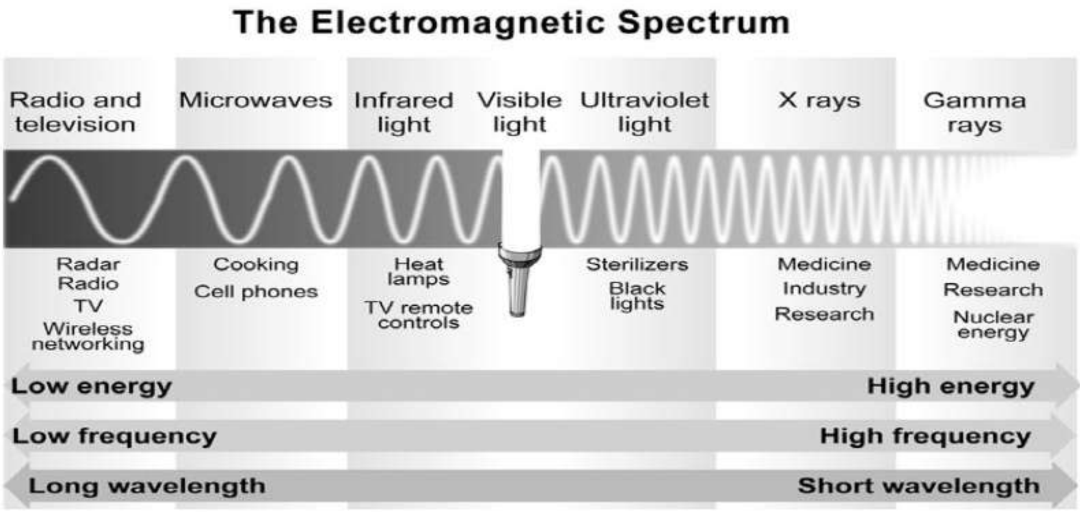
$n\lambda = \frac{xd}{L}$

(1) (10^{-6}) = x .1 x $10^{-3}/2$

X = 2 x 10^{-3}m

The electromagnetic spectrum

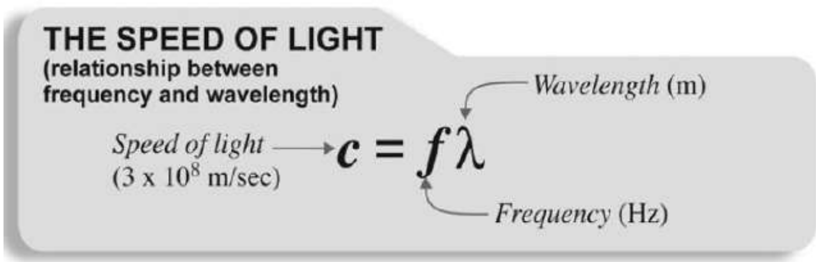
Radio waves, microwaves, visible light, and x-rays are familiar kinds of electromagnetic waves. All of these waves have characteristic wavelengths and frequencies. Wavelength is measured in meters. It describes the length of one complete oscillation. Frequency describes the number of complete oscillations per second. It is measured in hertz, which is another way of saying "cycles per second." The higher the wave's frequency, the more energy it carries.



Frequency, wavelength, and speed

In a vacuum, all electromagnetic waves travel at the same speed: $3\times10^{8}\text{m/s}$. This quantity is often called "the speed of light" but it really refers to the speed of all electromagnetic waves, not just visible light. It is such an important quantity in physics that it has its own symbol, *c*.

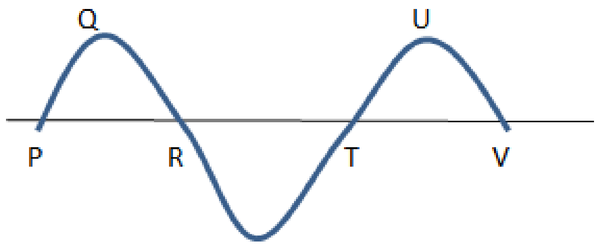
The speed of light is related to frequency *f* and wavelength λ by the formula given below.



The different colours of light that we see correspond to different frequencies. The frequency of red light is lower than the frequency of blue light. Because the speed of both kinds of light is the same, a lower frequency wave has a longer wavelength. A higher frequency wave has a shorter wavelength. Therefore, red light's wavelength is longer than blue light's.

4.2.16 EXERCISE WAVES

1. A waves travels along a piece of rope. The source of vibration **S** is on the left.

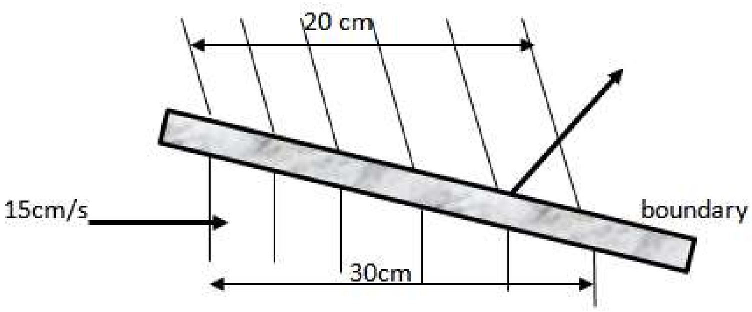


a) State the name given to the distance between positions R and V.
b) Which direction will position V move in at the instant shown?

2. Copy and complete the table using the formula $v = f\gamma$

Speed $(m\|s)$	**Frequency (Hz)**	**Wavelength (m)**
	20	**3**
330	**100**	
3x10^8	**1x10^6**	
3x10^8		**150**

3. Wave fronts are seen to cross a boundary from deep to shallow water as shown below. The arrow shows the direction of the waves as they move from deep water to shallow water.

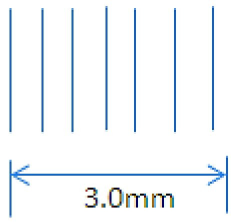


deep water

Use the information in the diagram above to answer the questions that follow.

i. Determine the wavelength of the waves in deep water
ii. Calculate the frequency of the waves in deep water
iii. Find the velocity of the waves in shallow water

4. The interference fringe pattern formed on a screen in a Young Double Slit experiment is shown in the figure below.

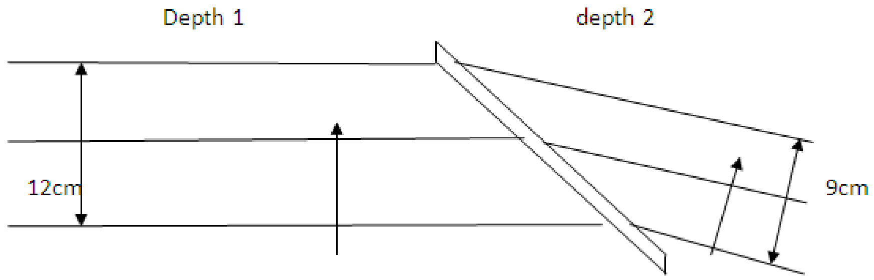


The screen is located 2.0m from the double slit. The slits are separated by a distance of 1.0mm.

   i. Calculate the wavelength of the light?
   ii. What would happen to the interference fringe pattern if the slits were moved closer to the screen?
   iii. Why does this experiment support the wave model of light rather than the particle model?

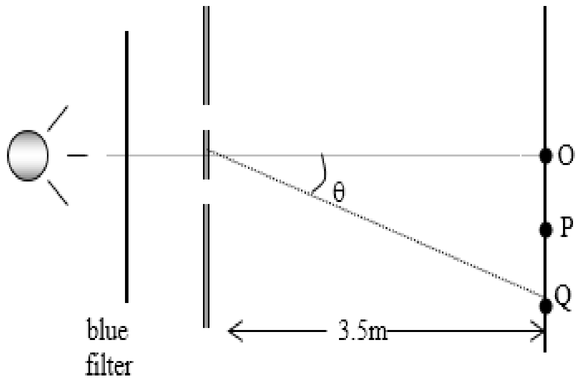
5. In a Young's double slit experiment, the monochromatic light used is of wavelength 600 nm.
   i. If the distance between the two slits is 0.50 mm and the screen distance from the double slit is 0.75 m, calculate the distance of the third bright band from the central maximum.
   ii. What is the path difference of the light from the double slits to the second dark band?

6. Plane waves of frequency 5Hz in a ripple tank pass from one depth of water into another across a boundary.



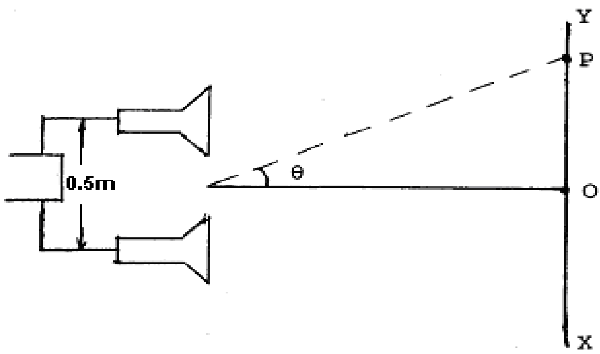
   a) Label depth 1 and depth 2 as shallow or deep region.
   b) What is the wavelength of waves in Depth 1?
   c) Calculate the speed of waves in depth 1.

7. The set up shown below is used to obtain an interference pattern on a screen 3.5m away from the slit. The slit separation is 2 mm.



   i. Explain the purpose of the blue filter in the above set up?
   ii. A bright band is formed at P. Briefly explain how a bright band is formed?
   iii. If the distance between O and P is 0.0012 m, calculate the wavelength of blue light?
   iv. Calculate the angle of θ, at Q (second bright fringe) from the centre of the slits.

8. Two speakers placed 0.5 m apart are connected to a source of sound waves of frequency 500 Hz. Take the speed of sound as 340 m/s.



   i. Calculate the wavelength of the sound.
   ii. Calculate the angle θ at which the point P is found.
   iii. Along the line XY in front of the speakers a series of loud and faint sound can be heard. State **ONE** way in which the distance between the loud sounds along the line can be decreased.

CHAPTER 5: ELECTRICITY

5.1 ELECTROSTATICS

Electrostatics is the study of electric charges, forces, and fields. The symbol for electric charge is the letter "*q*" and the SI unit for charge is the **coulomb (C).** The coulomb is a very large unit.

$$1\text{ C} = 6.25 \times 10^{18} \text{ electrons or}$$
$$1 \text{ electron has a charge of } 1.60 \times 10^{-19}\text{ C}$$

5.1.10 COULOMBS LAW

The electrostatic force was first studied in detail by Charles Coulomb around 1784. According to his observations he was able to show that two charged objects attract each other with a force that is proportional to the charge on the objects and inversely proportional to the square of the distance between them.

$$F \quad \alpha \quad \frac{q_1 q_2}{r^2}$$

where

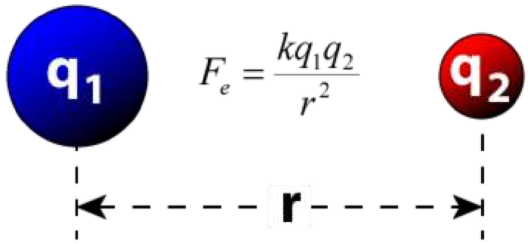
- q_1 is the charge on the one point-like object(C),
- q_2 is the charge on the second,(C),
- r is the distance between the two (m) and
- F is the magnitude of the electrostatic force between two point-like charges (N).

To make an equation out of this proportionality, a quantity called the **electrostatic constant, *k*** is inserted.

$$k = 9 \times 10^9 \text{ N.m}^2\text{C}^{-2}.$$

The magnitude of Coulomb's law can now be written as an equation:

$$\text{Electrostatic force} = \frac{(K_{electrostatics})(Charge1)(Charge2)}{(distance)^2}$$

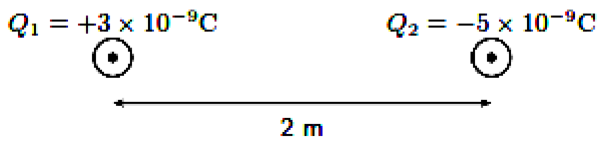


http://www.aplusphysics.com/courses/honors/estat/Coulomb.html

EXAMPLE

Two point-like charges carrying charges of $+3 \times 10^{-9}$ C and -5×10^{-9} C are 2 m apart. Determine the magnitude of the force between them and state whether it is attractive or repulsive.

Given: $q_1 = +3 \times 10^{-9}$ C
$q_2 = -5 \times 10^{-9}$ C
r = 2 m



Using Coulomb's Law we have

$$F = \frac{Kq_1q_2}{r^2} = \frac{(9\times10^9\,Nm^2C^{-2})(3\times10^{-9}\,C)(5\times10^{-9}\,C)}{(2m)^2} = \underline{3.37\times10^{-8}\,N}$$

Thus, the magnitude of the force is $3.37\times10^{-8}\,N$. However since both point charges have opposite signs, the force will be attractive.

EXAMPLE

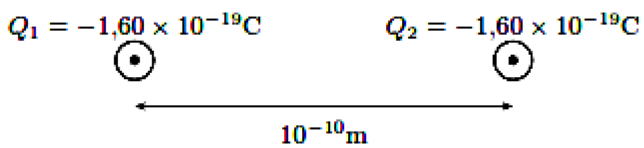
Determine the electrostatic force and gravitational force between two electrons 10^{-10} m apart (i.e. the forces felt inside an atom).

Soln: We are required to calculate the electrostatic and gravitational forces between two electrons, a given distance apart.
We can use:
$F_e = \frac{Kq_1q_2}{r^2}$ to calculate the electrostatic force and $F_g = \frac{Gm_1m_2}{r^2}$ to calculate the gravitational force.

Given: $q_1 = q_2 = 1.6 \times 10^{-19}$ C (charge of an electron)
$m_1 = m_2 = 9.1 \times 10^{-31}$ kg (mass of an electron)
$r = 1 \times 10^{-10}$ m

$K = 9 \times 10^9\ Nm^2C^{-2}$
$G = 6.67 \times 10^{-11}\ Nm^2kg^{-2}$

We can draw a diagram of the situation.



Electrostatic Force:

$$F_e = \frac{Kq_1q_2}{r^2} = \frac{(9\times10^9\,Nm^2C^{-2})(1.6\times10^{-19}\,C)(1.6\times10^{-19}\,C)}{(1\times10^{-10}\,m)^2} = \underline{2.3\times10^{-8}\,N}$$

Hence the magnitude of the electrostatic force between the electrons is $\underline{2.3\times10^{-8}\,N}$.
Since electrons carry the same charge, the force is repulsive.

Gravitational Force:

$$F_g = \frac{Gm_1m_2}{r^2} = \frac{(6.67\times10^{-11}Nm^2kg^{-2})(9.1\times10^{-31}kg)(9.1\times10^{-31}kg)}{(1\times10m)^2} = \underline{5.54\times10^{-51}N}$$

5.1.11 ELECTRIC FIELD

Electric Field is an area of influence around a charged object. The magnitude of the field is proportional to the amount of electrical force exerted on a positive test charge placed at a given point in the field.

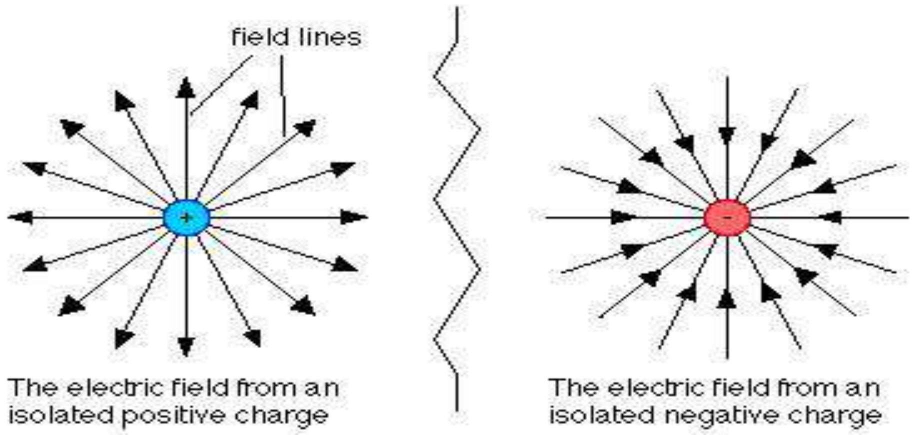
$$E = \frac{F}{q}$$

Where E = Electric Field (N/C)
F = Electrostatic Force (N)
q = Test Charge (C)

The SI unit of electric field is the **newton per coulomb (N/C).**

FIELD NEAR A POINT CHARGE

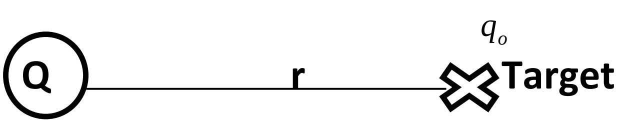
A point charge has around it a radial electric field. If the charge is positive the field is directed away from the charge. If the charge is negative the field is directed towards the charge.



http://physics.appstate.edu/laboratory/quick-guides/electric-fields

For a point charge (or other spherical charge distribution), the magnitude of the electric field can be written as

$$E = \frac{F}{q_0} = \frac{kq_oq}{q_0r^2} = \frac{kq}{r^2}$$



That is $$E = \frac{kq}{r^2}$$

Where

q is the charge on the surface of the object (C), and

r is the distance between the centre of the charged object and a small positive test charge, q_0, placed in the field (E).

EXAMPLE

Shirley pulls her wool sweater over her head, which charges her body as the sweater rubs against her cotton shirt.

(a) What is the electric field at a location where a 1.60×10^{-19} C - piece of lint experiences a force of 3.2×10^{-9} N as it floats near Shirley?

(b) What will happen if Shirley now touches a conductor such as a door knob?

Soln:

a). $E = \frac{F}{Q} = \frac{3.2 \times 10^{-9} N}{1.6 \times 10^{-19} C}$ = **$\underline{2 \times 10^{10}\ \text{N/C}}$**

b) She will reduce her charge in a process called **grounding,** in which excess electrons flow from her body into the ground and spread evenly over the surface of Earth.

EXAMPLE

A fly accumulates 3.0×10^{-10}C of positive charge as it flies through the air. What is the magnitude and direction of the electric field at a location 2.0 cm away from the fly?

$E = \frac{kq}{r^2} = \frac{(9.0 \times 10^9 Nm^2C^{-2})(3.0 \times 10^{-10} C)}{(0.020m)^2}$ = **<u>6800 N/C</u> away from the fly**.

EXAMPLE

Two charges of Q_1 = +3 nC(3×10^9C) and Q_2 = -4 nC(-4×10^9C) are separated by a distance of 40 cm. What is the electric field strength at a point that is 10 cm from Q_1 and 30 cm from Q_2? The point lies between Q_1 and Q_2.

Given: $Q_1 = 3 \times 10^{-9}$C
$Q_2 = -4 \times 10^{-9}$C
R_1 = 0.1 m
R_2 = 0.3 m

What is required: calculate the electric field, E at x :

We will use the equation: $E = \frac{kq}{r^2}$

We need to work out the electric field for each charge separately and then add them to get the resultant field.

Step 1: first solve for Q_1: $E = \frac{kq}{r^2} = \frac{(9.0\times10^{9}\,Nm^{2}C^{-2})(3.0\times10^{-9}\,C)}{(0.10m)^{2}} = 2700N/C$

Step2: solve for Q_2: $E = \frac{kq}{r^2} = \frac{(9.0\times10^{9}\,Nm^{2}C^{-2})(4.0\times10^{-9}\,C)}{(0.30m)^{2}} = 400N/C$

Step 2: We need to add the two electric field because both are in the same direction. The field is away from Q_1 and towards Q_2. Therefore,

E_{TOTAL} = 2700 N/C + 400 N/C = **<u>3100 N/C</u>**

ELECTRIC POTENTIAL ENERGY

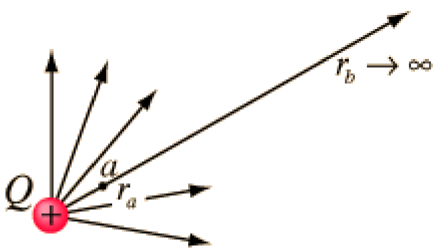
Potential Reference at Infinity

The general expression for the electric potential as a result of a point charge Q can be obtained by referencing to a zero of potential at infinity. The expression for the potential difference is:

$$V_a - V_b = kQ\left[\frac{1}{r_a} - \frac{1}{r_b}\right]$$

Taking the limit as $r_b \rightarrow \infty$ gives simply

$$V = \frac{kQ}{r} = \frac{Q}{4\pi\varepsilon_0 r}$$



For any arbitrary value of r. The choice of potential equal to zero at infinity is an arbitrary one, but is logical in this case because the electric field and force approach zero there. The electric potential energy for a charge q at r is then $E_{p(electric)} = \frac{kQ_1Q_2}{r}$

Where k is Coulomb's constant.

The electric potential energy of a charge is the energy it has because of its position relative to other charges that it interacts with. The potential energy of a charge Q_1 relative to a charge Q_2 a distance r away is calculated by:

$$E_{p(electric)} = \frac{kQ_1Q_2}{r}$$

EXAMPLE

What is the electric potential energy of a 7 nC charge that is 2 cm from a 20 nC ?

Given: $Q_1 = 7 \times 10^{-9}$ C

$Q_2 = 20 \times 10^{-9}$C

R = 0.02 m

What is required: Electric potential energy, E_p

We will use the equation: $E_{p(electric)} = \frac{kQ_1Q_2}{r} = \frac{(9\times10^{9}\,Nm^2C^{-2})(7\times10^{-9}\,C)(20\times10^{-9}\,C)}{(0.02m)}$

$E_{p(electric)}$ = 6.3 × 10^{-5} J

ELECTRICAL POTENTIAL DIFFERENCE

Potential Difference refers to the work done to move a positive test charge from one location to another.

Potential Difference = $\frac{Work}{Test\,Charge}$ or $$\boxed{V = \frac{W}{q}}$$

The SI unit for potential difference is the **volt (V),** which equals a **joule per coulomb (J/C).**

RECALL, the term "*work*" can be replaced with the term "electric potential energy," since to store energy in, or give energy to, an object, work must be done. Therefore, potential difference can also be defined as the electrical potential energy per unit test charge. **Voltage** is often used to mean **potential difference**.

EXAMPLE

What is the potential difference between two points in an electric field if it takes 600 J of energy to move a charge of 2 C between these two points?

Given: W = 600 J
Q = 2 C
Find: Potential difference, V:

We use $V = \frac{W}{Q} = \frac{600J}{2C}$ $V = 300V$

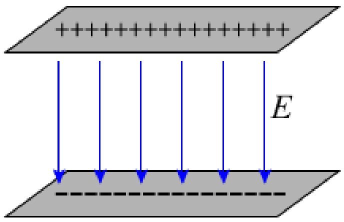
EXAMPLE

An **electron** in Akeneta's TV is accelerated toward the screen across a potential difference of 22 000 V. How much kinetic energy does the electron lose when it strikes the TV screen?

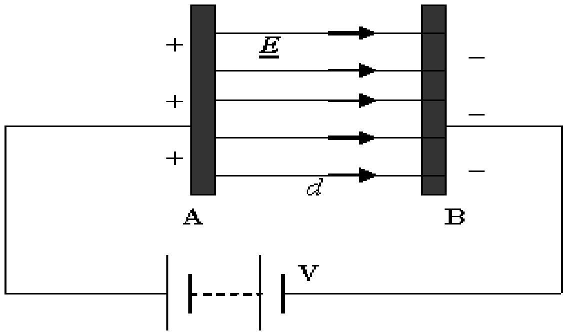
Given: Q_e = 1.6 × 10^{-19} C
V = 22 000 V
Find: E_k or W

Therefore, $V = \frac{W}{Q} \Longrightarrow W = VQ = (22000V)(1.6\times10^{-19}C) = 3.5\times10^{-15}J$

FIELD BETWEEN TWO PARALLEL CHARGED PLATES



When a potential difference is applied across a pair of parallel plates, a uniform electric field is established which is dependent on the separation of the plates.



The work done to move a charge, *q*, a distance *d* between the plates is: $W = F.d$

Since $F = Eq$, $\therefore$ $W = Eqd$ (**1**)

Also Voltage (potential difference), *V*, is $V = \frac{W}{q}$,

Therefore $W = Vq$ (**2**)

Equating equations (**1**) and (**2**) $Vq = Eqd$,

Thus, $$V = Ed$$

Where : V = potential difference (V)

E = electric field strength between the plates $\left(\frac{N}{C} = \frac{V}{m}\right)$

d = separation distance of the plates (*m*)

EXAMPLE

Two oppositely charged plates are separated a distance of 3 cm and attached to a potential difference of 12 V.

(a) Calculate the electric field strength between the plates.
(b) What is the force on an electron in the plates?
(Charge of an electron = 1.06 x 10^{-19} C)

Solution: (a) E = $\frac{V}{d}$ = $\frac{12V}{0.3m}$ = $\underline{400 \ \frac{N}{C}}$

(b) F = Eq = (400 N/C) (1.0 x 10^{-19} C) = $\underline{6.4 \times 10^{-17}\text{ N}}$

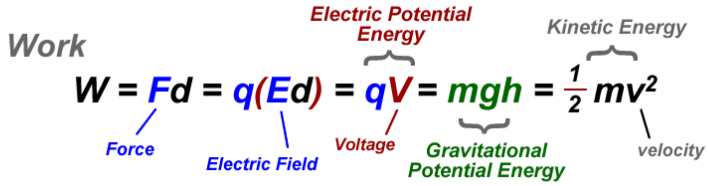
Points to Note:

- If the electric field E is uniform, the force on a charge is independent of the position of the charge in the field. (F = Eq)

- Electric field strength E is given by: **E** = $\frac{V}{d}$, units $\left(\frac{N}{C} = \frac{V}{m}\right)$

- Work done on a charge *q* is: ***W = Eqd***

- The work done on a charge becomes the E_K (kinetic energy) of the charge;



http://sdsu-physics.org/physics180/physics180B/Chapters/electric_potential.htm

EXAMPLE

A potential difference of 100 V is connected to two parallel plates. Calculate the velocity with which an **electron** leaving the negative plate strikes the opposite plate. (Charge, q = 1.6×10^{-19} C, mass, m = 9.11×10^{-31} kg)

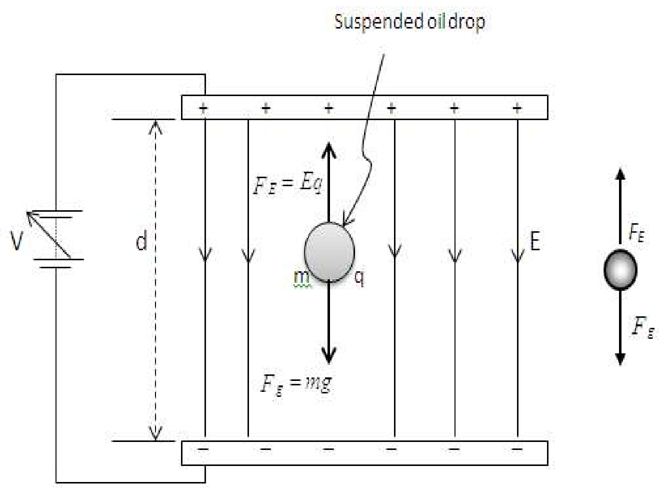
Work done = gain in E_K : $\boldsymbol{Eqd = \frac{1}{2} mv^2}$, $\boldsymbol{Vq = \frac{1}{2} mv^2}$

$$(100\text{V})(1.6\times10^{-19}\text{C}) = \tfrac{1}{2}(9.11\times10^{-31}\text{kg})v^2$$

$$v = 5.96 \times 10^6 \text{ m/s}$$

MILIKAN'S OIL DROP EXPERIMENT

A pair of parallel plates was set-up with a variable voltage. Oil drops were exposed to X – Rays and they dropped through the hole between the plates. The variable voltage was adjusted to bring the drops to equilibrium.



When an oil drop becomes stationary, the gravitational and electrical forces are equal. From this Millikan was able to quantify the value of the smallest charge, (i.e. the electronic charge, e^-).

Equating the electrical and gravitational forces;

$Eq = mg$

$\frac{Vq}{d} = mg$, $q = \frac{mgd}{V}$ (*m,d and V* are measurable quantities.)

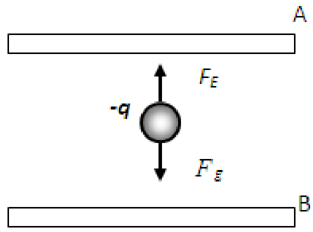
Millikan discovered that only whole number values of charge were found. This means that charge occurs in discrete values. That is multiples of the smallest charge.

$\boldsymbol{Q = ne^-}$ *unit* : Coulomb (C)

Where : n = number (n = 1,2,3,….)
e^- = 1.6 x 10^{-19} C

EXAMPLE

In a Millikan experiment set – up, an oil drop of mass 2.05 x 10^{-12} kg is suspended between plates A and B. The plates are at a separation of 5 cm and at a potential difference of 500 V.



(a) What is the polarity of A?

(b) Calculate the electric field strength, E.

(c) What is the charge on the oil drop?

Solution:

(a) The polarity of A should be **positive**. (+)

(b) $E = \frac{V}{d} = \frac{500V}{0.05m} = 10\ 000\ NC^{-1}$

(c) $F_E = F_g$ $Eq = mg$, $\boldsymbol{q} = \frac{mg}{E} = \frac{(2.05\times10^{-12}kg)(10m/s^2)}{10000N/C}$

$= 2.05 \times 10^{-15}$ C

Note

1. For charges being accelerated between a pair of parallel plates, the expression for velocity as the charge reaches the opposite plate is given by:

$Vq = ½ mv^2$, $v = \sqrt{\frac{2Vq}{m}}$

2. The expression for acceleration is given by:

Equating $F = ma$, and $F = Eq$, $a = \frac{Eq}{m}$

[$\boldsymbol{V}$ = potential difference (V), and $\boldsymbol{v}$ = velocity (m/s)]

THE ELECTRON VOLT

An electron volt (eV) is the energy acquired by an electron in moving through a Potential Difference of 1 volt.

$$1eV = 1.6x10^{-19} J$$

EXERCISES

COULOMBS LAW

1. Calculate the electrostatic force between two charges of +6nC and +1nC if they are separated by a distance of 2 mm.

2. When sugar is poured from the box into the sugar bowl, the rubbing of sugar grains creates a static electric charge that repels the grains, and causes sugar to go flying out in all directions. If each of two sugar grains acquires a charge of 3.0×10^{-11} C at a separation of 8.0×10^{-5} m, with what ***force*** will they repel each other?

3. Calculate the distance between two charges of +4nC and -3nC if the electrostatic force between them is 0.005 N.

4. Lusiana is dusting the house and raises a cloud of dust particles as she wipes across a table. If two 4.0×10^{-14} C pieces of dust exert an electrostatic force of 2.0×10^{-12} N on each other, ***how far apart*** are the dust particles at that time?

5. Calculate the charge on two identical spheres that are similarly charged if they are separated by 20 cm and the electrostatic force between them is 0.006 N.

6. Miriam uses hairspray on her hair each morning before going to school. The spray spreads out before reaching her hair partly because of the electrostatic charge on the hairspray droplets. If two drops of hairspray repel each other with a force of 9.0×10^{-9} N at a distance of 0.070 cm, what is the ***charge on each*** of the equally-charged drops of hairspray?

7. Two insulated metal spheres carrying charges of +6nC and -10nC are separated by a distance of 20 mm.
   a) What is the electrostatic force between the spheres?
   b) The two spheres are touched and then separated by a distance of 60 mm.
      What are the new charges on the spheres?
   c) What is the new electrostatic force between the spheres at this distance?

ELECTRIC FIELD

1. In an experiment, a positively charged oil droplet weighing 6.5×10^{-15} N is held stationary by a vertical electric field. If the electric field strength is 5.3×10^{3} N/C, what is the charge on the oil droplet?

   A. 1.2×10^{-18} C   B. 3.4×10^{-11} C   C. 4.1×10^{4} C   D. 8.2×10^{17} C

2. A flash of lightning between a cloud and the earth causes a potential difference of 10^{9} V which results in the movement of 40 C of charge in a time of 10^{-2} s.

i. The average current produced by a flash of lightning in ampere, would be

A. 0.4 B. 40 C. 400 D. 4 000

ii. The energy transferred in J, would be

A. 4 x 10^9 B. 4 x 10^{10} C. 4 x 10^{11} D. 4 x 10^{12}

3. When 10^{14} electrons are removed from a neutral metal sphere, the charge on the sphere becomes

A. 32 µC B. 16 µC C. -16 µC D. -32 µC

4. The work done in moving a charge of 450nC from one point to another to achieve a potential difference of 6V would be

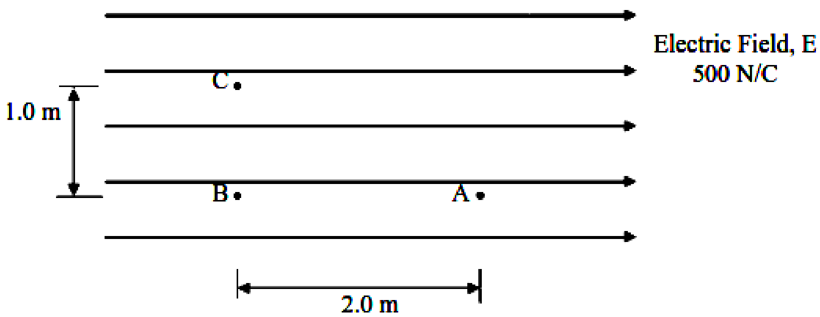
A. 6 nJ B. 75 nJ C. 450 nJ D. 2700 nJ

5. Calculate the potential difference between two parallel plates if it takes 5000 J of energy to move 25 C of charge between the plates?

6. Calculate the electric field between the plates of a capacitor if the plates are 20 mm apart and the potential difference between the plates is 300 V.

7. Calculate the electrical potential energy of a 6nC charge that is 20 cm from a 10nC charge.

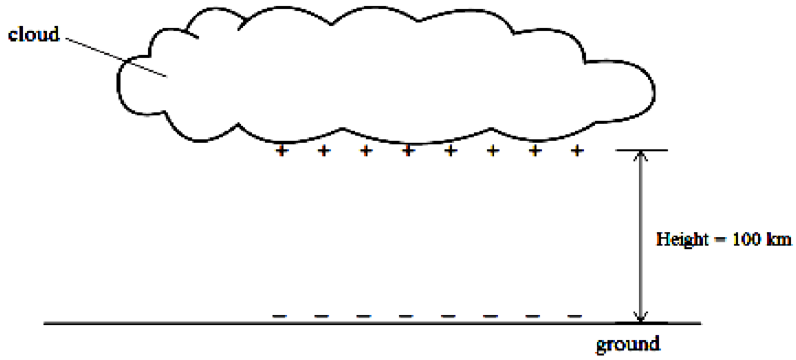
8. The figure below shows a 500 N/C uniform electric field. The distance between points A and B is 2.0 m while the distance between points B and C is 1.0 m.



Which of the following **best** describes the potential difference between the points A, B and C?

	Potential Difference between A and B	Potential Difference between B and C
A.	1 000 V	0 V
B.	1 000 V	500 V
C.	500 V	0 V
D.	500 V	1 000 V

9. A thunder cloud at 100 km above the ground contains positive charges. The potential difference between the cloud and the ground is 1000 V.



i. Draw the uniform electric field lines between the cloud and the ground.
ii. Calculate the uniform electric field strength between the cloud and the ground.
iii. If an electron moves from the ground towards the cloud, calculate the work done by the electric field on the electron. (Neglect the effect of gravity on the electron)

ENERGY CONSERVATION

1. This question is about an experiment designed to measure the charge on an electron. In this experiment, 'Millikan's Oil Drop Experiment', two parallel metal plates, 3.2×10^{-2} m apart, are connected to a 600 V power supply.

i. Calculate the electric field strength between the two plates.
ii. The electric field between the plates just supports the weight of an oil drop of mass 1.8×10^{-15} kg, which has acquired a charge due to a few excess electrons. Given that the oil drop is stationary, calculate the charge on the oil drop.
iii. What is the most likely number of excess electrons acquired by the oil drop?

2. Shown below are two closely spaced metal plates connected to a 240 V supply.

A **uniform** electric field exists between the plates.
a) Draw the electric field pattern between the two plates.
b) If the plates are 5 cm apart, find the work done on an electron travelling from the negative to the positive plate.

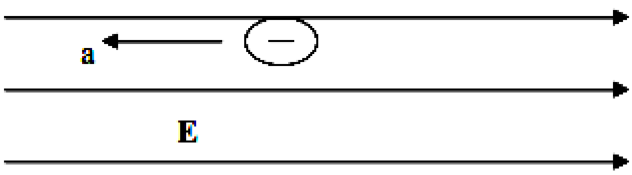
3. An oil drop of mass 5.20×10^{-10} kg is suspended between two parallel plates. The electric field between the plates is 520 N/C downwards.
(i) Explain why the charge on the oil drop is negative.
(ii) Determine the magnitude of charge on the oil drop.

4. An electron of charge 1.6×10^{-19} C passes through the deflecting plates of a cathode ray tube.
   a) The deflecting plates are maintained at a voltage of 45 V, and are 8.0 mm apart. Calculate the electric field strength between the plates.
   b) The charge on an electron is 1.6×10^{-19} C. Calculate the electric force on an electron between the plates.

5. State **one** significant conclusion from the Millikan oil drop experiment.

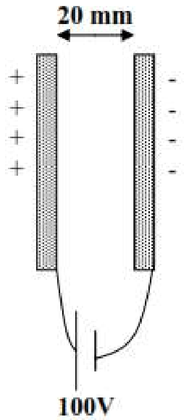
6. How many electrons are there in an oil drop that has a charge of 4.0 coulombs?

7. An electron is injected into a region of uniform electric field of magnitude E = 1×10^{5} N/C as shown below.



   Calculate the initial acceleration of the electron.

8. The terminals of a 100V battery are connected between two parallel metal plates, 20 mm apart as shown below.



(i) Find the electric field strength between the plates.

An electron is released from rest from the negative plate

(ii) Calculate the energy gained by the electron in moving between the plates.

(iii) Determine with what velocity the electron arrives at the positive plate.

(iv) What would be the acceleration of the electron ?

(v) How long does it take the electron to travel between the plates ?

5.2 CURRENT ELECTRICITY

CURRENT AND RESISTANCE

Current is the amount of charge that passes through an area in a given amount of time.

$$Current = \frac{Charge}{Time} \quad \text{or} \quad I = \frac{\Delta q}{\Delta t}$$

The SI unit for current is the **ampere (A),** which equals one **coulomb per second (C/s)**.

Resistance is an opposition to the flow of charge.

For a given source voltage, the resistance of a circuit determines how much charge will flow in the circuit. When charge passes through a resistance, some electrical energy is changed to other forms. This is produced by a potential difference (voltage) across the resistance.

$$Potential Difference = Current \times Resistance$$
$$V = IR$$

The SI unit for resistance is the **ohm (Ω)**, which equals one **volt per amp (V/A)**.

POWER

Power is the amount of work done in a given unit of time.

$$Power, P = \frac{W}{t} = \frac{\Delta qV}{\Delta t} = IV$$

The SI unit for electrical power is the **watt (W),** which equals one **joule per second (J/s).**

Since P = IV and $R = \frac{V}{I}$, Power dissipated in a conductor can also be expressed as

$$P = I^2R \qquad \text{or} \qquad P = \frac{V^2}{R}$$

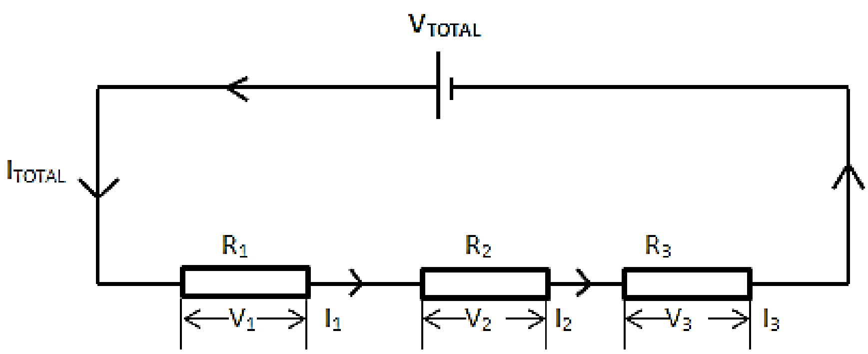
SERIES AND PARALLEL CIRCUITS

When multiple resistors are used in a circuit, the total resistance in the circuit must be found before finding the current. Resistors can be combined in a circuit in series or in parallel.

Resistors in Series

When connected in series, the **total resistance, *R*$_T$,** is equal to

$$\boxed{R_T = R_1 + R_2 + R_3 + \dots\dots\dots}$$



In series, the total resistance is always *larger* than any individual resistance.

Current in series resistors: In series circuits, charge has only one path through which to flow. Therefore, the current passing through each resistor in series is the same.

$$I_{TOTAL} = I_1 = I_2 = I_3$$

Voltage across series resistors: As charge passes through each of the resistors, it loses some energy. This means that there will be a **voltage drop** across each resistor. The sum of all the Voltages equals the Voltage across the battery, assuming negligible resistance in the connecting wires.

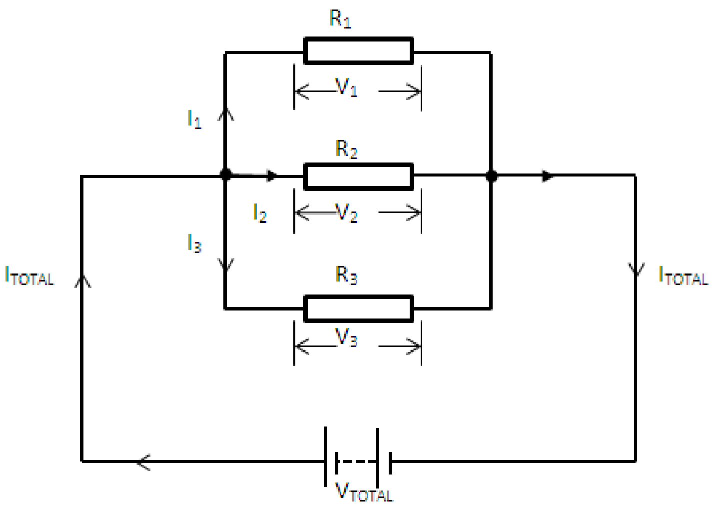
$$V_{TOTAL} = V_1 + V_2 + V_3$$

Resistors in Parallel

When connected in parallel, the total resistance, R_T, is equal to

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots\dots$$

Don't forget! After finding a common denominator and determining the sum of these fractions, flip over the answer to determine R_T.



In parallel circuits, the total resistance is always *smaller* than any individual resistance.

Current in parallel resistors: In parallel circuits, there is more than one possible path and current divides itself according to the resistance of each path. Since current will take the "path of least resistance," the smallest resistor will allow the most current through, while the largest resistor will allow the least current through. The sum of the currents in each parallel resistor equals the original current entering the branches.

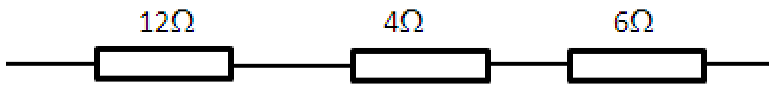
$$I_{TOTAL} = I_1 + I_2 + I_3$$

Voltage in parallel resistors: The potential difference across each of the resistors in a parallel combination is the same. If there are no other resistors in the circuit, it is equal to the potential difference across the battery, assuming negligible resistance in the connecting wires.

$$V_{TOTAL} = V_1 = V_2 = V_3$$

EXAMPLE

Find the total resistance of the three resistors connected in series.



$R_T = R_1 + R_2 + R_3 = 12\Omega + 4\Omega + 6\Omega = \underline{\mathbf{22\Omega}}$.

EXAMPLE

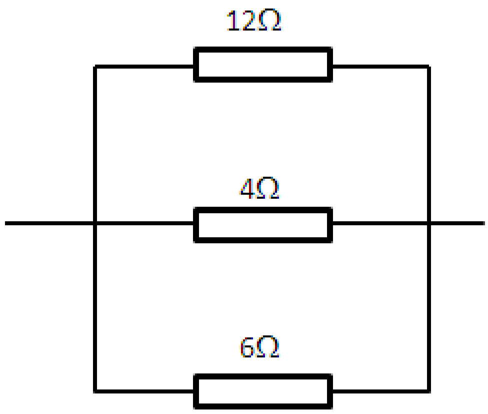
Find the total resistance of the same three resistors now connected in parallel.

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{R_T} = \frac{1}{12} + \frac{1}{4} + \frac{1}{6}$$

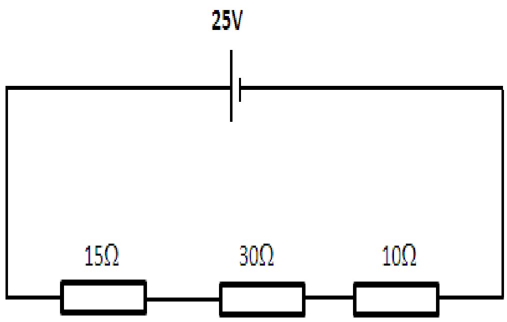
$$\frac{1}{R_T} = \frac{6}{12}\Omega = \frac{1}{2}\Omega$$

$$\underline{R_T = 2\Omega}$$



EXAMPLE

A circuit diagram is given below.



Find the:

(i) total resistance of the circuit
(ii) total current of the circuit
(iii) voltage drop through the
  a) 15Ω Resistor
  b) 30Ω Resistor
  c) 10Ω Resistor
(iv) power dissipated by the 30 Ω resistor

Soln:

(i) $R_{TOTAL} = R_1 + R_2 + R_3$

R_T = 15 Ω + 30 Ω + 10 Ω = 55 Ω

(ii) Total Current, from Ohm's Law: $I_T = \frac{V_T}{R_T} = \frac{25V}{55\Omega} \Rightarrow \underline{I_T = 0.455A}$

(iii) Voltage drop: since the current passing through each resistor in series is the same, therefore

a) $V_{drop}(15\ \Omega) = IR \Rightarrow V = (0.455A)\ (15\ \Omega) =$ **<u>6.825 Volts</u>**

b) $V_{drop}(30\ \Omega) = IR \Rightarrow V = (0.455A)\ (30\ \Omega) =$ **<u>13.65 Volts</u>**

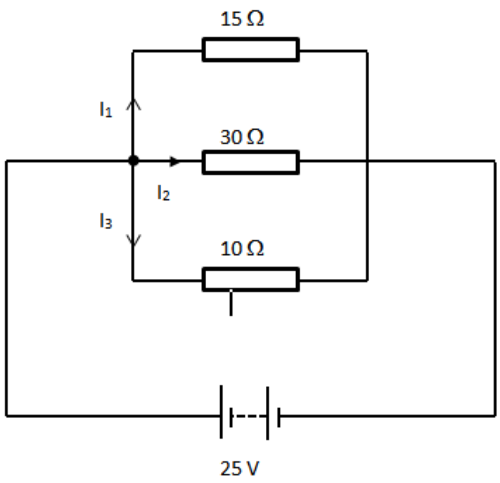
c) $V_{drop}(10\ \Omega) = IR \Rightarrow V = (0.455A)(10\ \Omega) =$ **4.55 Volts**

(iv) Power dissipated, $P = VI \Rightarrow P = (13.65\ V)(0.455\ A) =$ **6.21 Watts**

Or use $P = I^2R$ $P = (0.455\ A)^2(30\ \Omega) =$ **6.21 Watts**

EXAMPLE

A circuit diagram is given below.



Find the:

(i) total resistance of the circuit

(ii) total current of the circuit

(iii) current through the

a) 10 Ω resistor

b) 30 Ω resistor

c) 15 Ω resistor

(iv) power dissipated by the 30 Ω resistor

Soln:

(i) $$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \qquad \frac{1}{R_T} = \frac{1}{15} + \frac{1}{30} + \frac{1}{10} \qquad \frac{1}{R_T} = \frac{1}{5}$$

Therefore, $\underline{R_T = 5\Omega}$

(ii) Total Current, from Ohm's Law: $I_T = \frac{V_T}{R_T} = \frac{25V}{5\Omega} \Rightarrow \underline{I_T = 5A}$

(iii) Since the voltage across each of the resistors in a parallel combination is the same:

a) I (10Ω) = $\frac{V_T}{R} = \frac{25V}{10\Omega}$ $\underline{I(10\Omega) = 2.5A}$

b) I (30Ω) = $\frac{V_T}{R} = \frac{25V}{30\Omega}$ $\underline{I(30\Omega) = 0.833A}$

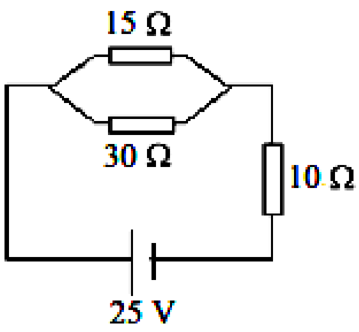
c) I (15Ω) = $\frac{V_T}{R} = \frac{25V}{15\Omega}$ $\underline{I(15\Omega) = 1.67A}$

(iv) Power dissipated, P = VI $\Rightarrow$ P = (25 V)(0.833 A) = **<u>20.83 Watts</u>**

Or use P = $\frac{V^2}{R}$ P = $\frac{25^2}{30}$ = **<u>20.83 Watts</u>**

EXAMPLE

1. For the circuit diagram shown below:



Find the:
(i) total resistance of the circuit
(ii) total current of the circuit
(iii) Voltage drop through the 10 Ω resistor
(iv) current through the
a) 15 Ω resistor
b) 30 Ω resistor
(v) power dissipated by the 30 Ω resistor

Soln:

(i) This circuit contains resistors in parallel that are then combined with a resistor in series. Always begin solving such a resistor combination by working from the inside out. In other words, first determine the equivalent resistance of the two resistors in parallel before combining this total resistance with the one in series.

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{15} + \frac{1}{30} \Rightarrow \frac{1}{R_P} = \frac{1}{10} \qquad R_P = 10\Omega$$

Now, combine this equivalent resistance with the resistor in series.

$$R_T = R_P + R_S = 10\Omega + 10\Omega = \underline{20\Omega}$$

(ii) *Total Current, from Ohm's Law:* $I_T = \frac{V_T}{R_T} = \frac{25V}{20\Omega} \Rightarrow \underline{I_T = 1.25A}$

(iii) V_{drop}(10Ω) = IR = (1.25A) (10 Ω) = **<u>12.5 V</u>**

(iv) Current through 15Ω and 30Ω resistor:

2. A is the current through the entire circuit. Use this current to find the voltage across the parallel combination. Remember, the voltage across resistors wired in parallel is the same regardless of which path is taken.

Therefore, Resistors in parallel, R_P = 10 Ω
Voltage through parallel circuit: V = IR_P = (1.25 A) (10 Ω) = 12.5 V

(a) Current through 15 Ω: $I = \frac{V}{R} = \frac{12.5V}{15\Omega}$ = **<u>0.833 A</u>**

(b) Current through 30 Ω: $I = \frac{V}{R} = \frac{12.5V}{30\Omega}$ = **0.417 A**

NOTE: 0.833 A + 0.417 A = 1.25 A = Total Current.

(v) Power, P = VI = (12.5 V) (0.417 A) = **5.213 W**

SAFETY DEVICES IN THE HOMES

In an electric circuit, fuses and circuit breakers act as safety devices. They prevent circuit overloads that can occur when too many appliances are turned on at the same time or when a short circuit occurs in one appliance.

A properly designed electrical system is very safe. However, when problems occur, electricity can generate dangerous heat and can be fatal to people and animals

FUSES

[http://www.a1telecom.com/theshop/radio-accessories/fuses/workman-agc-glass-tube-fuse-100-pack.html]

A fuse is inserted into a circuit to protect the device from receiving too much current when shorted. It is a device which contains a very thing conductor inside. Its function is that if the current exceeds more than needed, the fuse melts and breaks the circuit. Example: a Fuse rated 0.25A (250mA), will break if the current exceeds 250mA.

CIRCUIT BREAKER

Residual Current circuit breaker Mini Circuit Breaker

[http://www.made-in-china.com/showroom/meywon8/product-detaildMinfkrFbGhD/China-Mini-Circuit-Breaker-DZ47LE-63-.html]

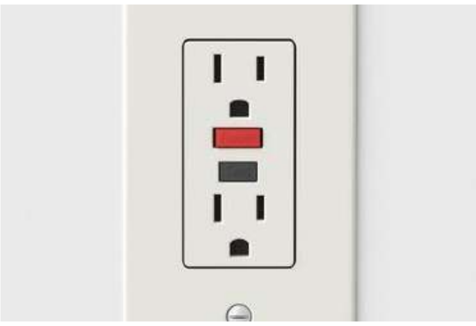
A circuit breaker has the same function as the fuse. If a surge of current is evident through a line, the circuit breaker "breaks" the line, opening the flow of current.

Circuit breakers are mechanical switches that open when an electrical fault is detected. Circuit breakers have replaced fuses in modern electrical systems. In a fuse, excess electricity or heat melts a metal strip, thus interrupting the flow of electricity.

The problem with fuses is that they only can be used once. A circuit breaker can be switched back on as part of trouble-shooting or when the fault has been fixed.

Simple domestic circuit breakers are designed to prevent damage to home wiring. Damage is usually caused by plugging too many high power devices, like electric heaters, appliances and high power lights, in to a circuit. When the circuit breaker senses the overload, it trips interrupting the flow of electricity and preventing damage to home wiring.

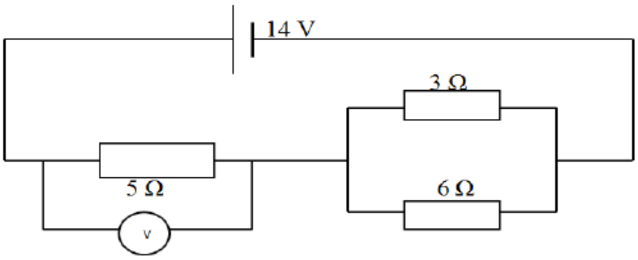
A current as small as 5 mA flowing through a person could result in electrocution. A **ground-fault interrupter** (GFI) in an electric outlet prevents such injuries because it contains an electronic circuit that detects small differences in current caused by an extra current path and opens the circuit. The earthing system is there as a safety ring around the installation.
Electric codes for buildings often require ground-fault interrupters to be used in bathroom, kitchen, and exterior outlets.



http://homerepair.about.com/od/termsgn/g/gloss_GFI.htm

EXERCISES

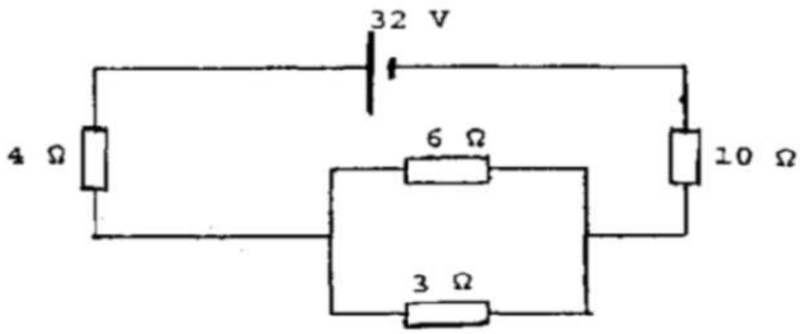
1. A circuit diagram is shown.



Calculate:

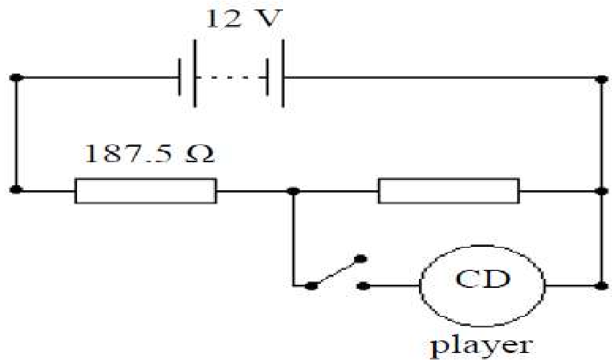
a) The total resistance of the circuit.
b) The current leaving the battery.
c) The reading on the voltmeter.
d) The current in the 3 Ω resistance

2. Consider the following circuit and answer the questions that follow:
a) Calculate the total resistance of the circuit.
b) What is the voltage drop across the 3 Ω resistors?
c) Calculate the energy dissipated in the 4 Ω *resistors* in 5 minutes.

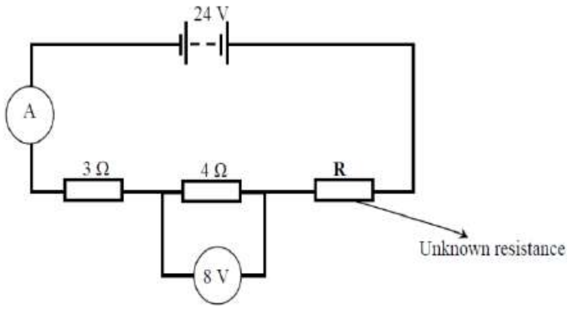


3. Mark has a battery-operated CD player that he wants to connect to his car battery. The voltage of his car battery is 12.0 V and his CD player is marked "4.5 V, 30 mA". He knows that he cannot connect it directly to the car battery, so he decides to connect it in a circuit with a switch initially open as shown in the diagram below.

(i) Calculate the resistance of the CD player.
(ii) Calculate the voltage across the 187.5Ω resistor if the CD player has the correct voltage across it when the switch is closed.



4. Refer to the diagram below and answer the questions that follow.



(i) Determine the reading on the ammeter.
(ii) Find the value of the unknown resistance, **R**.

CHAPTER 6: ELECTROMAGNETISM

Electromagnetism is the science of the properties and relationship between electric currents and magnetism. An electric current creates a magnetic field and a moving magnetic field will create a flow of charge. This relationship between electricity and magnetism has resulted in the invention of many devices which are useful to humans.

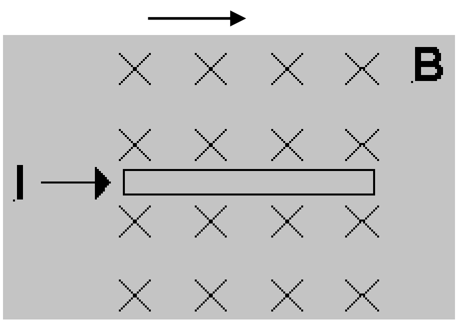
6.1 MOTOR EFFECT

When two magnets are close together, they affect each other and produce a force. The same happens when any two magnetic fields are close together. If a wire carrying a current is placed in a magnetic field a force is produced. This is called the **motor effect.** The direction of the force will depend on the direction of the magnetic field and the direction of the current in the field.

The direction of movement of a current carrying wire in a magnetic field can be determined using Fleming's Left Hand Rule. The current, magnetic field and force will always be at right angles to each other, so the wire will not move towards the poles.

When a current carrying conductor is placed in a magnetic field, a **FORCE** is produced except when it is placed parallel to the magnetic field.

Current + *Field* = *Force* (motion)



The magnitude of the force is given by:

$$F = BIL\sin\theta$$

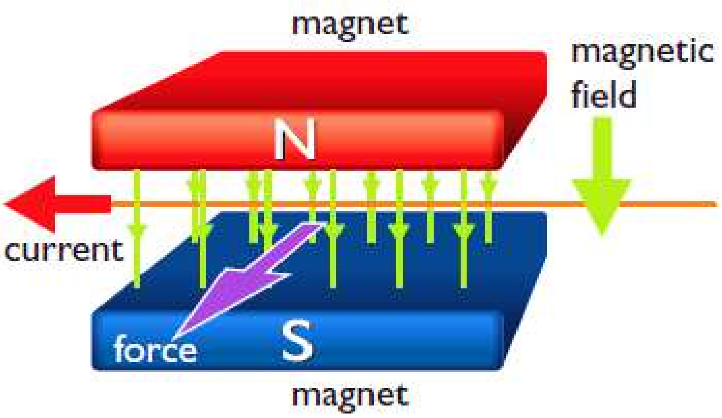
Where:

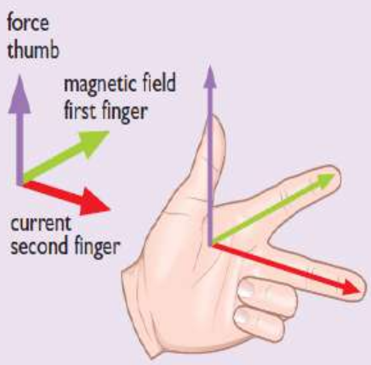
F= force
B = magnetic field strength (T)
I = current flowing (A)
L = Length of conductor (wire) in the field (m).
θ = angle the conductor makes with the magnetic field

An electrical motor is a device that converts electrical energy to mechanical energy. It works on the principle of the interactions between the magnetic fields of a permanent magnet and the field generated around a coil conducting electricity. The attractive and repulsive forces between the magnet and the coil create rotational motion.

LENGTH OF CONDUCTOR IN THE FIELD

When a current is passed through a wire placed in a magnetic field a force is produced which acts on the wire.





You need to be able to use **Fleming's Left Hand Rule** to work out the direction of the force that acts on the wire.

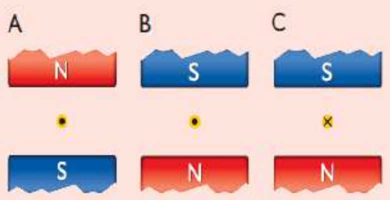
This is also called the **Motor Rule**.

The force acting upon the wire will make the wire move. The thumb of your left hand may be used to determine the direction of movement caused by the force on the wire. In order to remember what component the direction of the thumb shows remember: thu**m**b shows the direction of **m**ovement caused by the force on the wire.

(Alternatively, fumb → force!!)

Example

Use Fleming's Left Hand Rule to work out the direction of the force that will act on the conductors shown in the magnetic fields, below.



a) Field lines should be drawn going from N to S

b) A: ⟶ F
B: ⟵ F
C: ⟶ F

● Shows a conductor carrying current flowing **out of** the plane of the paper perpendicularly **towards** you.

✕ Shows a conductor carrying current flowing **into** the plane of the paper perpendicularly **away from** you.

The *size of the force* that acts on a current-carrying conductor placed at right angles to a magnetic field may be increased by either *increasing the strength of the magnetic field* or by *increasing the current* in the wire.

When a conductor carrying a current is placed in a magnetic field, the conductor will experience a force. The reason for this is that the current in the conductor creates a surrounding magnetic field which is either repelled or attracted to the field in which it is placed. The force depends on the following

- The magnetic field strength (density of the magnetic flux) = B [Tesla]
- The current passing through the wire = A [Amps]
- The length of the conductor in the field = L [meters]

When the conductor is placed 90^o to the magnetic field it experiences the maximum force $(\sin 90 = 1)$.

❖ $F = BIL$

Example

If a conductor of length 0.4m carrying a current of 10.6A is placed in a magnetic field stength of 0.003T, determine the force experienced by this conductor in Newtons.

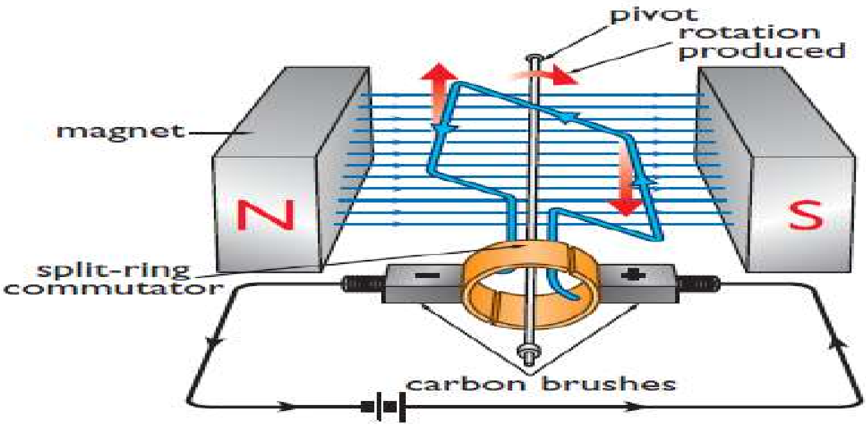
F = BIL

F = (0.03) (10.6) (0.4)

= 0.13N

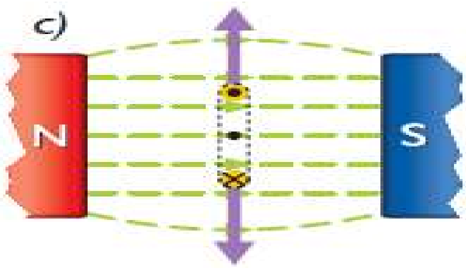
THE ELECTRIC MOTOR

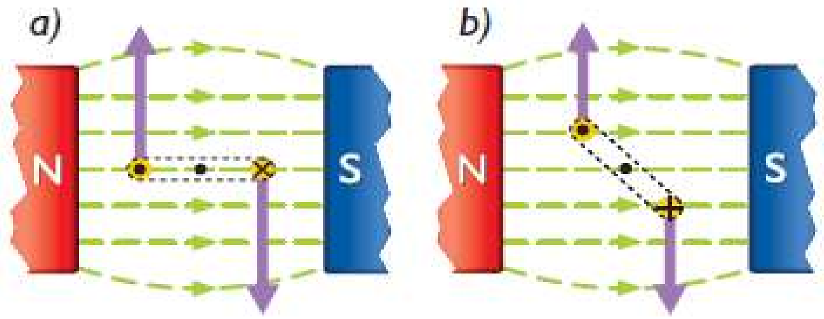
In its simplest form a DC motor consists of a single turn coil of wire that is free to rotate in a magnetic field about an axle. Carbon brushes make contact with the ends of the coil that are connected to a commutator so that a current can be passed through the coil



Simple electric Motor

The sequence of diagrams below shows the coil from an end-on view, making it easy to see how the forces acting on each side of the coil produce a turning effect about the axle. Diagram *c)* shows that the turning effect is zero when the coil is parallel to the permanent magnets (because the line of action of the forces passes through the axis of rotation). This might suggest that the coil stops in this position, but it will inevitably overshoot, and as soon as it does so, the commutator will reverse the direction of the current in the coil which means the coil will continue to spin.





- If the current is reversed, the motion will be in the opposite direction
- If the field is reversed, the motion will change direction again.

Example

State three ways in which you could change the design of a DC motor to make it spin faster for a given load.

Increase the strength of the magnetic field. Put more turns on the coil. Pass a larger current through the coil.

(But note that if the maximum design current for a motor is exceeded then the motor is likely to burn out.)

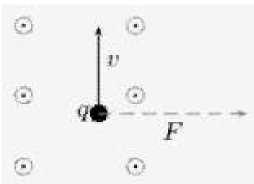
MOTION OF A CHARGED PARTICLE IN A MAGNETIC FIELD

When a charged particle moves through a magnetic field it experiences a force. For a particle that is moving at right angles to the magnetic field, the force is given by:

$$\boxed{F = Bvq}$$

Where F is the force

q is the charge on the particle,
v is the velocity of the particle and
B is the magnetic field through which the particle is moving



EXAMPLE

An electron travels at 150 ms^{-1} at right angles to a magnetic field of 80 000 T. What force is exerted on the electron?

ANSWER

We are required to determine the force on a moving charge in a magnetic field
F = qvB

We are given
$q = 1.6 \times 10^{-19}C$ (The charge on an electron)
$v = 150$ m/s
$B = 80\,000T$

Using

$F = qvB$
$= (1.6 \times 10^{-19}\ C)\ (150\ ms^{-1})\ (80\,000T)$
$= 1.92 \times 10^{-12}\ N$

The direction of the force exerted on a charged particle moving through a magnetic field is determined by using the Right Hand Rule.

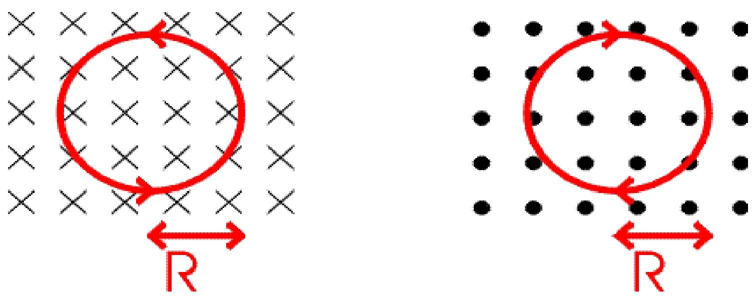
Point your fingers in the direction of the velocity of the charge and turn them towards the direction of the magnetic field. Your thumb will point in the direction of the force. If the charge is negative, the direction of the force will be opposite to the direction of your thumb.

CIRCULAR MOTION IN A MAGNETIC FIELD

Charged particles in a magnetic field feel a force perpendicular to their velocity. Since their movement is always perpendicular to the force, magnetic forces due no work and the particle's velocity stay constant. Since the force is ***F = qvB*** in a constant magnetic field, a charged particle feels a force of constant magnitude always directed perpendicular to its motion. The result is a circular orbit.

The diagram below represents constant magnetic field for two cases. On the left the magnetic field is pointed into the page while on the right the field lines are exiting the page. The *crosses* indicate the field is directed into the page. One can think of this as the tail of a feather as it travels away from view, whereas the *dots* represent the point of the approaching arrow.

The fact that the field is uniform is indicated by the equal spacing of the arrows. Using the right-hand rule one can see that a positive particle will have the counter-clockwise and clockwise orbits shown below.



The radius of the orbit depends on the charge and velocity of the particle as well as the strength of the magnetic field. The acceleration of a particle in a circular orbit is:

$$a = \frac{v^2}{R}$$

Using ***F = ma***, one obtains:

$$F = qvB = m\frac{v^2}{R} \quad \Rightarrow \quad R = \frac{mv}{qB}$$

Thus the radius of the orbit depends on the particle's momentum, ***mv***, and the product of the charge and strength of the magnetic field

6.20 GENERATOR

When a conductor moves through a magnetic field, there will be a generated voltage. The voltage generated in a length of wire, presuming that the entire length moves through a uniform field, is given below.

$$\boxed{V = BLv\sin\theta}$$

V- Voltage
B - Magnetic Field Strength
L - Length of conductor (in meters)
v - Velocity of conductor moving through the field
θ- is the angle (in degrees)

To increase the voltage or current generated:

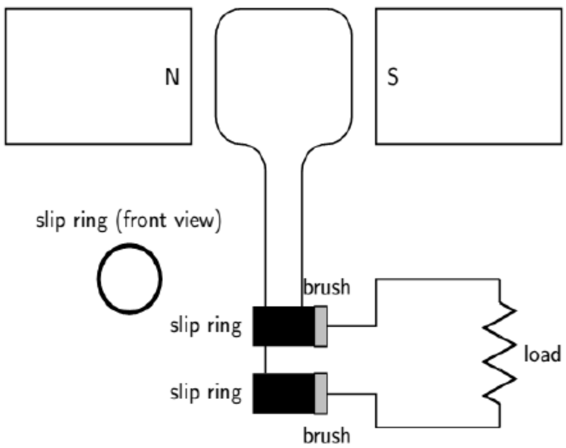
1. Spin the coil faster.
2. Put more loops on the coil.
3. Use a stronger magnetic field.
4. Use a coil with a larger area.

A generator converts mechanical energy into electrical energy.

AC GENERATOR

The principle of rotating a conductor in a magnetic field is used in electricity generators.

The layout of an AC generator is shown below. The conductor in the shape of a coil is connected to a ring. The conductor is then manually rotated in the magnetic field generating an alternating emf. The slip rings are connected to the load via brushes.



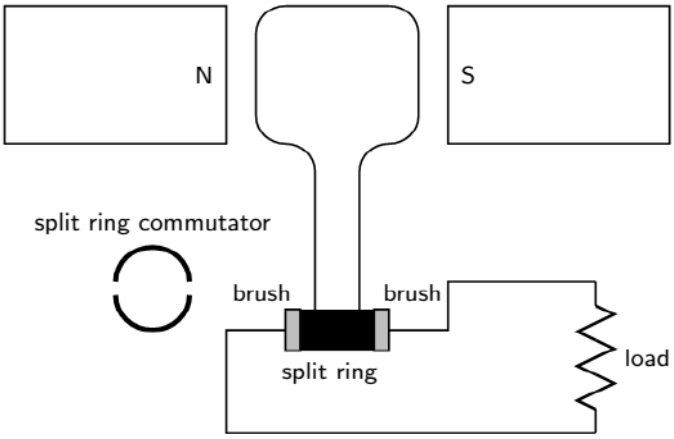
If a machine is constructed to rotate a magnetic field around a set of stationary wire coils with the turning of a shaft, AC voltage will be produced across the wire coils as that shaft is rotated, in accordance with Faraday's Law of electromagnetic induction. This is the basic operating principle of an AC generator.

In an AC generator the two ends of the coil are each attached to a slip ring that makes contact with brushes as the coil turns. The direction of the current changes with every half turn of the coil. As one side of the loop moves to the other pole of the magnetic field, the current in it changes direction. The two slip rings of the AC generator allow the current to change directions and become alternating current.

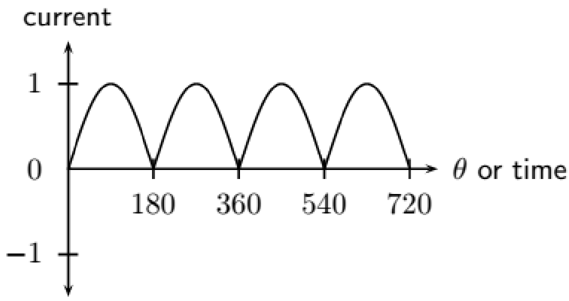
AC generators are also known as alternators. They are found in motor cars to charge the car battery.

DC GENERATOR

A DC generator is constructed the same way as an AC generator except that there is one slip ring which is split into two pieces, called a commutator, so the current in the external circuit does not change direction. The layout of a DC generator is shown in below. The split-ring commutator accommodates for the change in direction of the current in the loop, thus creating DC current going through the brushes and out to the circuit.



The shape of the emf from a DC generator is shown in Figure 28.5. The emf is not steady but is more or less the positive halves of a sine wave.



AC VERSUS DC GENERATORS

The problems involved with making and breaking electrical contact with a moving coil should be obvious (sparking and heat), especially if the shaft of the generator is revolving at high speed.

If the atmosphere surrounding the machine contains flammable or explosive vapors, the practical problems of spark-producing brush contacts are even greater.

An AC generator (alternator) does not require brushes and commutators to work, and so is immune to these problems experienced by DC generators. The benefits of AC over DC with regard to generator design are also reflected in electric motors. While DC motors require the use of brushes to make electrical contact with moving coils of wire, AC motors do not. In fact, AC and DC motor designs are very similar to their generator counterparts.

The AC motor being dependent upon the reversing magnetic field produced by alternating current through its stationary coils of wire to rotate the rotating magnet around on its shaft, and the DC motor being dependent on the brush contacts making and breaking connections to reverse current through the rotating coil every 1/2 rotation (180 degrees).

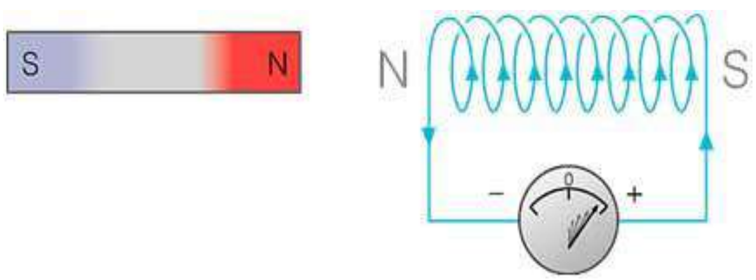
Lenz's Law

In 1834, German physicist Heinrich Friedrich Lenz (1804-1865) deduced a rule, known as *Lenz's law* which gives the polarity of the induced emf in a clear and concise fashion. The statement of the law is:

The polarity of induced emf is such that it tends to produce a current which opposes the change in magnetic flux that produced it.

EXAMPLE 1 Magnet is moving towards the coil.

When the North Pole end of the magnet is approaching the coil, the magnetic flux linking the coil will increases. According to Faraday's law of electromagnetic induction, when there is change in flux, an emf and hence current is induced in the coil and this current will create its own magnetic field. Now according to Lenz's law, this magnetic field created will oppose its own or we can say opposes the increase in flux through the coil and this is possible only if approaching coil side attains north polarity, as we know similar poles repel each other. Once we know the magnetic polarity of the coil side, we can easily determine the direction of the induced current by applying right hand rule. In this case, the current flows in anticlockwise direction.

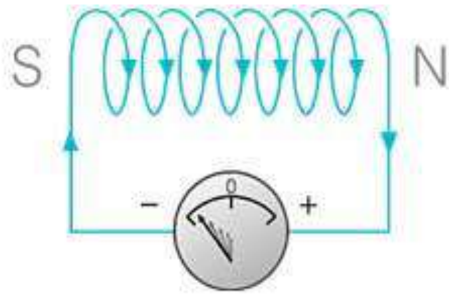


EXAMPLE 2 Magnet is moving away from the coil

When the north pole of the magnet is moving away from the coil, the magnetic flux linking to the coil will decreases. According to Faraday's law of electromagnetic induction, an emf and hence current is induced in the coil and this current will create its own magnetic field.

Now according to Lenz's law, this magnetic field created will oppose its own or we can say opposes the decrease in flux through the coil and this is possible only if approaching coil side attains south polarity, as we know dissimilar poles attract each other.

Once we know the magnetic polarity of the coil side, we can easily determine the direction of the induced current by applying right hand rule. In this case, the current flows in clockwise direction.

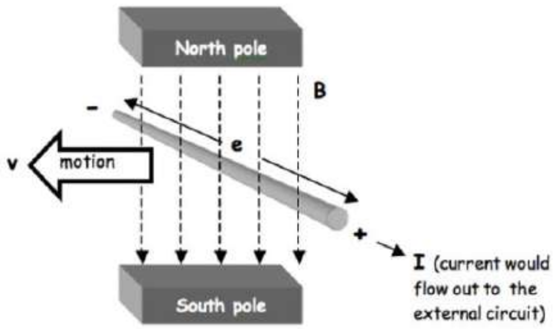


ELECTROMOTIVE FORCE (EMF) INDUCED BETWEEN THE ENDS OF THE CONDUCTOR

When a conductor moves through a magnetic field, an EMF is induced across it. If the conductor was connected to an external circuit, a current would flow just like a battery. The Emf generated depends upon the following

- The magnetic field strength = B [Tesla]
- The length of the conductor in the field = L [meters]
- The speed of the conductor = v [metres per second]

Consider a conductor in a magnetic field where the magnetic field flows from North to South Pole. If the conductor is moved through the field in the direction shown below, the emf will have the polarity shown. When the conductor is placed 90^o to the magnetic field it induces maximum emf $(\sin 90 = 1)$.



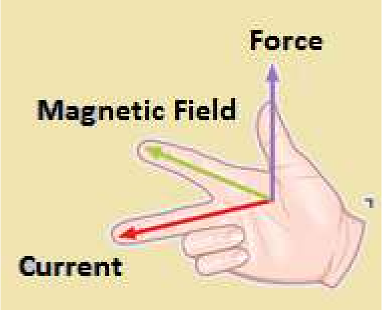
❖ $V = BLv$

Example

Calculate the emf induced across the ends of a wire of length 0.3m when it is moved through a magnetic field strength of 0.015T at a speed of 50 m/s.

$$V = BLv$$
$$= (0.015)\,(0.3)\,(50)$$
$$= 0.23V$$

If the conductor is connected to a closed circuit, the direction of the current flow can be found using Fleming's Right Hand Rule.



- If the motion is reversed, the current will be in the opposite direction.
- If the field is reversed, the current will also change direction again.

The EMF in a single conductor is small. However, it can be increased by moving the conductor at a higher speed or by making the field denser (increasing the magnetic field strength) by forming a coil with many turns. The total EMF is found by multiplying the EMF of a single conductor by the number of turns.

Example

A 200 turn coil has a radius of 0.12 m and length of 0.23m. It is placed in a magnetic field strength of 0.06T and rotated at 3000rpm. When the coil is in its vertical position at right angles to the field, calculate the EMF.

$$\boldsymbol{v = 2 \pi r N/60}$$
$$= (2)(\pi)(0.12)(3000/60)$$
$$= 37.70 \text{ m/s}$$

$$V = BLv$$
$$= (0.06)(0.23)(37.70)$$
$$= 0.52V$$

THE TRANSFORMER

Definition:

A transformer is an electrical device that uses the principle of induction between the primary coil and the secondary coil to either step-up or step-down voltage. A **step-up transformer** results in an increased voltage. A **step-down transformer** results in a decreased voltage.

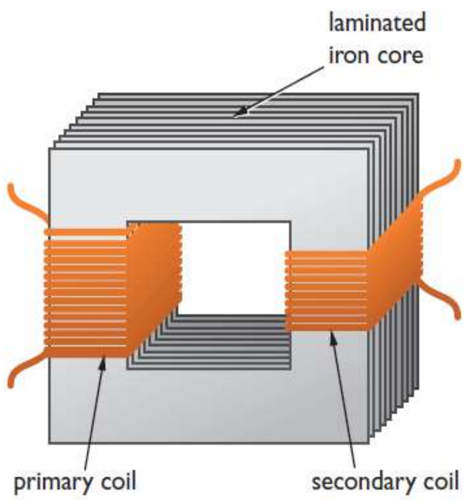
The essential features of a transformer are two coils of wire, called the primary coil and the secondary coil, which are wound around different sections of the same iron core to intensify the magnetic field in the primary.

When an alternating voltage is applied to the primary coil it creates an alternating current in that coil, which induces an alternating magnetic field in the iron core, thus creating a changing magnetic field that thread through the secondary. Thus, there is a changing magnetic flux in the secondary coil, which produces a current in that coil.

Since the magnetic field is changing at a given frequency, the current induced in the secondary coil is also an alternating one.

The function of a transformer is to change the size of an alternating voltage. This is done by having two separate coils with different numbers of turns.

Transformers consist of a core made from thin sheets of a magnetically soft material clamped together. Two separate coils of wire, insulated from one another, are tightly wound onto the core. Transformers are designed to perform the job of changing voltage with very little power loss → you may **assume that they are 100%** efficient.



$$\frac{\text{input (primary) voltage}}{\text{output (secondary) voltage}} = \frac{\text{primary turns}}{\text{secondary turns}}$$

$$\frac{V_p}{V_s} = \frac{n_p}{n_s}$$

If np > ns then the transformer steps down the input voltage; if ns > np then the transformer steps up the input voltage.

EXAMPLE

A transformer is designed to step down the mains voltage of 230 V to 11.5 V. If there are 1200 turns on the primary coil how many turns should be wound on the secondary coil?

Rearrange $\frac{Vp}{Vs} = \frac{n_p}{n_s}$ *to give* $n_s = \frac{V_s}{V_p} \times n_p$ *So,* $n_s = \frac{11.5\,V}{230V} \times 1200$

Therefore, $n_s = 60$ *turns*

In an **ideal transformer** no energy is lost and so the energy input is the same as the energy output per unit time. We can write this as:

Power in = Power out

$V_P I_P = V_S I_S$

TRANSMISSION OF ELECTRICAL ENERGY

Transformers are used in the transmission of electric energy over large distances. Transmission lines have low but not zero resistance. Power loss due to this resistance is given by the formula $P = I^2 R$, and this means that the power losses betweeen the power station and the consumers would be acceptably large. As transformers are close to 100% efficient.

Power input = power out **so** $V_p \times I_p = V_s \times I_s \rightarrow \frac{V_p}{V_s} = \frac{I_s}{I_p} \left(\frac{n_p}{n_s}\right)$

EXERCISES

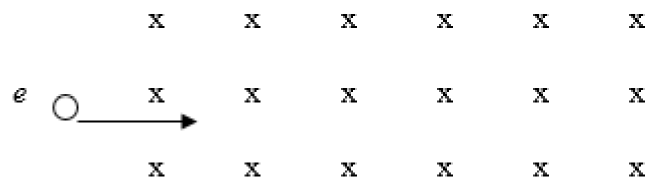
1. A square loop of aluminium wire is initially placed perpendicular to the lines of a constant magnetic field of 0.5 T. The area enclosed by the loop is 0.2 m^2. The loop is then turned through an angle of 90° so that the plane of the loop is parallel to the field lines. The turn takes 0.1 s. What is the induced emf in the loop?

2. A straight wire of mass 200 g and length 1.5 m carries a current of 2 A. It is suspended in mid-air by a uniform horizontal magnetic field B (Fig. 4.3). What is the magnitude of the magnetic field?

3. Synchrotron is used in Nuclear Physics to produce high speed protons. A strong magnetic field is used to keep the protons in a circular orbit. Consider a proton traveling at 5×10^8 m/s around a synchrotron with a 6 m diameter in a magnetic field of strength 0.08 T. (The charge of a proton is 1.6×10^{-19}C)

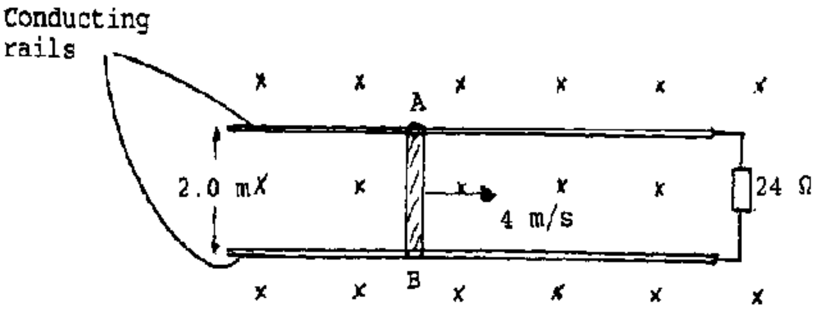
(i) Calculate the magnetic force on the proton.
(ii) Calculate the mass of the proton.

4. An electron enters a uniform magnetic field of intensity 2.5 N/A-m (Tesla) at right angles with a speed of 6.5×10^3 m/s as shown below:



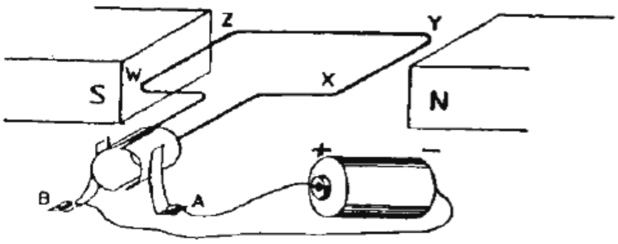
(i) State the direction of the magnetic force experienced by the electron.
(ii) Calculate the magnitude of the magnetic force experienced by the electron.
(iii) Explain why the electron will follow a circular path.
(iv) Determine the radius of the circular path followed by the electron in the magnetic field.

5. A thick copper rod AB moves horizontally to the right at a uniform speed of 4 m/s as shown below. The ends of the rod slide along and make good contact with the conducting rails. The ends of the rails are joined through a 24Ω resistor and a uniform magnetic field of 0.9 T is directed into the page.



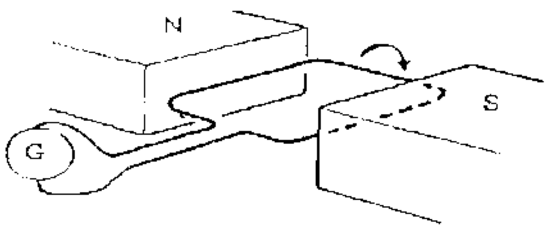
(i) Calculate the e.m.f induced across AB.
(ii) State the direction of the current and the polarity at A.
(iii) What is the current across the 24Ω resistor?
(iv) Explain why a force is required to keep the rod moving at constant speed.

6. The diagram given below shows a simple DC electric motor.



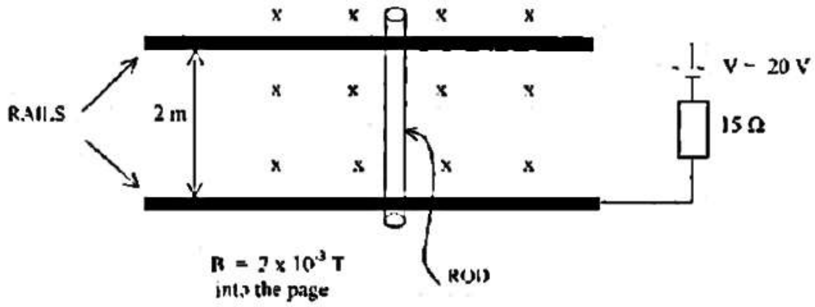
The square coil WXYZ is a single turn and has a side length of 0.1 m. The current through the coil is 4A and the uniform field strength between the magnetic poles is 0.5 T.
(i) Show the direction of the force on edge WZ and state direction of rotation of coil.
(ii) Calculate the force acting on the side XY.
(iii)What energy conversion takes place in an electric motor?

7. The figure given below illustrates the principle of a simple A.C. generator. The end of the coil is connected to a galvanometer.



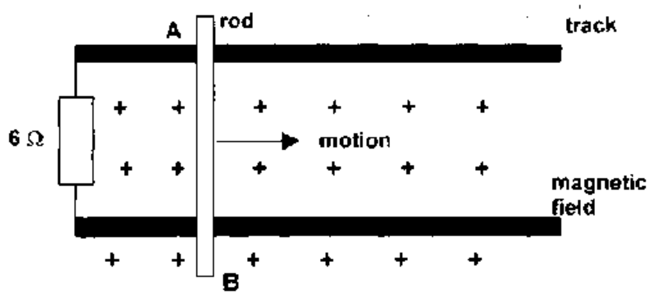
Considering the coil to be rotating in a clockwise direction as indicated,
i. Show the direction of the induced current in the coil
ii. State two ways in which the deflection in the galvanometer can be increased.

8. A metal rod slides to the left along horizontal parallel rails 2 m apart. The rod is connected in series to a power supply of 20 V and a resistor of 15 Ω as shown below.



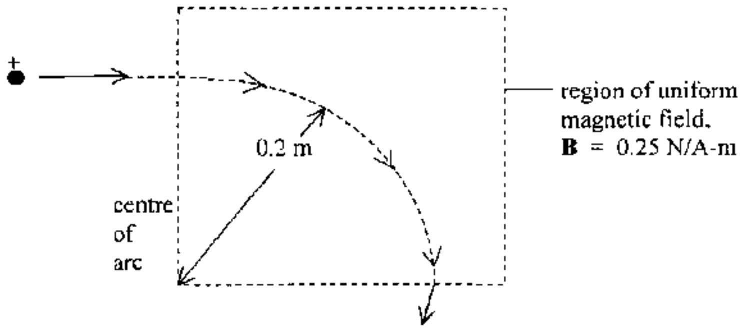
(i) Calculate the current in the rod.
(ii) Describe the motion of the rod when connected.
(iii) Determine the magnitude and the direction of the magnetic force on the rod.

9. A 1 .2 m rod **AB** is moved along a conducting track perpendicularly to a **2.5 T** magnetic field directed into the page. A circuit is complete with the connection of a **6 ohm** resistor as shown in the diagram.



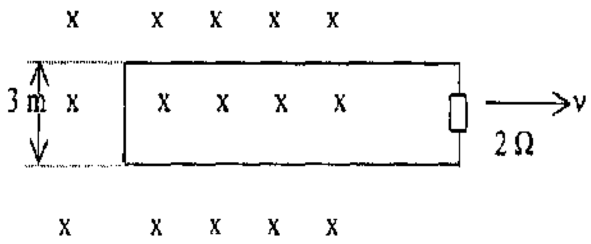
a) Which end of the rod, becomes positively charged as a result of the motion?
b) How fast should the rod be moved to produce a current of 5 A?

10. A positively charged particle of mass 4 x 10^{-20} kg travelling to the right at a speed of 5 x 10^4 m/s enters a region of uniform magnetic field. The force due to the magnetic field causes the particles to move in a circular path of radius 0.2 m.



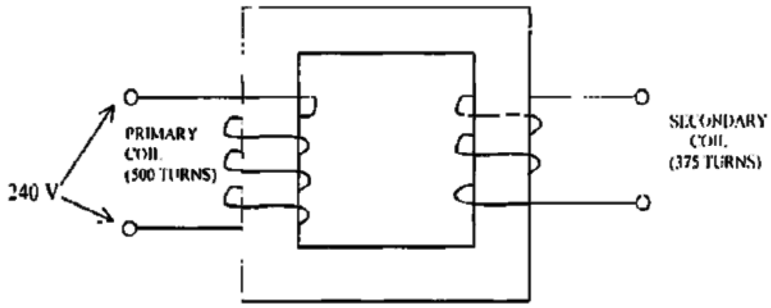
a) State the direction of magnetic field in the region. Is it upwards, downwards, to the right, to the left, into the page or out of the page?
b) What is the centripetal force on the particle?
c)If the magnetic field B = 0.25 T, calculate the charge on the particle that gives the force found in (ii) above.
d) What would happen to the path of the particle if the magnetic field in the region was weaker?

11. A rectangular loop of copper wire with a 2Ω resistor connected to it is pulled to the right through a magnetic field of strength 4 T.



(i) With what speed v should the loop be pulled so that there is an induced emf of 30 volts?
(ii) Calculate the energy dissipated in the 2Ω resistor in 30 seconds.
(iii) Explain where this energy comes from.

12. The diagram given below shows a transformer.



(i) State with a reason whether it is a step-up or a step-down transformer.
(ii) Calculate the output voltage of the transformer.

CHAPTER 7: ATOMIC PHYSICS

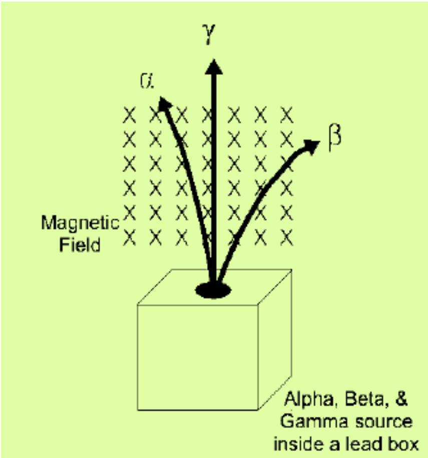
7.1 RADIOACTIVITY

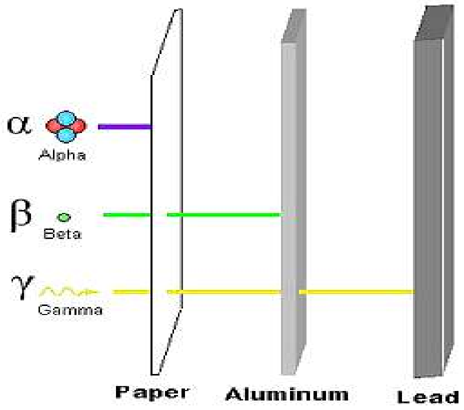
There are three distinct forms of radiation, originally divided up based on their ability to pass through certain materials and their deflection in magnetic fields.

Alpha (α): could barely pass through a single sheet of paper. Alpha particles are deflected as a positive particle in a magnetic field.

Beta (β): can pass through about 3mm of aluminum. Beta particles are deflected as a negative particle in a magnetic field.

Gamma (γ): can pass through several centimeters of LEAD! It is not deflected in a magnetic field.





Source: studyphysics.ca

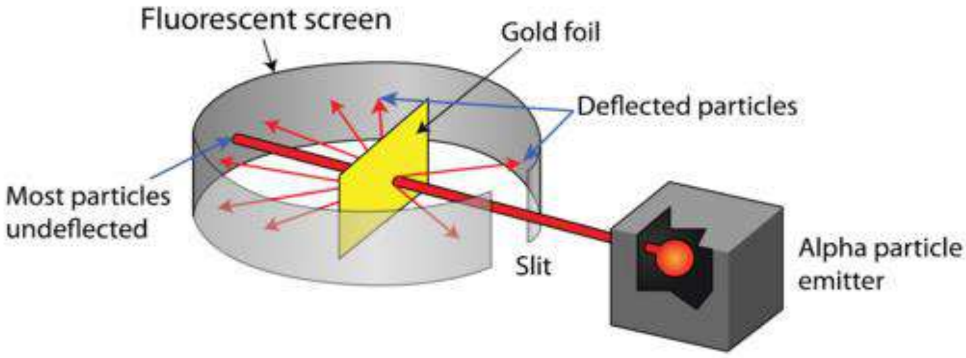
Radioactive decay, also known as **nuclear decay** or **radioactivity**, is the process by which a nucleus of an unstable atom loses energy by emitting ionizing radiation.

RUTHERFORD'S EXPERIMENTS

In 1911, Ernest Rutherford (1871–1937) and his students Hans Geiger and Ernest Marsden performed a critical experiment that showed that Thomson's model could not be correct. In this experiment, a beam of positively charged alpha particles (helium nuclei) was projected into a thin metallic foil such as the target shown in Figure below.

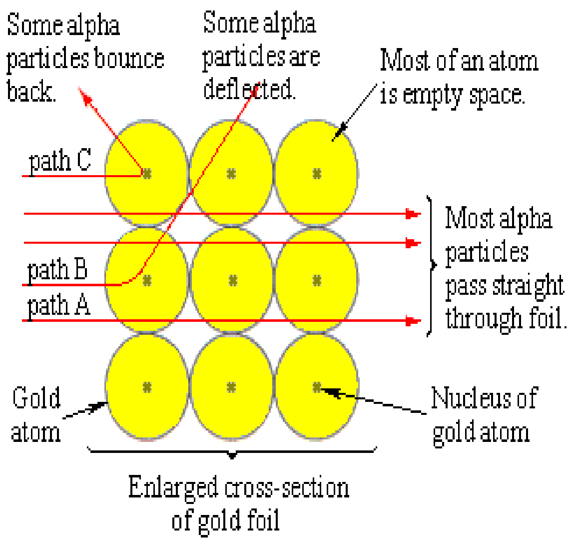
Most of the particles passed through the foil as if it was empty space, but some of the results of the experiment were astounding.

Many of the particles deflected from their original direction of travel were scattered through *large* angles. Some particles were even deflected backward, completely reversing their direction of travel!

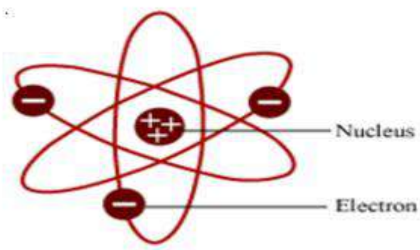


Rutherford made 3 observations:

- Most of the fast, highly charged alpha particles went whizzing straight through undeflected. This was the expected result for all of the particles if the plum pudding model was correct.
- Some of the alpha particles were deflected back through large angles. This was not expected.
- A very small number of alpha particles were deflected backwards! This was definitely not as expected. Rutherford later remarked "It was as incredible as if you fired a 15-inch shell at a piece of tissue paper and it came back at you.



Rutherford reasoned that the only way the alpha particles could be deflected backwards was if most of the mass in an atom was concentrated in a nucleus. He thus developed the planetary model of the atom which put all the protons in the nucleus and the electrons orbited around the nucleus like planets around the sun.



On the basis of these observations Rutherford made the following conclusions:

- Since most of the alpha particles passed straight through the gold foil without any deflection, most of the space within the atoms is empty.
- Since some of the alpha particles (which are big in size) were deflected by large angles or bounced backwards, they must have approached some positively charged region responsible for the deflection. This positively charged region is now called the nucleus.

- As very few alpha particles undergone the deflection, it was concluded that the volume occupied by the central region (nucleus) is very small.
- Since alpha particles which are relatively denser, were deflected by the central volume of charge, it shows that almost the complete mass of the atom must be within the central volume.

Drawbacks of Rutherford's Model

The Rutherford's atomic model explains the structure of an atom in a very simple way. But it suffers from the following drawbacks:

(i) An electron revolving around the nucleus gets accelerated towards the nucleus. An accelerating charged particle must emit radiation and lose energy. Thus, the electron in an atom must continuously emit radiation and lose energy and would slow down and will not be able to withstand the attraction of the nucleus. As a result it should follow a spiral path and ultimately fall into nucleus.

(ii) Rutherford model of atom does not say anything about the arrangement of electron in an atom.

Alpha Decay

During an alpha decay, a nucleus is able to reach a more stable state be allowing 2 protons and 2 neutrons to leave the nucleus. This will result in a smaller nucleus, which is often the more stable arrangement. Because 2 protons and 2 neutrons are really just helium-4, the particle that is emitted is really helium. Because this helium is not just regular helium floating around in the air, but is "born" from nuclear decay, we usually don't call it a helium atom. Instead we call it an alpha particle. Alpha particles come out of the nucleus as just nucleons without any electrons. So, each alpha particle has a charge of +2e

$$^{235}_{92}U \longrightarrow ^{231}_{90}Th + ^{4}_{2}He$$

A **helium nucleus**, the **alpha particle**, of 2 protons and 2 neutrons is **emitted** at high speed/kinetic energy **from the nucleus**.

Example: The iridium-168 isotope is known to go through alpha decays. Write out a decay equation that shows this process.

Solution $$^{168}_{77}Ir \rightarrow ^{4}_{2}\alpha + ^{164}_{75}Re$$

Beta$^-$ Decay e.g. the nuclear equation

In the beta negative decay, the neutron becomes a proton (which stays in the nucleus) and an electron that goes out (the beta particle).

Example: $$^{14}_{6}C \longrightarrow ^{14}_{7}N + ^{0}_{-1}e$$

Potassium-40 is known to go through beta positive decays. Write out the decay equation for this decay.

Solution $${}^{40}_{19}K \rightarrow {}^{0}_{1}\beta + {}^{40}_{18}Ar$$

Gamma emission (γ)**:** The **emission of gamma radiation** from a nucleus does not involve any change in the atomic (proton) number or mass number.

Gamma radiation can only be stopped by stuff like a few inches of lead. This is because unlike the other two forms of decay, gamma decays emit a form of EMR, not a particle which allows it to pass through anything but the densest of matter.

Example:

The argon-40 that was produced in Example 4 is in an excited state, so it releases a burst of gamma radiation. Write the equation for this.

Solution $${}^{40}_{18}Ar \rightarrow \gamma + {}^{40}_{18}Ar$$

HALF LIFE

The half-life of an element is the time it will take half of the parent atoms to trans mutate into something else (through alpha or beta decays, or another process) or it is the time it takes for half of a given amount to decay.

Example

Let say you have 100 g of radioactive C-14. The half-life of C-14 is 5730 years.

(a) How many grams are left after one half-life?

50g

(b) How many grams are left after two half-lives?

$$100gram \xrightarrow{\boxed{1}} 50gram \xrightarrow{\boxed{2}} 25gram$$

Example

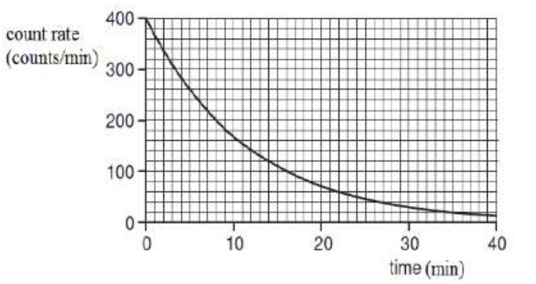
You have 160 g of an isotope with a half-life of 4 days. How much will be left after 16 days?

You can work this out by saying: $\frac{16days}{4days} = 4$ *Half lives*

$$\boxed{160\text{gram}} \xrightarrow{1} 80\text{gram} \xrightarrow{2} 40\text{gram} \xrightarrow{3} \quad \xrightarrow{4} 10\text{gram}$$

Example

Given below is the decay curve for a radioactive isotope that emits only β-particles. Use the graph to find the value of the half-life of the isotope. *The count rate drops from 400 to 200 counts a minute in 8 minutes, so the half-life is 8 minutes.*



7.2 PHOTOELECTRIC EFFECT

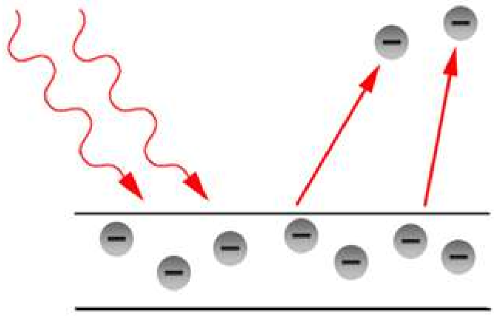
The photoelectric effect occurs when light above a certain frequency (the threshold frequency) is shone on metals like zinc, and this causes electrons to escape from the zinc. The escaping electrons are called photoelectrons.

It was shown in experiments that;

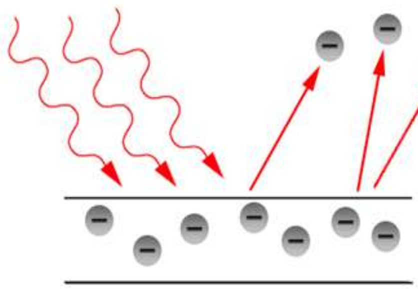
- the frequency of the light needed to reach a particular minimum value (depending on the metal) for photoelectrons to start escaping the metal
- the maximum kinetic energy of the photoelectrons depended on the frequency of the light not the intensity of the light

The above two observation can only be explained if the electromagnetic waves are emitted in packets of energy (quanta) called photons, the photoelectric effect can only be explained by the particle behaviour of light.

The diagram shows photons hitting the surface of a metal and photoelectrons being ejected.



Photons with their Photon Energy and at least the threshold frequency hit a metal. If the plate is Zinc, UV will nudge the photoelectrons off, if gamma rays hit the metal they will be whipped off with more force.
The surface photoelectrons absorb the energy and are emitted out of the metal with the excess energy in the form of Kinetic energy.



If the intensity increases so that there are now more photons, more photoelectrons are emitted. But each photon arriving at the surface has the same photon energy therefore each photoelectron emitted has the same kinetic energy.

Photon Energy (The Einstein relation)

Einstein assumed that each packet of light had a certain amount of energy. This energy must be proportional to its frequency.

Energy of a photon, $E = hf$

Where h is Planck's constant = 6.63×10^{-34} Js and f is the frequency of the light.

Using c = f λ we get

$$\mathbf{E = hc / \lambda}$$

Where c is the speed of the electromagnetic waves

Because of the law of conservation of energy we can see that:

The Photon Energy	=	The Work Function Energy	+	The Photoelectron's Kinetic Energy.
The Photoelectron's Kinetic Energy.	=	The Photon Energy	−	The Work Function Energy
E_K	=	hf	−	ϕ

Definitions:

Retarding / Stopping potential / Cut-off voltage *(V_{co})*

The potential applied to a photocell whereby the current in the circuit becomes zero. At this potential the electrons leaving the emitter plate have zero kinetic energy.

Threshold frequency *(f_o)*

The minimum frequency of light needed for photoelectric effect to occur.

Threshold wavelength *(λo)*

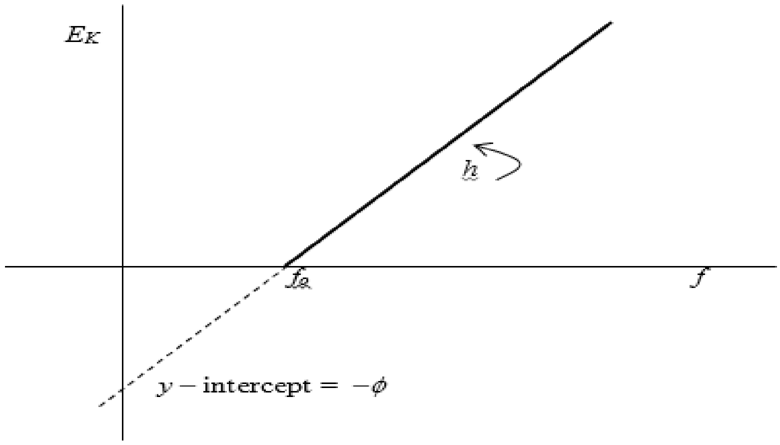
The maximum wavelength of light needed for photoelectric effect to occur.

Work function *(ϕ)*

The amount of energy needed for a photoelectron to eject from the metal surface.

At threshold frequency or wavelength the following relation can be used: $c = f_o\lambda_o$

A graph of ***kinetic energy*** (E_K) against ***frequency*** (f) of incident light in a photoelectric set-up.



Analysing the Einstein's equation we get: $\boxed{E_K = \boldsymbol{hf} - \phi}$

Which corresponds to the linear equation $y = mx + c$

It can be seen that from the graph that the slope represents the Plank's constant (h), and the *y-intercept* is the negative of the work function (ϕ).

It can also be deduced that the *x – intercept* is the threshold frequency (f_o).

Two other relations can be obtained are:

$\underline{\phi = hf_o}$ *and* $\underline{E_K = eV_{co}}$

In photoelectric effect the unit of energy used is called the ***electron-volt*** (*eV*)

$1\ eV = 1.602 \times 10^{-19}$ J , $1\ J = \dfrac{1}{1.602 \times 10^{-19} eV} = 6.24 \times 10^{18}$ eV.

The Einstein's equation can be further modified to give:

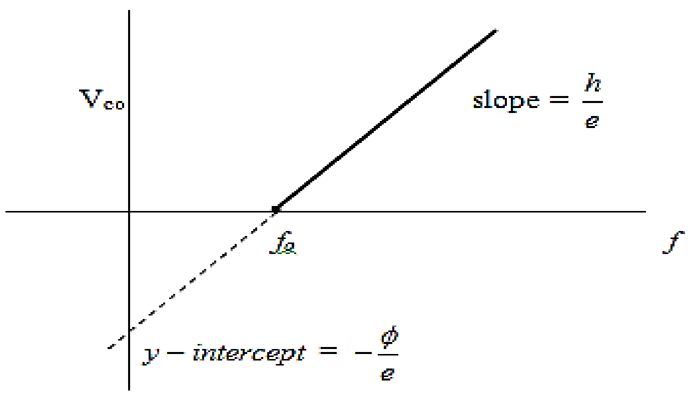
$E_K = hf - \phi$ (since, $E_K = eV_{co}$)

$eV_{co} = hf - \phi$ (dividing by e throughout gives)

$$V_{co} = \left(\frac{h}{e}\right) f - \frac{\phi}{e}$$

$$y = mx + c$$

Graph of V_{co} against kinetic energy (E_k)



Example

A radioactive material emits photons, each having energy of 1.6 x 10^{-13} J.

(A). Calculate the frequency of the electromagnetic radiation emitted by the radioactive material.

$$E = hf$$
$$f = E/h$$
$$= (1.6 \times 10^{-13}\ J)/\ (6.63 \times 10^{-34}\ )$$
$$= 241.33\ H_z$$

(B). Calculate the wavelength of the electromagnetic radiation.

$$c = f\lambda$$
$$\lambda = c/f$$
$$= 3\times 10^{8}/241.33$$
$$= 1.26 \times 10^{6}\ m$$

Example

A photosensitive metal has work function of 3.0 eV.

(A). What is the threshold frequency?

$$\phi = hf_o$$
$$f_o = \phi/h$$
$$= (3\times 1.6 \times 10^{-19})/\ (6.63\times 10^{-34})$$
$$= 7.24 \times 10^{14}\ H_z$$

(B). What is the cut off voltage used to reduce the photoelectric current to zero, if light of wavelength 450 nm is used?

$$E_K = eV_{co}$$
$$V_{co} = E_K/e$$
$$= 3eV/e$$
$$= 3\ V$$

EXERCISES

1. The photoelectric effect is the name given to the process where light waves striking the surface of a metal frees some electrons and produces an electric current. How is it possible for a light wave to liberate an electron from a piece of metal?
2. If all electromagnetic waves are made up of photons (discrete quanta), why don't we hear the effect of each distinct packet of energy when we listen to a radio (which is being effected by a radio wave)?
3. For biological organisms, more damage is done to cells by standing in front of a very weak (low power) beam of x-rays than in front of a much brighter red light. How does the photon concept explain this situation that an 18th century physicist would have found paradoxical?

4. In photoelectric effect experiments, no photoelectrons are produced when the frequency of the incident radiation drops below a cut-off value (which varies depending on the metal used in the experiment), no matter how bright or intense the light is. How can you explain this fact using a "particle" theory of light instead of a wave theory of light?

5. What is the energy of one quantum of 5.0×10^{14} Hz light?

6. A photon has 3.3×10^{-19} J of energy. What is the wavelength of this photon? What part of electromagnetic spectrum does it come from?

7. Which as more energy, a photon of violet light or a photon of red light from the extreme ends of the visible spectrum? How many times more energy does the bigger photon have?

8. What is the lowest frequency of light that can cause the release of electrons from a metal that has a work function of 2.8 eV?

9. The work function for a photoelectric material is 3.5 eV. The material is illuminated with monochromatic light with a wavelength of 300 nm. What is the cut off frequency for that particular material?

10. In studying a solid material for possible use in a solar cell (which turns light into electrical energy), material engineers shine a monochromatic blue light (λ = 420 nm) to produce photoelectrons. They measure the maximum kinetic energy of the emitted electrons to be 1.00×10^{-19} J. Predict what will happen when the engineers test the material with red light (λ = 700 nm). Will the light dislodge electrons from the material? If so, how much kinetic energy will those dislodged electrons have?

11. The threshold wavelength for emission from a metallic surface is 500 nm.
    a) What is the work function for that particular metal?
    b) Calculate the maximum speed of a photoelectron produced by each of the following wavelengths of light:

       (i) 400 nm
       (ii) 500 nm
       (iii) 600 nm.

Reference

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2) **Breithaupt, Jim. (2009). Physics for IGCSE: Printing International Ltd.**

3) **Castle, Trevor. (1990). Senior Physics: Octopus Publishing Group (NZ) Ltd.**

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APPENDIX

The following amendments are made in the text book

Sub-Topic	Amendments	Page Number
Forces	Added exercise questions	44
Moments	Added exercise questions	51
Full projectile	Added diagram	56
Projectile	Added exercise questions	59
Conservation of momentum in two-D	Added notes Added exercise questions	63 68
Newton law of gravitation	Added notes Added exercise questions	72 73
Albedo Greenhouse effect What cause the greenhouse effect Consequences of greenhouse effect Absorption graph of atmosphere	Added notes Added exercise questions	82
Heat energy	Added exercise	91
Static fluids	Added exercise	96
Geometrical optics	Added exercise	102
Waves	Added exercise	113
Electricity	Added exercise	133
Electromagnetism Length of conductor in the field The electric motor EMF induced between the ends of the conductor	Added notes Added exercise	136 146
Radioactivity Drawbacks of Rutherford's Model Half-life	Added diagram and notes Added notes	150 153